Quantum Gravity as a QFT, part 2: the analogy with the strong interactions

Roberto Percacci

SISSA, Trieste, Italy

Ubu, Brazil, April 2023

A new paradigm

Renormalizability not sufficient for the theory to be UV complete
not necessary for the theory to be predictive useful.

Accept that theory has finite range of validity.

Chiral perturbation theory is a paradigmatic example.

Analogy with strong interactions at low energy

Chiral field $U \in (SU(2)_L \times SU(2)_R)/SU(2)_V$

$$U = exp(\pi/f_\pi)$$

π=pion, f_π=pion decay constant

Metric in $\mathbb{R}^4$ = $g \in GL(4)/O(1,3)$

$$g = exp(h/m_P)$$

h=graviton, $m_P = \sqrt{8\pi G}$=Planck mass

Chiral vs. gravitational dynamics

Chiral action

$$S = \int dx \left[\frac{f_\pi^2}{4} tr(U^{-1}\partial U)^2 + \ell_1 tr((U^{-1}\partial U)^2)^2 + \ell_2 tr((U^{-1}\partial U)^2)^2 + O(\partial^6) \right]$$

Gravitational action

$$S = \int dx \sqrt{g} \left[2m_P^2 \Lambda + m_P^2 R + \ell_1 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \ell_2 R_{\mu\nu} R^{\mu\nu} + \ell_3 R^2 + O(\partial^6) \right]$$

$$R \sim \Gamma\Gamma \sim (g^{-1}\partial g)^2$$

Differences disappear for unimodular gravity.

General idea

Even though chiral theory is non-renormalizable, quantum (loop) corrections to pion cross sections can be calculated unambiguously.

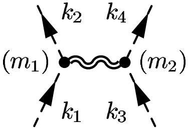
The same is true for gravity

Use the ratio $E/4\pi f_\pi$, resp. $E/4\pi m_P$, as expansion parameter.

General EFT recipe

- fix the level of precision that is required in the calculation
- fix the ratio E/M. This determines the order of the expansion that will be required
- at the given order in the expansion, determine all the terms in the action that can contribute to the given process. By power counting there can only be finitely many.
- if the fundamental theory is known, their coefficients can in principle be calculated
- otherwise, measure them with as many experiments.
- use the EFT Lagrangian to compute the amplitudes to the desired precision

Newtonian potential



$$\sim \frac{G m_1 m_2}{\mathrm{q}^2}$$

$$V(r) = \int \frac{d^3 q}{(2\pi)^3} \frac{G m_1 m_2}{\mathrm{q}^2} e^{i \mathrm{qr}} = -\frac{G m_1 m_2}{r}$$

Leading quantum correction

For dimensional reasons the leading quantum correction is

$$V(r) = -\frac{Gm_1 m_2}{r}\left[1 + \beta\frac{G\hbar}{r^2 c^3} + \dots\right]$$

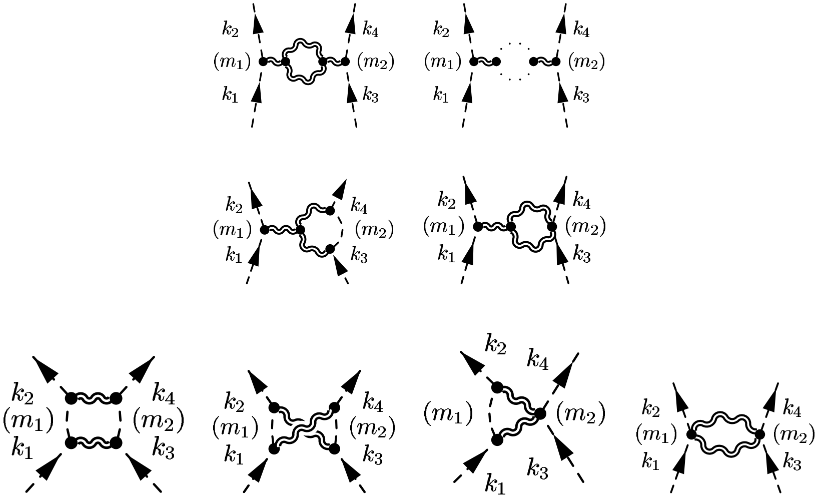
and

$$\int \frac{d^3q}{(2\pi)^3} \log\left(\frac{\mathrm{q}^2}{\mu^2}\right) e^{i\mathrm{qr}} = -\frac{1}{2\pi^2 r^3}$$

Clearly distinct from contributions of local counterterms that give analytic corrections to the amplitude. E.g.

$$\int \frac{d^3q}{(2\pi)^3} \ell e^{i\mathrm{qr}} = \ell\delta(r)$$

One loop graphs



(from Bjerrum Bohr, Donoghue, Holstein, 2002)

Evaluation

- J.F. Donoghue, P.R.L. 72, 2996 (1994); P.R.D50, 3874 (1994)
- H.W. Hamber, S. Liu, Phys. Lett. B357, 51 (1995)
- A. Akhundov, S. Bellucci, A. Shiekh, Phys. Lett. B395, 16 (1997)
- N.E.J. Bjerrum-Bohr (2002) Phys. Rev. D66, 084023
- I.B. Khriplovich, G.G. Kirilin (2002) J. Exp. Theor. Phys. 95, 981-986 (Zh. Eksp. Teor. Fiz. 95, 1139-1145 (2002))
- N.E.J. Bjerrum-Bohr, J.F. Donoghue, B.R. Holstein Phys. Rev. D68, 084005; Erratum-ibid.D71, 069904 (2005)
- N.E.J. Bjerrum-Bohr, J.F. Donoghue and B.R. Holstein Phys. Rev. D 67, 084033 (2003) [Erratum-ibid. D 71 (2005) 069903
- I.B. Khriplovich, G.G. Kirilin (2004) J. Exp. Theor. Phys. 98, 1063-1072

A prediction of quantum gravity

$$V(r) = -\frac{Gm_1 m_2}{r}\left[1 + \frac{41}{10\pi}\frac{G\hbar}{r^2 c^3} + \ldots\right]$$

Comes from non-local terms in the effective action, e.g.

$$\int dx \sqrt{g}\,[RF_1(\Box)R + R_{\mu\nu}F_2(\Box)R^{\mu\nu} + \ldots]$$

Local terms cannot be predicted

Lessons

- this is a quantum theory of gravity
- consistent and predictive below m_P
- it agrees with all experimental data
- open issues in the UV, IR, strong field...
- not a quantum theory of spacetime
- like pions, the metric may not be a fundamental field

Extension to the UV

In EFT, local terms in EA cannot be calculated, only nonlocal ones.

