VI Escola José Plínio Baptista de Cosmologia

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Lecture 1

Some motivations to go beyond GR

Dark energy models

Lecture 2

Ostrogradsky instability

Lecture 3

Screening mechanisms

Vainshtein, Chameleon, Symmetron, Dilaton

Lecture 4

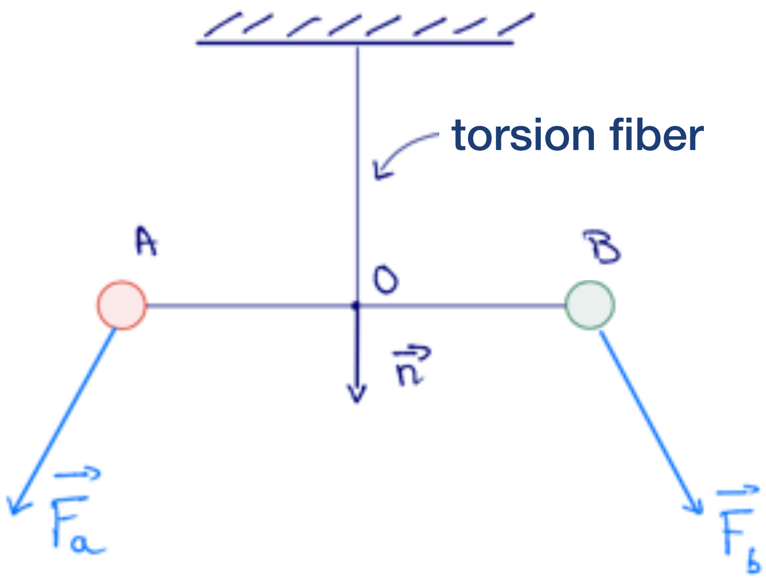
Some examples (brane models, DGP, Galileon, massive gravity, degravitation) and some constraints

General Relativity is based on some fundamental principles

- Weak Equivalence Principle
- Einstein Equivalence Principle
- Strong Equivalence Principle

According to Newton $m_g = m_i$

Eötvös experiment

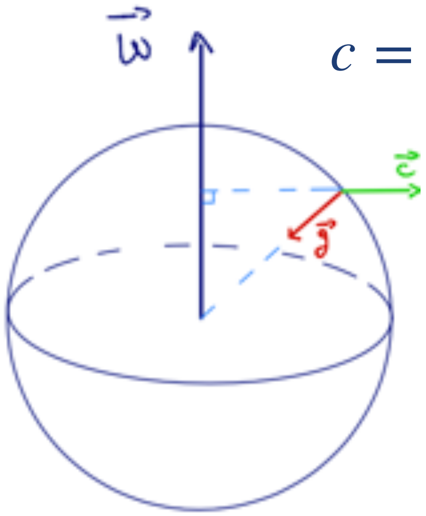


$$\vec{M} = \overrightarrow{OA} \times \vec{F}_a + \overrightarrow{OB} \times \vec{F}_b$$

$$M = \frac{1}{|\vec{F}_A + \vec{F}_B|}(\overrightarrow{OA} - \overrightarrow{OB}).(\vec{F}_A \times \vec{F}_B)$$

$$\vec{F}_A = m_{gA}\,.\,\vec{g} + m_{iA}\,\vec{c}$$

$$\vec{F}_B = m_{gB}\,.\,\vec{g} + m_{iB}\,\vec{c}$$



$$c = R_\oplus \omega^2 \sin\lambda$$

$$M \simeq \frac{m_{iA} m_{iB}}{m_{iA} + m_{iB}} \left[\boxed{\left(\frac{m_g}{m_i}\right)_A - \left(\frac{m_g}{m_i}\right)_B} \right] \frac{(\overrightarrow{OA} - \overrightarrow{OB}).(\vec{g} \times \vec{c})}{|\vec{g} + \vec{c}|}$$

Eötvös parameter η

$|\eta| < 10^{-15}$ MICROSCOPE arXiv:1712.01176

Einstein Equivalence Principle

If $m_g = m_i$ all forces which contributed to the mass, contributed similarly to m_g and m_i

$\Rightarrow$ binding energy between proton and neutron contributes similarly to m_g and m_i

$\Rightarrow$ all forces (except gravity) should contribute similarly to m_g and m_i

But these forces are described by special relativity

Einstein Equivalence Principle: Weak equivalence principle + In a reference frame where gravity is negligible, forces are described by special relativity

$\Rightarrow$ local Lorentz invariance

$\Rightarrow$ local position invariance

e.g.: electric force between two charged particles is the same at any position of space and in any moving reference frame

EEP can be tested by looking if fundamental constants, change in time or in space

Strong Equivalence Principle

The Strong Equivalence Principle generalizes to gravity the EEP, even gravity contributes similarly to m_g and m_i

So the gravitational constant should be the same everywhere in the Universe and at any epoch

Fifth force experiments …

$\boxed{\text{Modified gravity is usually equivalent to a violation of the (W/E/S)EP}}$

Jordan-Brans-Dicke theory

The main idea is to consider is not only the result of the metric but also a scalar field

Thus, the name Scalar-Tensor theories

$$L = \sqrt{-g}\left(\phi R - \frac{\omega}{\phi}\partial_\mu \phi \partial^\mu \phi\right) + L_m$$

plays the role of the gravitational constant

dimensionless parameter $\sim \mathcal{O}(1)$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\phi = \bar{\phi} + \delta\phi$$

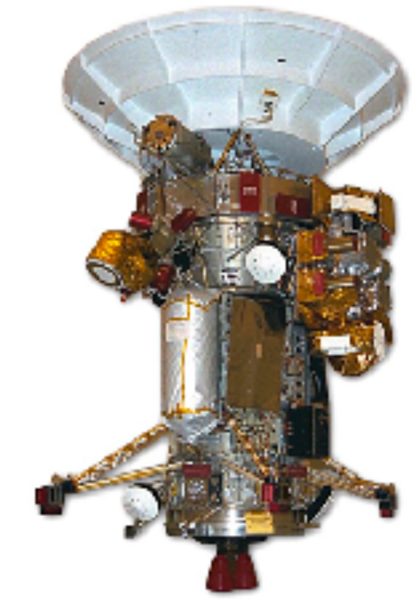
cosmological value

$$g_{00} = -1 + \frac{1}{\bar{\phi}}\frac{2\omega+4}{2\omega+3}\frac{2M}{r} = -1 + \frac{2G_{eff}M}{r}$$

γ

$$g_{ij} = \delta_{ij}\left(1 + \frac{1}{\bar{\phi}}\frac{2\omega+2}{2\omega+3}\frac{2M}{r}\right) = \delta_{ij}\left(1 + \boxed{\frac{\omega+1}{\omega+2}}\frac{2G_{eff}M}{r}\right)$$

$$\phi = \bar{\phi}\left(1 + \frac{2M}{(2\omega+3)\bar{\phi}r}\right)$$



$$\boxed{G_{eff} = \frac{1}{\bar{\phi}}\frac{2\omega+4}{2\omega+3}}$$

$$|\dot{G}_{eff}/G_{eff}| < (7.1 \pm 7.6) \times 10^{-14}\ \mathrm{yr}^{-1}$$

$$\omega > 40.000 \Leftarrow \quad |\gamma - 1| < 2.3\ 10^{-5}$$

LLR Class.Quant.Grav. 35 (2018) 3, 035015

constraint on the cosmological evolution of ϕ

ϕ should be constant?

Jordan-Brans-Dicke theory

$$L = \sqrt{-g}\left(\phi R - \frac{\omega}{\phi}\partial_\mu\phi\partial^\mu\phi\right) + L_m(\Psi_m; g_{\mu\nu})$$

$$L = \sqrt{-\tilde{g}}\left(\tilde{R} - \frac{\omega + 3/2}{\phi^2}\partial_\mu\phi\partial^\mu\phi\right) + L_m(\Psi_m; \tilde{g}_{\mu\nu}/\phi)$$

$$\tilde{g}_{\mu\nu} = \phi g_{\mu\nu}$$

redefinition of the metric

redefinition of the scalar field $\phi = e^{\beta\psi}$ $\beta = (3 + 2\omega)^{-1/2}$ $L = \sqrt{-\tilde{g}}\left(\tilde{R} - \frac{1}{2}\partial_\mu\psi\partial^\mu\psi\right) + L_m(\Psi_m; e^{-\beta\psi}\tilde{g}_{\mu\nu})$

Geodesic equation $\ddot{x}^\sigma + \Gamma^\sigma_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = 0 \iff \ddot{x}^\sigma + \tilde{\Gamma}^\sigma_{\mu\nu}\dot{x}^\mu\dot{x}^\nu \boxed{- \frac{\beta}{2}\left(2\dot{x}^\sigma\dot{x}^\mu\partial_\mu\psi + \partial^\sigma\psi\right)} = 0$

weak field approximation $\vec{F}_5 = \frac{\beta}{2}\vec{\nabla}\psi$

$\boxed{\text{Non-minimal coupling} \iff \text{Fifth force}}$

which is highly constrained

Einstein frame vs Jordan frame

Einstein vs Jordan frame

$$S = \int d^4x\sqrt{-g}\left[F(\phi)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right] + \boxed{S_m[g_{\mu\nu}, \Psi]}$$

Jordan frame

$$S = \int d^4x\sqrt{-\bar{g}}\left[\bar{R} - \frac{1}{2}\left(\frac{1+3F^{'2}}{F^2}\right)\bar{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right] + \boxed{S_m[F^{-1}g_{\mu\nu}, \Psi]}$$

Einstein frame

$$\boxed{\bar{m}(\phi) = \frac{m}{\sqrt{F}}}$$

They are an infinite number of frames, are they equivalent

Of course, it's just a field redefinition

But they are infinite debates on the physical frame, which one is physical

arXiv:astro-ph/9910176

Einstein vs Jordan frame

$$ds^2 = -\,dt^2 + a^2(t)d\mathrm{x}^2$$

$a(t)$ the Universe is expanding

How do we observe it?

We do not observe the expansion of the Universe

We observe cosmological redshift

$$E_{obs} = \frac{E_{lab}}{1+z}$$

So in the Jordan frame, we have expansion of the Universe

Let's work with conformal time $\quad ds^2 = a^2(\eta)(-d\eta^2 + d\mathrm{x}^2)$

And perform a conformal transformation $\quad g_{\mu\nu} = a^2\ \bar{g}_{\mu\nu}$

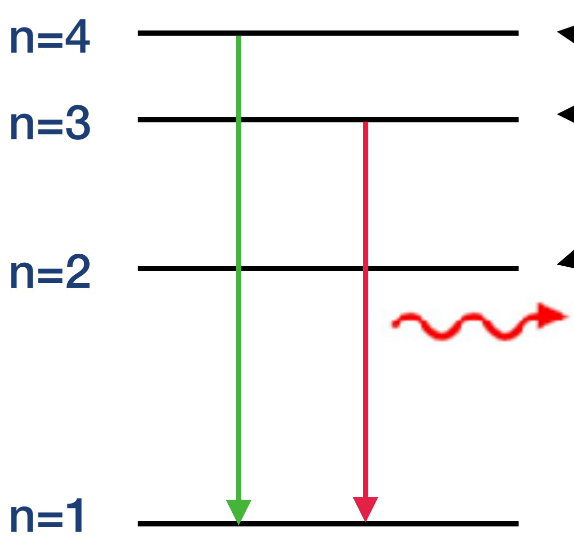
$$d\bar{s}^2 = -\,d\eta^2 + d\mathrm{x}^2$$

So in this frame, the Universe is static, we do not have expansion, we do not have redshift

$$\boxed{\text{Frames are not physically equivalent}}$$

All frames are physically equivalent

In the new frame, masses change with time $\bar{m}(\eta) = m \;\, a(\eta)$

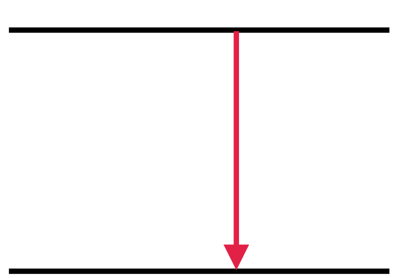


$$E_n = -\frac{m_e e^4}{2(4\pi\epsilon_0)^2\hbar^2}\frac{1}{n^2}$$

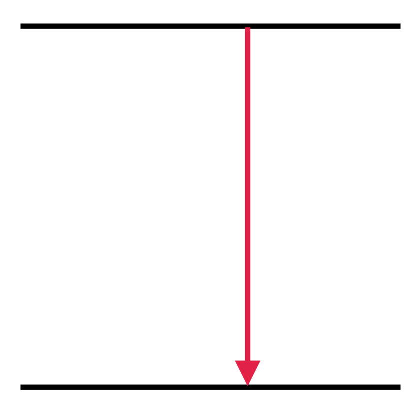
$E \equiv h\nu = E_{n=3} - E_{n=1}$

Energy levels change with time

The photon never lost energy,
our reference has changed



compared to



But we observe the same,
a redshift

$$E_{obs} = \frac{E_{lab}}{1+z}$$

When photon was emitted

In the laboratory, today

http://www2.yukawa.kyoto-u.ac.jp/~misao.sasaki/PDF/conformal1.pdf

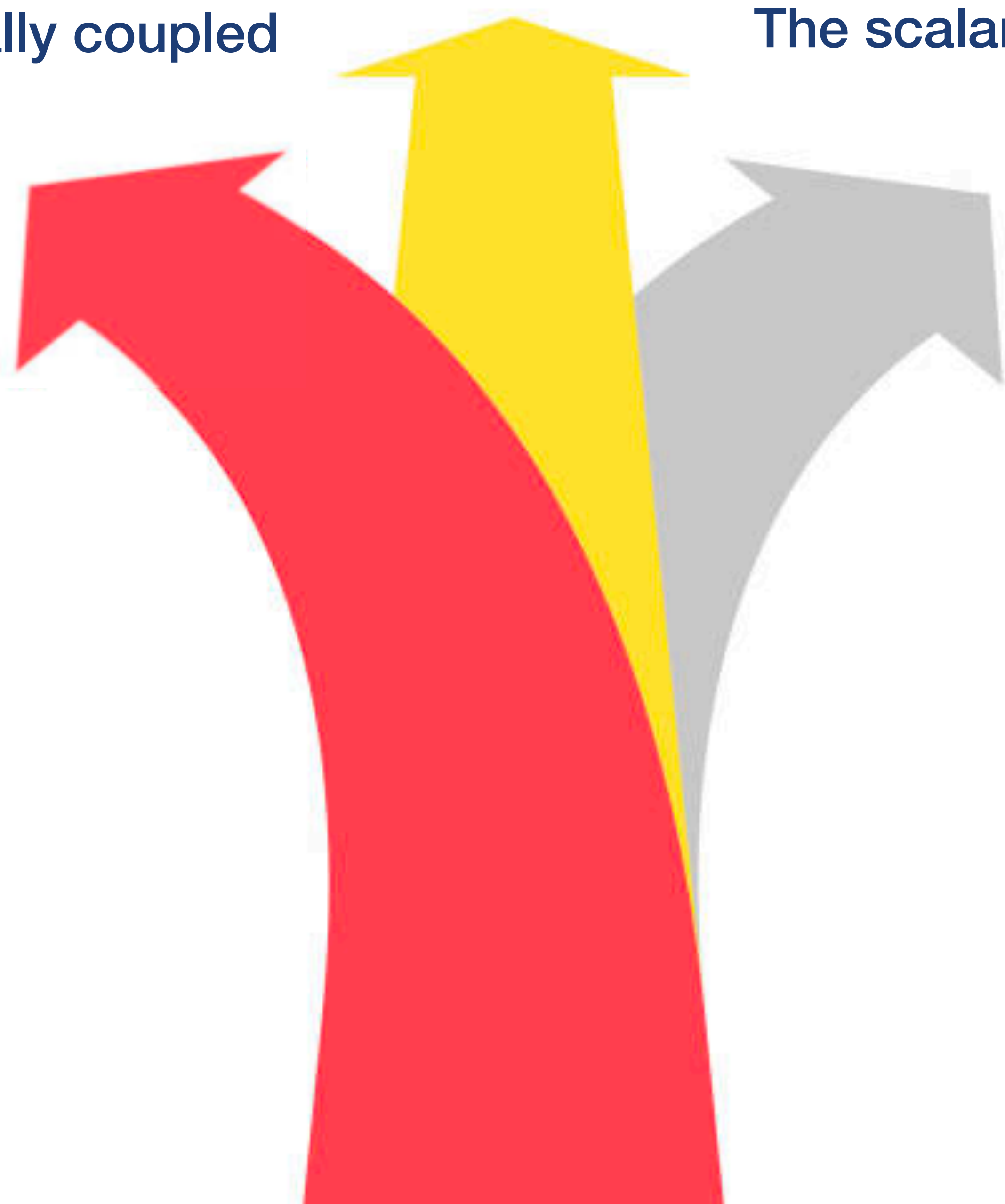
Three roads ahead

Non-minimal coupling $\Longleftrightarrow$ Fifth force

Standard Model

The scalar field is minimally coupled
Dark Energy
Inflation

The scalar field is non-minimally coupled
Modified Gravity



Dark Energy

Observational evidence of the acceleration of the Universe

$$ds^2 = -\,dt^2 + a(t)^2\left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right)$$

Friedmann equations

$$3H^2 = \kappa^2(\rho_m + \rho_r + \rho_\Lambda)$$

$$-2\dot{H} = \kappa^2(\rho_m + \frac{4}{3}\rho_r)$$

constant
could be related to vacuum energy

$$1 = \Omega_m + \Omega_r + \Omega_\Lambda$$

$$\Omega_i = \frac{\kappa^2\rho_i}{3H^2}$$

$$h^2 \equiv \frac{H^2}{H_0^2} = \frac{\kappa^2\rho_m}{3H_0^2} + \frac{\kappa^2\rho_r}{3H_0^2} + \frac{\kappa^2\rho_\Lambda}{3H_0^2} = \Omega_{m,0}a^{-3} + \Omega_{r,0}a^{-4} + \Omega_{\Lambda,0}$$

But $a(t) = \dfrac{1}{1+z}$ $\Rightarrow h^2 = \Omega_{m,0}(1+z)^3 + \Omega_{r,0}(1+z)^4 + \Omega_{\Lambda,0}$ $\quad \Omega_{m,0} + \Omega_{r,0} + \Omega_{\Lambda,0} = 1$

For an object of intrinsic luminosity L, the measured energy flux F defines the luminosity distance d_L to the object

$$d_L \equiv \sqrt{\frac{L}{4\pi F}} = (1+z)\int_0^z \frac{dz'}{H(z')}$$

$$d_L = \frac{1+z}{H_0}\int_0^z \frac{dz'}{\sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{r,0}(1+z)^4 + \Omega_{\Lambda,0}}} \qquad \Omega_{m,0} + \Omega_{r,0} + \Omega_{\Lambda,0} = 1$$

$$H_0 d_L = z + \left(\Omega_{\Lambda,0} + \frac{\Omega_{m,0}}{4}\right)z^2 + \mathcal{O}(z^3)$$

$$H_0 d_L = z + \left(\Omega_{\Lambda,0} + \frac{\Omega_{m,0}}{4}\right) z^2 + \mathcal{O}(z^3)$$

$$3H^2 = \kappa^2(\rho_m + \rho_r + \rho_\Lambda)$$

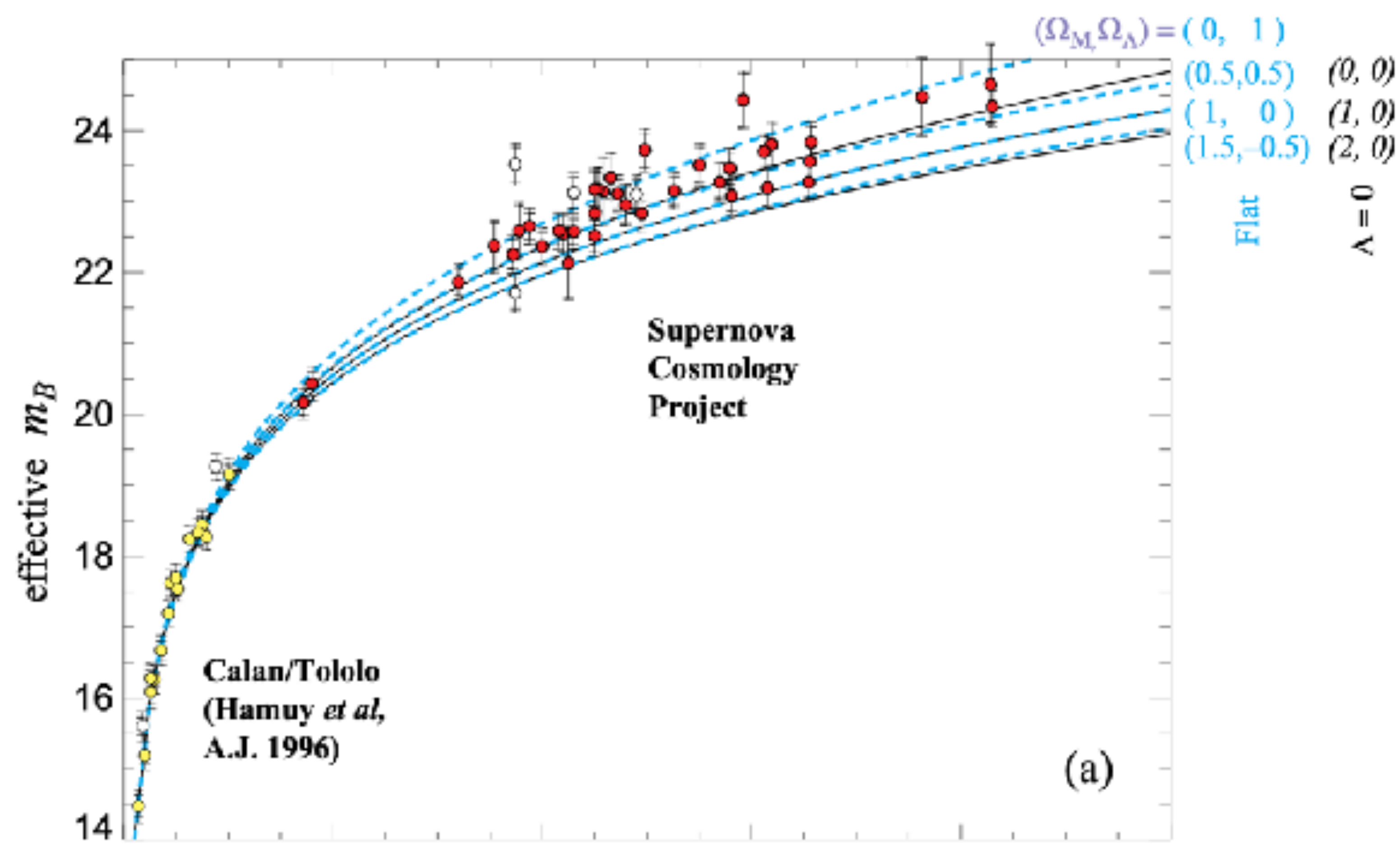
$$-2\dot{H} = \kappa^2(\rho_m + \frac{4}{3}\rho_r)$$

