Global anisotropies as large-scale perturbations

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Paradigm of ΛCDM model

- GR assumed valid at cosmological scales ($\gtrsim 100$ Mpc)
- Standard model of particles + Λ
- Cosmological Principle:
  - Maximally symmetric geometry
  - Trivial topology

What are the possible expanding geometries?

Next logical choice

Homog. & Isotropic

E.g. FLRW

Homog. & anisotropic

E.g. Bianchi

Inhom. & anisotropic

E.g. LTB

Observational Constraints on anisotropies

- CMB severelly constrains anisotropies
  - But mostly at $z \lesssim 1100$
  - Isotropic CMB $\not\Leftrightarrow$ Isotropic Universe (Ellis, Clarkson)
  - Observations are made from a privileged point
- LSS constraints are weaker
  - However, future probes are promising (e.g. Euclid, LSST)

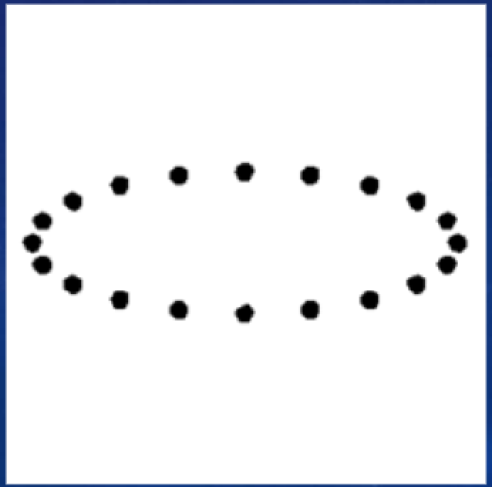
Moreover . . .

- CMB and LSS anomalies not fully understood
- Anisotropic models have an intimate connection with gravitational waves

Example (Pontzen & Challinor, CQG 2011) (Pereira & Pitrou, PRD 2019)

$$\beta'' + 2\mathcal{H}\beta' = 0 \leftarrow \text{Spacetime shear}$$

$$h'' + 2\mathcal{H}h' = 0 \leftarrow \text{Long GW}$$



This talk

- Quick intro to anisotropic models
- Signatures:
  - Early-time
  - Late-time
- Anisotropies × Large-wavelength perturbations
- Prospects and conclusions

Anisotropic Models

Quick intro

From homogeneous 3D spaces...

- Formulated by Luigi Bianchi (1898)
- 3D-spaces admitting 3 translational KVFs $\vec{\xi}_i$
- Lie algebra of KVF : $[\vec{\xi}_i, \vec{\xi}_j] = C^k_{ij}\,\vec{\xi}_k$

C^k_{ij} = const. of structure ← 10 inequivalent possibilities

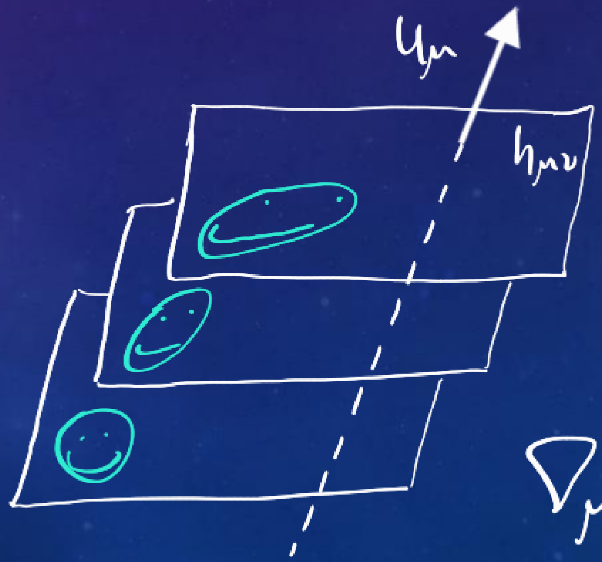
$$= N^{lk}\varepsilon_{ijl} + A_{[i}\delta^k_{j]}$$

...To anisotropic spacetimes

- Modern usage in cosmology due to Ellis & MacCallum (1969)
- We assume a global time-like vector $\vec{u}$ so that

$$g_{\mu\nu} = h_{\mu\nu} - u_\mu u_\nu \qquad \text{and} \qquad [\vec{u}, \vec{\xi}_i] = 0$$

$$\nabla_\mu u_\nu = H h_{\mu\nu}$$



spacetime shear

$$\nabla_\mu u_\nu = \sigma_{\mu\nu} + H h_{\mu\nu}$$

Models with isotropic limit

- Only 5 models have isotropic limit
- In these cases one can set (Pontzen & Challinor, CQG 2011)