Can make theory still more predictive by trying to extend it to the UV.

"UV complete" theories

This is because UV complete theories are of measure zero: UV complete theories correspond to RG trajectories that end at a fixed point, and the UV basin of attraction of a fixed point is generally expected to be finite dimensional.
Number of free parameters of a UV complete theory is equal to the dimension of this basin of attraction.

Fixed points can be free or interacting.
For a UV complete QFT of gravity, 2 possible scenarios.

Scenario I: gravity like strong interactions

Strong interactions are weak in the UV limit (asymptotic freedom)

They are strong around the GeV scale.
Can only be studied by heavy numerical methods.

Below the GeV scale, the lightest degrees of freedom are mesons.
Their interactions are weak and become free in the IR limit (χPT)

Stelle gravity

In the UV limit, HDG is asymptotically free. (Fradkin, Tseytlin, Avramidi and Barvinsky 1985)

Issues with ghosts/tachyons may perhaps be overcome (Holdom, Donoghue, Mannheim, Salvio, Strumia, Anselmi...)

Below Planck scale we have, GR which becomes free in the IR limit.

Near the Planck scale there is a strongly coupled regime.

Scenario II: Asymptotic safety

Start from the EFT, picture, with all possible Lagrangian terms allowed by the symmetries.

“Theory space” is parametrized by the coefficients of these monomials, made dimensionless by powers of the RG scale, e.g. $\tilde{G} = G\mu^2$.

AS happens if the EFT automatically mends itself:
instead of diverging, all these dimensionless parameters reach finite values, i.e. a fixed point, in the UV limit.

How is AS studied?

Define $\Gamma_k[\phi]$. There is a simple RG equation governing the k-dependence:

$$k\frac{d\Gamma_k}{dk} = \frac{1}{2}\mathrm{Tr}\left(\frac{\delta^2\Gamma_k}{\delta\phi\delta\phi} + R_k\right) k\frac{d\tilde{R}_{k\,k}}{dk}$$

Make an ansatz

$$\Gamma_k = \sum_i g_i(k)\mathcal{O}_i[\phi]$$

Inserting in the FRG we can read off

$$k\frac{dg_i}{dk} = \beta_i(g)$$

where β_i is the coefficient of $\mathcal{O}_i$ in the trace.

Gravity actions (Lorentzian signature)

$$
\begin{aligned}
S_{HDG} &= -\int d^4x\sqrt{-g}\Big[\mathcal{V} - Z_N R + \alpha R^2 + \beta R_{\mu\nu}^2 + \gamma R_{\mu\nu\rho\lambda}^2\Big], \\
&= -\int d^4x\sqrt{-g}\Big[\mathcal{V} - Z_N R + \frac{1}{2\lambda}C^2 + \frac{1}{\xi}R^2 - \frac{1}{\rho}E\Big],
\end{aligned}
$$

$C^2 = R_{\mu\nu\alpha\beta}^2 - 2R_{\mu\nu}^2 + \frac{1}{3}R^2$
$E = R_{\mu\nu\alpha\beta}^2 - 4R_{\mu\nu}^2 + R^2$

$$
Z_N = \frac{1}{16\pi G}\,,\ \mathcal{V} = 2\Lambda Z_N\,,
$$

$$
\frac{1}{\xi} = \frac{3\alpha+\beta+\gamma}{3}\,,\ \frac{1}{2\lambda} = \frac{\beta+4\gamma}{2}\,,\ -\frac{1}{\rho} = -\frac{\beta+2\gamma}{2}\,,\ \omega \equiv -\frac{3\lambda}{\xi},\quad \theta \equiv \frac{\lambda}{\rho}.
$$

Note that $S_E = -S_L$

4DG and AS

S_{4DG} used as truncation Ansatz for the coarse-grained EA.
Beta functions extracted from FRGE

One loop FRGE reproduces perturbative beta functions for the marginal couplings, plus nontrivial beta functions for Λ and G

A. Codello, R. P., Phys.Rev.Lett. **97** 22 (2006). M. Niedermaier, Nucl. Phys. B833, 226-270 (2010). K. Groh, S. Rechenberger, F. Saueressig, O. Zanusso, PoS EPS **-HEP2011** (2011) 124 [arXiv:1111.1743 [hep-th]]. N. Ohta, R.P. Class. Quant. Grav. **31** 015024 (2014); arXiv:1308.3398

FRG AT 1 LOOP

$$\beta_\lambda = -\frac{1}{(4\pi)^2}\frac{133}{10}\lambda^2$$

$$\beta_\omega = -\frac{1}{(4\pi)^2}\frac{25+1098\omega+200\omega^2}{60}\lambda$$

$$\beta_\theta = \frac{1}{(4\pi)^2}\frac{7(56-171\theta)}{90}\lambda$$

$$\beta_{\tilde\Lambda} = -2\tilde\Lambda + \frac{1}{(4\pi)^2}\left[\frac{1+20\omega^2}{256\pi\tilde G\omega^2}\lambda^2 + \frac{1+86\omega+40\omega^2}{12\omega}\lambda\tilde\Lambda\right] - \frac{1+10\omega^2}{64\pi^2\omega}\lambda + \frac{2\tilde G}{\pi} - q(\omega)\tilde G\tilde\Lambda$$

$$\beta_{\tilde G} = 2\tilde G - \frac{1}{(4\pi)^2}\frac{3+26\omega-40\omega^2}{12\omega}\lambda\tilde G - q(\omega)\tilde G^2$$

where $q(\omega) = (83+70\omega+8\omega^2)/18\pi$

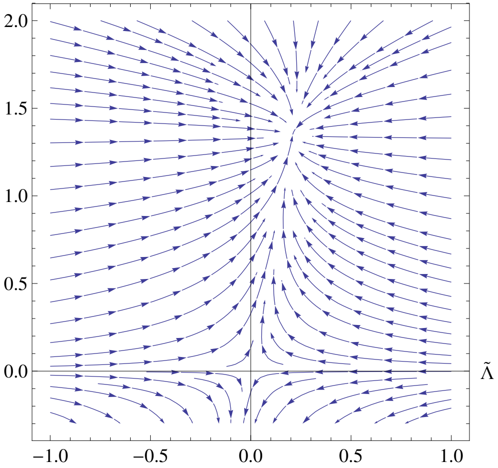
Flow in $\tilde{\Lambda}$–$\tilde{G}$ plane I

$$\beta_{\tilde{\Lambda}} = -2\tilde{\Lambda} + \frac{2\tilde{G}}{\pi} - q_* \tilde{G}\tilde{\Lambda}$$
$$\beta_{\tilde{G}} = 2\tilde{G} - q_* \tilde{G}^2$$

where $q_* = q(\omega_*) \approx 1.440$

$$\tilde{\Lambda}_* = \frac{1}{\pi q_*} \approx 0.221\,, \qquad \tilde{G}_* = \frac{2}{q_*} \approx 1.389\,.$$

Flow in $\tilde{\Lambda}$–$\tilde{G}$ plane II



FRG beyond 1 loop

Truncated FRGE on Einstein space gives nontrivial FP
D. Benedetti, P. F. Machado, F. Saueressig, Mod. Phys. Lett. A **24** (2009) 2233 [arXiv:0901.2984 [hep-th]]

Einstein background not enough because can read off beta function of only two combinations of λ, ξ and ρ.

