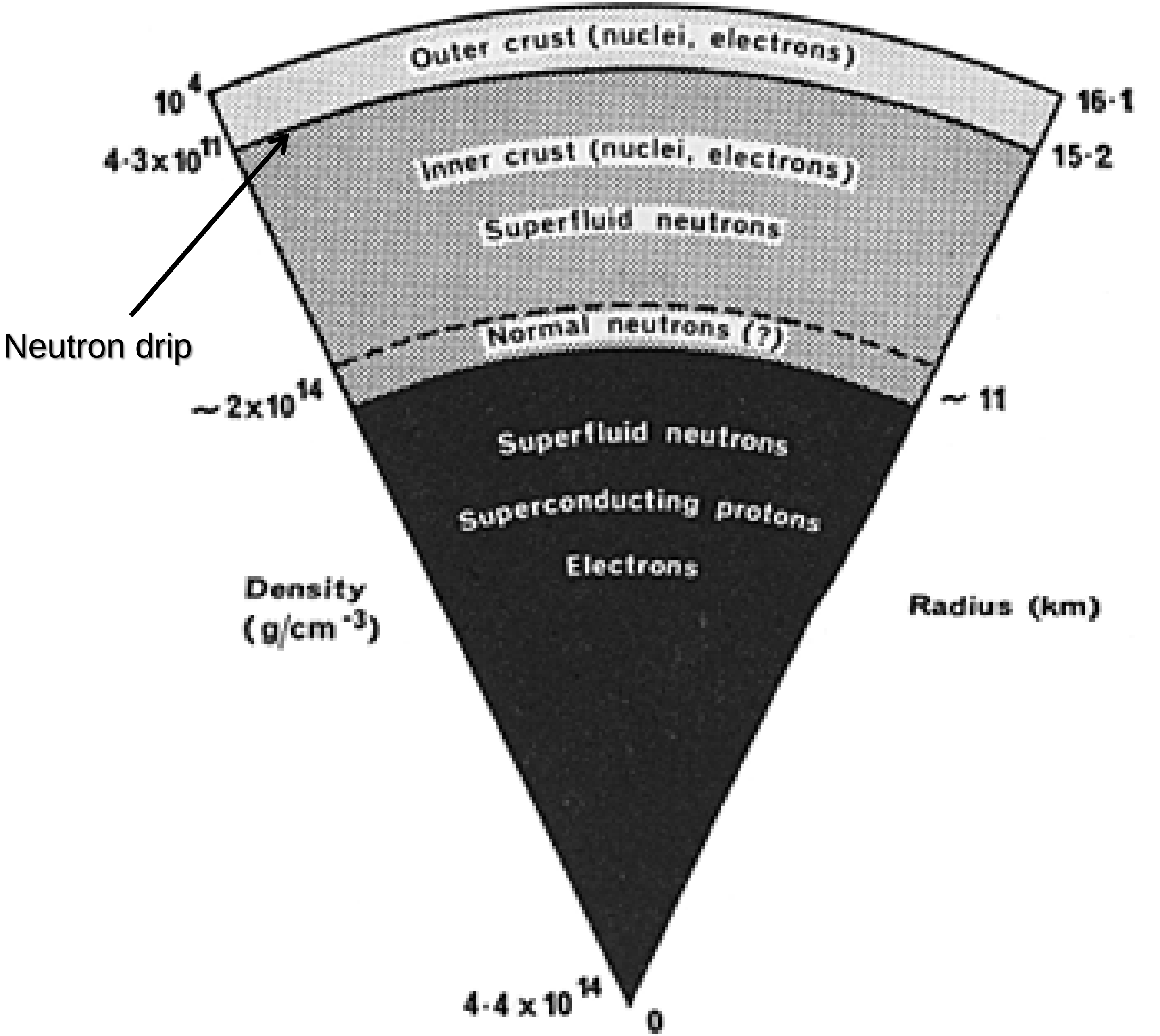
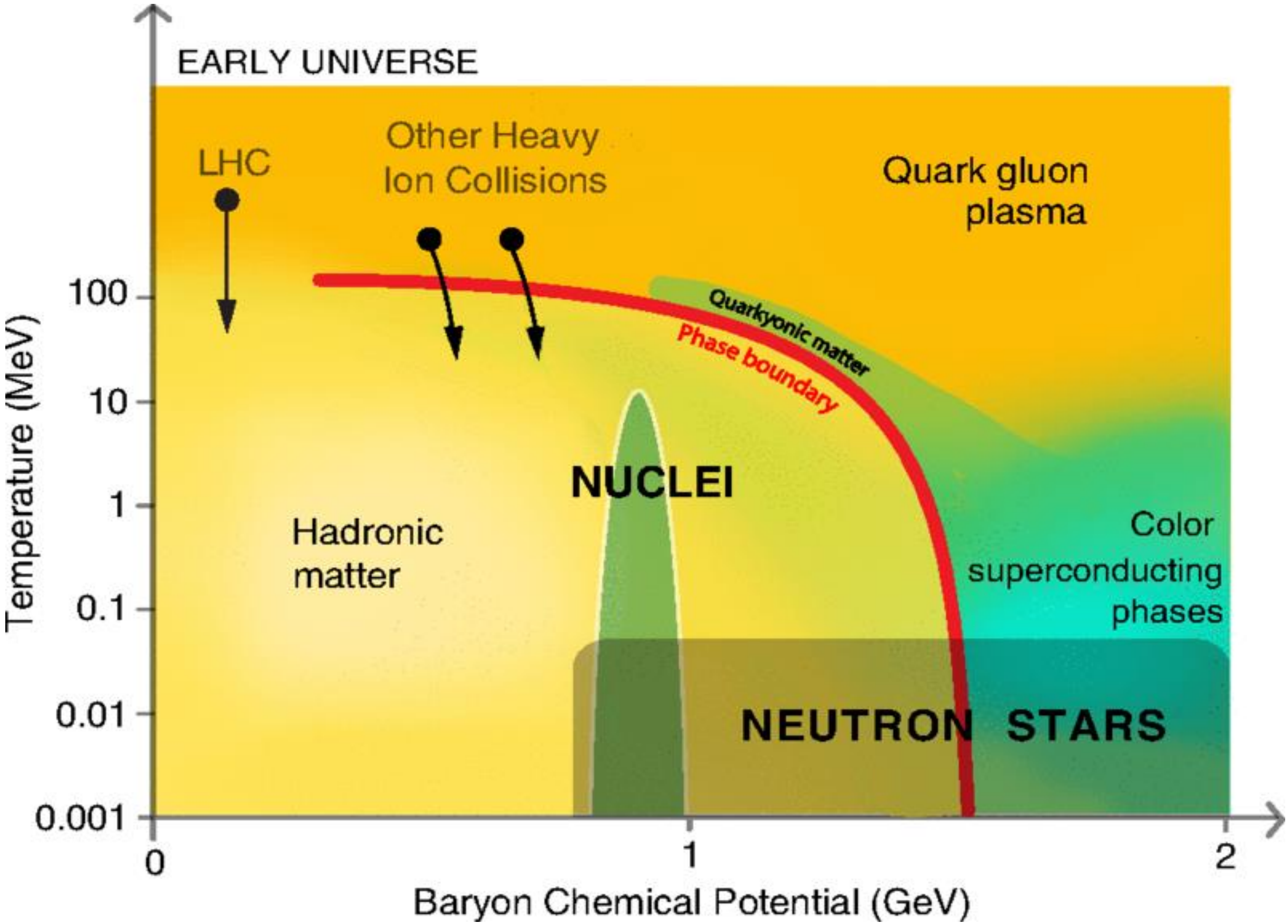
Physics of the Magnetosphere of Neutron Stars

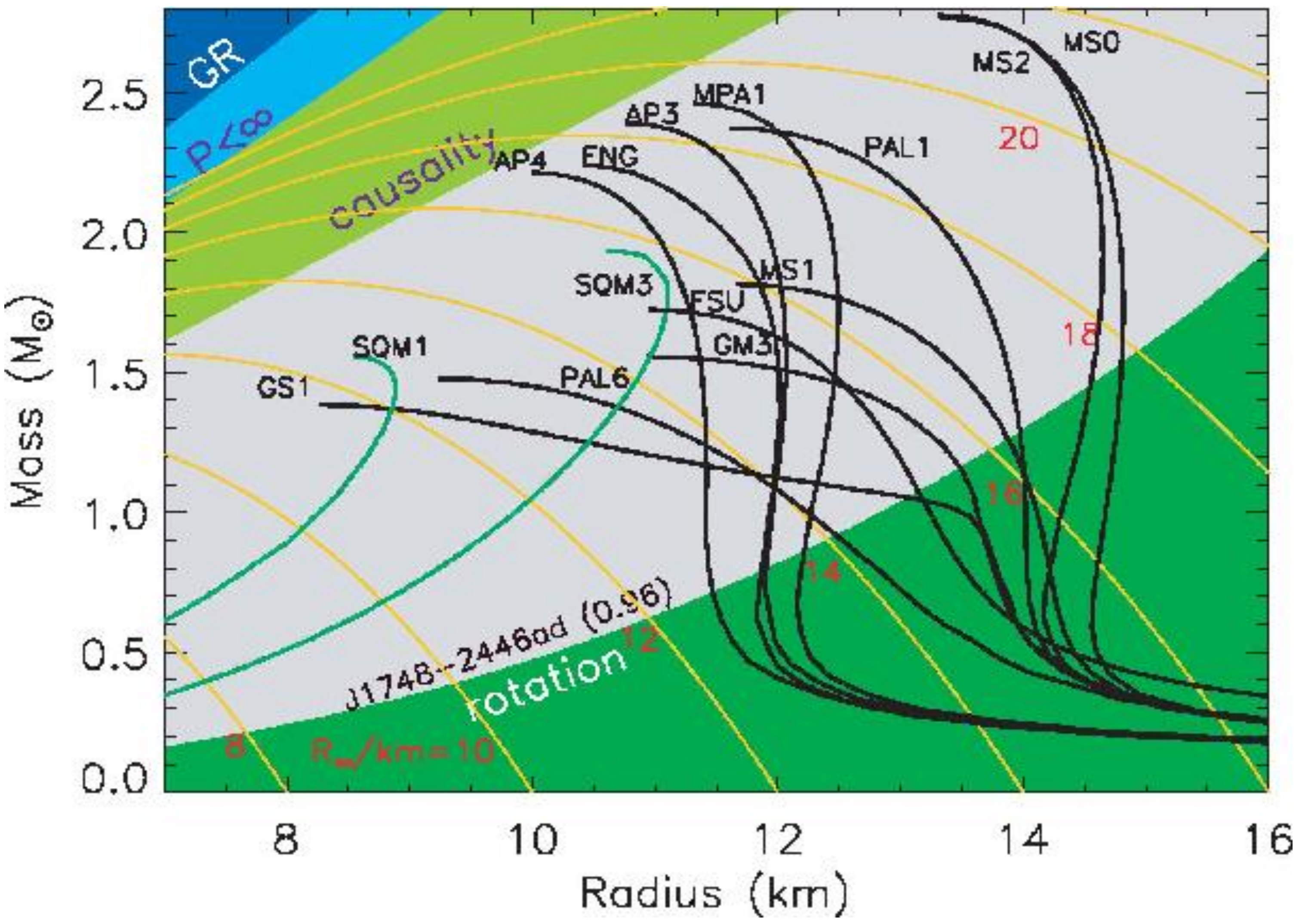
Recall about the internal structure of NS