- $\vec{\xi}_i = \vec{T}_i + \epsilon_i^j \vec{R}_j$ with $\vec{T}_i$ = translation ; $\vec{R}_i$ = rotation
- Search for ϵ_i^j such that $[\vec{\xi}_i, \vec{\xi}_j] = C_{ij}^k \vec{\xi}_k$

Type	A	N^1	N^2	N^3	$\frac{a^{2}\,{}^{(3)}R}{6} = K$	$\mathcal{M}^{(3)}$
I	0	0	0	0	0	$\mathbb{E}^3$
V	ℓ_c^{-1}	0	0	0	$-\ell_c^{-2}$	$\mathbb{H}^3$
VII_0	0	ℓ_s^{-1}	ℓ_s^{-1}	0	0	$\mathbb{E}^3$
VII_h	ℓ_c^{-1}	ℓ_s^{-1}	ℓ_s^{-1}	0	$-\ell_c^{-2}$	$\mathbb{H}^3$
IX	0	$2\ell_c^{-1}$	$2\ell_c^{-1}$	$2\ell_c^{-1}$	$+\ell_c^{-2}$	$\mathbb{S}^3$

Two free scales!

- Curvature ℓ_c
- Spiraling ℓ_s

Simplest case: Bianchi I

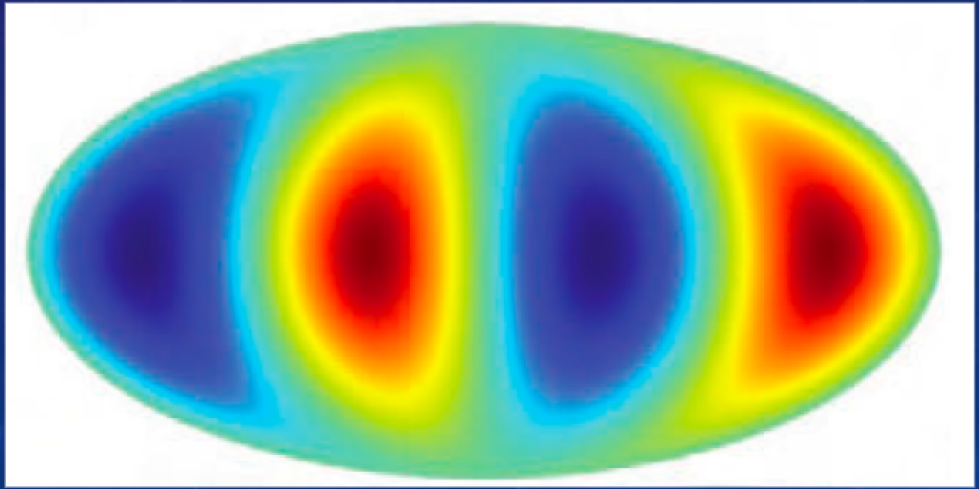
- Isometries are simple translations: $\vec{\xi}_1 = \partial_x$, $\vec{\xi}_2 = \partial_y$, $\vec{\xi}_3 = \partial_z$
- Metric:

$$ds^2 = -dt^2 + a^2 dx^2 + b^2 dy^2 + c^2 dz^2$$
$$= -dt^2 + S^2(t)\,\gamma_{ij}(t)\,dx^i dx^j$$

$$S = (abc)^{1/3} \qquad \sigma_{ij} = \frac{1}{2}\dot{\gamma}_{ij}$$

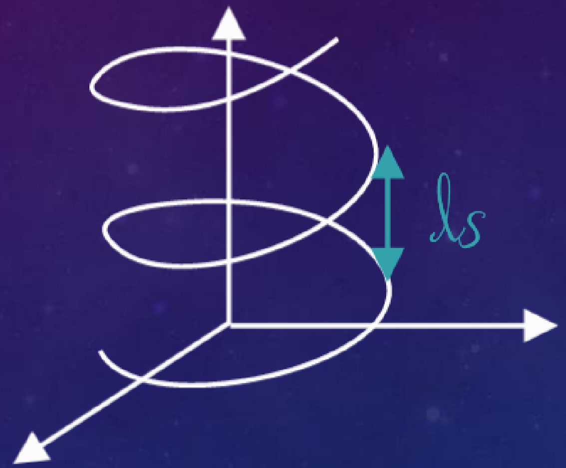
(Misner-Thorne-Wheeler)