- Truncate FRGE to 4DG action on arbitrary background
- use higher-derivative gauge fixing
- Expand to first order in Z_N.

K. Falls , N. Ohta, R.P., arXiv:2004.04126 [hep-th]

Also more recently without expanding in Z_N: S. Sen, C. Wetterich, M. Yamada, JHEP 03 (2022) 130, arXiv:2111.04696 [hep-th]

Beta functions (without cosmological term)

$$\beta_\lambda = -\frac{133}{160\pi^2}\lambda^2 + \tilde{Z}_N\lambda^3\frac{251\xi - 58\lambda}{120\pi^2\xi}$$

$$\beta_\xi = -\frac{5(72\lambda^2 - 36\lambda\xi + \xi^2)}{576\pi^2} + \tilde{Z}_N\frac{9720\lambda^3 - 1980\lambda^2\xi + 489\lambda\xi^2 - 14\xi^3}{6480\pi^2}$$

$$\beta_\rho = -\frac{49}{180\pi^2}\rho^2 + \tilde{Z}_N\lambda\rho^2\frac{233\xi - 58\lambda}{240\pi^2\xi}$$

$$\beta_{\tilde{Z}_N} = \left(-2 + \frac{(30\lambda - \xi)(4\lambda + \xi)}{192\pi^2\xi}\right)\tilde{Z}_N + \frac{-3168\lambda^2 + 654\lambda\xi + 35\xi^2}{1152\pi^2\xi(6\lambda + \xi)}$$
$$- \frac{72\lambda^2 - 84\lambda\xi + 65\xi^2}{192\pi^2(6\lambda + \xi)^2}\log\left(\frac{2}{3} - \frac{2\lambda}{\xi}\right) .$$

Fixed points

	λ_*	ξ_*	ω_*	$\tilde{Z}_{N*}$	$\tilde{\mathcal{V}}_*$	$\tilde{G}_*$	$\tilde{\Lambda}_*$
FP_1	0	0	-0.02286	0.00833	0.00649	2.388	0.3894
FP_2	24.91	-287	0.2603	0.01635	0.00457	1.217	0.1399
FP_3	28.24	175	-0.4825	0.01499	0.00693	1.327	0.2310
FP_4	0	-312	0	0.009222	0.00609	2.157	0.3303

all with $\rho_* = 0$
plus several others with $\lambda = 0$, $\xi \neq 0$
FP_2 has right signs for absence of tachyons

Scaling exponents

FP_1	4	2	0	0	0
FP_2	$2.352 + 1.677i$	$2.352 - 1.677i$	1.767	0	-3.200
FP_3	$2.327 + 1.521i$	$2.327 - 1.521i$	1.237	0	-5.277

Summary

- EH FP describes gravity in the IR
- Stelle FP (FP_1) possible UV completion
- There seem to be other (nontrivial) FP's supporting AS.

Important questions:

- can we trust the nontrivial FPs?
- can we flow from FP_1 to the nontrivial FP or vice-versa?
- can we flow from the UV fixed point to the EH FP?

Shift-invariant scalar

These issues not yet understood in gravity.

Consider a toy model

$$S[\phi] = \int d^4x \left[\frac{1}{2}Z_1(\partial\phi)^2 + \frac{1}{2}Z_2(\Box\phi)^2 + \frac{1}{4}g((\partial\phi)^2)^2\right]$$

D. Buccio and R.P., Renormalization group flows between Gaussian fixed points, JHEP 10 (2022) 113, e-Print: 2207.10596 [hep-th]

Has two Gaussian fixed points

$$g = Z_2 = 0\,, \quad S[\phi] = \frac{1}{2}\int d^4x(\partial\phi)^2\,, \quad [\phi] = 1\,, \qquad FP_1$$
$$g = Z_1 = 0\,, \quad S[\phi] = \frac{1}{2}\int d^4x(\Box\phi)^2\,, \quad [\phi] = 0\,, \qquad FP_2$$

Give rise to two separate perturbation theories.

Classical running of free theories

$$S[\phi] = \int d^4x \left[\frac{1}{2} Z_1 (\partial\phi)^2 + \frac{1}{2} Z_2 (\Box\phi)^2 \right]$$

Choosing $[\phi] = 1$, $Z_1 = 1$ and defining $\tilde{Z}_2 = Z_2 p^2$,
$\tilde{Z}_2$ goes from 0 in the IR to ∞ in the UV.

O.J. Rosten, arXiv:1106.2544

D. Benedetti, R. Gurau, S. Harribey and D. Lettera, JHEP 02 (2022) 147 [arXiv:2111.11792] .

Beta functions of dimful couplings

With field of dimension 1, the FRG gives

$$
\begin{aligned}
\partial_t Z_1 &= -\frac{(8-\eta_1)Z_1 + 16k^2 Z_2}{128\pi^2(Z_1 + k^2 Z_2)^2}\, g k^4 \,, \\
\partial_t Z_2 &= 0 \,, \\
\partial_t g &= \frac{(10-\eta_1)Z_1 + 20k^2 Z_2}{64\pi^2(Z_1 + k^2 Z_2)^3}\, g^2 k^4 \,.
\end{aligned}
$$

With dimensionless field the beta functions are the same, except for factors of k.

Parametrization 1

$$\tilde{g} = \frac{g k^4}{Z_1^2} \, , \qquad \tilde{Z}_2 = \frac{Z_2 k^2}{Z_1} \, .$$

$$\eta_1 = \frac{8\tilde{g}\left(1 + 2\tilde{Z}_2\right)}{\tilde{g} + 128\pi^2(1+\tilde{Z}_2)^2}$$

$$\beta_{\tilde{g}} \equiv \partial_t \tilde{g} = (4 + 2\eta_1)\tilde{g} + \frac{10 + 20\tilde{Z}_2 - \eta_1}{64\pi^2(1+\tilde{Z}_2)^3}\tilde{g}^2$$

$$\partial_t \tilde{Z}_2 = (2 + \eta_1)\tilde{Z}_2 \, .$$

FPs in chart 1

FP	$\tilde{Z}_{2*}$	$\tilde{g}_*$	η_{1*}	θ_1	θ_2
GFP_1	0	0	0	-4	-2
$NGFP_1$	0	-127.6	-0.90	4.40	-1.10
$NGFP_2$	0	-12505	8.90	43.60	-10.90
$NGFP_3$	-0.6	-1011	-2	-13.84	10.84

Also seen in

G. P. de Brito, A. Eichhorn and R. R. L. d. Santos, JHEP 11 (2021), 110 [arXiv:2107.03839 [gr-qc]].
C. Laporte, A. D. Pereira, F. Saueressig and J. Wang, [arXiv:2110.09566 [hep-th]].