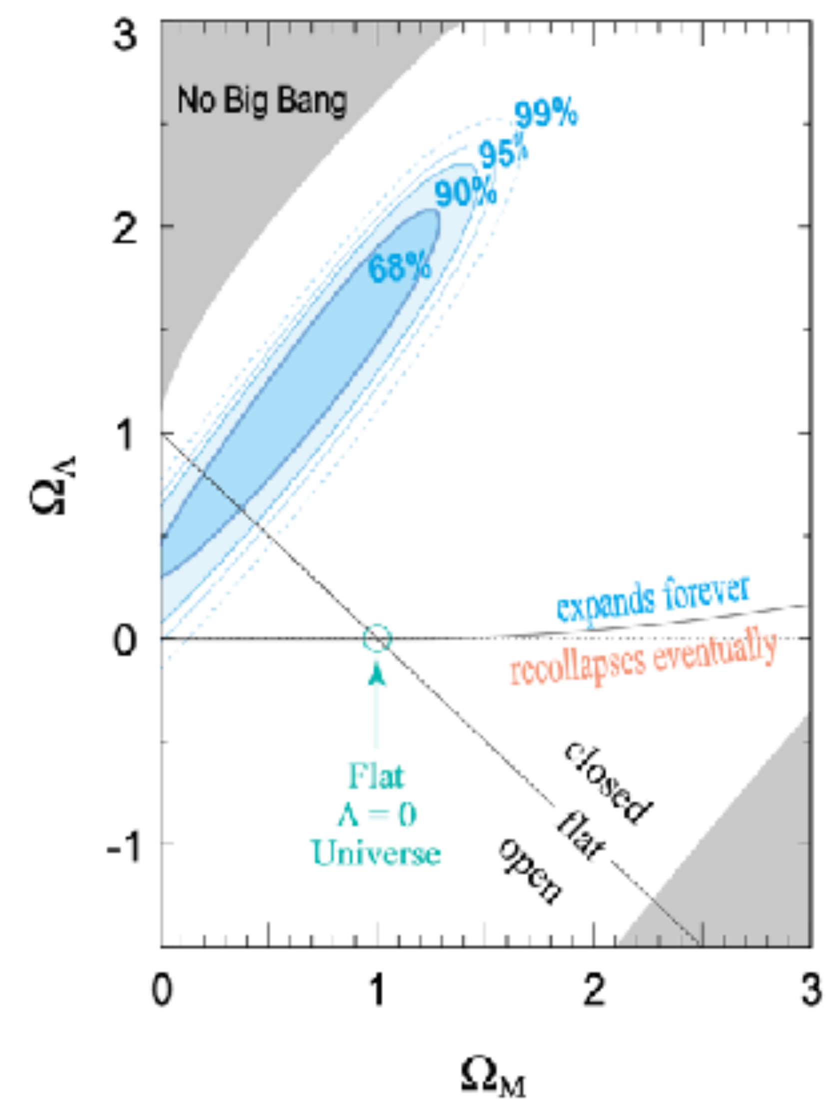
$$\Rightarrow \frac{\ddot{a}}{aH^2} = 2\left(\Omega_\Lambda + \frac{\Omega_m}{4} - \frac{1}{2}\right)$$

Acceleration means $\Omega_\Lambda + \frac{\Omega_m}{4} - \frac{1}{2} > 0$

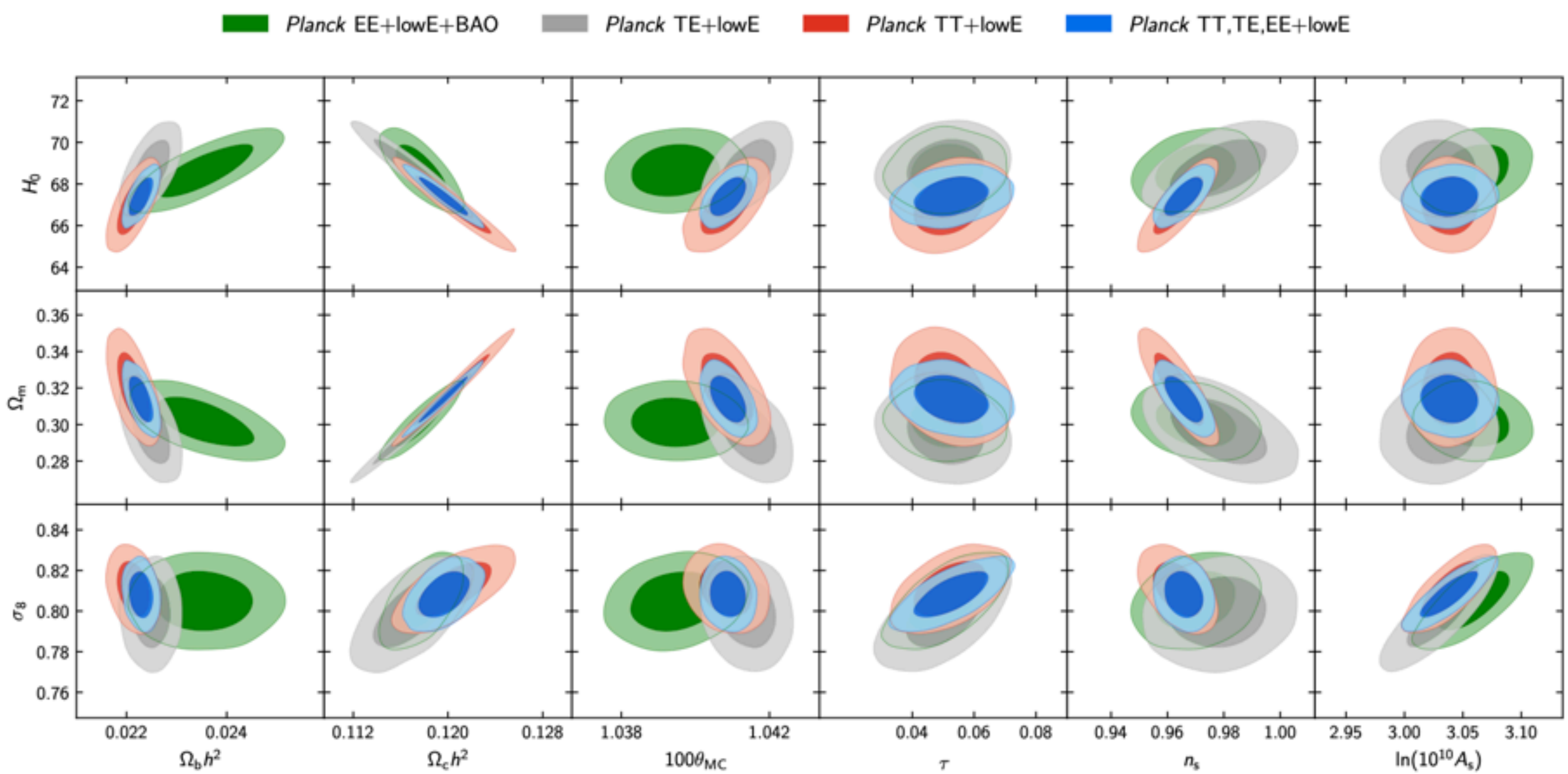
Supernova Search Team, A. Riess et al. arXiv:astro-ph/9805201 $\Omega_\Lambda + \frac{\Omega_m}{4} - \frac{1}{2} = 1 \pm 0.4$

Supernova Cosmology Project, S. Perlmutter et al. arXiv:astro-ph/9812133



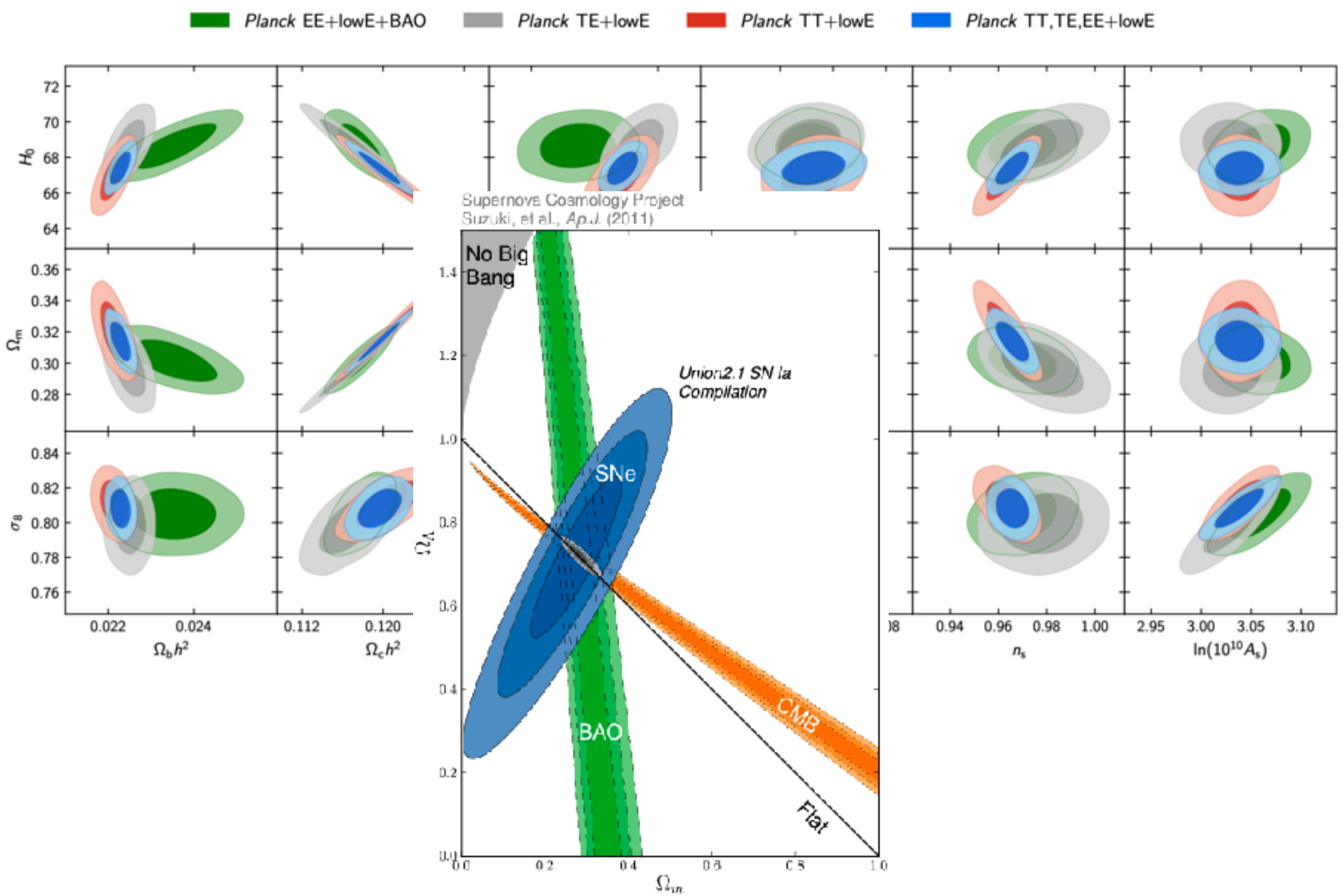


Planck 2018 arXiv:1807.06209



Dark energy

Planck 2018 arXiv:1807.06209



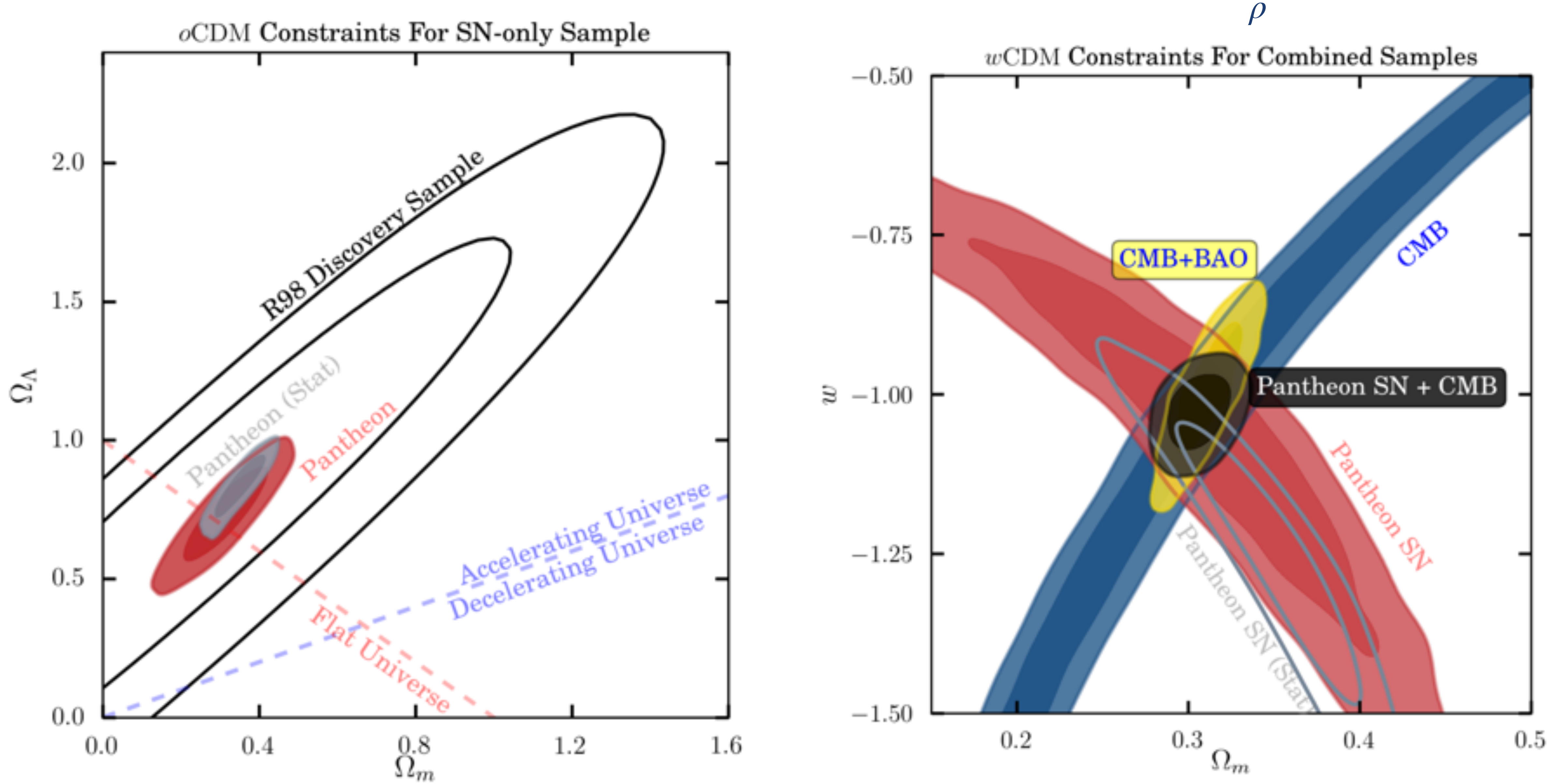
The Standard Model is Perfect

Why to look to something else

Are they any problems with the cosmological constant?

Observational

$$w \equiv \frac{P}{\rho}$$

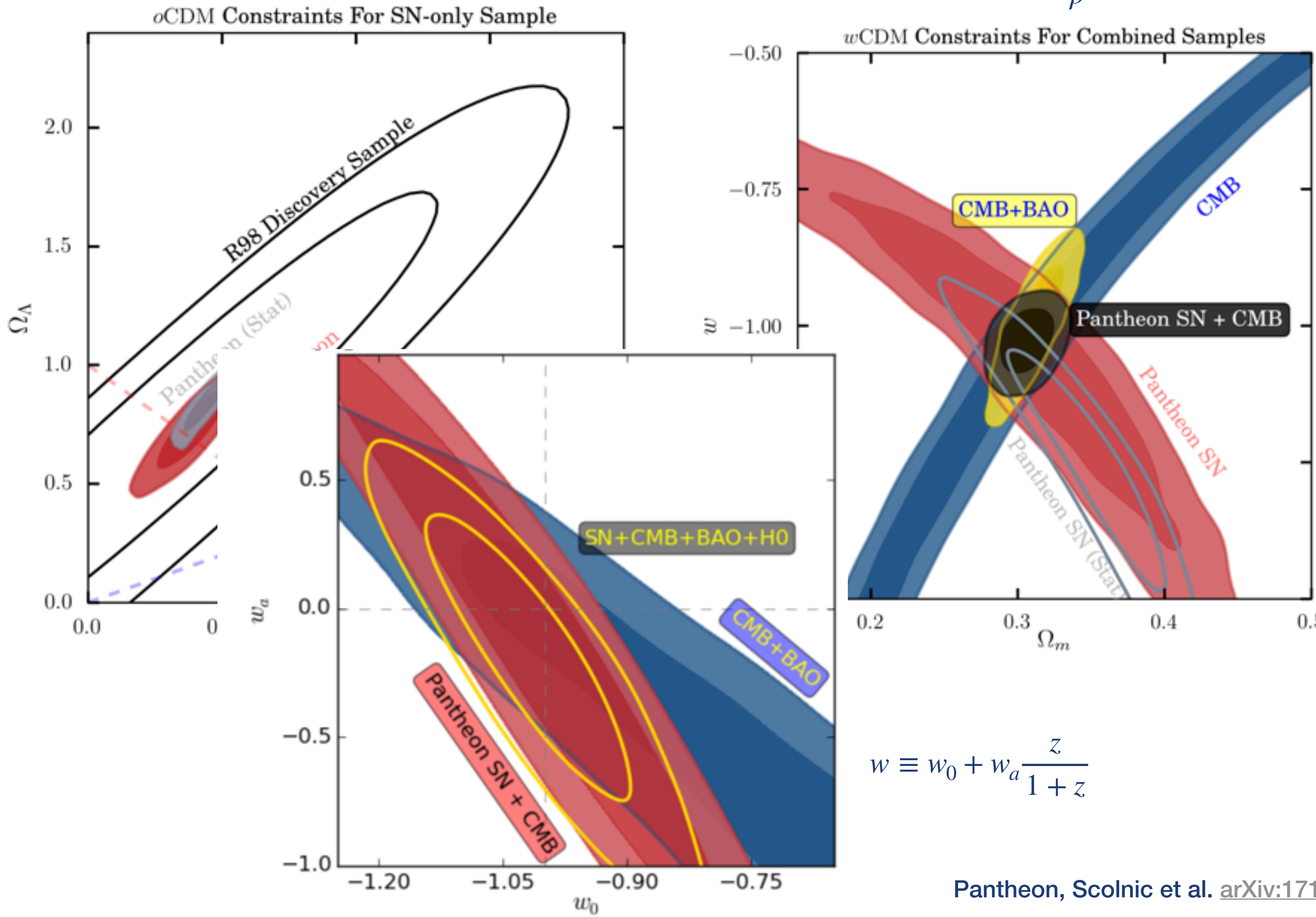


Pantheon, Scolnic et al. arXiv:1710.00845

Problems

Observational

$$w \equiv \frac{P}{\rho}$$



$$w \equiv w_0 + w_a \frac{z}{1+z}$$

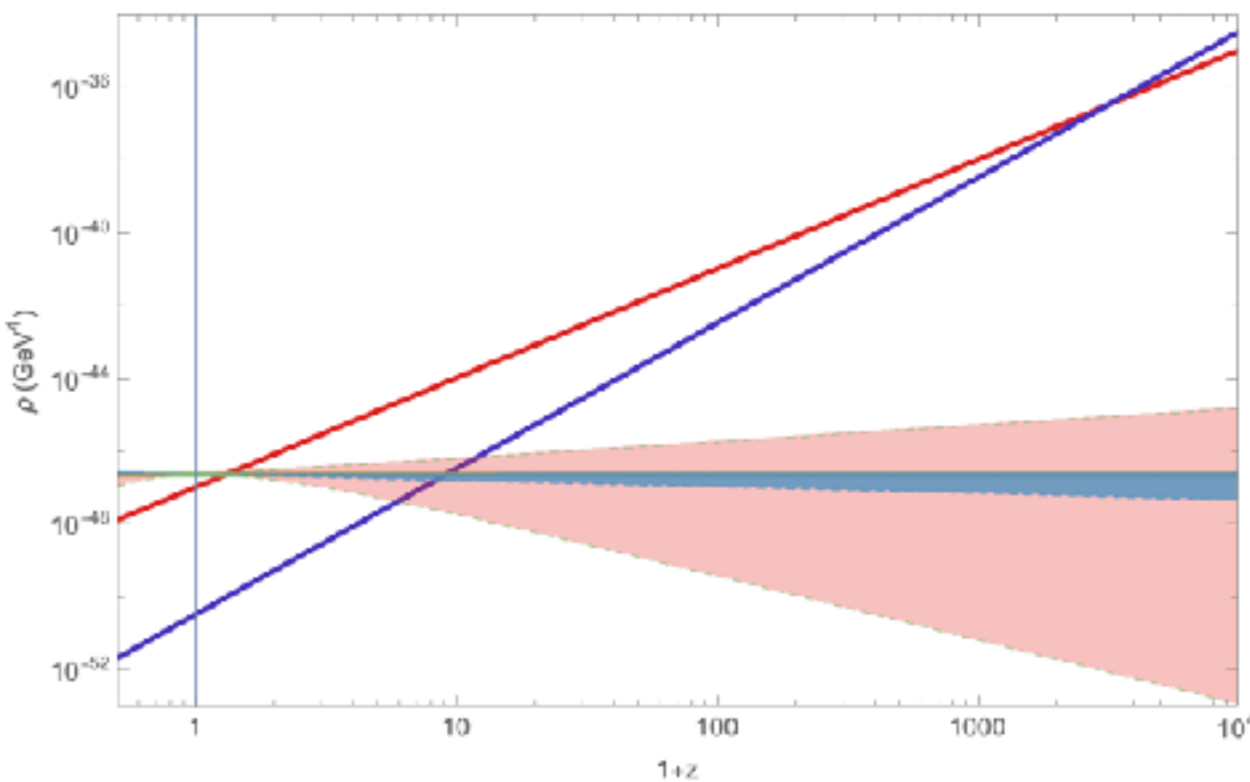
Pantheon, Scolnic et al. arXiv:1710.00845