- Quadrupolar CMB signal



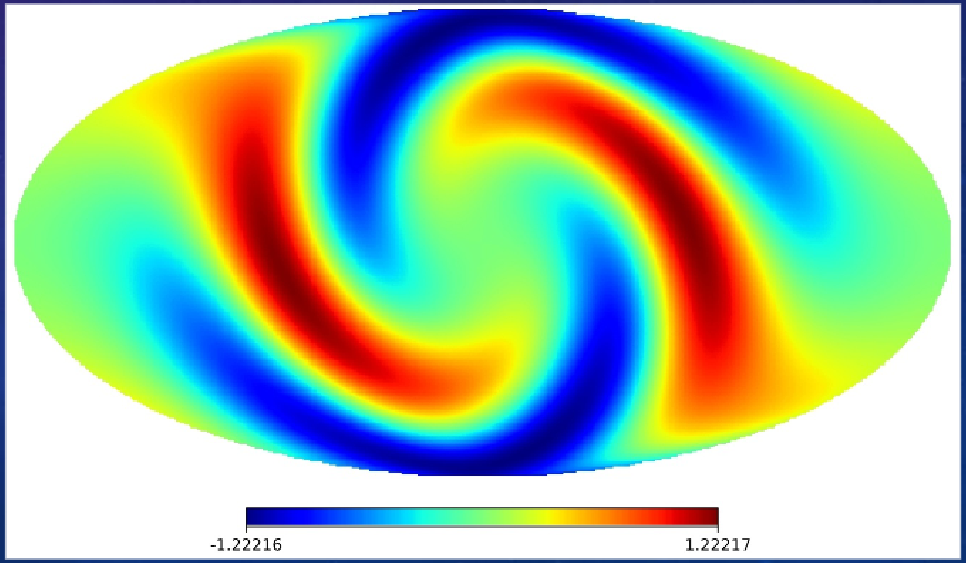
Second simplest case: Bianchi VII_0

- Isometries are richer:



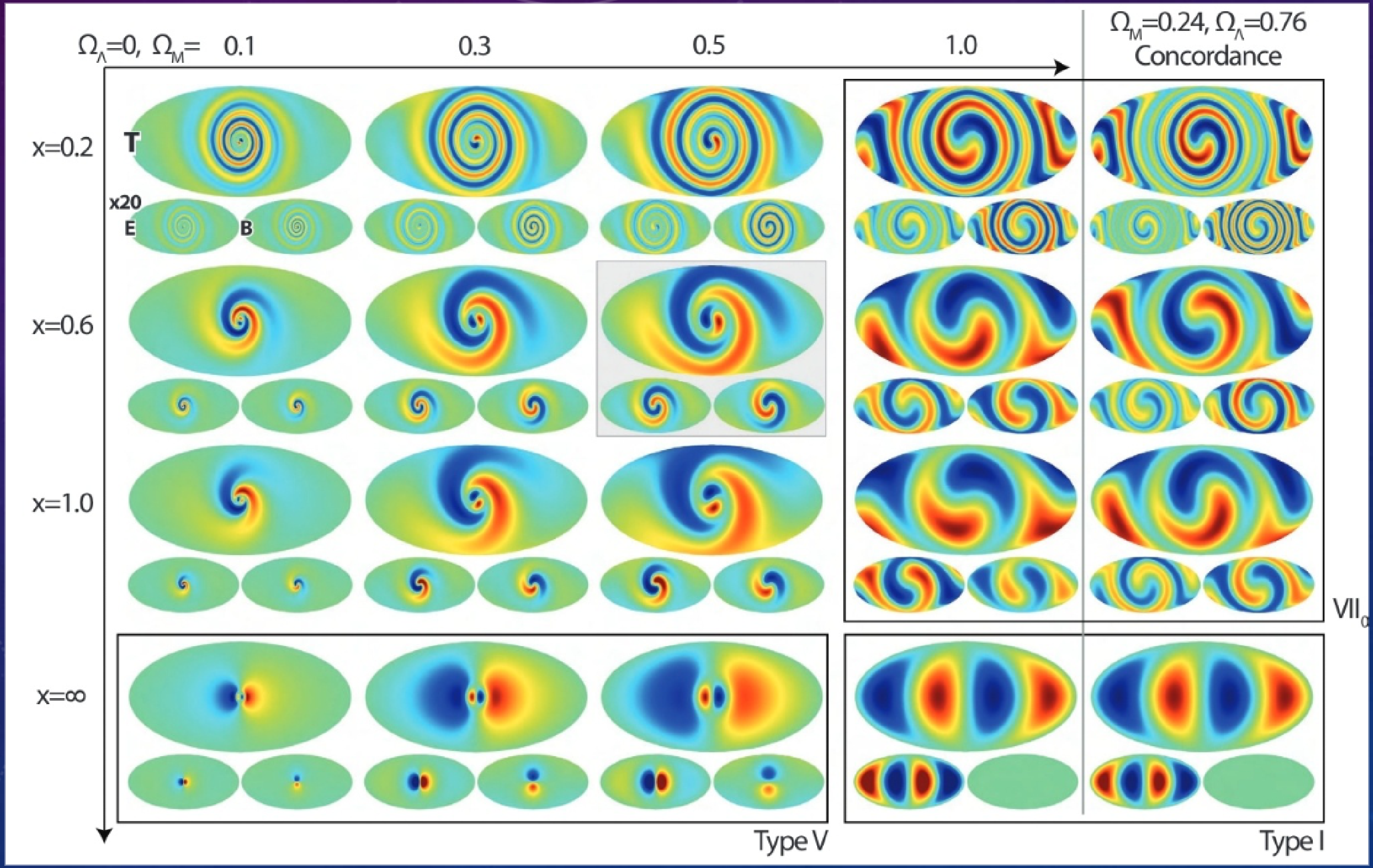
$$\vec{\xi}_1 = \partial_x \;,\; \vec{\xi}_2 = \partial_y \;;\; \vec{\xi}_3 = \partial_3 + l_s^{-1}(x\partial_y - y\partial_x)$$