EARLY UNIVERSE
LHC
Other Heavy
Ion Collisions
Quark gluon
plasma
100
10
1
0.1
0.01
0.001
Temperature (MeV)
Quarkyonic matter
Phase boundary
NUCLEI
Hadronic
matter
Color
superconducting
phases
NEUTRON STARS
0
1
2
Baryon Chemical Potential (GeV)

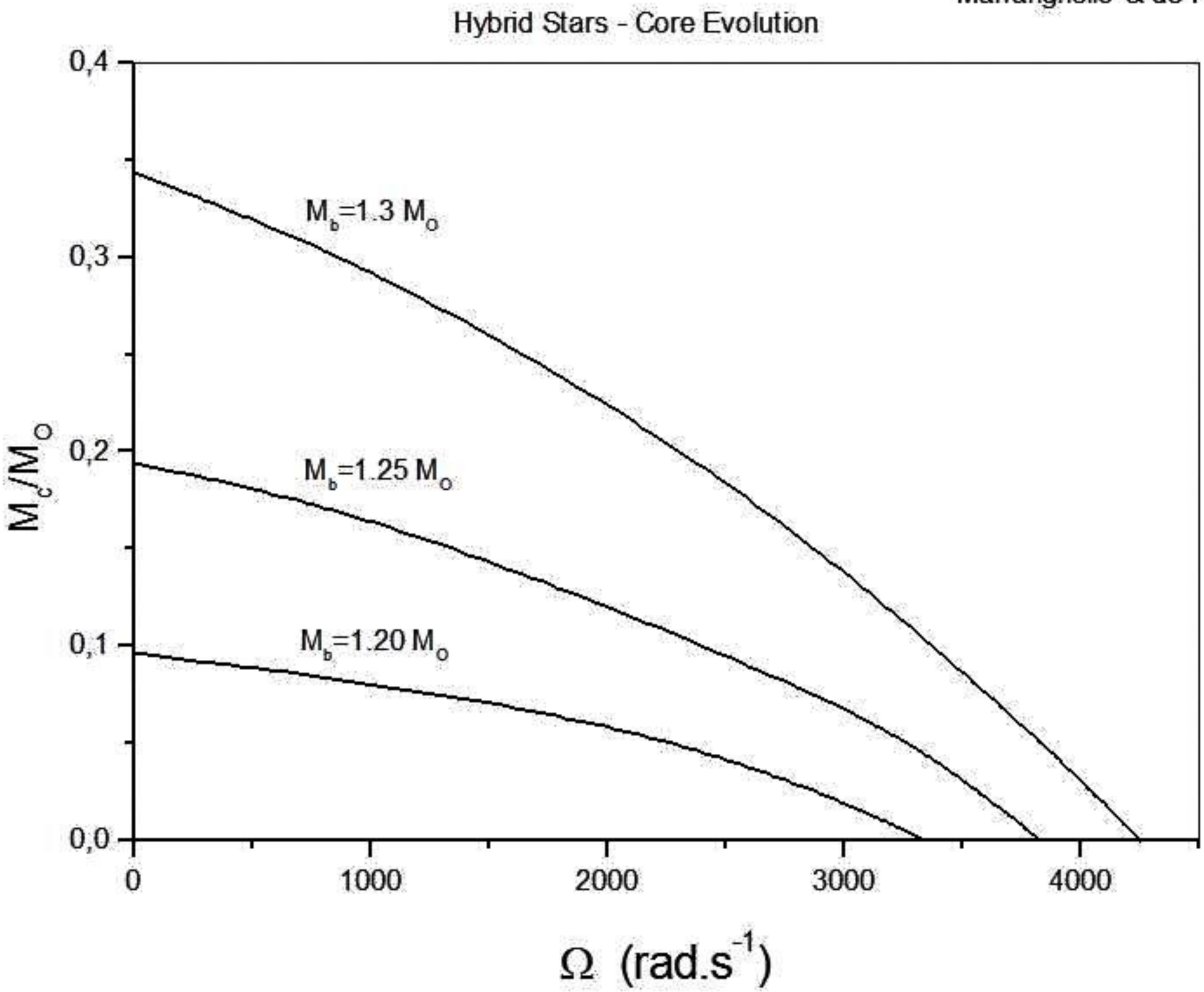
Mass-Radius Relation



Highest known accurate mass is that of **PSR J1614-2230**

$M = 1.97 \pm 0.04 M_{\odot}$ (Demorest et al 2010)

Marranghello & de Freitas Pacheco 2006



$M_g < 1.053\,M_\bullet$
(no quark cores)

$M_g > 1.26\,M_\bullet$
(born hybrid)

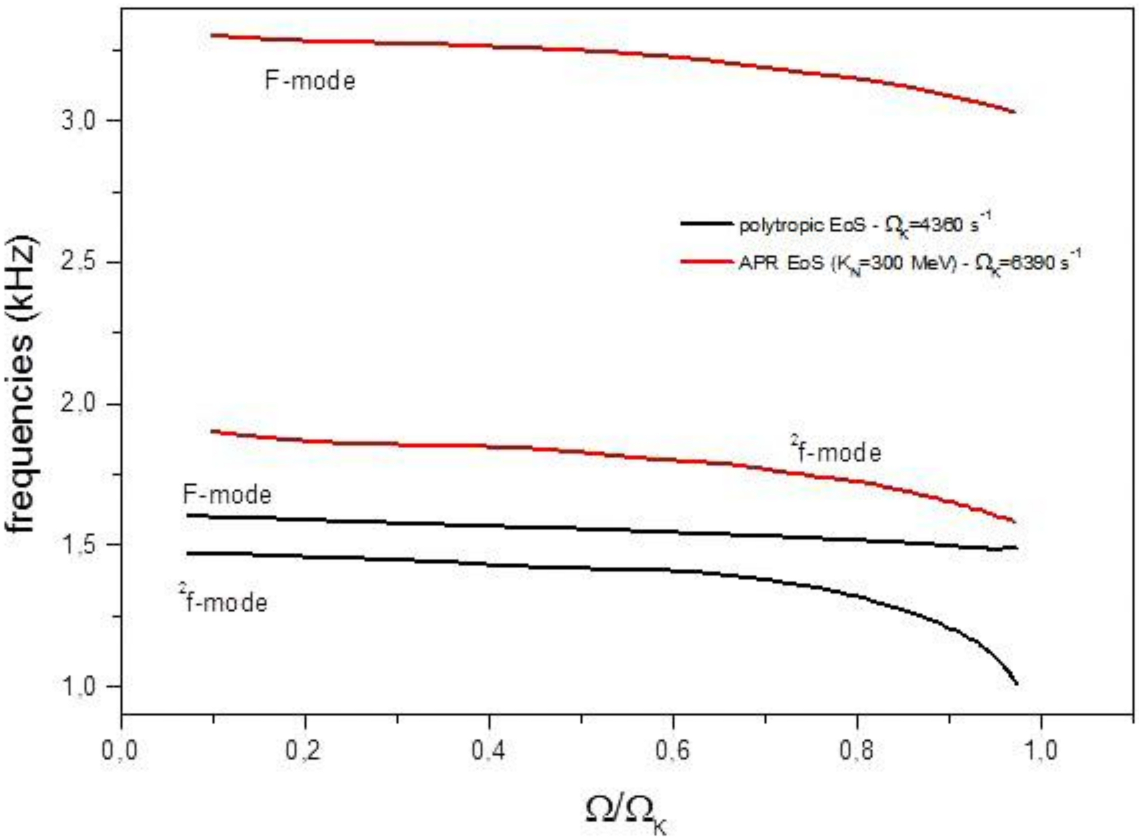
$1.05 < M_g/M_\bullet < 1.26$
(develop a core)

Quasi-Radial Modes

- Rotating stars – polar and axial modes are coupled – even the lowest ($\ell = 0$) quasi-radial mode radiates GWs (Stergioulas 2003)
- Slow rotation – Hartle & Friedman (1975); Datta et al. (1998)
- Fast rotation + Cowling approximation – Yoshida & Eriguchi (2001)
- Non-perturbative approach – relativistic hydrodynamics + field equations (Font et al. 2000, 2002)
- Fast rotation – perturbative approach including variations in the fluid variables + metric potential coefficients (Vincent 2008)

Quasi-Radial Frequencies

(Vincent & de Freitas Pacheco 2008)



Polytropic Models

* F-mode frequency < Q-mode frequency

* No crossing between F and Q modes

* Mode frequency decreases with Ω

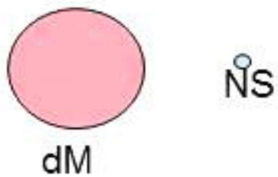
"Realistic" Models

* F-mode frequency > Q-mode frequency for M ~ 1.3-1.4 $M_\odot$

* F-Q crossing for higher masses

* For a given rest mass, frequencies are higher for "soft" equations of state

LMXB



- initial NS mass less than $1.05\,M_\odot$
- re-accelerated pulsar (P~1ms)
- accretion leads to a phase transition

Released energy → $E \sim 2\times10^{51}$ erg

Quasi-radial mode is excited!