C. Laporte, N. Locht, A. D. Pereira and F. Saueressig, [arXiv:2207.06749 [hep-th]].

Parametrization 2

For dimensionless field

$$S[\varphi] = \int d^4x \left[\frac{1}{2}\zeta_1(\partial\varphi)^2 + \frac{1}{2}\zeta_2(\Box\varphi)^2 + \frac{1}{4}\gamma((\partial\varphi)^2)^2\right]$$

comes from redefining the couplings

$$\phi = k\varphi \;, \quad Z_1 = k^{-2}\zeta_1 \;, \quad Z_2 = k^{-2}\zeta_2 \;, \quad g = k^{-4}\gamma \;.$$

The power counting is that of a renormalizable theory, with ζ_1 having the meaning of a mass. The natural variables for the parametrization of theory space are

$$\hat{\zeta}_1 = \frac{\zeta_1}{\zeta_2 k^2} \;, \qquad \hat{\gamma} = \frac{g}{\zeta_2^2} \;.$$

Beta functions in chart 2

$$\eta_2 = 0 \ ,$$

while the beta functions are

$$\beta_{\hat{\zeta}_1} = -2\hat{\zeta}_1 - \frac{8\hat{\gamma}(2+\hat{\zeta}_1)}{\hat{\gamma} + 128\pi^2(1+\hat{\zeta}_1)^2}$$

and

$$\beta_{\hat{\gamma}} = \frac{(2+\hat{\zeta}_1)(\hat{\gamma} + 640\pi^2(1+\hat{\zeta}_1)^2)}{32\pi^2(1+\hat{\zeta}_1)^3\left(\hat{\gamma} + 128\pi^2(1+\hat{\zeta}_1)^2\right)}\hat{\gamma}^2 \ .$$

FPs in chart 2

FP	$\hat{\zeta}_{1*}$	$\hat{\gamma}_*$	η_{2*}	θ_1	θ_2
GFP_2	0	0	0	2	0
$NGFP_3$	-1.67	-2807	0	-13.84	10.84

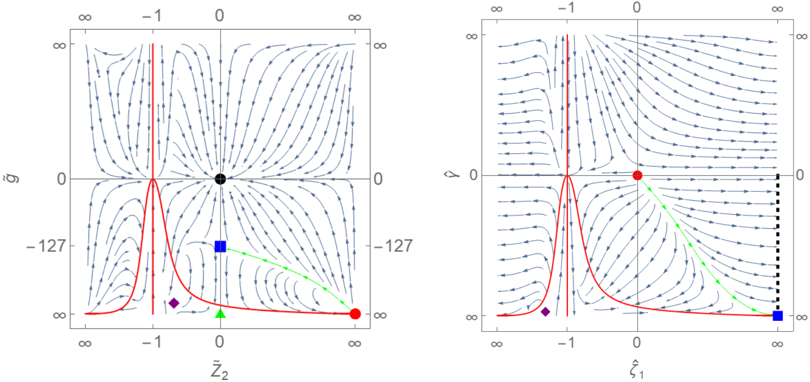


Figure: Left: flow in chart 1. Right: flow in chart 2.

Coordinate transformation

$$\hat{\zeta}_1 = \frac{1}{\tilde{Z}_2}\,, \qquad \hat{\gamma} = \frac{\tilde{g}}{\tilde{Z}_2^2} \qquad \textit{or conversely} \quad \tilde{g} = \frac{\hat{\gamma}}{\hat{\zeta}_1^2}\,.$$

Behavior of the dimensionful couplings

For $k \to \infty$ one finds

$$Z_1 = \frac{4Z_2 k^2}{11 \log k}$$

where Z_2 is fixed and arbitrary, and

$$g = -\frac{64\pi^2 Z_2^2}{11 \log k}$$

Thus for $k \to \infty$, g goes to zero.

For large momentum it is natural to work with dimless field. Then one find that

$$\zeta_1 = \frac{4\zeta_2 k^2}{11 \log k} \ ,$$

and

$$\gamma = -\frac{64\pi^2 \zeta_2^2}{11 \log k}$$

Properties

Recall that in chart 1, $\partial_t \tilde{Z}_2 = (2 + \eta_1)\tilde{Z}_2$.
For $k \to \infty$ we have $\eta_1 \to -2$, which can be reabsorbed in the definition of the field. Then the dimension of the field is

$$1 + \frac{1}{2}\eta_1 \to 0$$

The FRG correcty interpolates the dimension of the fields in the flow between the two FP's.

There are trajectories that remain perturbative for all k.
The separatrix lies at the other extreme.

Mass of the ghost increases when trajectory becomes less perturbative.
Goes to infinity for the AS trajectory.

Generalizations

$$\phi\,\Box^k\phi \qquad\qquad GFP_k$$

with $0 < k < \infty$, have been studied perturbatively as CFT's M. Safari, A. Stergiou, G. P. Vacca and O. Zanusso, JHEP 02 (2022), 034 [arXiv:2112.01084 [hep-th]].

- Every GFP defines a perturbative expansion and a chart.
- There is only one GFP in each chart.
- The other GFP's are in the closure of the domain of the chart.
- There seem to be flows between all these, lowering k.

The simplest case ($1 \to 0$)

Flow from GFP_1 to GFP_0:

$$S[\phi] = \int d^4x \left[\frac{1}{2} Z_1 (\partial\phi)^2 + \frac{1}{2} Z_0 \phi^2 + \frac{1}{4}\lambda\phi^4\right]$$

$$\beta_\lambda \sim \lambda^2$$

AF for $\lambda < 0$.

Symanzik's asymptotically free theory *ante litteram*

Furthermore...

The cutoff k is an artificial construct, in general does not have a direct physical interpreation.

Specific running couplings can acquire physical meaning when a system is characterized by a single scale and that scale behaves like a mass in the two point function.
Then we can identify k with that scale.