- CMB quadrupole:



CMB anisotropies

$$X = \frac{1}{\sqrt{\Omega_K}}$$



(Pontzen, PRD 2009)

Signatures

(Bianchi - I)

Background dynamics (I and VII_0)

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu} + \pi_{\mu\nu}$$

Einstein Eqs:

$$H^2 = \frac{\rho}{3} + \sigma^2$$

$$\frac{\ddot{a}}{a} = -\frac{(\rho + 3p)}{6} - \frac{\sigma^2}{3}$$

$$\dot{\sigma}_{ij} + 3H\sigma_{ij} = \pi_{ij}$$

Solution

$$\sigma_{ij} = \frac{C_{ij}}{a^3} + \frac{1}{a^3}\int^a \frac{s^2 \pi_{ij}}{H} ds$$

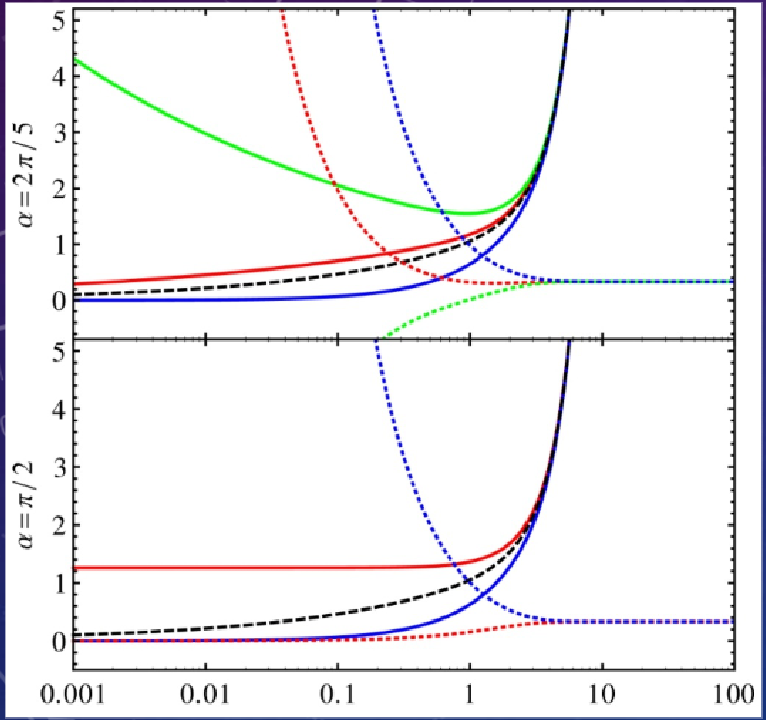
primordial

late-time

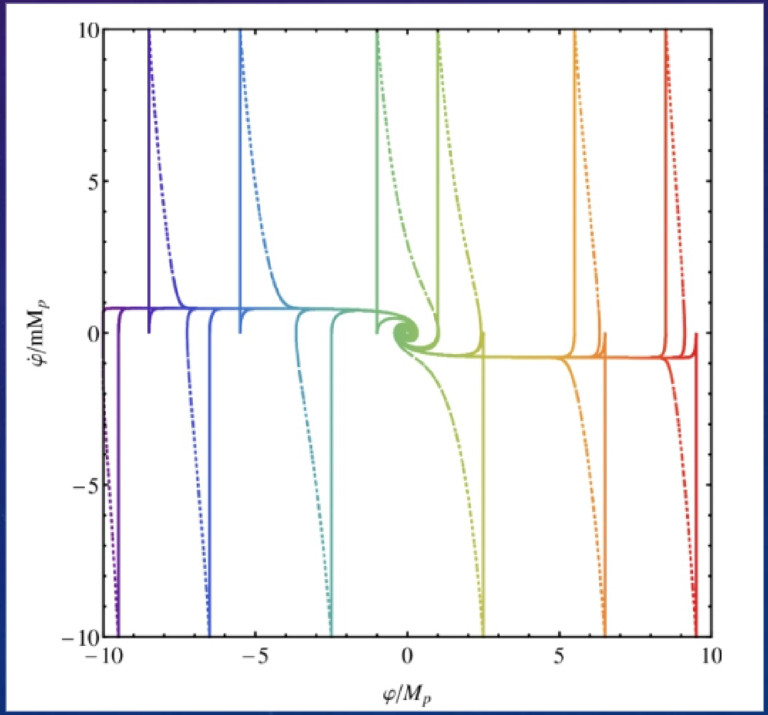
Early anisotropies

- Shear evolves as $\sigma \sim a^{-3}$ → early anisotropic phase

- "de Sitter" solution
- chaotic inflation



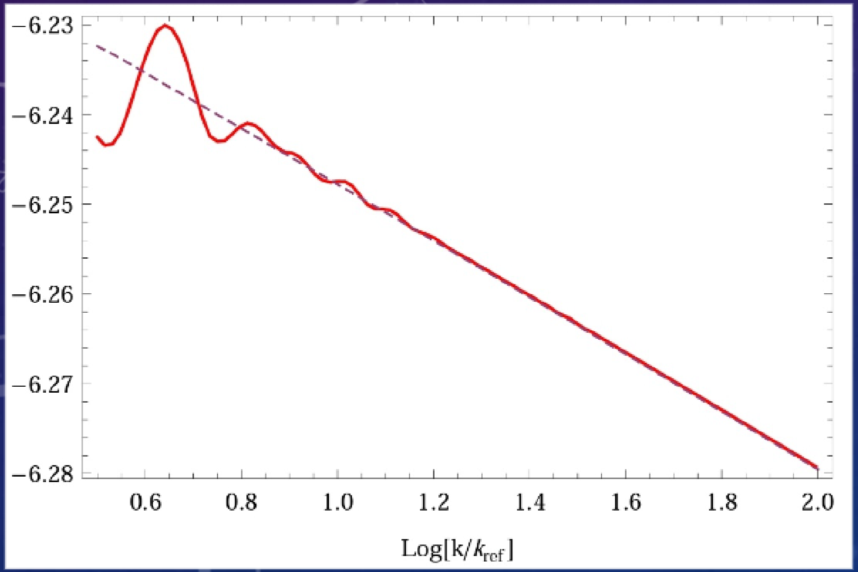
(Pereira, Pitrou, Uzan JCAP 2007)