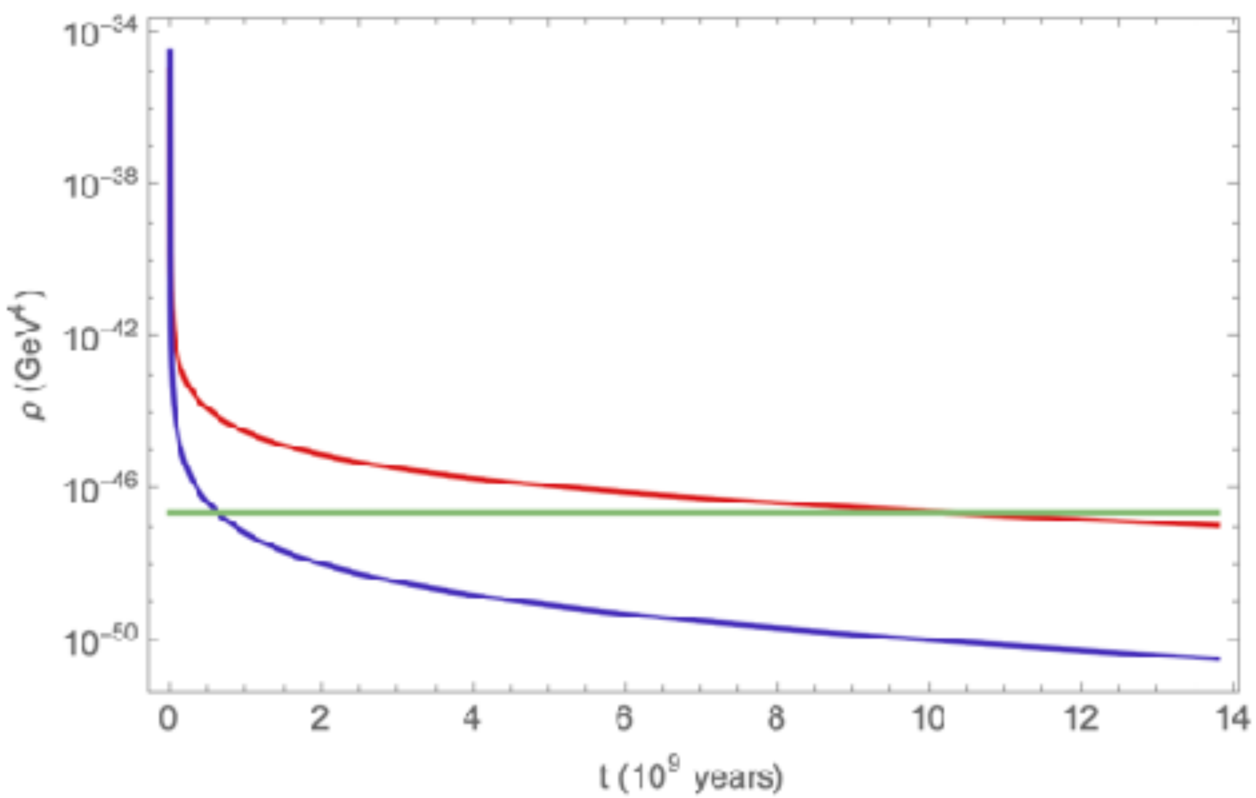
Theoretical

No positive cosmological constant in String Theory

Cosmological constant problem?

Coincidence problem?





Curiosity

Testing GR

Quintessence

Dark energy

The simplest model: Quintessence

$$L = \sqrt{-g}\left(R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right) + L_m(\Psi_m; g_{\mu\nu})$$

Applying this model to cosmology, we get an additional fluid

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \qquad P = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$$H^2 = \frac{\kappa^2}{3}\left(\rho_m + \rho_r + \frac{1}{2}\dot{\phi}^2 + V\right) \qquad \Rightarrow 1 = \Omega_m + \Omega_r + x^2 + y^2 \qquad x = \frac{\kappa\dot{\phi}}{\sqrt{6}H} \quad y = \frac{\kappa\sqrt{V}}{\sqrt{3}H}$$

$$\dot{H} = -\frac{\kappa^2}{2}\left(\rho_m + \frac{4}{3}\rho_r + \dot{\phi}^2\right)$$

If we neglect radiation, we have

$$\lambda = -\frac{V'}{\kappa V}$$

$$2x' = 3x^3 + \sqrt{6}y^2\boxed{\lambda} - 3x(1 + y^2)$$

$$2y' = y(3 - \sqrt{6}x\lambda + 3x^2 - 3y^2)$$

If $V(\phi) = V_0 e^{-n\kappa\phi} \Rightarrow \lambda = n$

Critical points

$(0{,}0)$ matter era

$(\pm 1{,}0)$ kinetic era $w = 1$

$\left(\frac{n}{\sqrt{6}}, \sqrt{1 - \frac{n^2}{6}}\right)$ era during which $w = -1 + n^2/3$, exists if $n^2 < 6$ and stable if $n^2 < 3$

$\left(\sqrt{\frac{3}{2n^2}}, \sqrt{\frac{3}{2n^2}}\right)$ scaling solution, exists if $n^2 > 3$, for which $w = 0$

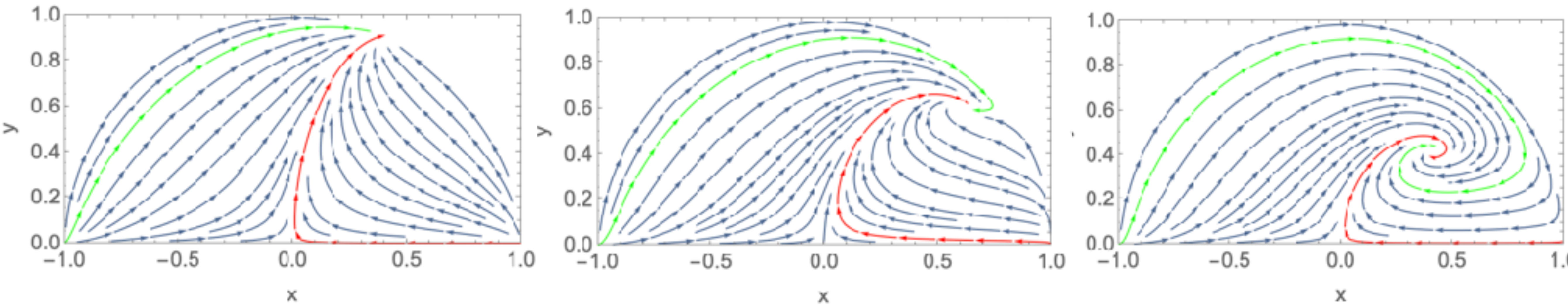
Dark energy

$(0{,}0)$ matter era

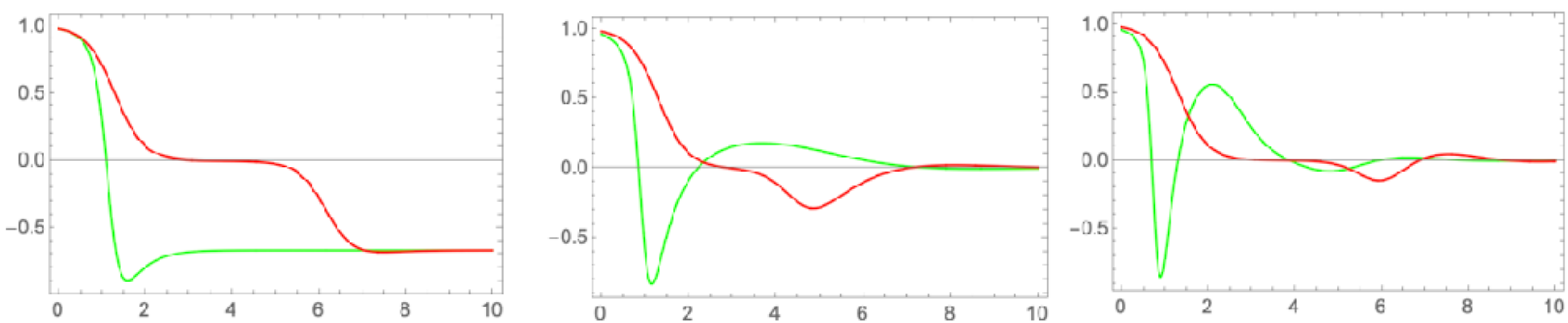
$(\pm 1{,}0)$ kinetic era $w = 1$

$\left(\frac{n}{\sqrt{6}}, \sqrt{1 - \frac{n^2}{6}}\right)$ era during which $w = -1 + n^2/3$, exists if $n^2 < 6$ and stable if $n^2 < 3$, produces acceleration only if $n^2 < 2$

$\left(\sqrt{\frac{3}{2n^2}}, \sqrt{\frac{3}{2n^2}}\right)$ scaling solution, exists if $n^2 > 3$, for which $w = 0$



$n = 1$ $\qquad$ $n = 2$ $\qquad$ $n = 3$



Scaling solution

We would like the scaling solution to replace matter, so $\lambda \gg 1$ but that would eliminate acceleration

If λ is not constant, then we need an equation for it

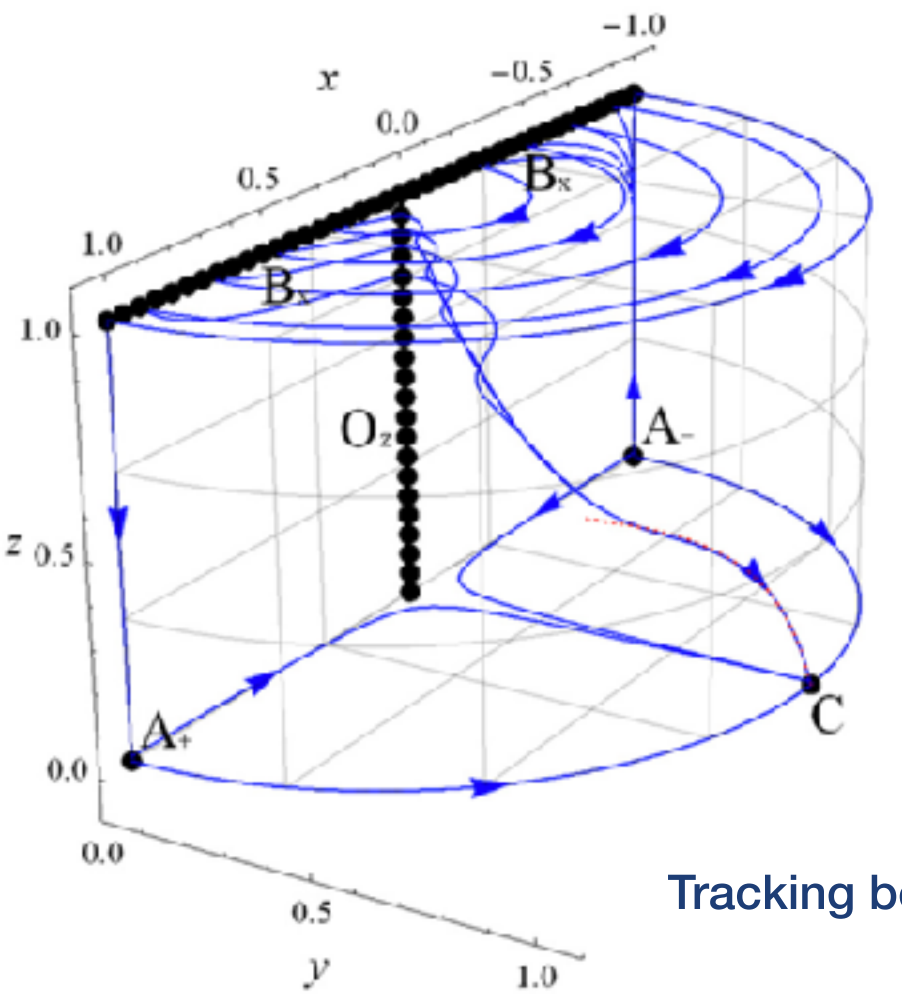
$$2x' = 3x^3 + \sqrt{6}y^2\lambda - 3x(1+y^2)$$
$$2y' = y(3 - \sqrt{6}x\lambda + 3x^2 - 3y^2)$$
$$\lambda' = \sqrt{6}\lambda^2 x(1-\Gamma)$$

$$\Gamma = \frac{VV''}{V'^2}$$

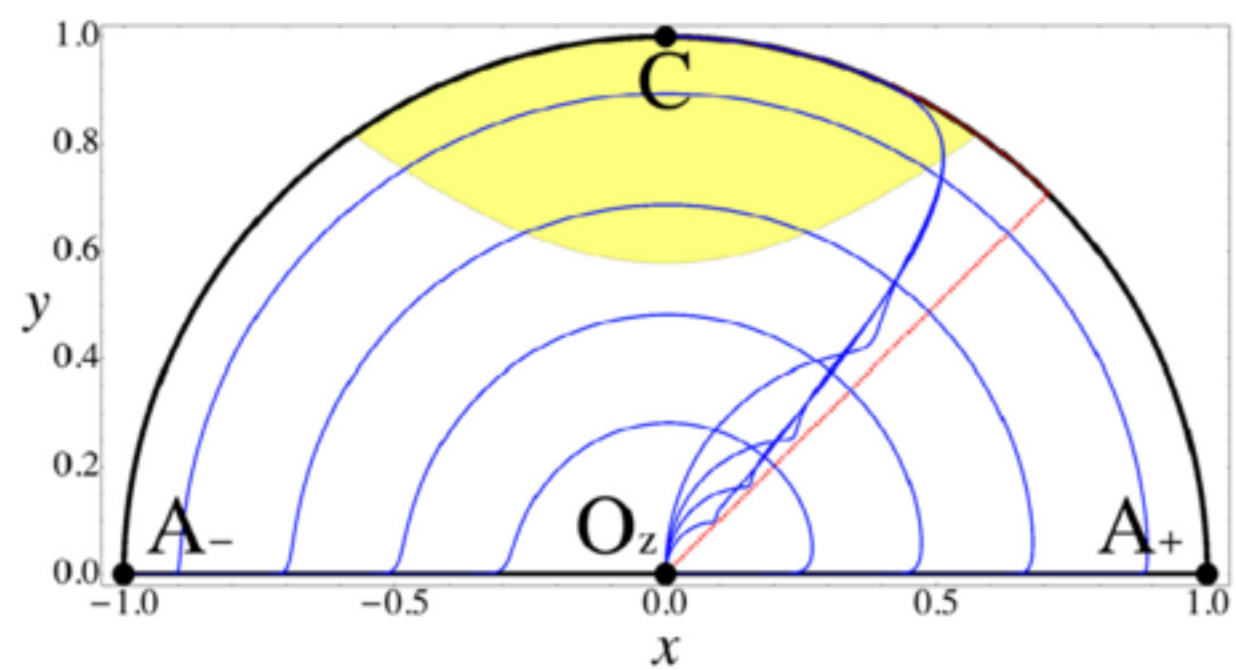
One of the first model considers the so-called Ratra-Peebles potential Phys.Rev.D 37 (1988) 3406

But $\lambda \in [0,\infty[$ so we need to compactify the variable $z = \dfrac{\lambda}{1+\lambda}$

$$V(\phi) = \frac{M^{4+n}}{\phi^n} \qquad \Gamma = 1 + \frac{1}{n}$$



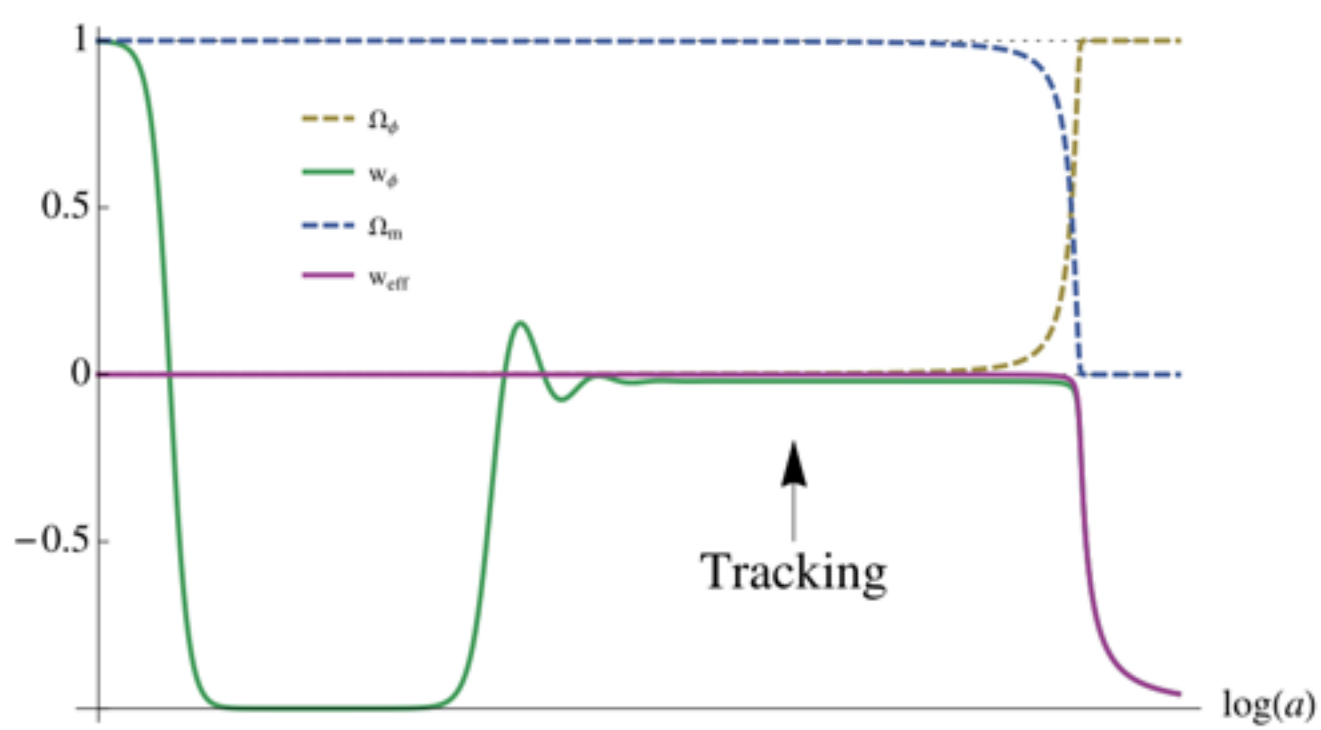
Tracking behavior



arXiv:1712.03107

Dark energy

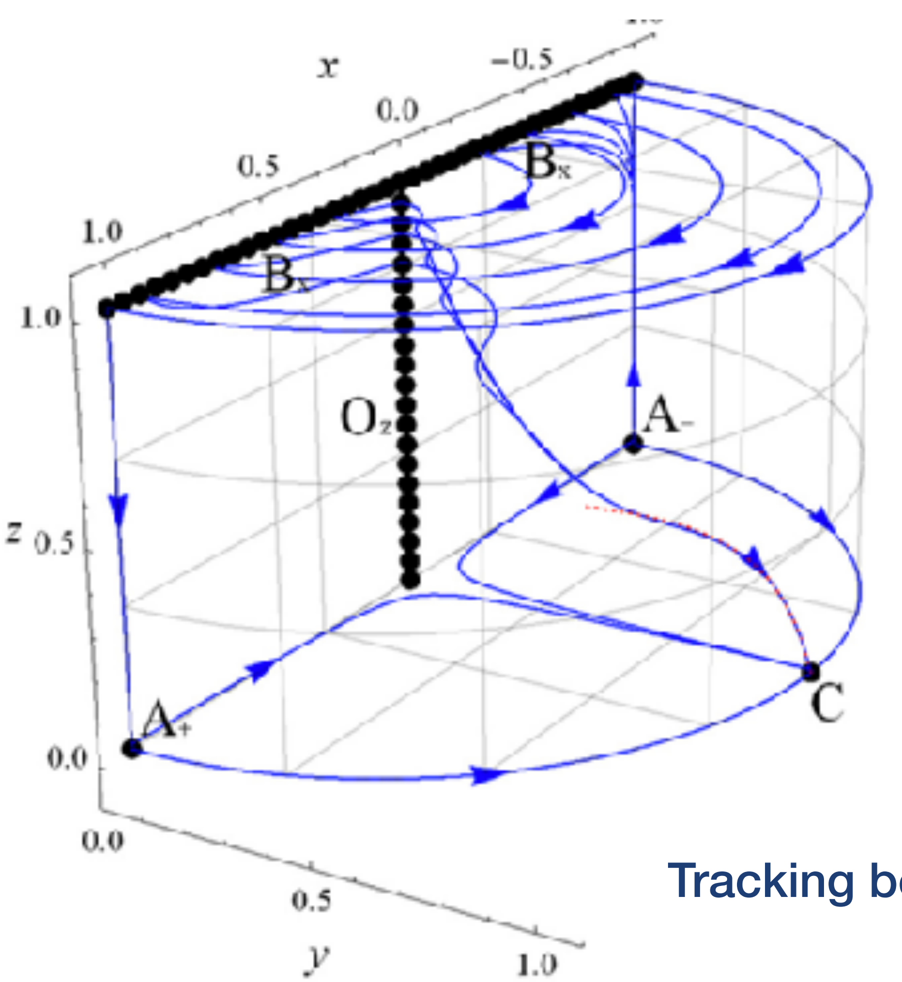
If λ is not constant, then we need an equation for it