Example: Coleman-Weinberg potential

In a massless scalar theory with quartic interactions, expanding around $\bar{\phi}$, the propagator goes like

$$\frac{1}{-q^2 + \lambda\bar{\phi}^2}$$

In this case it is justified to identify the cutoff $k = \bar{\phi}$ and the effective potential is

$$V = \frac{1}{4!}\lambda(\bar{\phi})\bar{\phi}^4$$

Example: running of quartic coupling

Define $\lambda(k) = \Gamma^{(4)}|_{s=t=u=-k^2}$

$$
\begin{aligned}
i\Gamma^{(4)} &= -i\lambda + \frac{\lambda^2}{2}\int_{|q|=\Lambda}\frac{d^4q}{(2\pi)^4}\left\{\frac{1}{((q+p_1+p_2)^2-m^2)\,(q^2-m^2)}\right. \\
&\qquad \left. + (p_2 \to -p_3) + (p_1 \to -p_4)\right\} \\
&= -i\lambda + i\frac{\lambda^2}{32\pi^2}\int_0^1 dx\left(\log\frac{\Lambda^2}{m^2 - x(1-x)\,s} - 1\right) + (s \to t) + (s \to u)
\end{aligned}
$$

where $s = (p_1+p_2)^2$, $t = (p_1-p_3)^2$, $u = (p_1-p_4)^2$

Example

$$
\begin{aligned}
k\partial_k\lambda &= \frac{3\lambda^2}{16\pi^2}\int_0^1 dx\,\frac{x(1-x)\,k^2}{m^2+x(1-x)\,k^2} \\
&= \frac{3\lambda^2}{16\pi^2}\left[1-\frac{4m^2\,\mathrm{arctanh}\left(\sqrt{\frac{k^2}{4m^2+k^2}}\right)}{\sqrt{k^2(4m^2+k^2)}}\right] .
\end{aligned}
$$

Exactly the same can be obtained from the FRG.
Note: for $k \gg m$ this reduces to the universal beta function

$$
k\partial_k\Gamma^{(4)} = \frac{3\lambda^2}{16\pi^2}\left[1+o\left(\frac{m^2}{k^2}\right)\right]
$$

while for $k \ll m$ the scalar decouples and the beta function goes to zero. The form of the threshold is not universal.

Example: thermal partition function

Thermal partition function of a scalar boson gas in d dimensions at temperature T= Euclidean partition function on $\mathbb{R}^d \times S^1$ with a periodic coordinate of period $1/T$

The 1-loop effective action is

$$\Gamma^{(1)} = \frac{V}{2} \sum_{n=-\infty}^{\infty} \int \frac{d^d q}{(2\pi)^d} \log \left[q^2 + (2\pi T n)^2 \right]$$

For each n T appears as an effective mass and therefore can be seen as an IR cutoff.

Taking the derivative with respect to T yields:

$$T\partial_T\Gamma^{(1)} = V\sum_{n=-\infty}^{\infty}\int\frac{d^dq}{(2\pi)^d}\frac{E_n^2}{E_n^2+q^2} = 2\pi^{d/2}\,\Gamma\left(1-\frac{d}{2}\right)\zeta(-d)VT^d$$

(In even dimension the Gamma function has a simple pole which is compensated by the simple zero of the Riemann Zeta function.)
Putting $d=3$ we get the usual result

$$T\partial_T\Gamma^{(1)} = -\frac{\pi^2VT^3}{30} \quad\Longrightarrow\quad \log Z = -\Gamma^{(1)} = \frac{\pi^2VT^3}{90}$$

The same result comes from the FRG by identifying $k=T$ *independent of details of the cutoff.* A. Baldazzi, R.P., V. Skrinjar, "Quantum fields without Wick rotation", Symmetry (2019) 11(3), 373

Example: curved space generalization

A scalar field coupled to a static metric on $\mathbb{R} \times \Sigma$ gives:

$$\Gamma^{(1)} = -\int d^3x\sqrt{g}\left(\frac{\pi^2 T^3}{90} + \frac{T}{144}R\right)$$

We have an induced Hilbert term.
Also in this case $T\partial_T\Gamma$ is the same function as $k\partial_k\Gamma$ with the identification $k = T$ (Again independent of details of the cutoff).

Amplitudes

In NLSM, gravity and similar models, the running of dimensionful couplings with k does not reflect the dependence of scattering amplitudes on external momenta.

Compute scattering amplitudes at one loop and compare with the solutions of the FRG flow.

Downgrade FRG to one loop. Not much difference.

Amplitude for $Z_2 = 0$

$$
\begin{aligned}
&-\frac{i\gamma}{2}\left(s^2+t^2+u^2\right)\\
&+\frac{i\left(41(s^4+t^4+u^4)+2(s^2t^2+t^2u^2+u^2s^2)\right)}{1920\pi^2M^8\epsilon}\\
&-\frac{i}{28800\pi^2M^8}\Bigg\{15\Bigg[s^2\left(41s^2+t^2+u^2\right)\log\left(\frac{s}{4\pi\mu^2}\right)\\
&\qquad\qquad\qquad\qquad+t^2\left(s^2+41t^2+u^2\right)\log\left(\frac{t}{4\pi\mu^2}\right)\\
&\qquad\qquad\qquad\qquad+u^2\left(s^2+t^2+41u^2\right)\log\left(\frac{u}{4\pi\mu^2}\right)\Bigg]\\
&-(1301-615\gamma)(s^4+t^4+u^4)-2(46-15\gamma)(s^2t^2+t^2u^2+u^2s^2)\Bigg\}\\
&\qquad+O\left(\epsilon\right)
\end{aligned}
$$

Amplitude for $Z_1 = 0$

$$
\begin{aligned}
&-\frac{i\gamma}{2}\left(s^2+t^2+u^2\right)\\
&+\frac{i5\gamma^2\left(s^2+t^2+u^2\right)}{64\pi^2\epsilon}\\
&+\frac{i\gamma^2}{192\pi^2}\Bigg[\left(14-15\gamma_E\right)\left(s^2+t^2+u^2\right)+\log\left(\frac{4\pi\mu^2}{-s}\right)\left(13s^2+t^2+u^2\right)\\
&+\log\left(\frac{4\pi\mu^2}{-t}\right)\left(s^2+13t^2+u^2\right)+\log\left(\frac{4\pi\mu^2}{-u}\right)\left(s^2+t^2+13u^2\right)\Bigg]+O\left(\epsilon\right)
\end{aligned}
$$

A.A. Tseytlin, Comments on 4-derivative scalar theory in 4 dimensions, e-Print: 2212.10599 [hep-th]

General amplitude

Contains

$$\frac{Z_1^3}{Z_2^5 s}, \quad \frac{Z_1^2}{Z_2^4}, \quad \frac{Z_1 s}{Z_2^3}, \quad \frac{\sqrt{4Z_1 - sZ_2}}{Z_2^{5/2}}, \quad \frac{s^2}{Z_2^2},$$

$$\frac{\sqrt{4Z_1 - sZ_2}}{Z_1 Z_2^{3/2}}, \quad \frac{s^3}{Z_1 Z_2}, \quad \frac{\sqrt{4Z_1 - sZ_2}}{Z_1^2 Z_2^{1/2}}, \quad \frac{s^4}{Z_1^2}.$$

Apparently the last four terms are divergent in the limit $Z_1 \to 0$, but their coefficients cancel in the limit and the earlier result is reproduced. This is the form of the amplitude at high energy.