F-mode frequency – 3.0 kHz
Q-mode frequency – 1.6 kHz

Maximum distance probed by the detector

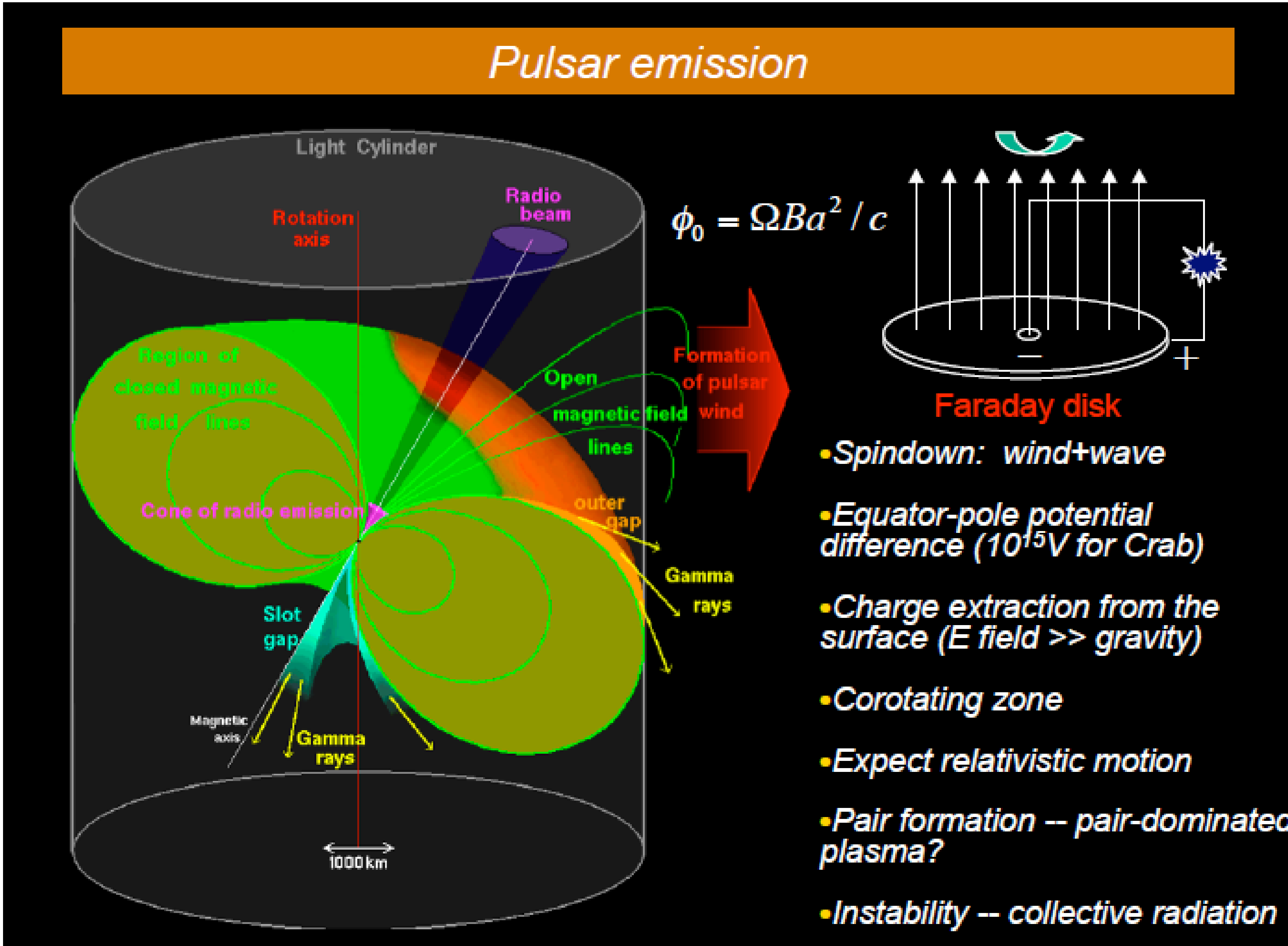
$$D_{\max} = \frac{2}{\pi\sqrt{5}} \frac{1}{(S/N)} \left(\frac{G E_{ef}}{c^3 f^2 S_n(f)} \right)^{1/2}$$

case	detector	D_{max} (kpc) F-mode	D_{max} (kpc) Q-mode
1	ad-LIGO	170	600
1	EGO	1290	4500
2	ad-LIGO	52	830
2	EGO	400	6250

Why to study magnetic field effects in NS ?

- Magnetic dipole radiation drains the rotational energy of pulsars
- Magnetic fields affect the spectrum of the emitted radiation
- Large magnetic fields affect the internal structure of the neutron star and its shape in particular
- Magnetic fields modify the beta equilibrium in the inner crust and hence the equation of state in these regions
- Magnetic fields induce electric fields able to accelerate particles at high energies

Sketch of the pulsar magnetosphere and emission mechanisms



Magnetic field regimes

Weak regime: $H < H_{crit}$ $\longrightarrow$ critical field $\longrightarrow$ Larmor's radius = Bohr's radius

$$r_L = \frac{mc^2}{eH} = a_0 = \frac{\hbar^2}{Z^2 m e^2} \Rightarrow H_{crit} = \frac{Z^2 m^2 e c^2}{\hbar^2} \approx 3.2 \times 10^{11} Z^2 \ G$$

Magnetic white dwarfs are in this regime

Strong regime: $H_{crit} < H < H_{Sch}$ where H_{Sch} is the Schwinger field

The majority of the pulsars are in this regime. The Schwinger field is derived from the condition $\longrightarrow$

$$\hbar \omega_H = mc^2 \Rightarrow H_{Sch} = \frac{m^2 c^3}{\hbar e} = 4.4 \times 10^{13} \ G$$

Schwinger regime: $H > H_{Sch}$ $\longrightarrow$ quantum effects are important

Magnetars belong to this high field regime

Magnetosphere of pulsars

Magnetic fields participate on the equilibrium of the outer atmosphere of pulsars

In the limit of very high conductivity, the hydrostatic equilibrium is described by

$$\vec{\nabla}\cdot\left(P+\frac{B^2}{8\pi}\right)-\frac{1}{4\pi}\vec{B}\cdot\vec{\nabla}\vec{B}+\rho\vec{g}=0$$

Magnetic field cannot be neglected if $\longrightarrow$ $B \geq \sqrt{\left(\frac{8\pi g}{\kappa}\tau_{eff}\right)}$

For a typical NS: g ~ 1.9 x 10^{14} cm/s^2 and κ ~ 0.2 cm^2/g $\longrightarrow$ B ~ 1.5 x 10^8 G

The magnetic field affects not only the equilibrium but also the radiative transfer, since the opacity is not the same in the direction of the field lines and perpendicular to them

Thomson scattering in anisotropic media

Equation of motion $\longrightarrow$ $$\frac{d\vec{V}}{dt} = \frac{e\vec{E}e^{i\omega t}}{m} + \frac{e}{mc}\vec{V}\times\vec{B}$$

Use the ansatz $\longrightarrow$ $$\vec{V} = a_*\vec{E}_{0||}e^{i\omega t} + b_*\vec{E}_{0\perp}e^{i\omega t} + c_*(\vec{E}_{0\perp}\times\vec{B})e^{i\omega t}$$

Replacing in the eq. of motion

$$i\omega\left[a_*\vec{E}_{0||} + b_*\vec{E}_{0\perp} + c_*\left(\vec{E}_{0\perp}\times\vec{B}\right)\right] = \frac{e\vec{E}_{0||}}{m} + \frac{e\vec{E}_{0\perp}}{m} + b_*\frac{e}{mc}(\vec{E}_{0\perp}\times\vec{B}) - c_*\frac{e}{mc}B^2\vec{E}_{0\perp}$$

Identifying the terms

$$a_* = \frac{e}{i\omega m} \qquad b_* = \frac{e}{m}\frac{i\omega}{(\omega_H^2 - \omega^2)} \qquad c_* = \left(\frac{e}{m}\right)^2 \frac{1}{c\left(\omega_H^2 - \omega^2\right)}$$

Hence the solution is

$$\vec{V} = \frac{e}{m}\left[\frac{\vec{E}_{0||}}{i\omega} + \frac{i\omega}{\left(\omega_H^2 - \omega^2\right)}\vec{E}_{0\perp} + \frac{e}{mc}\frac{\left(\vec{E}_{0\perp}\times\vec{B}\right)}{\left(\omega_H^2 - \omega^2\right)}\right]e^{i\omega t}$$

To compute the scattering cross section, the dipolar emission approximation will be adopted – In this case, the scattered radiation rate must be equivalent of the emission rate by an electric dipole

$$\sigma \cdot c \cdot \frac{E_0^2}{8\pi} = \frac{2}{3}\frac{e^2}{c^3}\left\langle \dot{V}^2 \right\rangle$$

Case A – wave propagating perpendicular to the magnetic field (along the z-axis). The propagation vector is along the x-axis and the E-field has two components (polarization components) along the z-axis and the y-axis respectively

$$\vec{E}_{0\perp} = E_0 \cos\varphi \vec{j} \quad and \quad \vec{E}_{0||} = E_0 \sin\varphi \vec{k}$$

Replace in the solution for the electron velocity and compute the time derivative

$$\dot{\vec{V}} = \frac{eE_0}{m}\left[\sin\varphi - \frac{\omega^2 \cos\varphi}{\left(\omega_H^2 - \omega^2\right)} + \frac{ieB}{mc}\frac{\omega\cos\varphi}{\left(\omega_H^2 - \omega^2\right)}\right]e^{i\omega t}$$

Squaring and rearranging the terms $\longrightarrow$ $$\langle \dot{V}^2 \rangle = \langle \dot{V}\dot{V}^* \rangle = \frac{e^2}{2m^2} E_0^2 \, f(W,\varphi)$$

Where $\longrightarrow$ $$f(W,\varphi) = \left[\sin^2\varphi + \frac{W^2 \left(W^2+1\right)}{(1-W^2)^2} \cos^2\varphi - \frac{W^2 \sin 2\varphi}{\left(1-W^2\right)} \right] \quad with \quad W = \frac{\omega}{\omega_H}$$