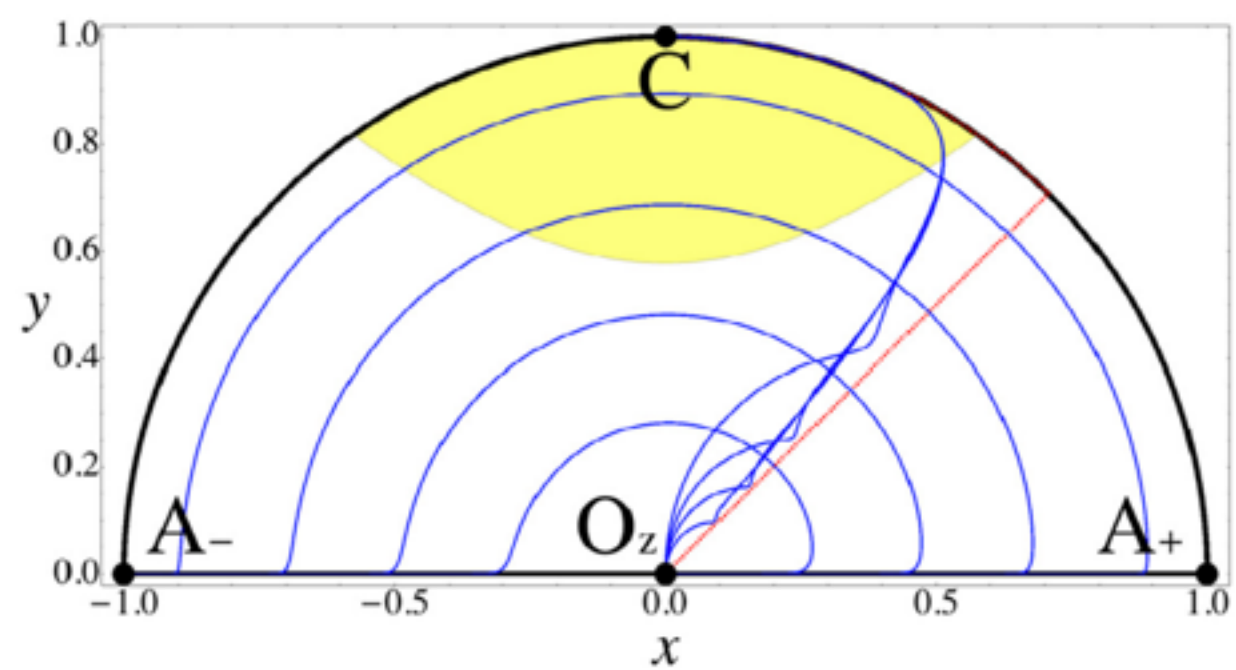


One of the first model con… (1988) 3406

But $\lambda \in [0,\infty[$ so we neec…

$$V(\phi) = \frac{M^{4+n}}{\phi^n} \qquad \Gamma = 1 + \frac{1}{n}$$





Tracking behavior

arXiv:1712.03107

Potentials which satisfy the constraint $\Gamma = \dfrac{VV''}{(V')^2} \geq 1$ have a de Sitter attractor

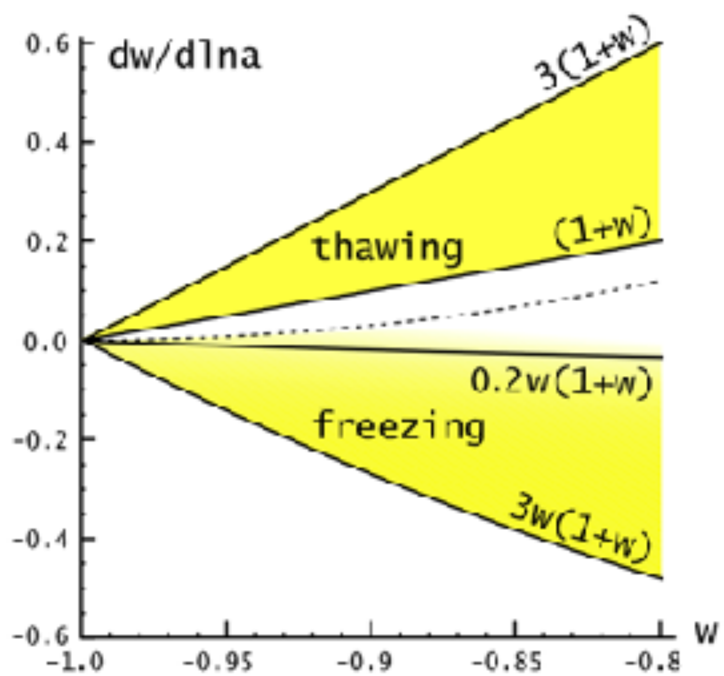
So they are more interesting for dark energy, $\Gamma < 1$ are used for inflation

$$w = \frac{P}{\rho} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}$$

Freezing models

w gradually decreases to the attractor $w = -1$

$$V(\phi) \sim \phi^{-n}$$



Thawing models

The scalar field has been frozen by the friction term $H\dot{\phi}$ in the equation for the scalar field

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0$$

w starts around -1 and evolves when the mass of the scalar field becomes larger than the Hubble function H

$$V(\phi) \sim V_0 + \phi^n$$

$$V(\phi) \sim \cos^2 \phi$$

arXiv:astro-ph/0505494

Problems with quintessence

In principle it could be detected with cosmological observations

Background metric is different from ΛCDM

LLS constraints

$$\delta_m = \delta\rho_m/\rho_m \qquad \delta_m'' + \frac{1}{2}(1 - 3w\Omega_\phi)\delta_m' - \frac{3}{2}\Omega_m\delta_m = 0$$

So far nothing has been observed

In order to obtain an acceleration at the right moment, one needs to fine-tune M

Doesn't solve the cosmological constant problem

Might solve the coincidence problem

Not coupled to other fields, so difficult to detect locally

Quintessence

$$X = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

$$L = \sqrt{-g}\left(R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right) \equiv \sqrt{-g}\left(R + X - V(\phi)\right)$$

K-essence

$$L = \sqrt{-g}\left(R + K(\phi, X)\right)$$

arXiv:hep-th/9904075

arXiv:astro-ph/9912463

arXiv:astro-ph/0004134

One motivation is to have acceleration without potential

It also can be motivated by higher order corrections to the action

arXiv:hep-th/9211021 Low energy effective string theory, dilaton comes with terms such as X^2

arXiv:hep-th/0203211 Unstable mode D-branes

$$w = \frac{P}{\rho} = \frac{K}{2XK_{,X} - K}$$

If $K = -X + \frac{X^2}{M^4}$ known as Ghost condensate arXiv:hep-th/0312099

$$w = \frac{1 - X/M^4}{1 - 3X/M^4}$$

we have an acceleration if $1/2 < X/M^4 < 2/3$ today if $M \simeq 10^{-3}\ eV$

Let's consider the perturbation $\phi = \phi(t) + \delta\phi$ in a flat background, the Hamiltonian at second order is

$$H^{(2)} = \left(K_{,X} + 2XK_{,XX}\right)\frac{(\delta\dot{\phi})^2}{2} + K_{,X}\frac{(\nabla\delta\phi)^2}{2} - K_{,\phi\phi}\frac{(\delta\phi)^2}{2}$$

Stability conditions $\quad K_{,X} + 2XK_{,XX} > 0 \quad K_{,X} > 0$

Speed propagation of the perturbations is $c_s^2 = \dfrac{K_{,X}}{K_{,x} + 2XK_{,XX}}$

Which is positive under the stability conditions and subliminal under the condition $K_{,XX} > 0$

arXiv:hep-th/9904176

The causal structure is modified

Variation wrt ϕ

$$\Big(\underbrace{K_{,X}g^{\mu\nu} - K_{,XX}\partial^\mu\phi\partial^\nu\phi}_{=\tilde{G}^{\mu\nu}(\phi,X)}\Big)\partial_{\mu\nu}\phi - 2XK_{,\phi X} + K_{,\phi} = 0$$

$= \tilde{G}^{\mu\nu}(\phi, X)$ effective metric

This equation is hyperbolic if $\tilde{G}^{\mu\nu}$ has Lorentzian signature (Leray theorem, Wald p.251)

$\Leftrightarrow c_s^2 > 0 \qquad$ Hyperbolicity is identical to hamiltonian stability

Considering fluctuations around some given solution $\phi = \phi_0 + \delta\phi$

$$\frac{1}{\sqrt{-G}}\partial_\mu\left(\sqrt{-G}G^{\mu\nu}\partial_\nu\delta\phi\right) + M^2_{eff}\delta\phi = 0$$

arXiv:0708.0561

$$G^{\mu\nu} = \frac{c_s}{K^2_{,X}}\tilde{G}^{\mu\nu}, \quad G = \det(G^{-1}_{\mu\nu})$$

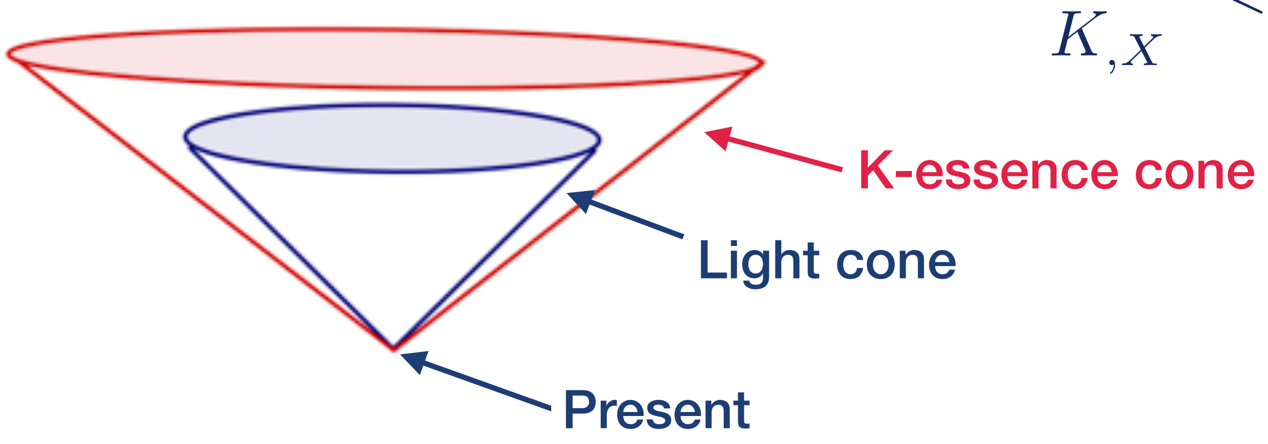
The fluctuations around the solution propagate in an emergent geometry

$$ds^2 = G^{-1}_{\mu\nu}dx^\mu dx^\nu$$

This influence cone is larger than those one determined by the metric $g_{\mu\nu}$ if

$$\frac{K_{,XX}}{K_{,X}} < 0$$

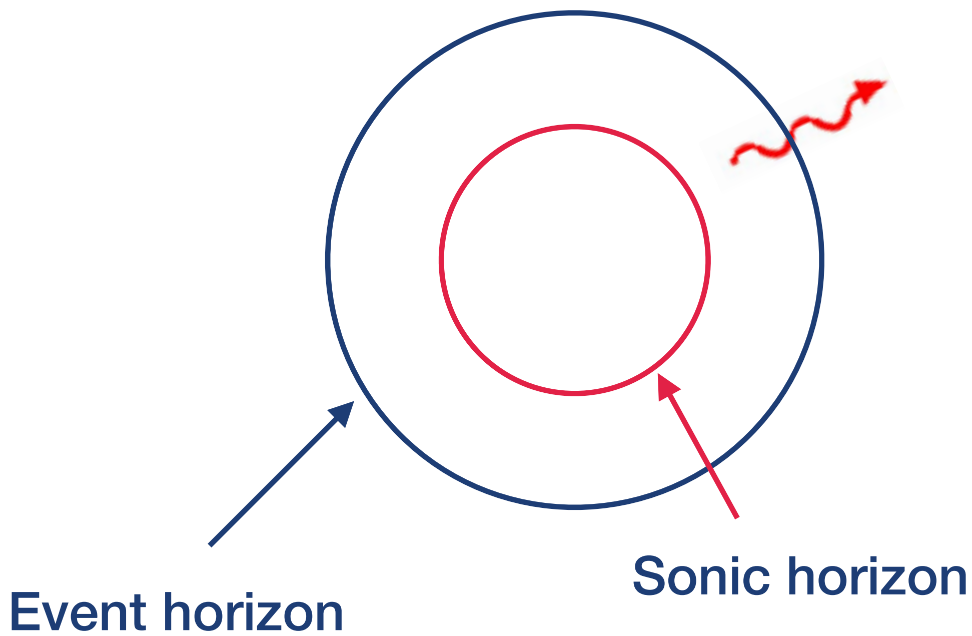
arXiv:gr-qc/0511158



Beyond General Relativity

Let's consider $$K(\phi, X) = X + \alpha X^2 \equiv -\frac{1}{2}(\partial\phi)^2 + \frac{\alpha}{4}(\partial\phi)^4$$