Comparison

For $Z_2 = 0$ no corrections to the $s^2 + t^2 + u^2$ terms.
Agrees with the fact that g becomes constant in the IR.

At high energy the amplitude collapses to the case $Z_1 = 0$.
The $s^2 + t^2 + u^2$ terms get log corrections.
The beta function computed from the amplitude

$$\mu \frac{\partial \gamma}{\partial \mu} = \frac{5\gamma^2}{16\pi^2}$$

does agree with the FRG at one loop.

Summary

- gravity is the most perfect example of EFT
- UV completion of gravity as a QFT is possible
- AF and AS may not be that different
- physics non completely clear, even in the scalar model

Quantum Gravity as a QFT, part 3: the electroweak analogy

Roberto Percacci

SISSA, Trieste, Italy

Ubu, Brazil, April 2023

MAG - notation

	coefficients	cov. der.	curvature
Independent conn.	$A_\mu{}^a{}_b$	D_μ	$F_{\mu\nu}{}^a{}_b$
LC conn.	$\Gamma_\mu{}^a{}_b$	∇_μ	$R_{\mu\nu}{}^a{}_b$

What is the gauge group?

Not a physically meaningful question.
It is related to the type of frames we use.

Normally the gauge transformations are the diffeomorphisms.
This corresponds to using coordinate frames.

For some purposes it may be advantageous to use the smallest gauge group still compatible with locality: the volume–preserving diffeomorphisms (unimodular gravity).

Can enlarge the gauge group introducing local frame rotations.
Since in general the holonomy of the connection is $GL(4)$, it makes sense to present the theory in a general $GL(4)$ gauge.
This corresponds to working with arbitrary frames in the tangent bundle.

Gravity

In GR we have a frame field (local linear bases) $\theta^a{}_\mu$ and a metric g_{ab}. They are nonlinear objects

- metric $g_{ab} \in GL(4)/O(1,3)$, (signature $-,+,+,+$),
- frame field $\theta^a{}_\mu \in GL(4)$, (det$\theta \neq 0$),

They carry nonlinear realizations of $GL(4)$.
Think of them as Goldstone bosons.

$$g_{\mu\nu}(x) = \theta^a{}_\mu(x)\,\theta^b{}_\nu(x)\,g_{ab}(x)$$

Transformations under $GL(4)$

Linear changes of basis=$GL(4)$ gauge transformations

$$\begin{aligned} \theta &\mapsto \Lambda^{-1}\theta \\ g &\mapsto \Lambda^T g \Lambda \end{aligned}$$

Two “unitary gauges” for $GL(4)$:

- $\theta^a_\mu = \delta^a_\mu$: coordinate frames - breaks $GL(4)$ to $\{\mathbf{1}\}$
  metric formulation - breaks $GL(4)$ to $\{\mathbf{1}\}$
- $g_{ab} = \eta_{ab}$: orthonormal frames - breaks $GL(4)$ to $O(1,3)$
  vierbein formulation

not enough freedom to fix both.

Torsion, Nonmetricity, Curvature

Let A_{abc} be an independent connection

$$\begin{aligned} T_\mu{}^a{}_\nu &= \partial_\mu \theta^a{}_\nu - \partial_\nu \theta^a{}_\mu + A_\mu{}^a{}_b\, \theta^b{}_\nu - A_\nu{}^a{}_b\, \theta^b{}_\mu \\ Q_{\lambda ab} &= -\partial_\lambda g_{ab} + A_\lambda{}^c{}_a\, g_{cb} + A_\lambda{}^c{}_b\, g_{ac} \\ F_{\mu\nu}{}^a{}_b &= \partial_\mu A_\nu{}^a{}_b - \partial_\nu A_\mu{}^a{}_b + A_\mu{}^a{}_c A_\nu{}^c{}_b - A_\nu{}^a{}_c A_\mu{}^c{}_b \end{aligned}$$

T, Q are the covariant derivatives of the Goldstone bosons

Cartan view of MAG

MAG as a theory of frame, metric, connection.

$$S(\theta^a{}_\mu, g_{ab}, A_\mu{}^a{}_b)$$

Powers of T, Q, F and their D-covariant derivatives.

The Levi-Civita connection

Unique connection with $T = Q = 0$

$$\Gamma_{abc} = \frac{1}{2}\left(E_{cab} + E_{abc} - E_{bac}\right) + \frac{1}{2}\left(C_{abc} + C_{bac} - C_{cab}\right)$$

where $E_{cab} = \theta^{-1}{}_c{}^{\lambda}\,\partial_\lambda g_{ab}$
$C_{abc} = g_{ad}\,\theta^d{}_\lambda\left(\theta^{-1}{}_b{}^{\mu}\,\partial_\mu\theta^{-1}{}_c{}^{\lambda} - \theta^{-1}{}_c{}^{\mu}\,\partial_\mu\theta^{-1}{}_b{}^{\lambda}\right)$

Its curvature is the Riemann tensor

$$R_{\mu\nu}{}^a{}_b = \partial_\mu\Gamma_\nu{}^a{}_b - \partial_\nu\Gamma_\mu{}^a{}_b + \Gamma_\mu{}^a{}_c\Gamma_\nu{}^c{}_b - \Gamma_\nu{}^a{}_c\Gamma_\mu{}^c{}_b$$

Distortion, disformation, contortion

Any connection A can be split uniquely in

$$A = \Gamma + \Phi$$

("post–Riemannian decomposition") where

$$\phi_{\alpha\beta\gamma} = L_{\alpha\beta\gamma} + K_{\alpha\beta\gamma} \ ,$$

$$\begin{aligned} L_{\alpha\beta\gamma} &= \frac{1}{2}\left(Q_{\alpha\beta\gamma} + Q_{\gamma\beta\alpha} - Q_{\beta\alpha\gamma}\right) \ , \\ K_{\alpha\beta\gamma} &= \frac{1}{2}\left(T_{\alpha\beta\gamma} + T_{\beta\alpha\gamma} - T_{\alpha\gamma\beta}\right) \ . \end{aligned}$$

conversely

$$T_{\alpha\beta\gamma} = \phi_{\alpha\beta\gamma} - \phi_{\gamma\beta\alpha} \ , \qquad Q_{\alpha\beta\gamma} = \phi_{\alpha\beta\gamma} + \phi_{\alpha\gamma\beta}$$