Replacing in the dipole-scattering relation $\longrightarrow$ $$\sigma = \frac{8\pi}{3} \frac{e^4}{m^2 c^4} \, f(W,\varphi) = \sigma_T \, f(W,\varphi)$$

Wave polarized in the direction of the B-field $\longrightarrow$ $\varphi = \pi/2 \;\; \Rightarrow \;\; f(W,\varphi) = 1 \qquad \sigma = \sigma_T$

Wave polarized perpendicular to the B-field $\longrightarrow$ $\varphi = 0 \;\Rightarrow\; f(W,\varphi) = \dfrac{W^2(1+W^2)}{(1-W^2)^2}$

$$If \;\; \omega \gg \omega_H \; (W \gg 1) \;\; \Rightarrow \;\; f(W,\varphi) \simeq 1$$

$$If \;\; \omega \ll \omega_H \; (W \ll 1) \;\; \Rightarrow \;\; f(W,\varphi) \simeq W^2 \;\; \Rightarrow \;\; \sigma \simeq \sigma_T W^2$$

Case B – wave propagating in the direction of B-field – propagation vector along the B-field and the electric field is perpendicular to the B-field

The solution for the derivative of the velocity is

$$\dot{\vec{V}} = \frac{eE_0}{m}\left[-\frac{W^2}{(1-W^2)}\vec{i} + \frac{eB}{mc}\frac{i\omega}{\left(\omega_H^2 - \omega^2\right)}\vec{j} \right] e^{i\omega t}$$

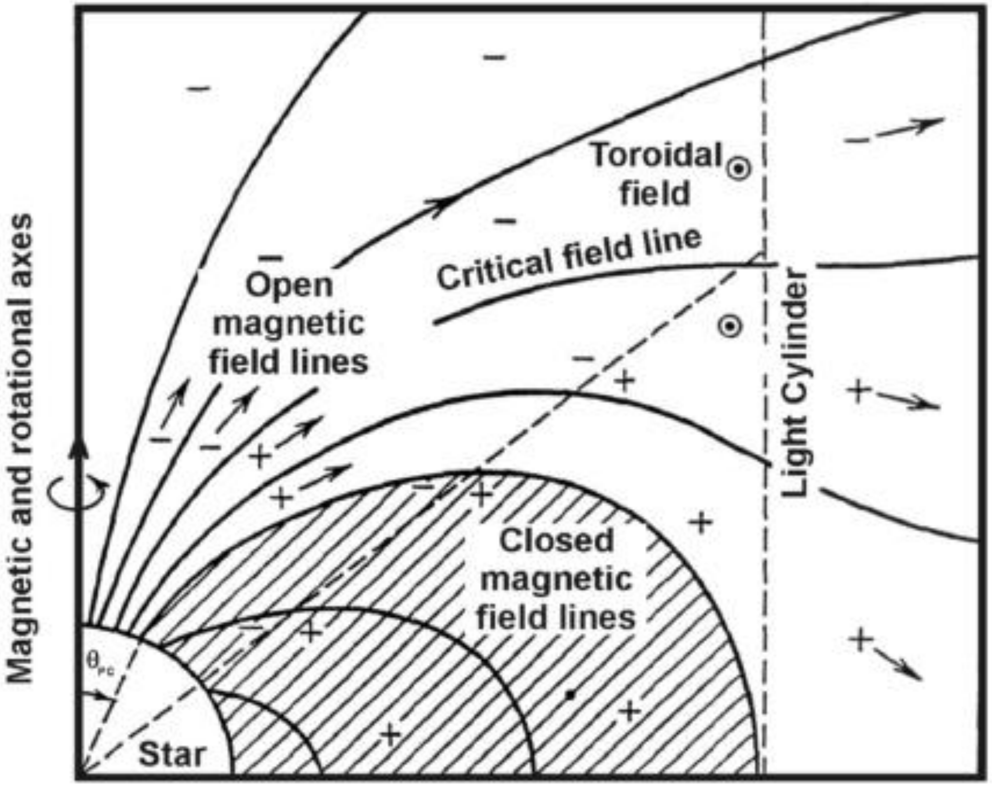
Computing the square one obtains that now the dispersion function is simply

$$f(W) = \frac{W^2(1+W^2)}{(1-W^2)^2}$$

Thus $\longrightarrow$ $\omega \gg \omega_H \Rightarrow f(W) \simeq 1 \quad and \quad \omega \ll \omega_H \Rightarrow f(W) \simeq W^2$

Polarized radiation is expected from the direction perpendicular to the magnetic field

Goldreich-Julian Model



* Aligned magnetic and spin axes

- Magnetosphere has Infinite conductivity

$$\vec{E} = -\frac{1}{c}\vec{V}\times\vec{B} = -\frac{1}{c}\vec{\Omega}\times\vec{r}\times\vec{B}$$

- Goldreich-Julian charge

$$\rho_{GJ} = \frac{1}{4\pi}\vec{\nabla}\cdot\vec{E} = -\frac{\vec{\Omega}\cdot\vec{B}}{2\pi c}$$

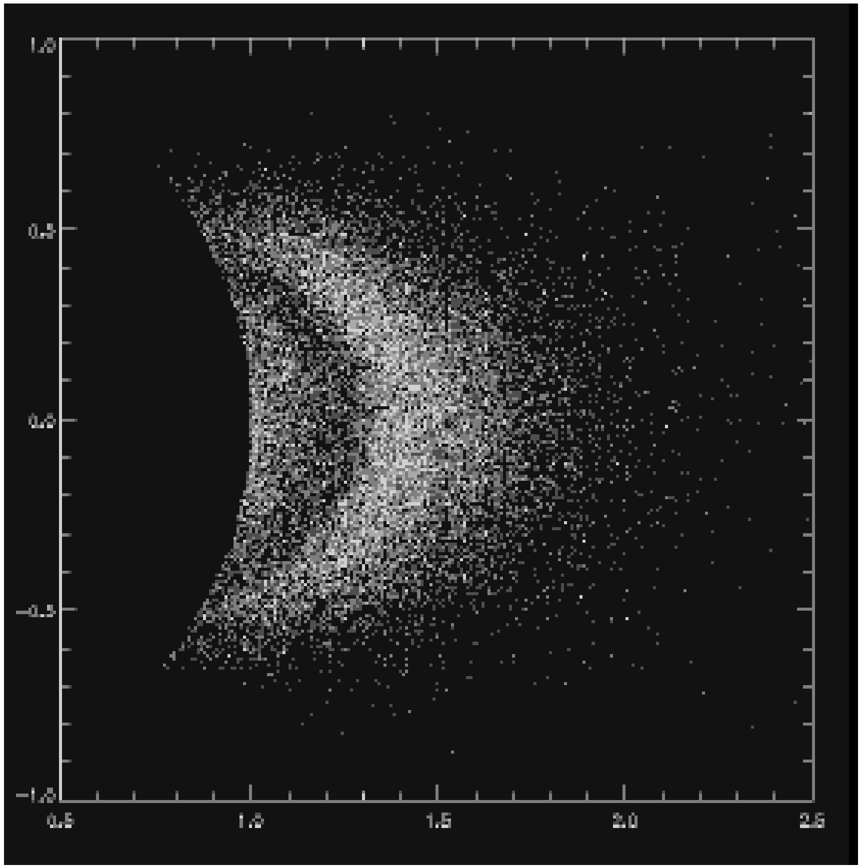
- Energy loss – Poynting flux across the light cylinder

$$L \simeq 4\pi r_{LC}^2 \times \frac{c}{4\pi} S_r$$

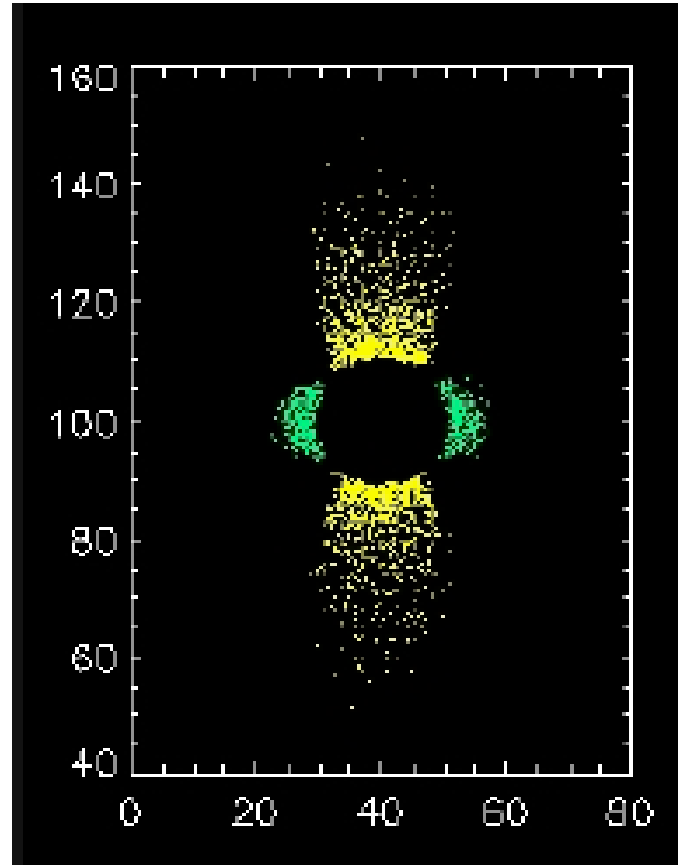
R_{PC} – radius defining the polar region from where the open field lines emerge