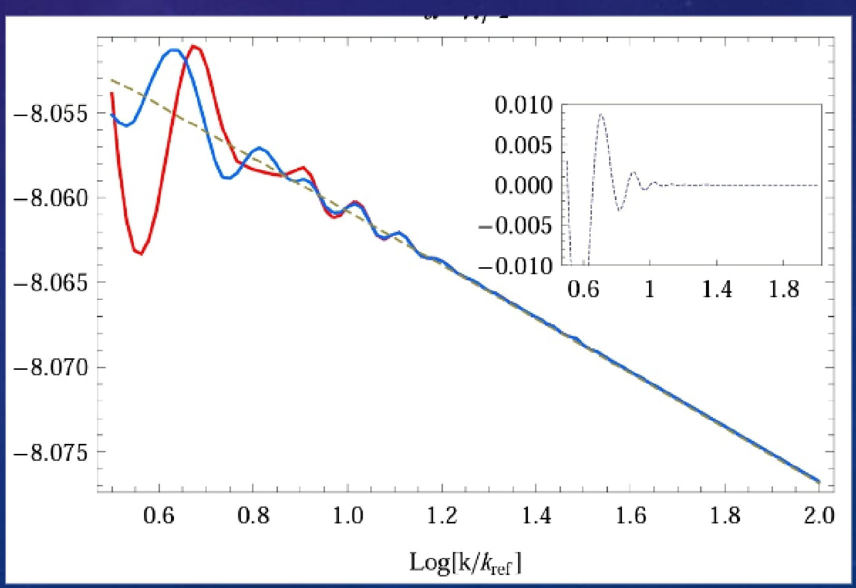


(Pitrou, Pereira, Uzan JCAP 2008)

CMB signatures from inflation

- Non-diagonal covariance matrix: $\langle a_{\ell m} a_{\ell' m'} \rangle \neq \delta_{\ell\ell'}\delta_{mm'}$
- Non-zero scalar × tensor correlations
- Polarizations × and + of GW with different dynamics
- Anisotropic power spectrum: $P(\vec{k}) = P_{iso}(k)\left[1 + \sum_{\ell m} r_{\ell m} Y_{\ell m}(\hat{k})\right]$





Constraints from CMB: $\frac{\sigma}{H_0} \lesssim 10^{-6}$

(Saadeh et al PRL 2016)

Late anisotropics

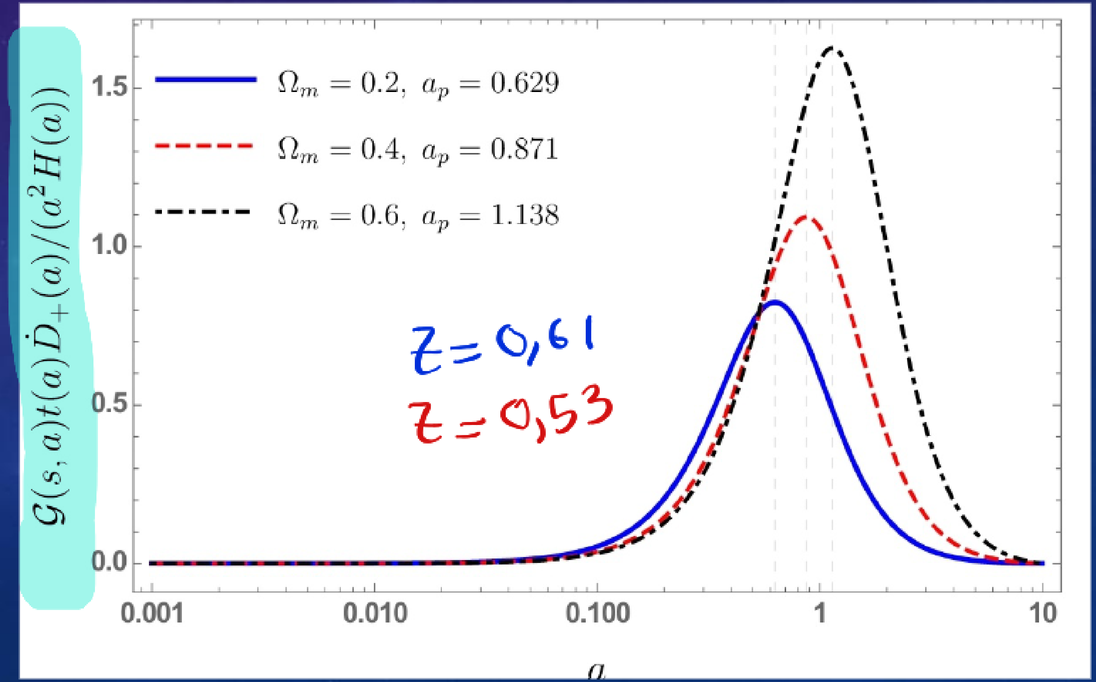
- Anisotropic dark energy?
- linear density contrast: $\delta_1'' + \mathcal{H}\delta_1' - \frac{3}{2}\mathcal{H}^2\Omega_m\delta_1 = -2\sigma(\eta,k)\delta_1'$