Einstein view of MAG

Consequently

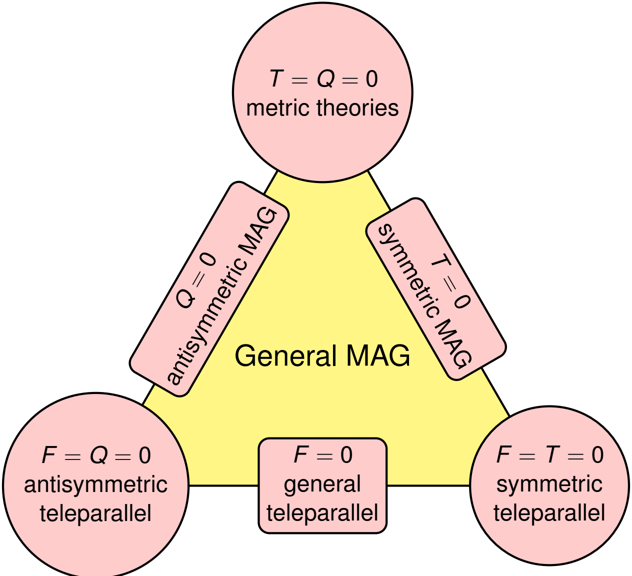
$$S(\theta^a{}_\mu, g_{ab}, A_\mu{}^a{}_b) = S(\theta^a{}_\mu, g_{ab}, \Gamma_\mu{}^a{}_b + \phi_\mu{}^a{}_b) = S'(\theta^a{}_\mu, g_{ab}, \Phi_\mu{}^a{}_b)$$

Powers of T, Q, R and their ∇-covariant derivatives.
Now T, Q are independent fields.

Any MAG is equivalent to a metric theory of gravity coupled to some "matter" fields.

Calculations easier in Einstein-variables.

Main kinematical sublasses of MAGs



Dynamical equivalences: GR

Up to total derivatives, R is equivalent to

$$\mathbb{T} = \frac{1}{4} T_{abc} T^{abc} + \frac{1}{2} T_{abc} T^{acb} - T^{b}{}_{ba} T_{c}{}^{ca}$$

for the antisymmetric MAG,

$$\mathbb{Q} = \frac{1}{4} Q_{abc} Q^{abc} - \frac{1}{2} Q_{abc} Q^{bac} - \frac{1}{4} Q^{b}_{ab} Q^{ac}{}_{c} + \frac{1}{2} Q^{b}_{ab} Q_{c}{}^{ca}$$

for the symmetric MAG and

$$\mathbb{G} = \mathbb{T} + \mathbb{Q} - Q_{abc} T^{abc} - Q_{ab}{}^{b} T_{c}{}^{ca} + Q^{b}{}_{ba} T_{c}{}^{ca} \ .$$

for the general MAG.

Every metric theory has teleparallel equivalents.

Some actions for MAG

Poincaré Gauge Theory (PGT) view $A_{\mu[ab]}$ and $\theta^a{}_\mu$ as gauge fields
and $F_{\mu\nu[ab]}$ and $T_\mu{}^a{}_\nu$ as field strength.
this motivates

$$\mathcal{L} = a^{TT}TT + c^{FF}FF$$

In teleparallel case $\mathcal{L} = f(\mathbb{T})$ or $f(\mathbb{Q})$.

Let us be systematic and order Lagrangians according to their dimension. (T and Q have dimension one, F and R have dimension 2).

General action of MAG

In Cartan-variables and including terms of dimension 2 and 4:

$$
\begin{aligned}
\mathcal{L} = {} & a^{F} F + a^{TT} TT + a^{TQ} TQ + a^{QQ} QQ \\
& + c^{FF} FF \\
& \quad + c^{FT} FDT + c^{FQ} FDQ + c^{TT} (DT)^2 + c^{TQ} DTDQ + c^{QQ} (DQ)^2 \\
& + c^{FTT} FTT + c^{FTQ} FTQ + c^{FQQ} FQQ \\
& + c^{TTT} TTDT + \ldots + c^{QQQ} QQDQ \\
& + c^{TTTT} TTTT + \ldots + c^{QQQQ} QQQQ \, ,
\end{aligned}
$$

Similar classification in Einstein variables. Large number of invariants.

Counting independent invariants of dimension 4

Antisymmetric (Christensen 1979)

R^2	$(\nabla T)^2$	$R\nabla T$	RT^2	$T^2\nabla T$	T^4	Total
3	9	2	14	31	33	92

Symmetric

R^2	$(\nabla Q)^2$	$R\nabla Q$	RQ^2	$Q^2\nabla Q$	Q^4	Total
3	16	4	22	59	69	173

General

R^2	$(\nabla\phi)^2$	$R\nabla\phi$	$R\phi^2$	$\phi^2\nabla\phi$	ϕ^4	Total
3	38	6	56	315	504	922

Further kinematical restrictions: Weyl theory as MAG

In a symmetric MAG with

$$Q_{\mu\alpha\beta} = b_\mu g_{\alpha\beta}$$

the quadratic MAG action collapses to

$$S(g) = \int d^4x\sqrt{g}\left[\frac{g_1 + 12g_4}{32\pi G g_4} b_\mu b^\mu + \frac{g_3}{4} F_{\mu\nu}F^{\mu\nu} + \frac{1}{16\pi G}(2\Lambda - R)\right]$$

which, introducing a Stückelberg field χ can be rewritten

$$S = \int d^4x\sqrt{g}\left[\frac{g_1 + 12g_4}{2} D_\mu\chi D^\mu\chi + g_2\chi^4 + \frac{g_3}{4} F_{\mu\nu}F^{\mu\nu} - g_4\chi^2\mathcal{R}^{(s)}\right].$$

Weyl gauge field+dilaton

or equivalently

$$S = \int d^4x\sqrt{g}\left[\frac{g_1}{2}D_\mu\chi D^\mu\chi + g_2\chi^4 + \frac{g_3}{4}F_{\mu\nu}F^{\mu\nu} - g_4\chi^2\mathcal{R}^{(b)}\right] .$$

$$D_\mu\chi = (\partial_\mu + b_\mu)\chi.$$

D.M. Ghilencea, JHEP 03 (2019) 049, arXiv: 1812.08613 [hep-th]
D. Sauro, O. Zanusso, Class.Quant.Grav. 39 (2022) 18, 185001, arXiv: 2203.08692 [hep-th]

Analogy with electroweak interactions

Central question in the physics is status of symmetries.