$\alpha < 0$ arXiv:1106.2054



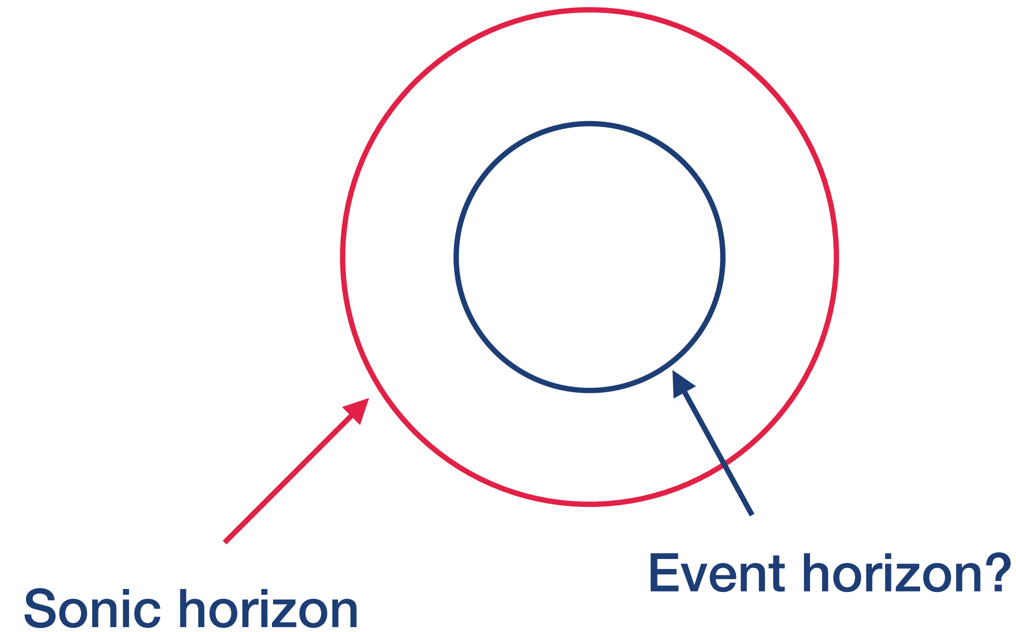
Superluminal propagation

But this model violates "positivity bounds" $(\alpha < 0)$

arXiv:1904.05874

Conditions to be UV completed

$\alpha > 0$ arXiv:2003.1373



The code crashes before the formation of the event horizon

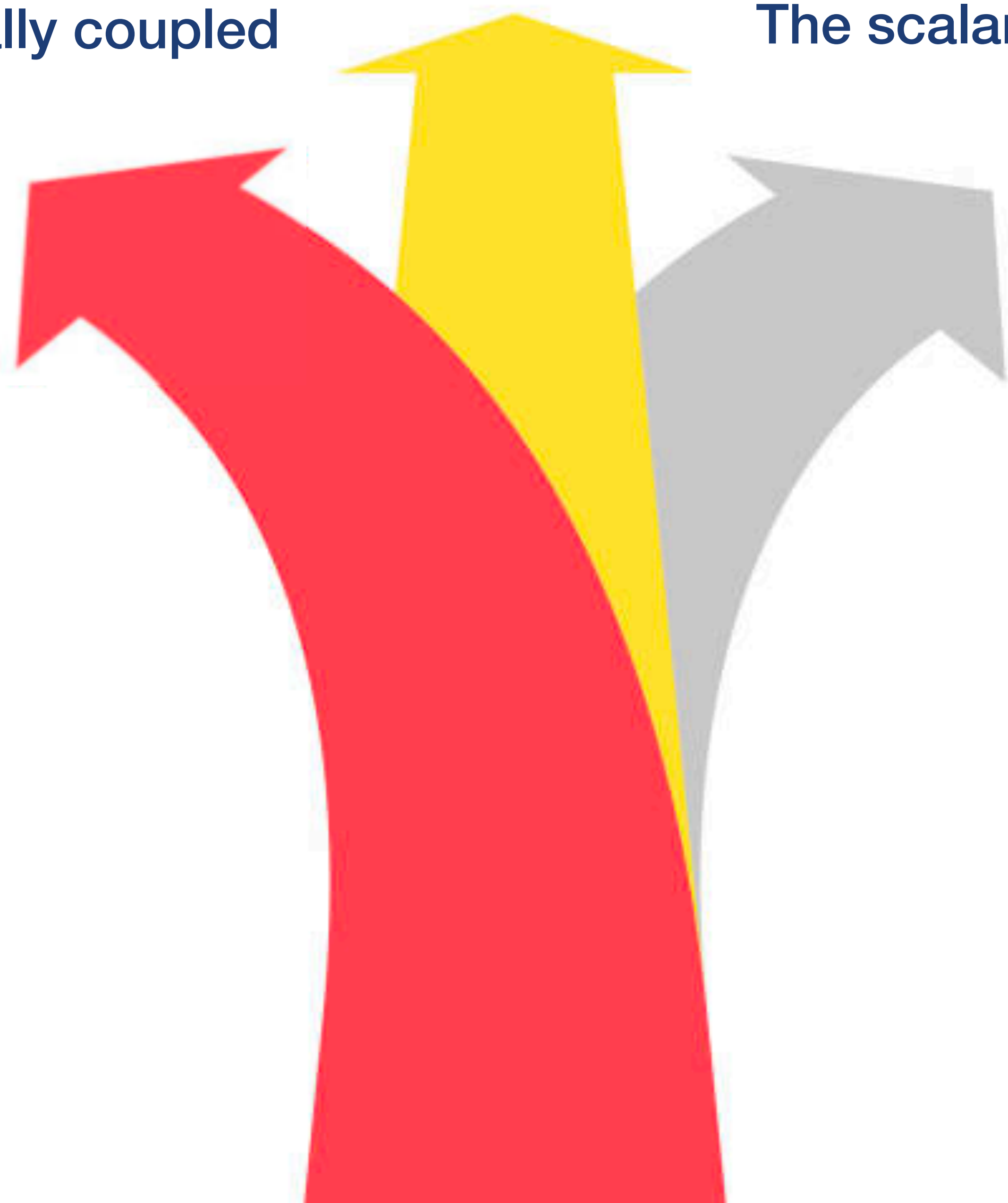
Three roads ahead

Non-minimal coupling $\Longleftrightarrow$ Fifth force

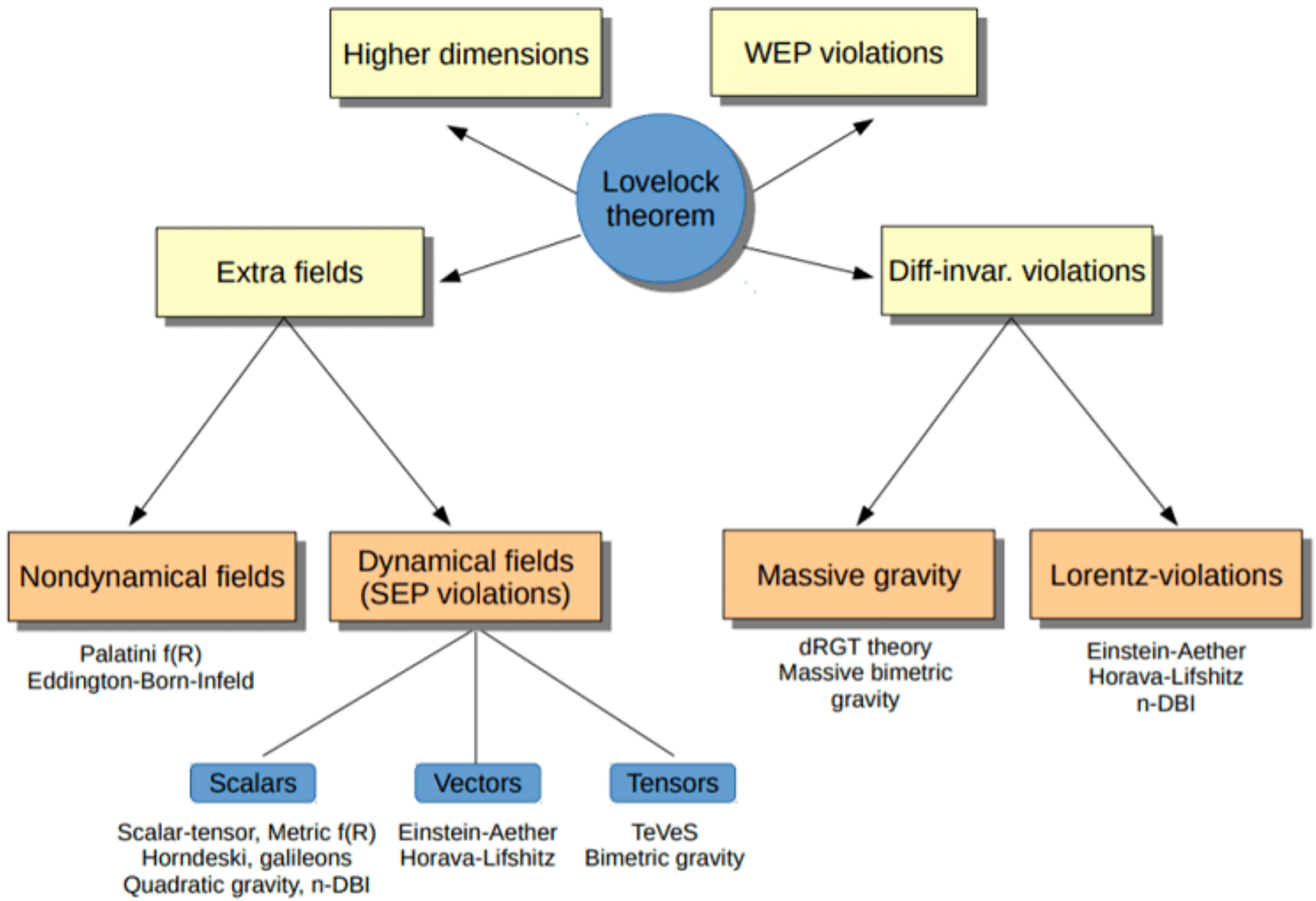
Standard Model

The scalar field is minimally coupled
Dark Energy
Inflation

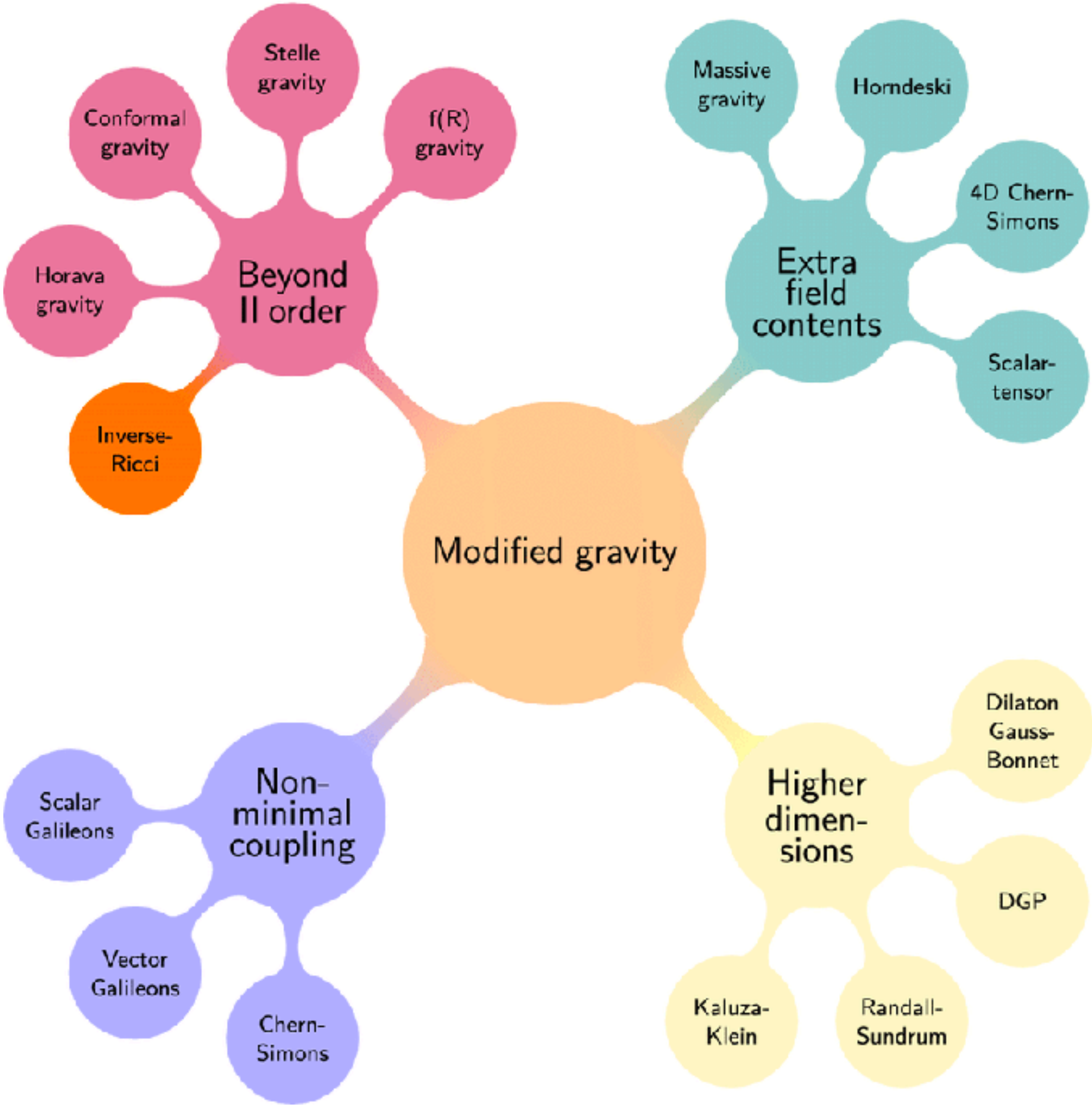
The scalar field is non-minimally coupled
Modified Gravity



Modified gravity

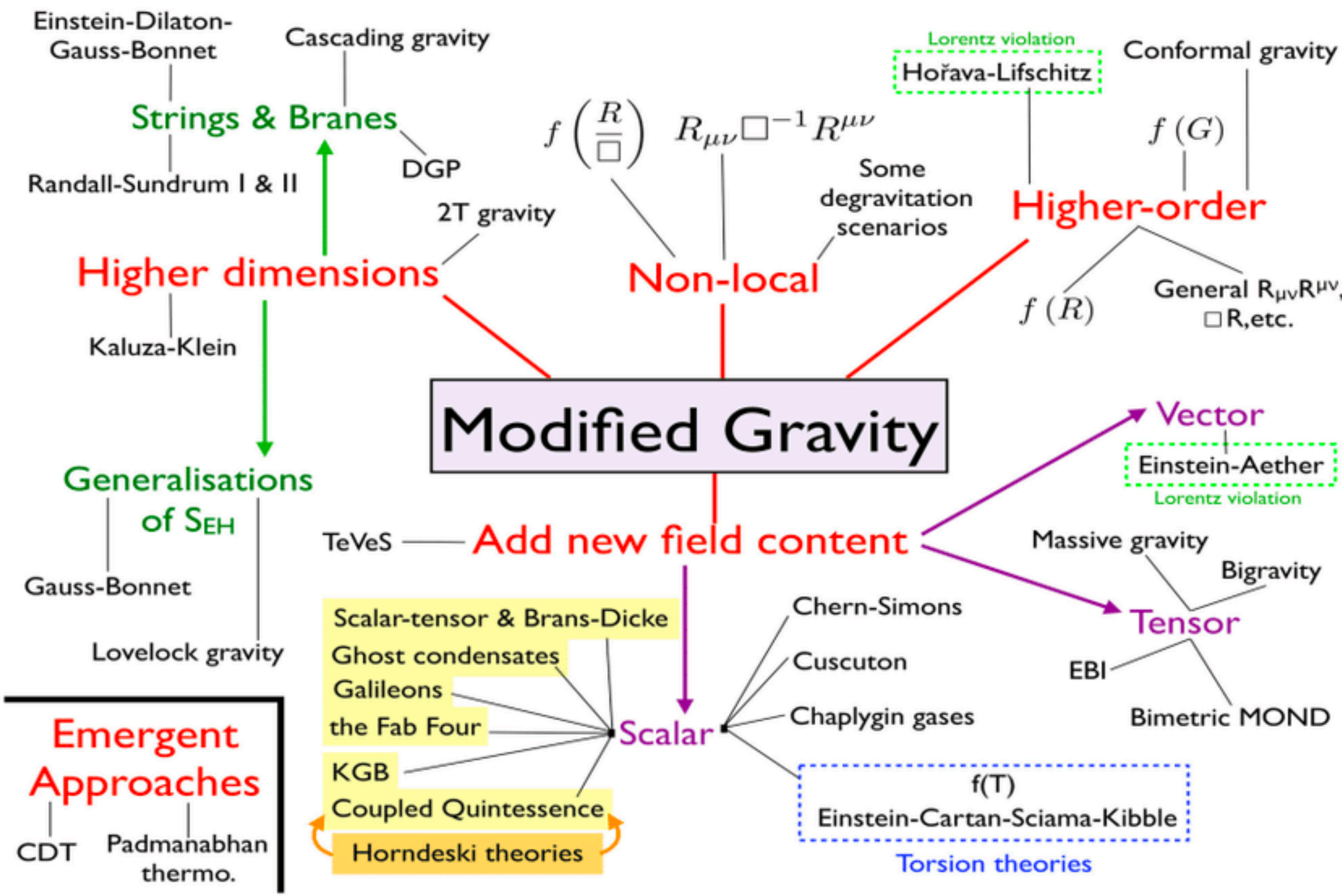


arXiv:1501.07274



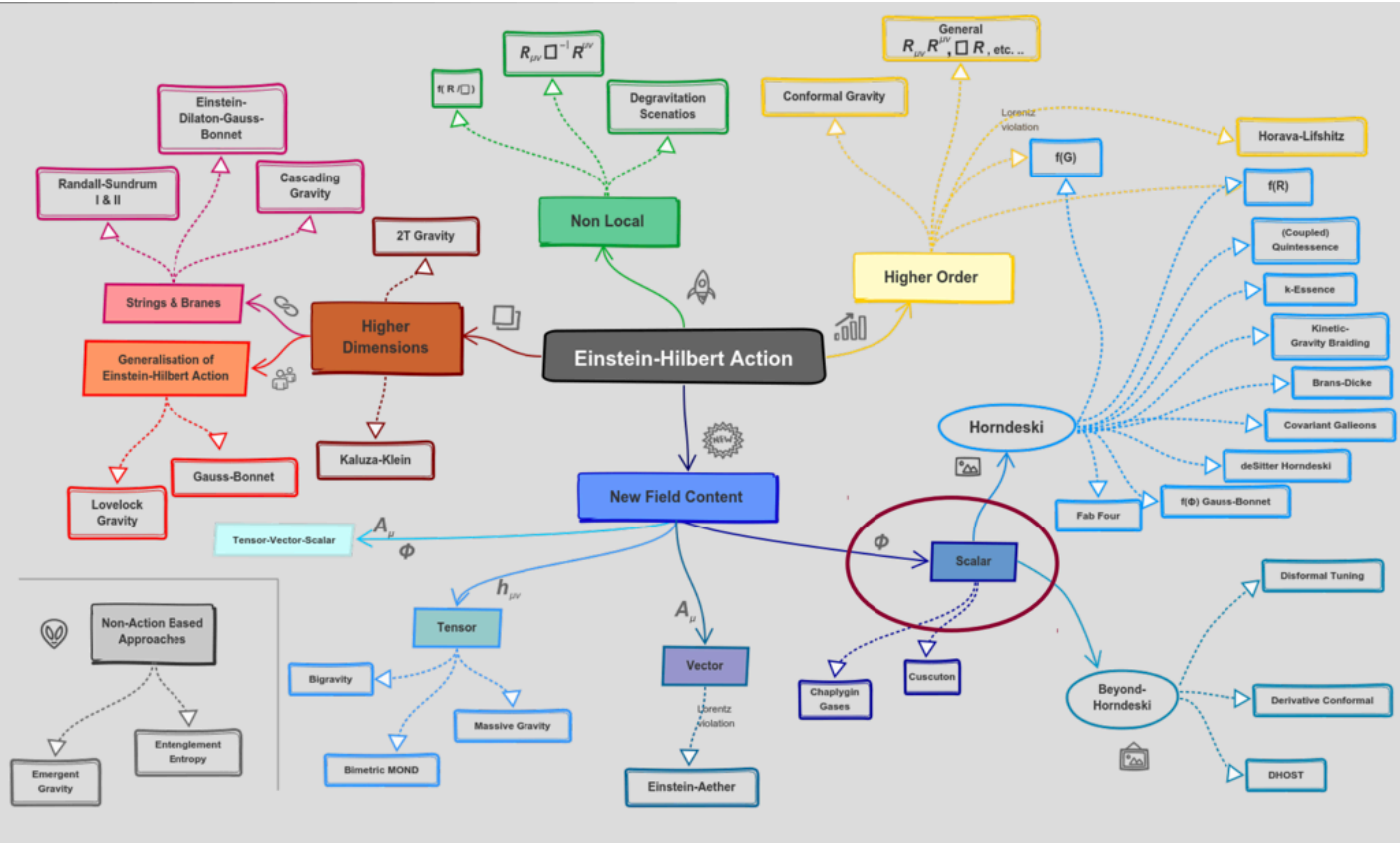
Modified gravity
Beyond II order
Stelle gravity
Conformal gravity
f(R) gravity
Horava gravity
Inverse-Ricci
Extra field contents
Massive gravity
Horndeski
4D Chern-Simons
Scalar-tensor
Non-minimal coupling
Scalar Galileons
Vector Galileons
Chern-Simons
Higher dimen-sions
Dilaton Gauss-Bonnet
DGP
Kaluza-Klein
Randall-Sundrum

Modified gravity



Tessa Baker

Modified gravity



Ostrogradski ghost

Ostrogradsky ghost

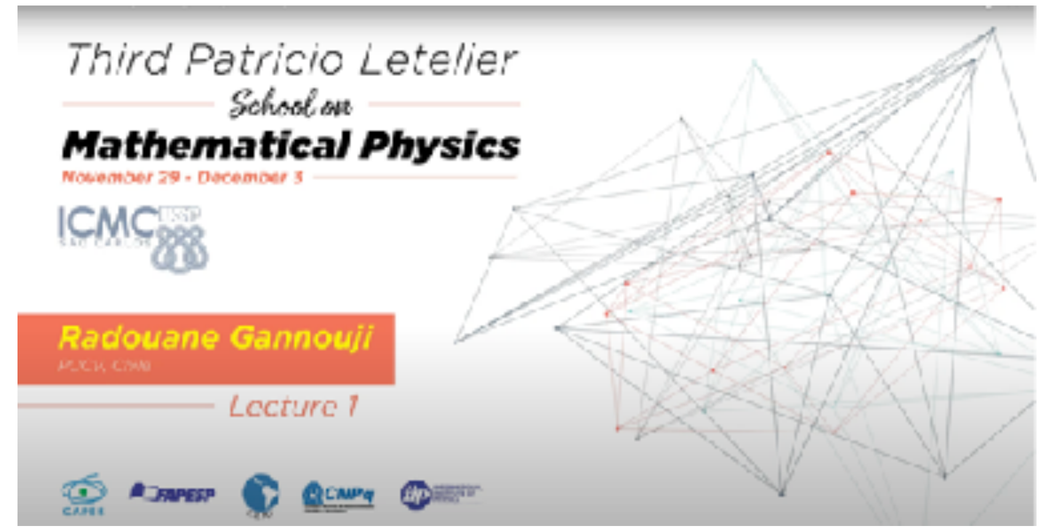
A bit of propaganda

Third Patricio Letelier - School on Mathematical Physics

R. P. Woodard arXiv:1506:02210

A. Ganz, K. Noui arXiv:2007.01063

K. Aoki, H. Motohashi arXiv:2001.06756



$$S = \int dt\ L(q, \dot{q}) \quad \Rightarrow \quad \delta S = \int dt \left[\frac{\partial L}{\partial q}\delta q + \frac{\partial L}{\partial \dot{q}}\delta \dot{q}\right]$$

$$= \int dt \left[\frac{\partial L}{\partial q}\delta q - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)\delta q\right] = 0$$

$$\Leftrightarrow \frac{\partial L}{\partial q} - \boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)} = 0$$

$$\boxed{\frac{\partial^2 L}{\partial \dot{q}^2}}\ddot{q} + \frac{\partial^2 L}{\partial q \partial \dot{q}}\dot{q}$$

$\neq 0$ nondegeneracy condition

$$\boxed{\text{If } \frac{\partial^2 L}{\partial \dot{q}^2} \neq 0 \qquad \ddot{q} = f(q, \dot{q})}$$

$$p = \frac{\partial L}{\partial \dot{q}}(q, \dot{q}) \quad \Rightarrow \quad \dot{q} = F(q, p)$$

$$\Rightarrow H = p\dot{q} - L = pF(q, p) - L(q, F(q, p))$$

$$S = \int dt\ L(q, \dot{q}, \ddot{q}) \quad \Rightarrow \quad \delta S = \int dt\ \left[\frac{\partial L}{\partial q}\delta q + \frac{\partial L}{\partial \dot{q}}\delta \dot{q} + \frac{\partial L}{\partial \ddot{q}}\delta \ddot{q}\right]$$

$$= \int dt\ \left[\frac{\partial L}{\partial q}\delta q - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)\delta q + \frac{d^2}{dt^2}\left(\frac{\partial L}{\partial \ddot{q}}\right)\delta q\right] = 0$$

$$\Leftrightarrow \frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) + \boxed{\frac{d^2}{dt^2}\left(\frac{\partial L}{\partial \ddot{q}}\right)} = 0$$

$$\boxed{\frac{\partial^2 L}{\partial \ddot{q}^2}}\ddddot{q} + \cdots$$

$\neq 0$ nondegeneracy condition

$$\boxed{\text{If } \frac{\partial^2 L}{\partial \ddot{q}^2} \neq 0 \qquad \ddddot{q} = F(q, \dot{q}, \ddot{q}, \dddot{q})}$$

Ostrogradsky ghost

$$X_1 = q \qquad P_1 = \frac{\partial L}{\partial \dot{q}} - \frac{d}{dt}\left(\frac{\partial L}{\partial \ddot{q}}\right)$$