$$R_{PC} \simeq R\sqrt{\frac{\Omega R}{c}}$$

Aligned Rotator – numerical solution of magnetohydrodynamic equations

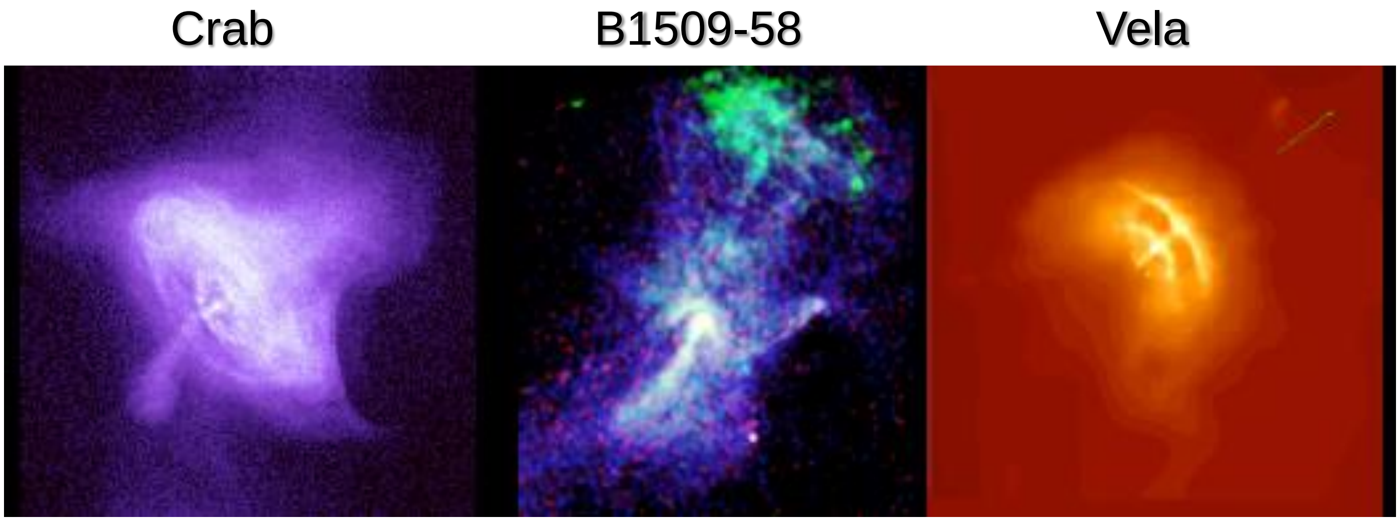


2D-model - charge distribution – non rotating star (Spitkovsky 2005)



2D charge distribution –rotating star (Spitkovsky 2005)

The Sturrock model – Pulsar winds



The model assumes that in the open field lines zone the charge density is given by the Goldreich-Julian value – The emitted particle rate is

$$\frac{dN_p}{dt} = 2 \times \pi R_{PC}^2 \times c \times \frac{\Omega \cdot B}{2\pi Zec} \approx \frac{\Omega^2 B R^3}{Zec} \quad s^{-1}$$

Since $\longrightarrow$ $< W > \approx \frac{Ze\Omega B R^2}{3c}$ Rate of energy loss $\longrightarrow$ $L_w = \frac{1}{3c^2}\Omega^3 B^2 R^5$

Consider a (spherical) neutron star uniformly magnetized:

the internal magnetic field $\longrightarrow$ $\vec{B}_{in} = B_0 \vec{k}$ and the internal electric field $(\sigma \longrightarrow \infty)$

$$\vec{E}_{in} = -\frac{\vec{\Omega} \times \vec{r} \times \vec{B}}{c} = \frac{\Omega B_0 r}{c}\left[\sin^2\theta\,\vec{e}_r + \sin\theta\cos\theta\,\vec{e}_\theta\right]$$

The external fields are $\longrightarrow$ $B_{r,ex} = \frac{2\mu}{r^3}\cos\theta \quad and \quad B_{\theta,ex} = \frac{\mu}{r^3}\sin\theta$

and $\longrightarrow$ $E_{r,ex} = -\frac{3}{2}\frac{\Omega B_0 R^5}{cr^4}\left[\cos^2\theta - \frac{1}{3}\right] \quad and \quad E_{\theta,ex} = -\frac{\Omega B_0 R^5}{cr^4}\cos\theta\sin\theta$

Particles accelerated along the pole ($\theta = \pi/2$)

$$c\frac{dW}{dr} \approx ZecE_{r,ex} = -\frac{Ze\Omega B_0 R^5}{r^4} \quad \Rightarrow W \approx \frac{1}{3c} Ze\Omega B_0 R^2$$

The braking index problem

Definition $\longrightarrow$ $$n = \frac{\ddot{\Omega}\Omega}{\dot{\Omega}^2}$$

Depends on the measurement of the second derivative of the rotation period

Pulsar	Period (sec)	dP/dt (10^{-15})	Braking index
B0531+21	0.033	421	2.509
B0540-69	0.050	479	2.140
B0833-45	0.089	124	1.400
B1509-58	0.150	1490	2.837
J1846-02	0.325	7083	2.650
J1119-61	0.408	4021	2.910
J1734-33	1.170	2280	0.900

The dynamics of the pulsar is fixed by the magnetic dipole radiation that carries angular momentum and energy, which is compensated by angular momentum and rotational energy losses.

Radiation losses $\longrightarrow$ $$L_{rad} = \frac{2}{3}\frac{\mu^2\Omega^4}{c^3}\sin^2 i \quad with \quad \mu = \frac{1}{2}BR^3$$

Voir Landau-Lifshitz "Classsical Theory of Fields"

Rotational Energy losses $\longrightarrow$ $$\frac{d}{dt}\left(\frac{1}{2}I\Omega^2\right) = I\Omega\frac{d\Omega}{dt}$$

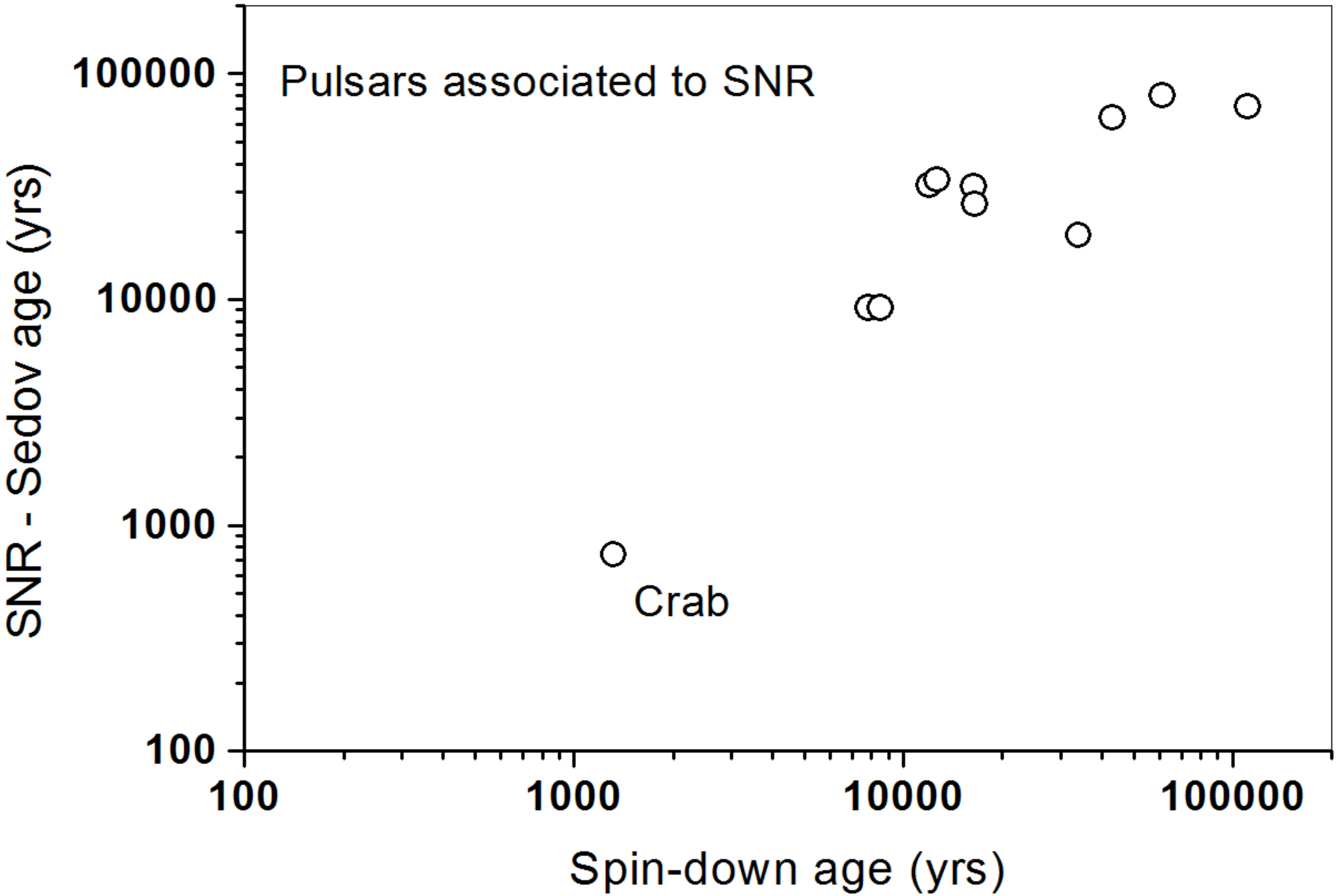
From these eqs $\longrightarrow$ $$\frac{d\Omega}{dt} = -K\Omega^3 \quad with \quad K = \frac{2}{3}\frac{\mu^2}{Ic^3}\sin^2 i$$

and $$\frac{d^2\Omega}{dt^2} = -3K\Omega^2\frac{d\Omega}{dt} = 3K^2\Omega^5$$ hence $$n = \frac{\ddot{\Omega}\Omega}{\dot{\Omega}^2} = 3$$

A pure magnetic dipole field cannot explain the observations!

Notice that since $L_w \propto \Omega^3$ the expected braking index is n = 2 – More studies!