Solutions

$$\delta_1 = D_+(a)\left(1 + Q_{ij}\hat{k}^i\hat{k}^j\right)\delta_{iso}$$

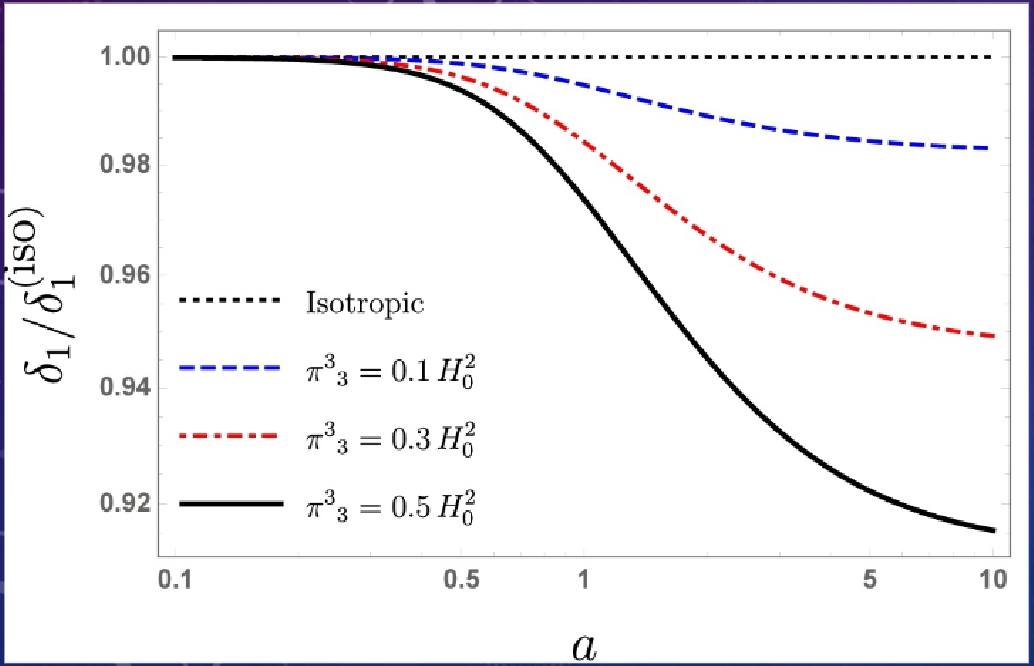
$$Q_{ij} = -\frac{2}{D_+}\int^a ds\, \frac{G(a,s)\dot{D}}{s^2 H}\sigma_{ij}$$

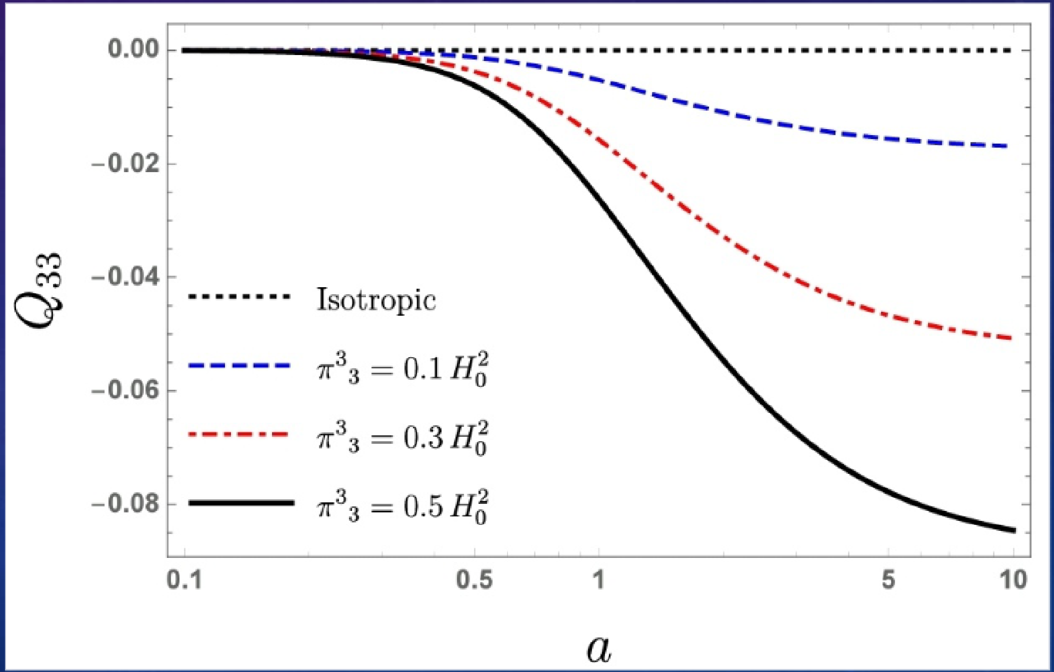
(Almeida et. al. JCAP 2022)



General properties

- Direction dependent spectrum and bispectrum
- Quadrupoles are degenerated with RSD.