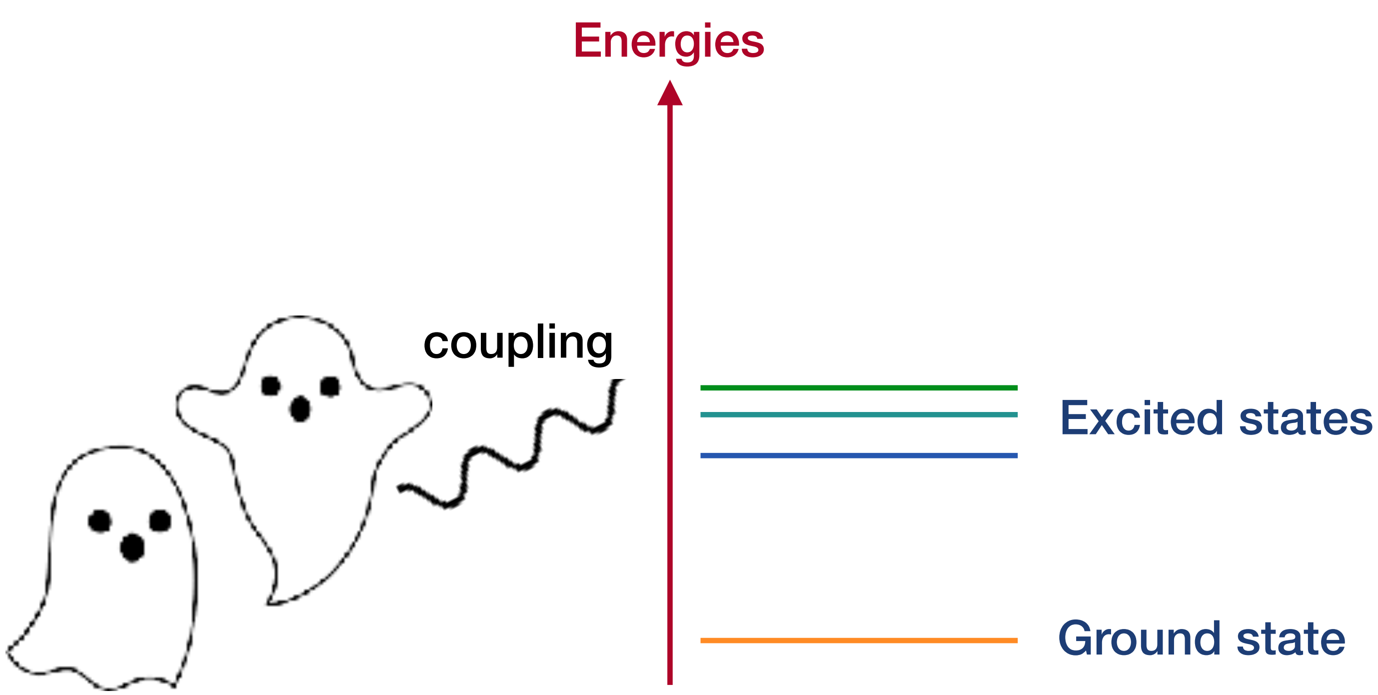
$$X_2 = \dot{q} \qquad P_2 = \frac{\partial L}{\partial \ddot{q}} \qquad \Longrightarrow \qquad \ddot{q} = G(q, \dot{q}, P_2)$$

$$H = P_1 \dot{X}_1 + P_2 \dot{X}_2 - L = P_1 X_2 + \boxed{P_2 G(X_1, X_2, P_2) - L(X_1, X_2, P_2)}$$

Linear in P_1

Function of (X_1, X_2, P_2)

$$\Longrightarrow \text{Unbounded Hamiltonian}$$



If we have a Lagrangian $L(q, \dot{q}, \ddot{q}, \cdots, q^{(N)})$

non-degenerate $\dfrac{\partial^2 L}{\partial {q^{(N)}}^2} \neq 0, \quad N \geq 2$

H will be unbounded from below and above

Ostrogradsky ghost

$$L = \ddot{q}\dot{q} + \alpha\dot{q}^2 + \beta q^2 \qquad \text{which is degenerate } \frac{\partial^2 L}{\partial \ddot{q}^2} = 0$$

$$\Leftrightarrow \quad L = \frac{1}{2}\frac{d}{dt}\left(\dot{q}^2\right) + \alpha\dot{q}^2 + \beta q^2$$

$$\Leftrightarrow \quad L = \alpha\dot{q}^2 + \beta q^2$$

$$L = \ddot{q}^2 + \alpha\dot{q}^2 + \beta q^2 \qquad \text{which is nondegenerate } \frac{\partial^2 L}{\partial \ddot{q}^2} \neq 0 \qquad \Rightarrow \boxed{\ddddot{q} - \alpha\ddot{q} + \beta q = 0}$$

$$\mathscr{L} = -2\dot{q}\dot{Q} - Q^2 + \alpha\dot{q}^2 + \beta q^2 \equiv \mathscr{L}(q, \dot{q}, Q, \dot{Q}) \quad \Rightarrow \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{Q}}\right) = \frac{\partial L}{\partial Q} \quad \Rightarrow \ddot{q} = Q$$

$$\Rightarrow \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = \frac{\partial L}{\partial q} \quad \Rightarrow \ddot{Q} - \alpha\ddot{q} + \beta q = 0$$

$$\Rightarrow \boxed{\ddddot{q} - \alpha\ddot{q} + \beta q = 0}$$

$$\mathscr{L} = -2\dot{q}\dot{Q} - Q^2 + \alpha\dot{q}^2 + \beta q^2 = \alpha\left(\dot{q} - \frac{\dot{Q}}{\alpha}\right)^2 - \frac{\dot{Q}^2}{\alpha} - Q^2 + \beta q^2$$

If $\alpha \neq 0$ $\qquad X = q - \frac{Q}{\alpha}$

$$= -\frac{1}{2}\left[(\dot{q} + \dot{Q})^2 - (\dot{q} - \dot{Q})\right] - Q^2 + \beta q^2$$

If $\alpha = 0$ $\qquad X = q - Q,\ \ Y = q + Q$

If $\alpha \neq 0$ $\qquad \mathscr{L} = \alpha\dot{X}^2 - \frac{\dot{Q}^2}{\alpha} - Q^2 + \beta\left(X + \frac{Q}{\alpha}\right)^2$

If $\alpha = 0$ $\qquad \mathscr{L} = -\frac{\dot{Y}^2}{2} + \frac{\dot{X}^2}{2} - \left(\frac{X - Y}{2}\right)^2 + \beta\left(\frac{X + Y}{2}\right)^2$

Ostrogradsky ghost

$$L(q, \dot{q}, \ddot{q}, x, \dot{x}) \quad \Rightarrow \quad det\begin{pmatrix} \frac{\partial^2 L}{\partial \ddot{q}^2} & \frac{\partial^2 L}{\partial \ddot{q}\partial \dot{x}} \\ \frac{\partial^2 L}{\partial \ddot{q}\partial \dot{x}} & \frac{\partial^2 L}{\partial \dot{x}^2} \end{pmatrix} \neq 0 \quad \text{non-degeneracy condition}$$

$$L = \alpha \ddot{q}^2 + \beta \dot{q}^2 + \gamma q^2 + \kappa \dot{x}^2 + \delta x^2 + 2\mu \dot{x}\ddot{q} \qquad \text{degeneracy condition} \quad \alpha = \frac{\mu^2}{\kappa}$$

$$\begin{aligned} L &= \frac{\mu^2}{\kappa}\ddot{q}^2 + \beta \dot{q}^2 + \gamma q^2 + \kappa \dot{x}^2 + \delta x^2 + 2\mu \dot{x}\ddot{q} \\ &= \kappa\left(\dot{x} + \frac{\mu}{\kappa}\ddot{q}\right)^2 + \beta \dot{q}^2 + \gamma q^2 + \delta x^2 \\ &= \kappa \dot{X}^2 + \beta \dot{q}^2 + \gamma q^2 + \delta\left(X - \frac{\mu}{\kappa}\dot{q}\right)^2 \end{aligned}$$

$\boxed{\text{Coupling (in the right way) a possibly problematic field to a normal field gives a way to avoid Ostrogradsky ghost with higher derivatives}}$

In summary, to avoid the Ostrogradsky ghost, you should

construct theories with first order derivatives in the action

have higher derivatives but appropriately coupled to another field

Ostrogradsky ghost

$$S = \int d^4x\sqrt{-g}\,\left(F(\phi, X)R + C^{\mu\nu\rho\sigma}\nabla_{\mu\nu}\phi\,\nabla_{\rho\sigma}\phi\right)$$

$$C^{\mu\nu\rho\sigma} = \frac{1}{2}\alpha_1\,(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}) + \alpha_2\,g^{\mu\nu}g^{\rho\sigma} + \frac{1}{2}\alpha_3\,(\phi^\mu\phi^\nu g^{\rho\sigma} + \phi^\rho\phi^\sigma g^{\mu\nu})$$
$$+\frac{1}{4}\alpha_4(\phi^\mu\phi^\rho g^{\nu\sigma} + \phi^\nu\phi^\rho g^{\mu\sigma} + \phi^\mu\phi^\sigma g^{\nu\rho} + \phi^\nu\phi^\sigma g^{\mu\rho}) + \alpha_5\,\phi^\mu\phi^\nu\phi^\rho\phi^\sigma$$

ADM decomposition

And calculate the det of the kinetic matrix

Horndeski

Beyond Horndeski

DHOST

Ostrogradsky ghost

Horndeski

G. W. Horndeski, Int.J.Theor.Phys. 10 (1974) 363–384

$$L_2 \equiv G_2(\phi, X)$$
$$L_3 \equiv G_3(\phi, X)\,\Box\phi$$
$$L_4 \equiv G_4(\phi, X)\,R - 2G_{4,X}(\phi, X)(\Box\phi^2 - \phi^{\mu\nu}\phi_{\mu\nu})$$
$$L_5 \equiv G_5(\phi, X)\,G_{\mu\nu}\phi^{\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X)(\Box\phi^3 - 3\,\Box\phi\,\phi_{\mu\nu}\phi^{\mu\nu} + 2\,\phi_{\mu\nu}\phi^{\mu\sigma}\phi^{\nu}{}_{\sigma})$$

Beyond Horndeski

$$\tilde{g}_{\mu\nu} = C(X, \phi)g_{\mu\nu} + D(X, \phi)\,\phi_\mu\,\phi_\nu$$

M. Zumalacárregui, J. García-Bellido, arXiv:1308.4685

Degrees of freedom do not change if we do an invertible transformation

$C(C - XC_X + 2X^2D_X) \neq 0$ G. Domènech et al., arXiv:1507.05390

$$L_4^{\mathrm{bH}} \equiv F_4(\phi, X)\epsilon^{\mu\nu\rho}{}_{\sigma}\,\epsilon^{\mu'\nu'\rho'\sigma}\phi_\mu\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'}$$
$$L_5^{\mathrm{bH}} \equiv F_5(\phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\phi_\mu\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'}\phi_{\sigma\sigma'}$$

with constraints on F_4 and F_5

Ostrogradsky ghost

DHOST — D. Langlois, K. Noui, arXiv:1510.06930

$$L_1^{(2)} = \phi_{\mu\nu}\phi^{\mu\nu}\,, \qquad L_2^{(2)} = (\Box\phi)^2\,, \qquad L_3^{(2)} = (\Box\phi)\phi^\mu\phi_{\mu\nu}\phi^\nu$$

$$L_4^{(2)} = \phi^\mu\phi_{\mu\rho}\phi^{\rho\nu}\phi_\nu\,, \qquad L_5^{(2)} = (\phi^\mu\phi_{\mu\nu}\phi^\nu)^2$$

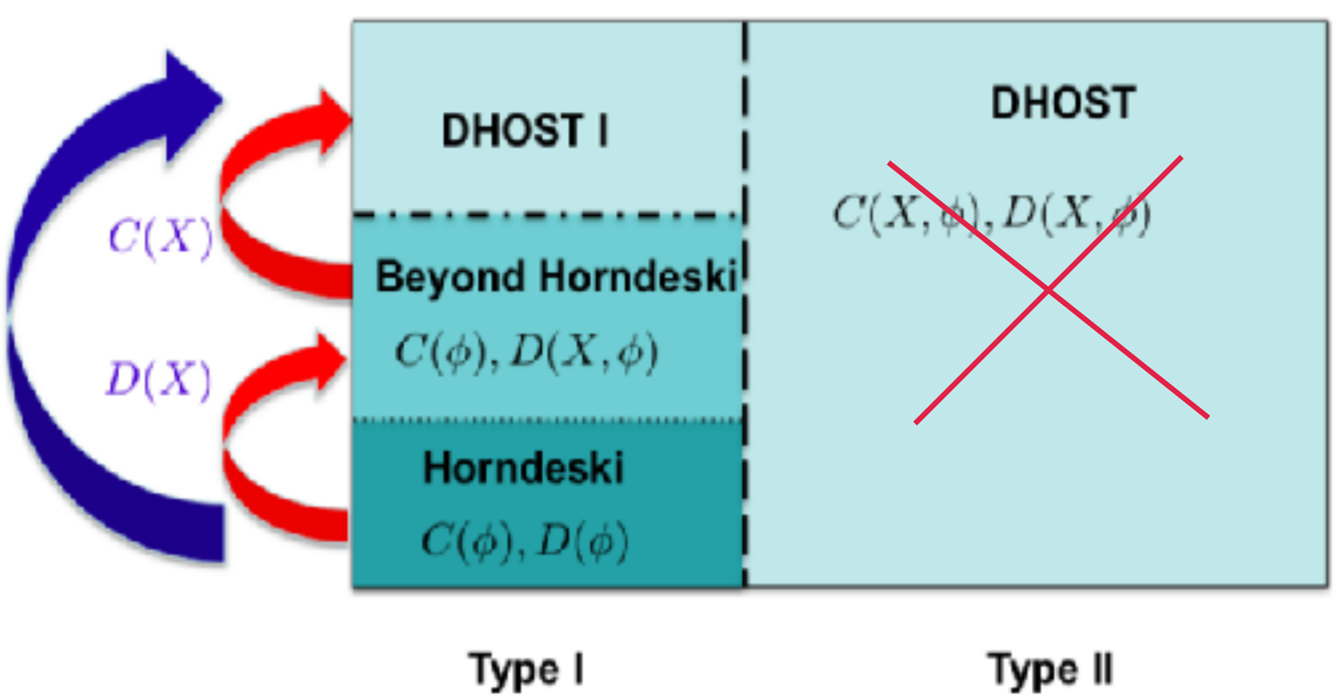
$$L_1^{(3)} = (\Box\phi)^3\,, \quad L_2^{(3)} = (\Box\phi)\,\phi_{\mu\nu}\phi^{\mu\nu}\,, \quad L_3^{(3)} = \phi_{\mu\nu}\phi^{\nu\rho}\phi^\mu_\rho\,,$$

$$L_4^{(3)} = \left(\Box\phi\right)^2\phi_\mu\phi^{\mu\nu}\phi_\nu\,, \quad L_5^{(3)} = \Box\phi\,\phi_\mu\phi^{\mu\nu}\phi_{\nu\rho}\phi^\rho\,, \quad L_6^{(3)} = \phi_{\mu\nu}\phi^{\mu\nu}\phi_\rho\phi^{\rho\sigma}\phi_\sigma\,,$$

$$L_7^{(3)} = \phi_\mu\phi^{\mu\nu}\phi_{\nu\rho}\phi^{\rho\sigma}\phi_\sigma\,, \quad L_8^{(3)} = \phi_\mu\phi^{\mu\nu}\phi_{\nu\rho}\phi^\rho\,\phi_\sigma\phi^{\sigma\lambda}\phi_\lambda\,,$$

$$L_9^{(3)} = \Box\phi\left(\phi_\mu\phi^{\mu\nu}\phi_\nu\right)^2\,, \quad L_{10}^{(3)} = \left(\phi_\mu\phi^{\mu\nu}\phi_\nu\right)^3\,.$$

Represent all the possible contractions (at second and third order) of the second-order derivatives $\phi_{,\mu\nu}$ with the metric $g_{\mu\nu}$ and the scalar field gradient $\phi_{,\mu}$



D. Langlois et al. arXiv:1703.03797

D. Langlois, arXiv:1811.06271

Screening mechanisms

Screening mechanism

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[R - Z^{\mu\nu}(\phi, \partial\phi, \cdots)\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] + S_m\left[e^{2\beta\phi}g_{\mu\nu}, \Psi_m\right]$$

$$\frac{d^2x^\mu}{d\tau^2} + \tilde{\Gamma}^\mu_{\rho\sigma}(e^{2\beta\phi}g_{\mu\nu})\frac{dx^\rho}{d\tau}\frac{dx^\sigma}{d\tau} = 0 \qquad \frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\rho\sigma}\frac{dx^\rho}{d\tau}\frac{dx^\sigma}{d\tau} + \beta\left[2\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}\partial_\nu\phi + \partial^\mu\phi\right] = 0$$

Newtonian limit $\quad \dfrac{d^2x^\mu}{dt^2} = -\,\partial^\mu\Phi - \beta\partial^\mu\phi$

Unnatural solutions, as Brans-Dicke $\quad \beta = 1/\sqrt{2\omega + 6} \ll 1$

Screening the Fifth force because of the potential (V): Chameleon mechanism $\quad f(R)$, Brans-Dicke with $V(\phi)$…

Screening the Fifth force because of the kinetic term (Z): Vainshtein mechanism $\quad$ DGP, massive gravity…

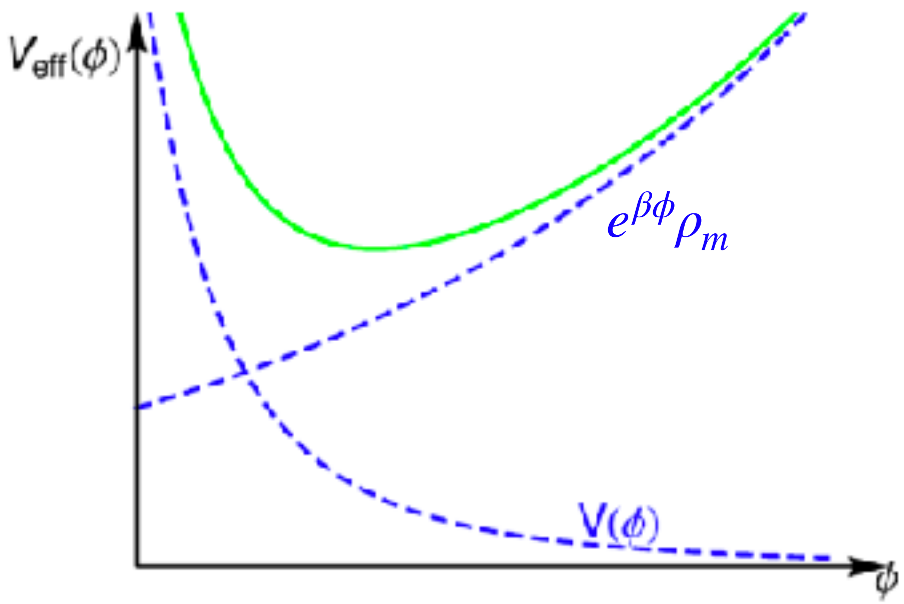
Chameleon mechanism

$$S = \int d^4x\sqrt{-g}\left[R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] + S_m\left[e^{2\beta\phi}g_{\mu\nu}, \Psi_m\right]$$

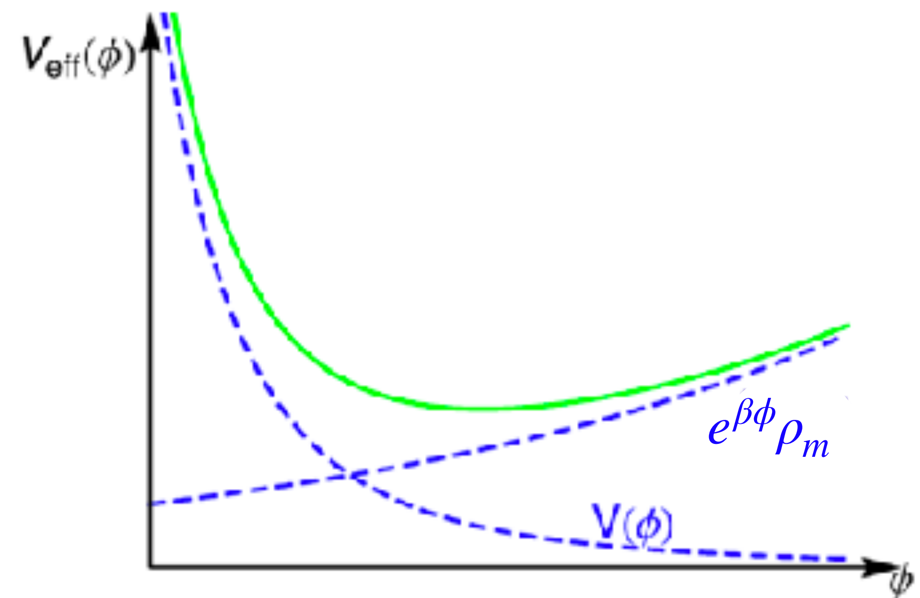
J. Khoury, A. Weltman
arXiv:astro-ph/0309300, arXiv:astro-ph/0309411

$$\Box\phi = V_{,\phi} + \beta e^{\beta\phi}\rho_m$$

$$\Box\phi = V_{eff,\phi} \quad \text{with} \quad V_{eff}(\phi) = V(\phi) + e^{\beta\phi}\rho_m$$



Large ρ_m



Small ρ_m

$$m^2_{eff}(\rho) = \frac{d^2V_{eff}}{d\phi^2} \equiv V''(\phi) + \beta^2 e^{\beta\phi}\rho_m$$

Chameleon mechanism

If we consider a specific model:

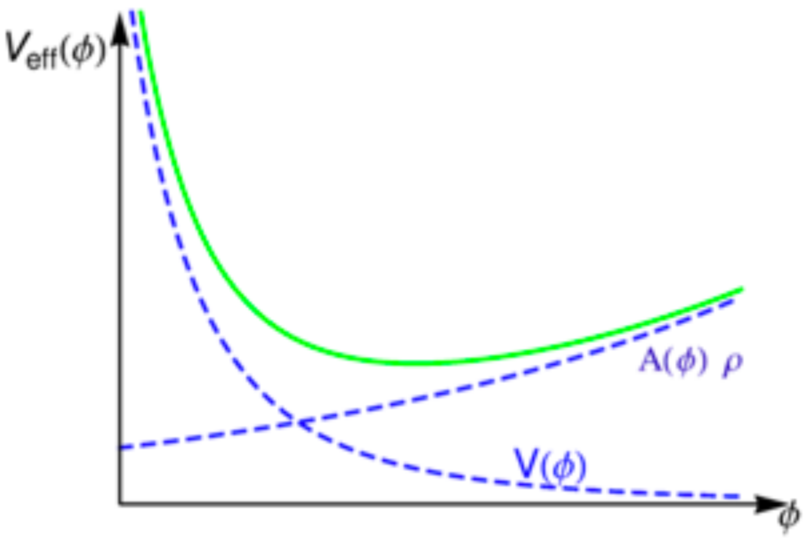
$$V(\phi) = \frac{M^5}{\phi} \qquad\qquad V_{eff}(\phi) = V(\phi) + e^{\beta\phi}\rho_m$$

For analytical simplicity, I will assume $e^{\beta\phi} = 1 + \beta\phi$ $\qquad V_{eff}(\phi) = \frac{M^5}{\phi} + (1 + \beta\phi)\rho_m$

The field sits at the minimum of the effective potential

$$\frac{dV_{eff}(\phi)}{d\phi} = 0$$

Which means $\phi_{min} = \left(\frac{M^5}{\beta\rho_m}\right)^{1/2}$



The effective mass becomes

$$m_{eff}^2 = V''_{eff}(\phi_{min}) = 2M^{-5/2}(\beta\rho_m)^{3/2}$$

Scalar field is light in empty environments and therefore produce a long range force $\quad \lambda = m_{eff}^{-1}$

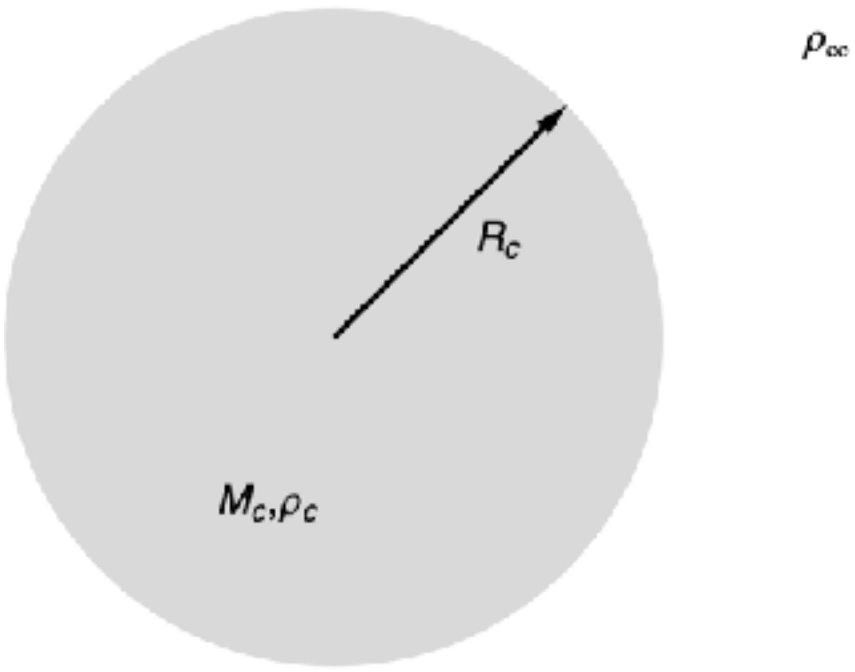
But is heavy in dense environments, like in the lab, so produce a short range force

$$\Box\phi = V_{,\phi} + \beta e^{\beta\phi}\rho_m$$

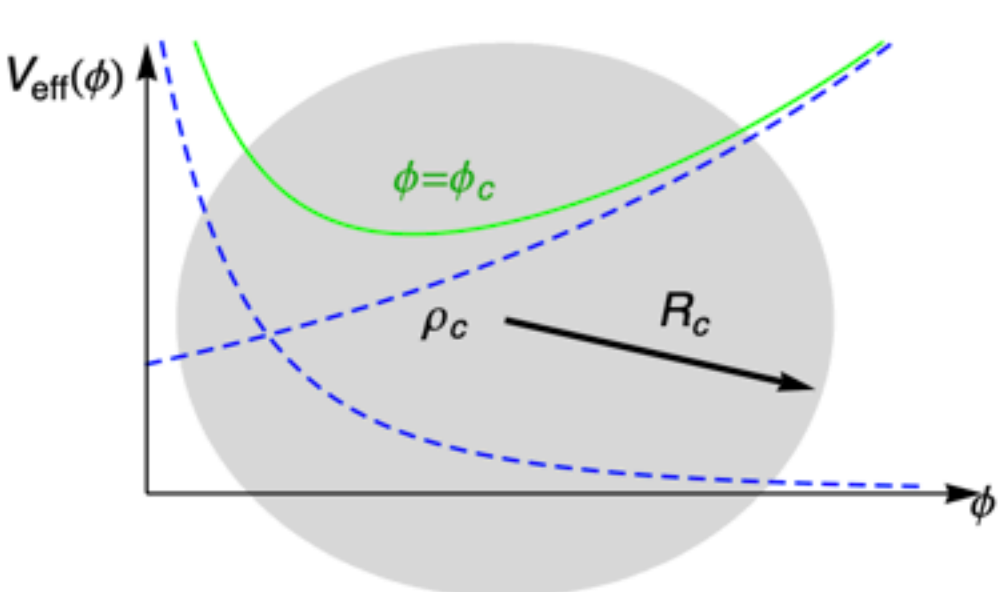
$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = V_{,\phi} + \beta e^{\beta\phi}\rho_m$$

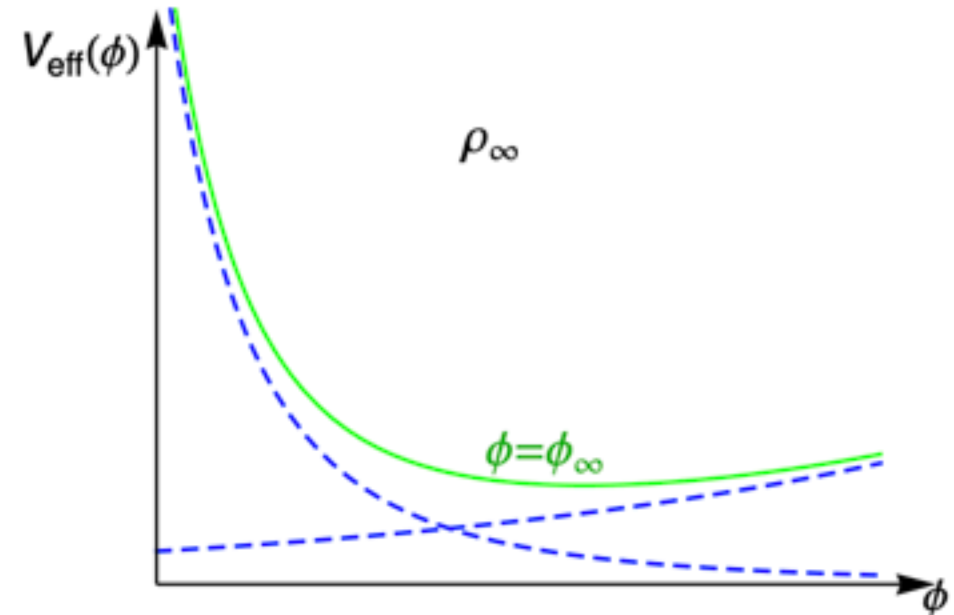
Boundary conditions:

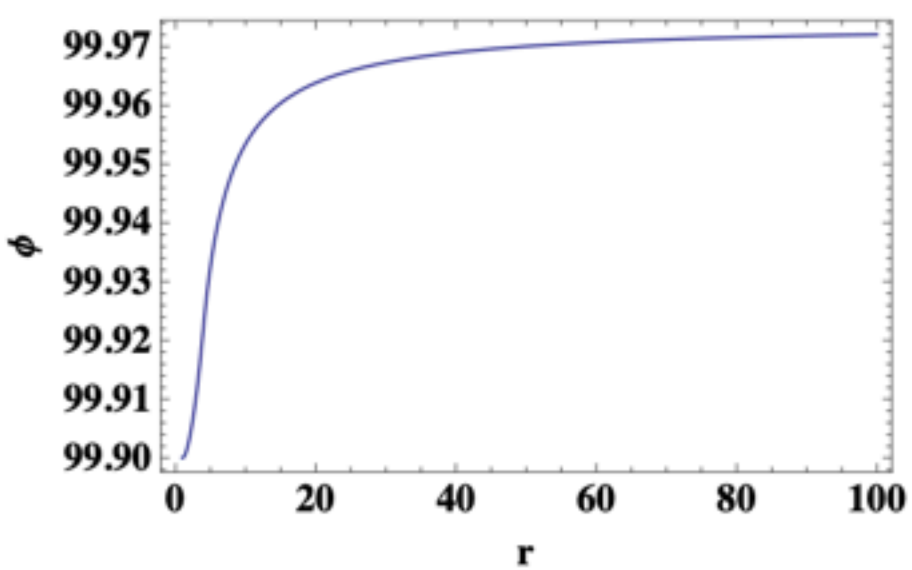
$$\phi'(r=0) = 0 \qquad \phi(\infty) = \phi_\infty$$



First case: the contrast density between the exterior and interior of the spherical object is small



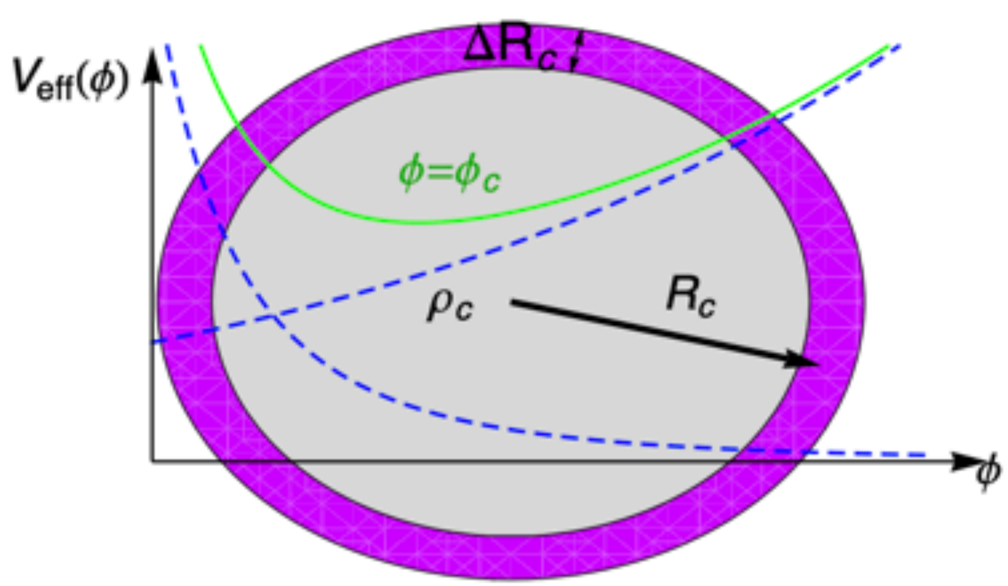


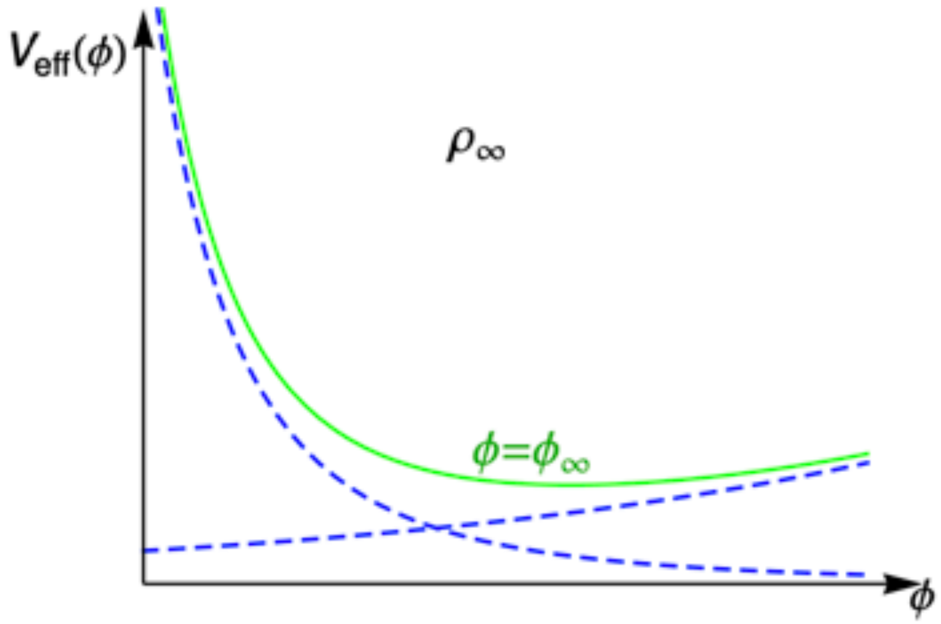


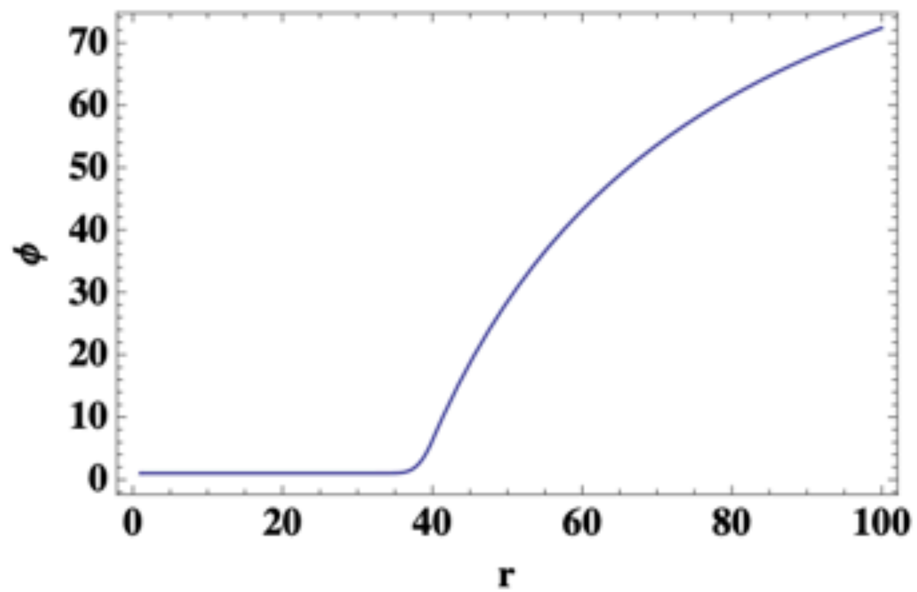
$$\phi(r) \simeq \phi_\infty - \frac{\beta}{4\pi}\frac{M_c e^{-m_\infty(r-R_c)}}{r}$$

Chameleon mechanism

Second case: the contrast density between the exterior and interior of the spherical object is large







$$\phi(r) \simeq \phi_\infty - \left(3\frac{\Delta R_c}{R_c}\right)\frac{\beta}{4\pi}\frac{M_c e^{-m_\infty(r-R_c)}}{r}$$

$$\frac{\Delta R_c}{R_c} \simeq \frac{\phi_\infty - \phi_c}{6\beta\Phi} \qquad \Phi = \frac{GM_c}{R_c}$$

But because fifth force $\propto \phi'(r)$, the fifth force could be suppressed if $\dfrac{\Delta R_c}{R_c} \ll 1$ known as the thin-shell condition

Chameleon mechanism

But because fifth force $\propto \phi'(r)$, the fifth force could be suppressed if $\dfrac{\Delta R_c}{R_c} \ll 1$

Let's consider that the system; earth, moon, sun, have a thin-shell

We want to know, if the thin-shell is strong enough to suppress any fifth force over the earth and the moon

In order not to have a violation of the equivalence principle

$$\left| \frac{a_{moon} - a_{earth}}{a_{moon} + a_{earth}} \right| < 10^{-13}$$

$$a_{\oplus} \simeq \frac{GM_{\odot}}{r^2} \left[1 + 18\beta^2 \epsilon^2_{\mathrm{th},\oplus} \frac{\Phi_{\oplus}}{\Phi_{\odot}} \right] ,$$

$$a_{\mathrm{Moon}} \simeq \frac{GM_{\odot}}{r^2} \left[1 + 18\beta^2 \epsilon^2_{\mathrm{th},\oplus} \frac{\Phi^2_{\oplus}}{\Phi_{\odot}\Phi_{\mathrm{Moon}}} \right] ,$$

$$\epsilon_{\mathrm{th},\oplus} = \frac{\Delta R}{R}$$ for the earth

$$\epsilon_{\mathrm{th},\oplus} < \frac{8.8\ 10^{-7}}{\beta}$$

$$\epsilon_{\mathrm{th},\oplus} < \frac{8.8\ 10^{-7}}{\beta}$$

$$V(\phi) = \frac{M^5}{\phi} \quad \Rightarrow M < 10^{-3}\ eV$$

Energy scale for dark energy

Chameleon mechanism

- Small galaxies should move faster than large ones in general
- Stars within unscreened galaxies may show the effects of modified gravity
  - Enhanced gravitational force makes stars of a given mass brighter and hotter than in GR
  - More ephemeral since they consume their fuel at a faster rate
- Internal motions of objects inside an unscreened galaxy
  - majority of stars inside an unscreened galaxy are themselves screened
  - diffuse gas inside the same galaxy likely remains unscreened because the gravitational potential at a gas cloud should be dominated by the potential of the galaxy rather than that of the gas cloud itself
  - Therefore, at the same radius from galactic center, gas clouds should move with an acceleration that is larger than that of the stars by a factor of $1 + 2\beta^2$, which is $4/3$ for $f(R)$

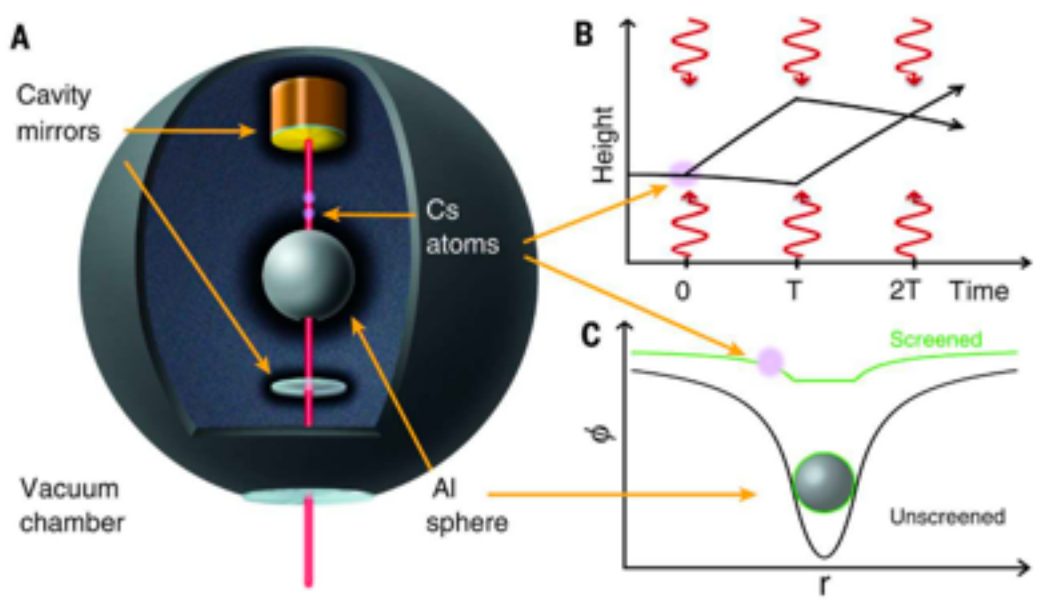
Chameleon mechanism

Atomic particles in an ultra-high vacuum chamber can mimic cosmos conditions

The experiment is realized inside a vacuum chamber in order to reduce the screening effect

An atom beam is split in two, one half of it is subjected to some field, and then the beams are combined again

From the resulting interference pattern one can extract the force that acted on the beams



This result rules out chameleons that would reproduce the observed acceleration of the cosmos

P. Hamilton et al. arXiv:1502.0388

Other screening mechanisms

Other mechanisms

Almost similar to chameleon mechanism, but with a bit different idea

$$S = \int d^4x \sqrt{-g}\left[R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] + S_m\left[A(\phi)^2 g_{\mu\nu}, \Psi_m\right]$$

Symmetron mechanism Kurt Hinterbichler, Justin Khoury arXiv:1001.4525

$$V(\phi) = -\frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 \qquad A(\phi) = 1 + \frac{\phi^2}{2M^2} \qquad V(\phi) = \frac{1}{2}\left(\frac{\rho_m}{M^2} - \mu^2\right)\phi^2 + \frac{\lambda}{4}\phi^4$$