Association – Pulsars and SNRs



Possible solution – the magnetic dipole migrates (Allen & Horvath 1997; Regimbau & de Freitas Pacheco 2001) due to surface tectonics (Link et al. 1998)

In this case, the evolution of the rotation period is given by

$$\frac{d\Omega}{dt} = -K_*\Omega^3 \sin^2 i(t) \qquad with \qquad K_* = \frac{2\mu^2}{3Ic^3}$$

It results for second derivative

$$\frac{d^2\Omega}{dt^2} = 3K_*^2\Omega^5 \sin^4 i(t) - 2K_*\Omega^3 \sin i(t) \cos i(t) \frac{di(t)}{dt}$$

and for the braking index $\longrightarrow \quad n = 3 - \frac{2}{K_*\Omega^2} \cot i(t) \frac{di(t)}{dt}$

If *i(t)* increases with time (*di/dt* > 0) observations can be reproduced since ***n* < 3**

Final configuration – orthogonality between magnetic dipole and spin axes

Pulsar population synthesis – motivations

a) to recover the pulsar population hidden by selection effects

b) true statistical parameters

- i) initial rotation period distribution
- ii) magnetic field distribution
- iii) number of active pulsars and birthrate

c) gravitational wave background - event rates

Simulations

- Espace distribution (birth place) p(R,Z) of massive stars (pop. I) – spiral arms effects

- Orbits in the Galaxy – local circular velocity model) + natal kick - bimodal (Cordes & Chernoff 1997)

- Distances from dispersion measure ($DM = \int n_e ds$) interstellar electron density from Taylor & Cordes (1993)

- Initial period distribution – Gaussian (P_o , σ_{Po})

- Magnetic field distribution - log-normal (log B, σ_{logB})

Rotation Period Evolution

$$"Classical"\ magnetic\ breaking \rightarrow P = P_0(1 + \frac{t}{\tau_B})^{1/2}$$

$$with \rightarrow \tau_B = \frac{3Ic^3}{4\Omega_0^2\mu^2}$$

$$Magnetic\ dipole\ "migration"\ (Link, Franco\ \&\ Epstein\ 1998)$$

$$\sin^2\alpha(t) = (1 - n_0 e^{-t/t_\alpha}) \rightarrow P = P_0\left[(1 + \frac{t}{\tau_B} - n_0\frac{t_\alpha}{\tau_B}(1 - e^{-t/t_\alpha}\right]^{1/2}$$

$$Including\ field\ dissipation\ (exponential\ decay)$$

$$P = P_0\left[1 + \frac{t_D}{2\tau_B}(1 - e^{-2t/t_D}) - \frac{n_0 t_D t_\alpha}{(2t_\alpha + t_D)}(1 - e^{-(2t_\alpha + t_D)t/t_D t_\alpha})\right]^{1/2}$$

$$or,\ for\ a\ power-law\ decay$$

$$P = \left[P_0^{2/(1+\beta)} + \frac{2}{(1+\beta)}\frac{t}{t_*}\right]^{(1+\beta)/2} \rightarrow with\ \ \beta \approx -3.86\ and\ \ B \propto t^{\beta/2}$$

Selection Effects

- After time ***t***, the pulsar should be in the sky area (l, b) covered by a given survey

- ''Flux density'' $\propto L_\upsilon/D^2 > S_\upsilon(A, T_s, n_e)$

$$L_\nu = A\left(\frac{dP}{dt}\right)^\alpha e^{-\beta P}$$

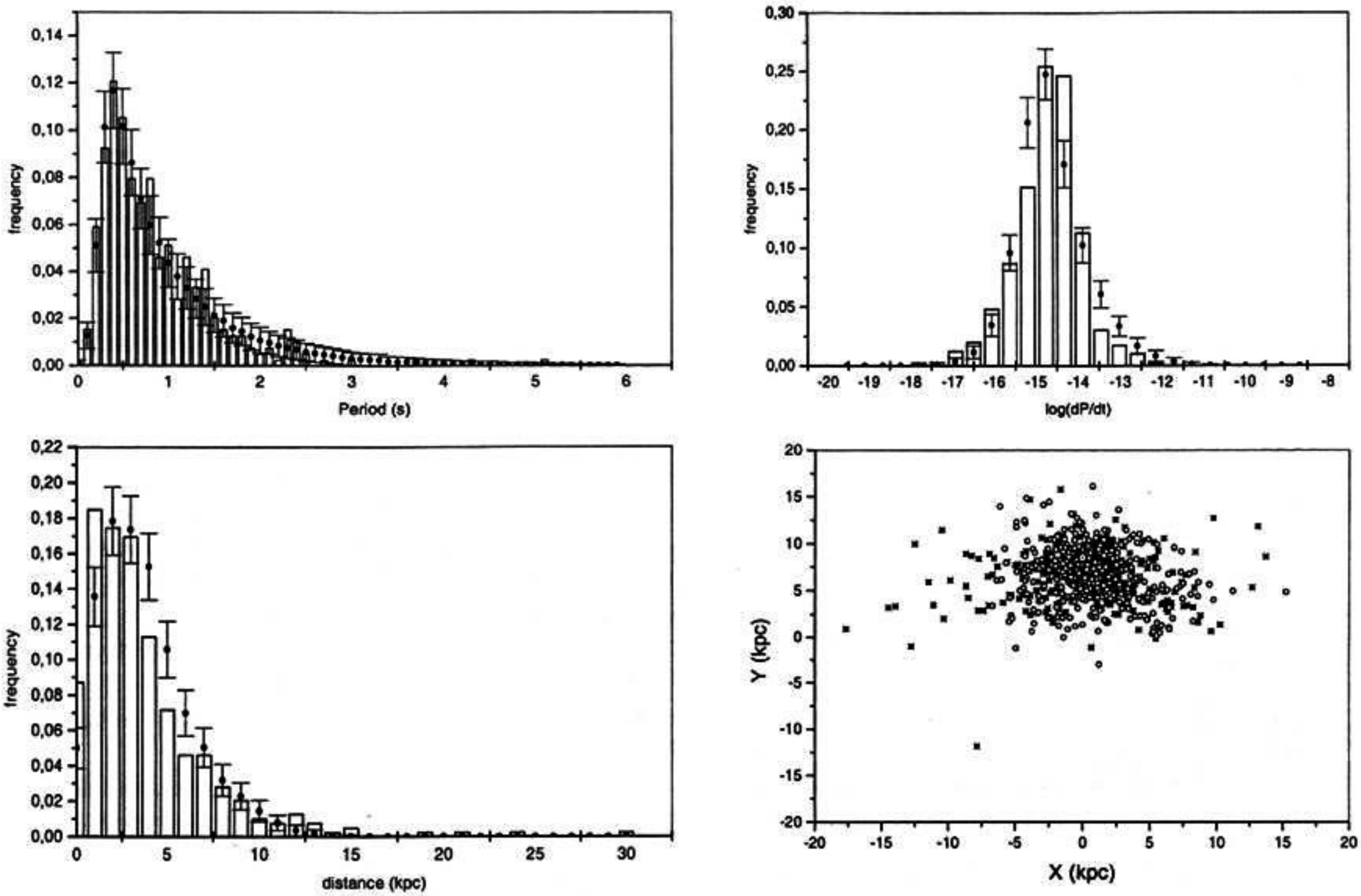
$$at\ \ 400\,Hz \rightarrow A = 1.23x10^6; \alpha = 0.25; \beta = 0.75$$

$$at\ \ 1.4\,kHz \rightarrow A = 4.67x10^3; \alpha = 0.15; \beta = 0.37$$

- Emission beam width ($\propto P^{-1/2}$)