(Almeida et. al. JCAP 2022)

Anisotropies from large-wavelength perturbations

What about signatures from other observables?

- One needs perturbation theory (à la Bardeen) in given Bianchi model → hard to get!
- ... But anisotropies are known to be small...
- Is there a simpler route?

Yes, there is!

- A closer inspection reveals that

$$ds^2 = -dt^2 + a^2\gamma_{ij}(t)\,dx^i dx^j$$

general Bianchi

$$ds^2 = -dt^2 + a^2 h_{ij}(t,\vec{x})\,dx^i dx^j$$

Perturbed FLRW in synchronous gauge

"$\lim_{\vec{k}\to 0} h_{ij} \to \gamma_{ij}$"

Example: tensor perturbations

- For small anisotropies $\gamma_{ij}(t) \approx \delta_{ij} + \beta_{ij}(t)$
- For small perturbations $h_{ij}(t,\vec{x}) \approx \delta_{ij} + E_{ij}(t,\vec{x})$

$$E_{ij}^{(\pm)}(t,\vec{k}) = E_{ij}^{(\pm)}(t)\, e^{i\vec{k}\cdot\vec{x}} \longrightarrow \lim_{k\to 0} E_{ij}^{(\pm)}(t,\vec{k}) = E_{ij}^{(\pm)}(t)$$

But as we have seen:

$$\beta_{ij}'' + 2H\beta_{ij}' = 0 \longleftarrow \text{deterministic}$$

$$E_{ij}'' + 2H E_{ij}' = 0 \longleftarrow \text{stochastic}$$

General tensor case

- $\beta_{ij}'' + 2H\beta_{ij}' + S_i{}^k{}_j{}^l \beta_{kl} = 0$

Identification between const. of structure and Fourier modes on curved spaces

- $E_{ij}'' + 2H E_{ij}' + (\Delta^2 - 2K) E_{ij} = 0$

	Homog. limit
I	$k \to 0$
VII_0	$k \to 2/l_s$
IX	$k \to 3/l_c$
V	$k \to i/l_c$
VII_h	$k \to 1/l_s + i/l_c$

(Pereira & Pitrou, PRD 2019)

Supercurvature modes

- Observables are exponded in terms of eigenfunctions

$$\nabla^2 Q = -q^2 Q \qquad \nabla^2 = \frac{1}{\sqrt{g}} \partial_i \left(\sqrt{g}\, \partial^i \right)$$

Laplace-Beltromi operator

- Separation of variables leads to

$$q^2 = k^2 + l_c^{-2}$$

(Hiperbolic space $\mathbb{H}^3$)

- Spectrum
  - $q^2 \geq l_c^{-2}$ or $k^2 \geq 0$ ← sub-curvature
  - $q^2 \geq 0$ or $-l_c^2 \leq k^2 \leq 0$ ← super-curvature

(Lyth & Woszczyna PRD 1995)

General recipe

One can thus adopt the following recipe to build cosmological observables in Bianchi models

1. Compute your favorite observable using linear perturbation theory in FLRW

2. Replace Bessel functions by hyperspherical Bessel functions

3. Take the appropriate k limit

4. Compute multiple moments as line-of-sight integral

$$\frac{{}_s\mathcal{O}_{\ell m}}{2\ell+1} = \int_0^{\eta_0} d\eta \sum_{\ell' \geq m} {}_s\alpha_\ell^{(\ell' m)}(\chi; k)\, S_{\ell'}^m(\eta, \vec{k})$$

radial function

Source

- This recipe is directly implementable in Class!

  (Pereira, Pitrou, Vicente, in progress...)

- Paves the way to constrain late-time anisotropies with diverse data (eg. Number counts, weak lensing, ...)

Conclusions

- To every perturbed FLRW observable there is a superposed non-stochastic, long-wavelength, anisotropic modulation
- Relation between supercurvature modes and (non) gaussianity not fully explored.
- Such signatures can be described by a simple line-of-sight integral
- Future probes (e.g. Euclid) will allow us to probe late-time geometry of the universe

Thank you!