Dilaton mechanism T. Damour, A. M. Polyakov arXiv:hep-th/9401069

$$V(\phi) = V_0 e^{-\phi/M_{Pl}} \qquad A(\phi) = 1 + \frac{(\phi - \phi_0)^2}{2M^2} \qquad V(\phi) = V_0 e^{-\phi/M_{Pl}} + \frac{(\phi - \phi_0)^2}{2M^2}\rho_m$$

Vainshtein mechanism

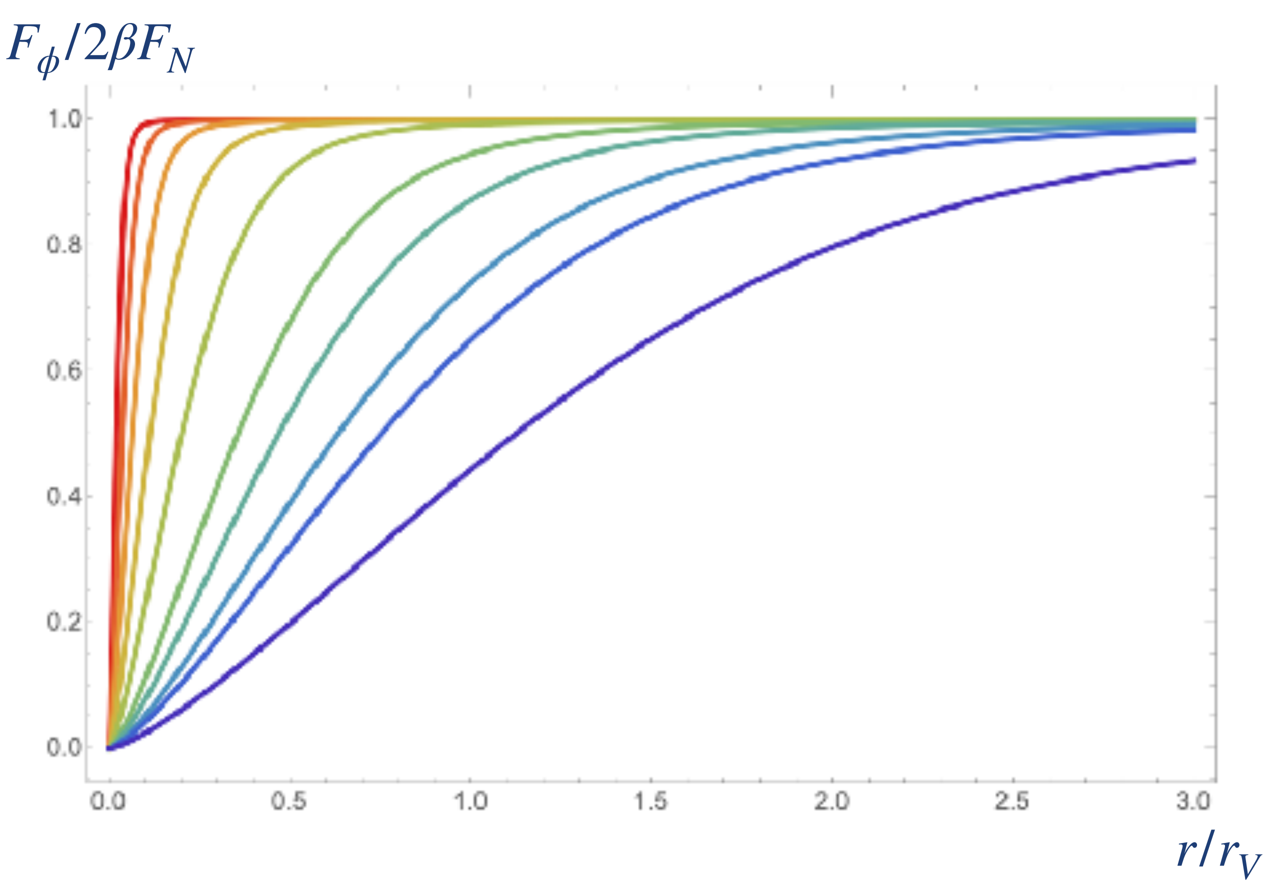
Vainshtein mechanism

$$S = \int d^4x\sqrt{-g}\left[R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{\alpha}{4M^4}(\partial_\mu\phi\partial^\mu\phi)^2\right] + S_m\left[A(\phi)^2 g_{\mu\nu}, \Psi_m\right]$$

$$\Box\phi - \frac{\alpha}{M^4}\nabla_\mu\left[(\partial_\alpha\phi\partial^\alpha\phi)\partial^\mu\phi\right] = A'(\phi)\rho_m \qquad A(\phi) = 1 + \frac{\beta}{M_{Pl}}\phi$$

Newtonian limit for spherically symmetric solution

$$\phi' - \frac{\alpha}{M^4}\phi'^3 = \frac{1}{4\pi r^2}\frac{\beta m}{M_{Pl}} \qquad F_\phi = \frac{\beta}{M_{pl}}\phi'$$



$$r = r_V = \frac{1}{M}\left(\frac{\beta m}{M_{Pl}}\right)^{1/2}$$

Vainshtein radius

Vainshtein mechanism

DBI type of model
$$S = \int \mathrm{d}^4x\sqrt{-g}\left[R + \Lambda^4\sqrt{1 - \frac{(\partial\phi)^2}{\Lambda^4}} + \beta\phi T\right]$$

$$\phi' = \frac{\Lambda^2}{\sqrt{1 + 16\pi^2(r/r_V)^4}} \qquad F_5 \simeq 1/r^2 \quad r \gg r_V \qquad F_5/F_N \simeq (r/r_V)^2 \quad r \ll r_V$$

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[R - \frac{1}{2}Z^{\mu\nu}(\phi, \partial\phi, \cdots)\partial_\mu\phi\partial_\nu\phi\right] + S_m\left[A(\phi)^2 g_{\mu\nu}, \Psi_m\right]$$

$$Z_{\mu\nu} = g_{\mu\nu} + \frac{1}{M^3}\partial_{\mu\nu}\phi + \frac{1}{M^6}(\partial_{\mu\nu}\phi)^2 + \cdots$$

At low energies $Z_{\mu\nu} \simeq g_{\mu\nu} = \mathcal{O}(1)$

At high energies $Z_{\mu\nu} \gg 1$

$$-\frac{1}{2}Z^{\mu\nu}(\phi, \partial\phi, \cdots)\partial_\mu\phi\partial_\nu\phi = -\frac{1}{2}\partial_\mu\psi\partial^\mu\psi \qquad \phi = \frac{\psi}{\sqrt{Z}} \quad \Rightarrow A(\phi) = A\left(\frac{\psi}{\sqrt{Z}}\right)$$

When $Z \gg 1,\ A(\phi) \rightarrow A(0) = 1$ the field decouples from matter

The field is strongly coupled to itself and becomes weakly coupled to external sources

Vainshtein mechanism

Many of these non-linearities come from higher dimensional theories

Gauss-Bonnet

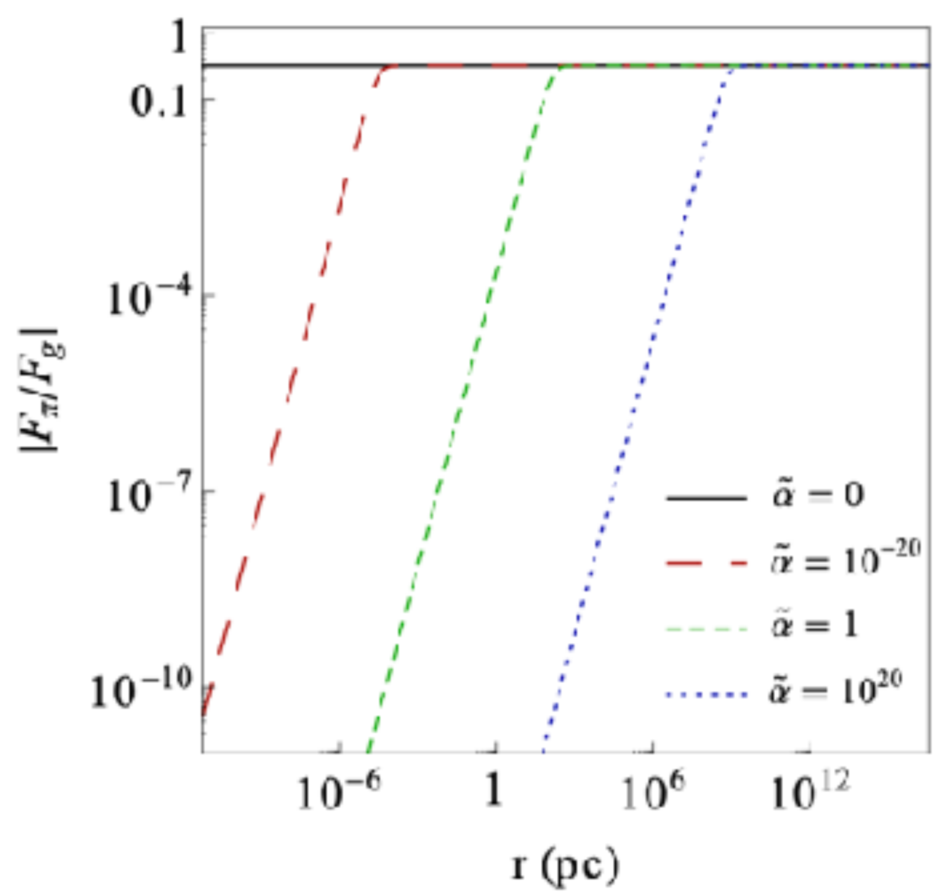
$$S = \int \mathrm{d}^5x\sqrt{-h}\left[R + \alpha(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma})\right]$$

Ansatz

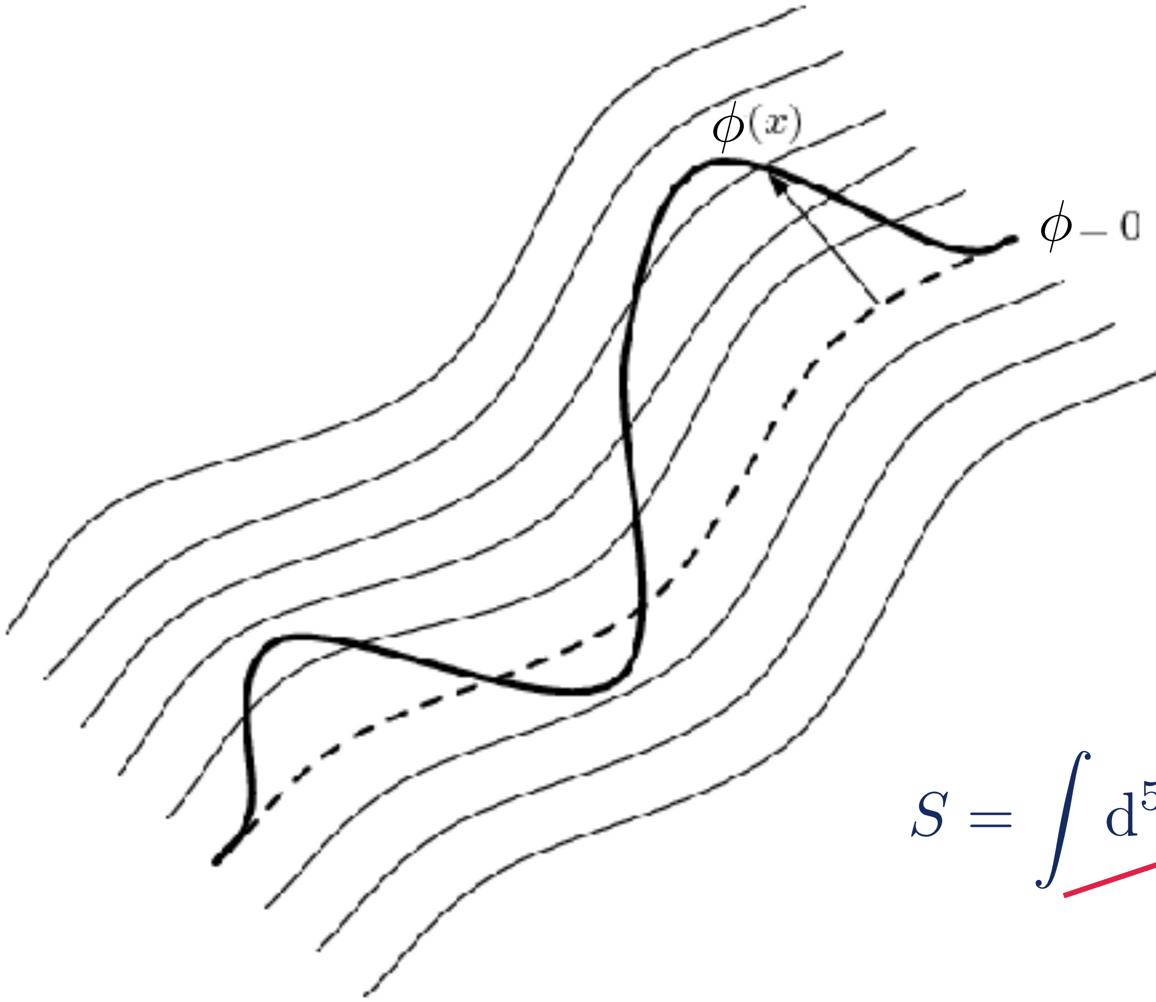
$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu + e^{\phi(x^\mu)}dw^2$$

R.G., M. Sami arXiv:1107.1892

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 + \alpha e^{\phi/\sqrt{3}}\left(R_{GB} + \frac{4}{3}G_{\mu\nu}\phi^\mu\phi^\nu - \frac{1}{\sqrt{3}}(\partial\phi)^2\Box\phi\right)\right] + S_m\left[e^{-\phi/\sqrt{3}}g_{\mu\nu};\Psi_m\right]$$



Vainshtein mechanism



ϕ measures the brane position with respect to some chosen foliation

$$X^\mu = x^\mu, \quad X^4 = \phi(x^\mu)$$

Induced metric $\quad g_{\mu\nu} = \eta_{AB} e^A_\mu e^B_\nu = \eta_{\mu\nu} + \partial_\mu\phi\partial_\nu\phi$

Extrinsic curvature $\quad K_{\mu\nu} = -\frac{\partial_{\mu\nu}\phi}{\sqrt{1+(\partial\phi)^2}}$

$$S = \cancel{\int \mathrm{d}^5X\sqrt{-h}\,{}^{(5)}\mathcal{R}} + 2\int \mathrm{d}^4x\sqrt{-g}K + \int \mathrm{d}^4x\sqrt{-g}\Lambda$$

$$\sqrt{-g} = \sqrt{1+(\partial\phi)^2} \qquad K = \Box\phi - \frac{\partial_{\mu\nu}\phi\partial^\mu\phi\partial^\nu\phi}{1+(\partial\phi)^2}$$

$$S = 2\int \mathrm{d}^4x\sqrt{-g}K + \int \mathrm{d}^4x\sqrt{-g}\Lambda$$

$$S = \int \mathrm{d}^4x\left[2\sqrt{1+(\partial\phi)^2}\Box\phi - 2\frac{\partial_{\mu\nu}\phi\partial^\mu\phi\partial^\nu\phi}{\sqrt{1+(\partial\phi)^2}} + \Lambda\sqrt{1+(\partial\phi)^2}\right]$$

$(\partial\phi)^2 \ll 1$

$$\boxed{S = \Lambda\int \mathrm{d}^4x\left[1 + \frac{1}{2}(\partial\phi)^2 + \frac{2}{\Lambda}(\partial\phi)^2\Box\phi\right]}$$

Galileon

arXiv:1103.5745

Origin of the Vainshtein mechanism

Massive gravity

Considering the linear theory of gravity, we can add a mass term

$$S=\int \mathrm{d}^4x\left[-\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu}+\partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda}-\partial_\mu h^{\mu\nu}\partial_\nu h+\frac{1}{2}\partial_\lambda h\partial^\lambda h-\frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu}-h^2)\right]$$

$$\Box h_{\mu\nu}-\partial_\lambda\partial_\mu h^\lambda{}_\nu-\partial_\lambda\partial_\nu h^\lambda{}_\mu+\eta_{\mu\nu}\partial_\lambda\partial_\sigma h^{\lambda\sigma}+\partial_\mu\partial_\nu h-\eta_{\mu\nu}\Box h-m^2(h_{\mu\nu}-\eta_{\mu\nu}h)=0$$

$$(\Box-m^2)h_{\mu\nu}=0,\quad \partial^\mu h_{\mu\nu}=0,\quad h=0$$

Wave equation for 5 propagating propagating dof

Solution for a point source

$$\phi=-\frac{4}{3}\frac{GM}{r}e^{-mr},\qquad \psi=-\frac{2}{3}\frac{GM}{r}e^{-mr}\delta_{ij}$$

We can rescale the gravitational constant $\quad G\to\frac{3}{4}G$

$$\phi=-\frac{GM}{r}e^{-mr},\qquad \psi=-\frac{GM}{2r}e^{-mr}\delta_{ij}$$

Even in the limit $m\to 0$, predictions will be different than GR

This is the vDVZ (van Dam, Veltman, Zakharov) discontinuity

Stückelberg trick

Formalism to expose the origin of this discontinuity

Taking $m \to 0$ in the equations does not yield to a smooth limit, because degrees of freedom are lost

To find the correct limit, the trick is to introduce new fields into the theory in a way that does not alter the theory and then the limit can be found in which no degrees of freedom are gained or lost

$$S = \int \mathrm{d}^4x \Big[-\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h \partial^\lambda h - \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2) + h_{\mu\nu}T^{\mu\nu} \Big]$$

The first part of the action is invariant under linear diffeomorphism

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

Let us do the following transformation $\quad h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu$

It is not sufficient to decouple all degrees of freedom, so we do $A_\mu \to A_\mu + \partial_\mu \phi$ and $h_{\mu\nu} \to h_{\mu\nu} + \phi \eta_{\mu\nu}$

$$S = \int \mathrm{d}^4x \Big[-\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h \partial^\lambda h - \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2) - \frac{1}{2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 + h_{\mu\nu}T^{\mu\nu} + \phi T \Big]$$

$$S = \int \mathrm{d}^4x \Big[-\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h \partial^\lambda h - \frac{1}{2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(\partial\phi)^2 + h_{\mu\nu}T^{\mu\nu} + \phi T \Big]$$

Origin of the Vainshtein mechanism

Why so much interest

$$S = \int \mathrm{d}^4x \Big[-\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h \partial^\lambda h - \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2)\Big] + S_m$$

$$\Box h_{\mu\nu} - \partial_\lambda\partial_\mu h^\lambda{}_\nu - \partial_\lambda\partial_\nu h^\lambda{}_\mu + \eta_{\mu\nu}\partial_\lambda\partial_\sigma h^{\lambda\sigma} + \partial_\mu\partial_\nu h - \eta_{\mu\nu}\Box h - m^2(h_{\mu\nu} - \eta_{\mu\nu}h) = -T_{\mu\nu}$$

In the case $T_{\mu\nu} = \Lambda\eta_{\mu\nu}$ and taking double derivative

$$\partial^{\mu\nu}h_{\mu\nu} - \Box h = 0$$

Which means $R = 0$

So the curvature scalar is zero in presence of a cosmological constant, this would be an example of degravitation

But the theory has a discontinuity

Vainshtein proposed that non-linearities could solve the problem

Origin of the Vainshtein mechanism

Massive general relativity

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

$$S = \int \mathrm{d}^4x \left[\sqrt{-g}R - \sqrt{-\bar{g}}\frac{m^2}{2}\boxed{\bar{g}^{\mu\alpha}\bar{g}^{\nu\beta}(h_{\mu\nu}h_{\alpha\beta} - h_{\mu\alpha}h_{\nu\beta})}\right]$$

But the theory has now 6 degrees of freedom and the additional one is a ghost (Boulware-Deser ghost)

Fortunately, there exist one potential U without a ghost, dRGT model

It has Vainshtein screening

Doesn't admit flat FLRW solution — D'Amico et al. arXiv:1108.5231

Exist open FLRW solution — Gumrukcuoglu et al. arXiv:1109.3845

Exist isotropic/inhomogeneous solutions — Gratia et al. arXiv:1205.4241

But all these solutions contain a ghost — F. Kuhnel arXiv:1208.1764

Because of these problems, massive gravity has been extended

Background metric $\bar{g}_{\mu\nu}$ is dynamical, bi-metric theory — S. F. Hassan, R. A. Rosen arXiv:1109.3515

Multi metric theory — K. Hinterbichler, R. A. Rosen arXiv:1203.5783

The mass is not constant but it's a scalar field, quasi-dilaton massive gravity (D'Amico et al. 2013)

The model has bouncing, turnaround or cyclic solutions — R.G. et al. arXiv:1304.5095

To date, there is no (extended) massive gravity model which possesses a completely satisfactory self-acceleration solution

But it opens the door to degravitation
a large cosmological constant does not strongly curve space-time

N. Arkani-Hamed et al. arXiv:hep-th/0209227

Any theory that exhibits degravitation must reduce to a theory of massive/resonance gravity (continuum of massive gravitons) at linearized level

G. Dvali et al. arXiv:hep-th/0703027

Vainshtein mechanism

Vainshtein mechanism seems stronger than chameleon

In time-dependent situations, Vainshtein mechanism has been found to be less efficient than around static sources

Y-Z Chu, M. Trodden arXiv:1210.6651)

Possible way to find a degravitation model

No satisfactory model for the moment

Obrigado