- Pulse smearing effects by the interstellar plasma

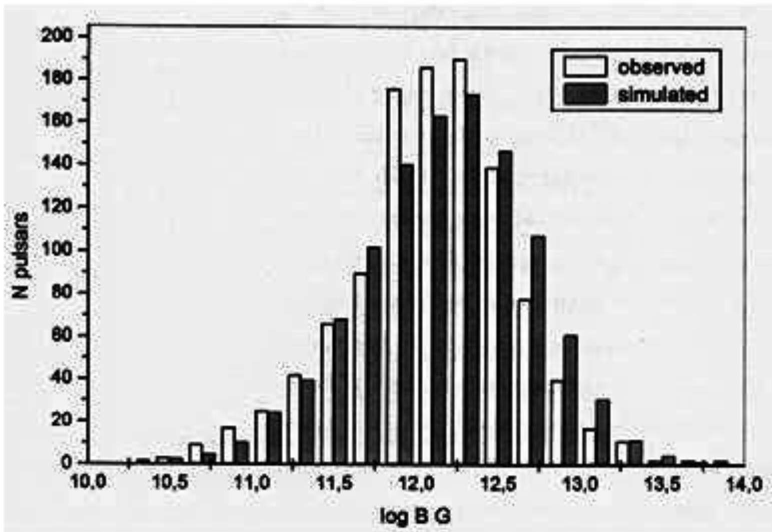
Determination of Parameters

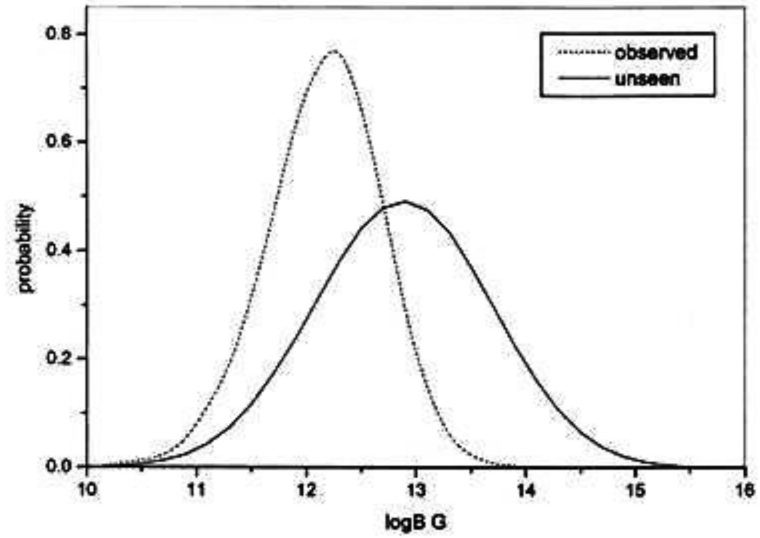


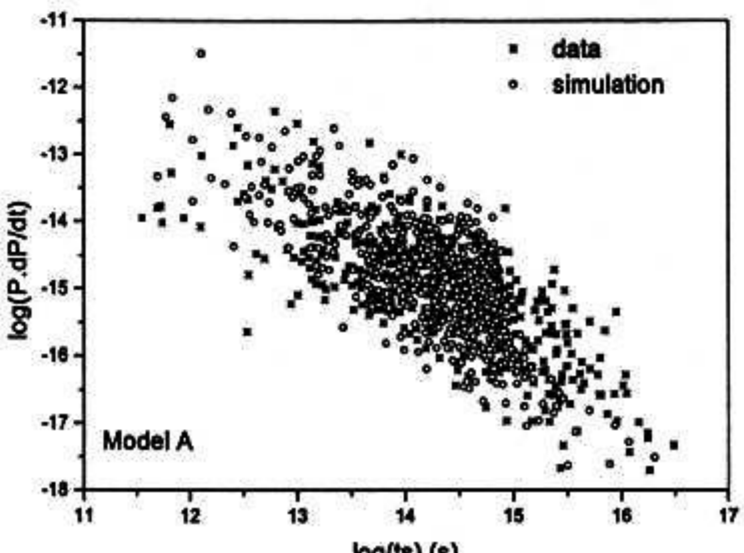
Population Parameters

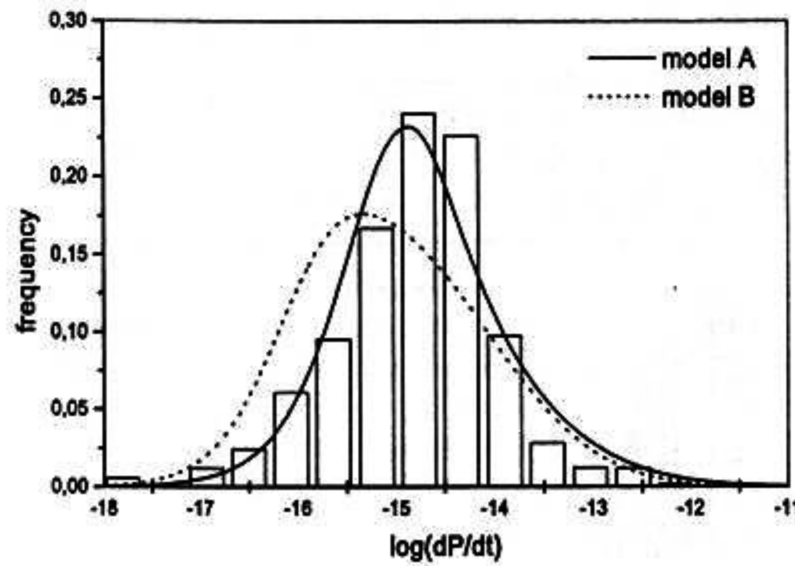
- Mean period – Po = 240±20 ms ; dispersion = 80±20 ms
- Magnetic Field - <logB> = 13.0±0.4 and $\sigma_{\log B}$ = 0.8±0.2
- Magnetic dipole axis migration timescale - $t_\alpha = (10 \pm 4) \times 10^3$ years
- Number of active pulsars in the Galaxy : 253 000
- Birthrate : one every 88 years (mean lifetime of 22 Myr)
- Single-to-binary ratio = 347 (required to explain the properties of PSR B2303+46 and PSR J0737-3039B)

Magnetic Field Properties









The unexpected guest: PSR1640-4631

n = 3.15±0.03

Possible solutions:

1) Migration of the magnetic dipole axis toward the spin axis (Elksi et al. 2016)

2) Magnetic field decay (Blandford & Romani 1988)

3) Mass quadrupole – emission of GW (de Araujo et al. 2016; Chen 2016)

GW emission rate: $$L_{gra} = \frac{32G}{5c^5}\varepsilon^2 I_{zz}^2 \Omega^6 \quad with \quad \varepsilon = \frac{(I_{xx} - I_{yy})}{I_{zz}}$$

Period evolution ⟶ $$I_{zz}\Omega \frac{d\Omega}{dt} = -L_{dip} - L_{gra}$$

Rotation period evolution ⟶ $$\frac{d\Omega}{dt} = -K_1\Omega^3 - K_2\Omega^5$$

derivating: $$\frac{d^2\Omega}{dt^2} = 3K_1^2\Omega^5 + 8K_1K_2\Omega^7 + 5K_2^2\Omega^9$$

Braking index ⟶ $$n = \frac{(3+8\zeta+5\zeta^2)}{(1+\zeta)^2}$$

where ⟶ $$\xi = \frac{K_2}{K_1}\Omega^2 = \frac{48}{5}\frac{G\varepsilon^2 I_{zz}^2}{\mu^2 c^2 \sin^2 i} = 2.8\times10^4\Omega^2\left(\frac{\varepsilon}{B_{12}}\right)^2$$

For PSR1640-4631:

$$I_{zz} = 10^{45}\,gcm^2;\ \mu = 5\times10^{29}\,Gcm^3;\ i = 18^o.5 \pm 3^o.0;\ \Omega = 30.41s^{-1}$$

Observations ⟶ ζ = 0.081 ⟶ $$\varepsilon = 5.6\times10^{-5}B_{12}$$

Ellipticity upper limits derived from LIGO

$$\varepsilon = (I_{xx} - I_{yy}) / I_{zz}$$

Owen (2005) → 'solid strange stars'

$$\varepsilon \approx 10^{-4}$$

Konno et al. (2000) → highly magnetized stars

Abbott et el. 2007 – runs 3 & 4 - LIGO

Pulsar	f_{gw} (Hz)	ϵ_{max}	ϵ_{LIGO}
Crab	59.9	7.5×10^{-4}	**2.6×10^{-4} (S5)**
J0621+10	69.3	1.9×10^{-7}	2.4×10^{-4}
J0737-30	88.1	8.1×10^{-7}	3.2×10^{-5}
J0711-68	364.2	9.0×10^{-9}	3.6×10^{-6}
J0751+18	574.9	3.2×10^{-9}	1.7×10^{-6}
J1024-07	387.4	9.1×10^{-9}	1.0×10^{-6}
J1744-11	490.9	4.4×10^{-9}	1.2×10^{-6}
J2124-33	405.6	9.0×10^{-9}	7.1×10^{-7}

$$\varepsilon_B = g \frac{B^2 R^4}{G M^2} \sin^2 \alpha = 1.9 \times 10^{-8} g B_{14}^2 R_{10}^4 M_{1.4}^{-2} \sin^2 \alpha$$

The Crab Pulsar

(*) The magnetic breaking age (~1330yr) is higher than the pulsar age (~950yr)

(*) Gravitational radiation contributes also to energy losses (Ostriker & Gunn 1969)

$$\frac{d\left(\frac{1}{2}I_{zz}\Omega^2\right)}{dt} = -\frac{B^2R^6\sin^2\alpha}{6c^3}\Omega^4 - \frac{32G\varepsilon^2 I_{zz}^2}{5c^5}\Omega^6$$

Equation of motion

$$t = \frac{\tau_m}{2}\left[x^2 - 1 - \frac{1}{\eta}\ln\left(\frac{1+\eta x^2}{1+\eta}\right)\right]$$

solution, with →

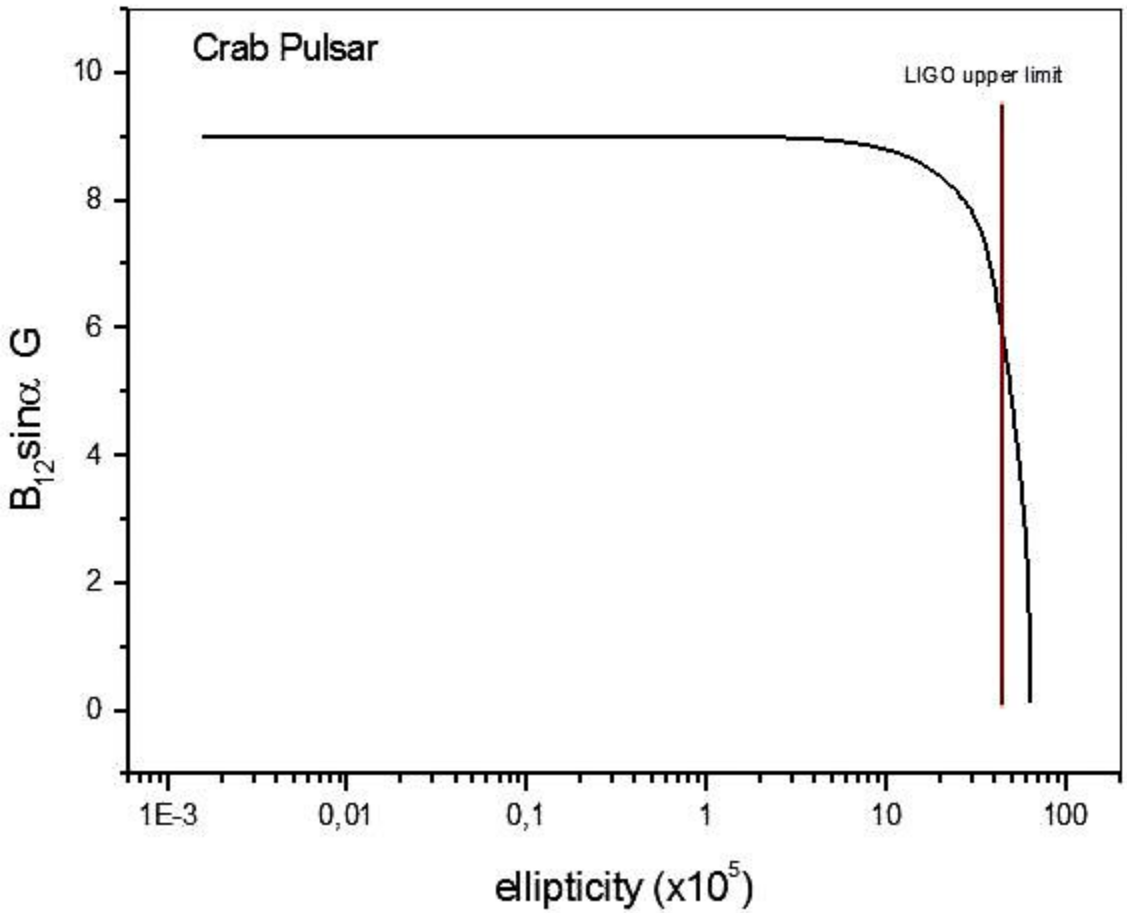
$$\begin{cases} x = \frac{P}{P_0}\ ;\ \tau_m = \frac{6I_{zz}c^3}{B^2R^6\Omega_0^2\sin^2\alpha} \\ \eta = \frac{\tau_{gw}}{\tau_m}\ ;\ \tau_{gw} = \frac{5c^5}{32GI_{zz}\varepsilon^2\Omega_0^4} \end{cases}$$

from observations → $age \equiv t_a = t(x_a) \quad t_* = (P/\dot{P})_{today}$

$$\left(\frac{2t_a}{t_*}\right) = \left(\frac{1+\eta x_a^2}{\eta x_a^2}\right)\left[x_a^2 - 1 - \frac{1}{\eta}\ln\left(\frac{1+\eta x_a^2}{1+\eta}\right)\right] = 0.7157$$

numerical solution gives pairs (η, x_a)

The Crab Pulsar



Initial Period

$P_0 = 24.9 \pm 0.6$ ms

Magnetic Field

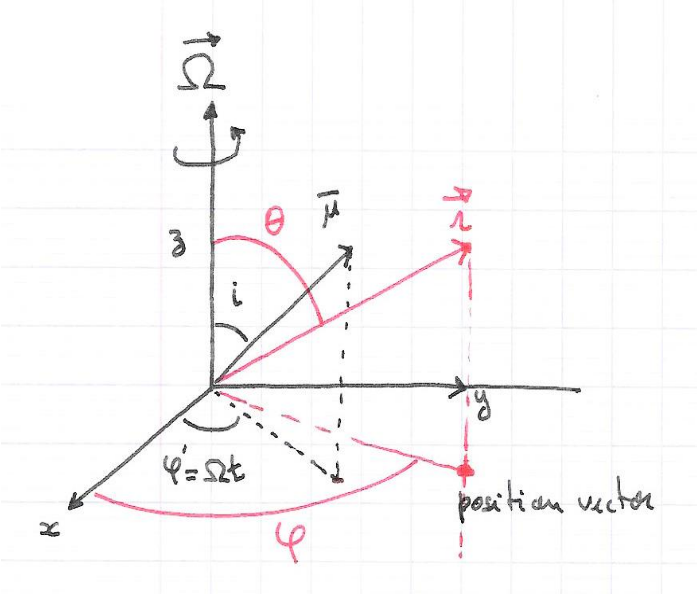
$B\sin i = (8 \pm 1) \times 10^{12}$ G

If distortion is magnetic

$10 < g < 1000$

$1.5\times10^{-9} < \epsilon < 1.5\times10^{-7}$

Electric Fields



An electric dipole is associated to the rotating magnetic dipole

$$\vec{d} = \frac{1}{c}\vec{u} \times \vec{\mu}$$ (see Jackson p. 389)

The corresponding electric potential is

$$\Phi = \frac{\vec{d} \cdot \vec{r}}{r^3} = \frac{(\vec{u} \times \vec{\mu}) \cdot \vec{r}}{cr^3} = \frac{(\vec{r} \times \vec{u}) \cdot \vec{\mu}}{cr^3}$$

Develop to obtain $\longrightarrow$ $$\Phi = \frac{\Omega R \mu}{cr^2}\left[\sin i \cos i \sin\theta \cos(\Omega t - \varphi) - \cos\theta \sin^2 i\right]$$

$$\Phi_{pole} = 0 \;\leftrightarrow\; \Phi_{eq} = \frac{\Omega R \mu}{cr^2}\sin i$$

The resulting electric fields can accelerate particles to very high energies

Particle Acceleration

Potential along the equator of the magnetic pole $\longrightarrow$ $\Phi_{pol}(r) = -\frac{\Omega \mu R}{cr^2} \sin i$

Energy acquired by particles $\longrightarrow$ $\frac{dW}{dt} = ZecE_r = Zec \frac{\partial \Phi_{pol}}{\partial r}$

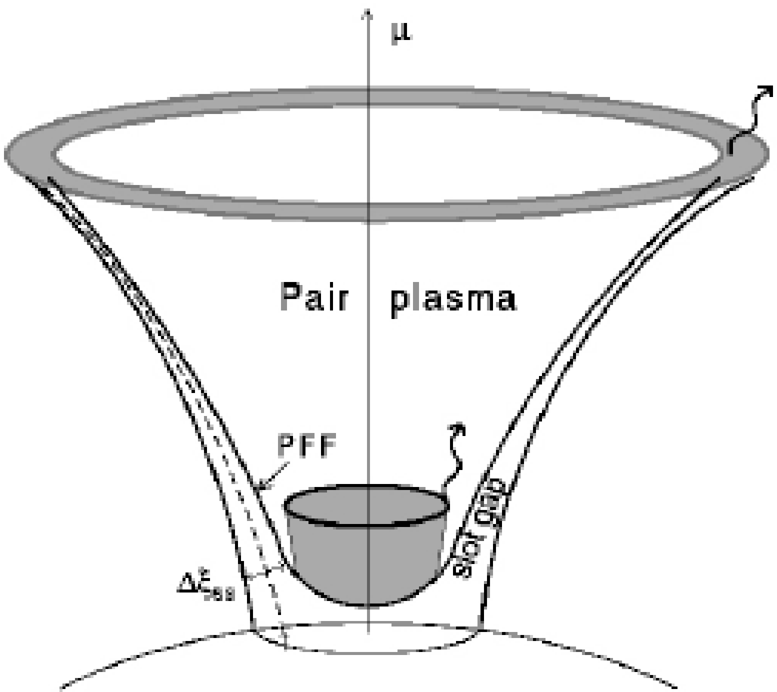
hence $\longrightarrow$ $\frac{dW}{dt} = c \frac{dW}{dr} = \frac{Zec\,\Omega R^4 B \sin i}{cr^3}$

Integrating, one obtains for the maximum (radiation losses were not taken into account) particle energy

$$W_{\max} = \frac{Ze\,\Omega B R^2 \sin i}{2c} = 9.5 \times 10^{17}\, Z R_{10}^2 B_{12} \left(\frac{33ms}{P} \right) \sin i \ eV$$

No acceleration for an aligned magnetic dipole!

Polar cap electrodynamics



In the polar caps, differences between the local (surface) charge density and the Julian-Goldreich charge produce an electric field along the magnetic field lines

The E-field present in the "gap" is limited by the production of electron-positron pairs by curvature radiation. These pairs produce a "screening" effect.

The formation of a "gap" requires fields larger than 10^{13} G (Gil & Mitra 2001)

Leptons accelerated in the "gap" may emit neutrinos via the process (Karminker & Yakovlev 1993)

$$e^-(e^+) + B \rightarrow e^-(e^+) + \nu\bar{\nu} + B$$

Energy of an electron at a distance z from the surface (Harding & Muslimov 2001)

$$E \simeq 0.02 \frac{B_{14}}{P^{3/2}} z^2 \quad MeV$$

Assume that inside the gap the charge density is given by the Goldreich-Julian corotating charge, corrected by relativistic effects . Then, the number of relativistic electrons with energies between E and E+dE is

$$\frac{dN(E)}{dE} dE = \frac{\varsigma}{\alpha} \frac{\vec{\Omega} \cdot \vec{B}}{2\pi ec} \pi r_p^2 \frac{dz}{dE} dE$$

where ⟶ $\zeta = 1 - \left(\frac{r_g}{R}\right)\left(\frac{I_{zz}}{MR^2}\right) \quad \alpha = \sqrt{\left(1 - \frac{r_g}{R}\right)} \quad r_p = \left(\frac{2}{3}\right)^{3/4} R \left(\frac{\Omega R}{c}\right)^{1/2}$

From these eqs. ⟶ $\frac{dN(E)}{dE} = 5.7 \times 10^{21} B_{14}^{1/2} R_{10}^{3} P^{-5/4} E^{-1/2} \quad MeV^{-1}$

Let $W_{\nu\bar{\nu}}(E,\hbar\omega)$ be the probability per unit of time and per energy interval for an electron of energy E to emit a neutrino par of total energy $\hbar\omega$

Then, the neutrino production rate per energy interval is

$$Q_{\nu\bar{\nu}}(\hbar\omega) = \int W_{\nu\bar{\nu}}(E,\hbar\omega)\,\frac{dN(E)}{dE}\,dE$$

The mean neutrino pair energy is $\longrightarrow$ $\hbar\omega \simeq E\,\dfrac{\chi}{(1+\chi)}$ with $\chi = \hbar\omega_L\,\dfrac{E^2}{(mc^2)^3}$

When $B > 10^{14}$ G we are in the regime $\chi >> 1$ if the electron energy $E > 5$ MeV

In this regime, the energy of the pair is comparable to the electron energy and the emission probability reduces to

$$W_{\nu\bar{\nu}}(E,\hbar\omega) = \frac{G_F^2(mc^2)^6}{216\pi^3(\hbar c)^7}\,\frac{c\chi^2}{E}\,g^2(\lg\chi - 1.96)\,\delta(E-\hbar\omega)$$

with $\longrightarrow$

$$\begin{cases} g^2 = g_A^2 + g_V^2 \;\Rightarrow\; g_A = \dfrac{1}{2} \text{ and } g_V = \dfrac{1}{2} + 2\sin^2\theta_W \;(\textit{electron neutrino}) \\ g_A = -\dfrac{1}{2} \text{ and } g_V = -\dfrac{1}{2} + 2\sin^2\theta_W \;(\textit{others}) \end{cases}$$

Computing the production rate (constrained by the condition 1<< χ << $(M_W/m)^3$)

$$Q_{\nu\bar{\nu}}(\hbar\omega) = 3.9\times10^{17} B_{14}^{5/2} R_{10}^{3} P^{-5/4} (\hbar\omega)^{5/2} \left[\lg(8.68 B_{14} (\hbar\omega)^{2} - 1.96 \right] \; s^{-1} MeV^{-1}$$

Therefore the expected neutrino pair luminosity in the range 0.01-20 GeV

$$L_{\nu\bar{\nu}} \simeq 1.3\times10^{32} B_{14}^{5/2} R_{10}^{3} P^{-5/4} \quad ergs^{-1}$$

A magnetar with a period of 10s at a distance of 1 kpc will produce a neutrino flux of 6.1 x 10^{-14} erg/(cm2s) below the sensibility of present neutrino detectors. (Present sensibility – Ice Cube, Antares, ~ two orders of magnitude lower)

Final Considerations

- Pulsars are sources of different energetic processes: they accelerate particles, produce energetic winds which affect the dynamics of the SN remnant

- They are probes for nuclear interaction (EoS) and some NS may even have deconfined matter in their cores

- NS can be also probes for modified gravitational theories

- Rotating NS are potential GW sources that could be detected by the future generation of laser interferometers