Basic properties of many systems determined by Higgs mechanism.
This is the case for superconductivity and for the electroweak interactions.

Superconductivity

Macroscopic properties of superconductor follow from simple assumption that e.m. $U(1)$ is in Higgs phase.

There is a complex scalar field $\phi = \rho e^{i\varphi}$ with nonzero VEV. IR physics only depends on its phase φ (Goldstone boson). It transforms under $U(1)$ as

$$\varphi(x) \to \varphi(x) + \alpha(x)$$

and has a covariant derivative $D_\mu \varphi = \partial_\mu \varphi - A_\mu$.

The main feature of the superconducting state is Meissner effect

$$D_\mu \varphi = 0 \quad \Rightarrow \quad B = 0\ .$$

EWSB

Goldstone bosons $\sigma \in (SU(2)_L \times U(1)_Y)/U(1)_Q$ coupled to $SU(2)_L \times U(1)_Y$-YM fields A_μ.

$$\mathcal{L} = -\frac{f^2}{2} D\sigma^2 \qquad \text{where} \qquad D\sigma = \partial\sigma + A^a K_a(\sigma)$$

In unitary gauge $\sigma = \sigma_0$

$D\sigma_0 = \sum_{a \in (SU(2)_L \times U(1)_Y)/U(1)_Q} A^a K_a(\sigma_0)$

$$\mathcal{L} = -\frac{1}{2} f^2 \sum_{a \in (SU(2)_L \times U(1)_Y)/U(1)_Q} A^a A^a$$

Low Energy EFT

- for $p \ll m_\rho$, $\rho = \rho_0$
- for $p \ll m_A$, $D\sigma = 0$
  or $F^a_{\mu\nu}\big|_{a \in (SU(2)_L \times U(1)_Y)/U(1)_Q} = 0$

analog of Meissner effect

Gravitational Higgs mechanism in Cartan view

In Cartan-variables

$$S_G(\theta, g, A) \;=\; m^2 \int d^4x \sqrt{|g|}\left[T_{...}T^{...} + Q_{...}Q^{...} + T_{...}Q^{...}\right]$$

expanding around flat background: $A = 0$, $\theta = \mathbf{1}$, $g = \eta$

$$\begin{aligned} T_\mu{}^a{}_\nu &= A_\mu{}^a{}_\nu - A_\nu{}^a{}_\mu \\ Q_{\mu ab} &= A_{\mu ab} + A_{\mu ba} \end{aligned}$$

kinetic term of Goldstone bosons becomes

$$S_G = m^2 \int d^4x \,\sqrt{|g|}\, A_{...}A^{...}$$

In general a non-degenerate quadratic form

The Palatini term

$$S_P(A, g, \theta) = \frac{m_P^2}{2} \int d^4x \, \sqrt{|g|} \, F(A)_{ab}{}^{ab}$$

also gives

$$\frac{m_P^2}{2} \int d^4x \, \sqrt{|g|} \, \left(A_a{}^{ac} A_{bc}{}^{b} - A_{bac} A^{acb} \right)$$

(a degenerate quadratic form)

Gravitational Higgs mechanism in Einstein view

Using the post-Riemannian expansion
$A = \Gamma + \Phi$

$$\begin{aligned} T_{\mu}{}^{a}{}_{\nu} &= \Phi_{\mu}{}^{a}{}_{\nu} - \Phi_{\nu}{}^{a}{}_{\mu} \\ Q_{\mu ab} &= \Phi_{\mu ab} + \Phi_{\mu ba} \end{aligned}$$

therefore (in any gauge and without assumptions on the background)

$$S_G = \int d^4x \sqrt{|g|}\, \Phi_{\dots} \Phi^{\cdots}$$

Macroscopic gravity

If mass matrix is nondegenerate, assuming all masses are $O(m)$, at $p \ll m$

$$\Phi = 0 \iff T = Q = 0 \iff A = \Gamma(\theta, g)$$

independent of the detailed form of the action.

To summarize

MAG below the Planck scale looks like a low energy EFT where Higgs phenomenon occurs at Planck scale.
Metric is the order parameter.

The conditions $T = 0$ and $Q = 0$ are the same as $D\varphi = 0$ (Meissner effect).

Core questions for quantum theory of spacetime:

- what is the dynamical origin of the Planck scale?
- why is there a nondegenerate metric?
- is the metric fundamental or “emergent”?

Scenarios

Natural to assume that all masses are $m \sim m_P$. No new d.o.f.s in the domain of validity of the EFT. Then MAG as an EFT is essentially indistinguishable from GR as an EFT.

Still has somewhat increased explanatory power: explains dynamically the conditions $T = Q = 0$.

If there are eigenvalues $m \ll m_P$ there may be new propagating d.o.f.s in the domain of validity of the EFT. What kind of states?

Spin$^{\text{parity}}$ states

$\delta g_{\alpha\beta}$ or $\delta\theta_{\alpha\beta}$

	s	a
TT	$2^+_4, 0^+_5$	1^+_4
TL	1^-_7	1^-_8
LL	0^+_6	-

$\delta A_{\alpha\beta\gamma}$ or $\Phi_{\alpha\beta\gamma}$

	ts	hs	ha	ta
TTT	$3^-, 1^-_1$	$2^-_1, 1^-_2$	$2^-_2, 1^-_3$	0^-
$TTL + TLT + LTT$	$2^+_1, 0^+_1$	-	-	1^+_3
$\frac{3}{2}LTT$	-	$2^+_2, 0^+_2$	$1^+_2,$	-
$TTL + TLT - \frac{1}{2}LTT$	-	1^+_1	$2^+_3, 0^+_3$	-
$TLL + LTL + LLT$	1^-_4	1^-_5	1^-_6	-
LLL	0^+_4	-	-	-

Spin projector formalism

The quadratic action for the variables A, h

$$S^{(2)} = \frac{1}{2}\int \frac{d^4q}{(2\pi)^4}\Big(A^{\lambda\mu\nu}\,\mathcal{O}^{(AA)}_{(C)}{}_{\lambda\mu\nu}{}^{\tau\rho\sigma}\,A_{\tau\rho\sigma} + 2A^{\lambda\mu\nu}\,\mathcal{O}^{(Ah)}_{(C)}{}_{\lambda\mu\nu}{}^{\rho\sigma}\,h_{\rho\sigma} + h^{\mu\nu}\,\mathcal{O}^{(hh)}_{(C)}{}_{\mu\nu}{}^{\rho\sigma}\,h_{\rho\sigma}\Big)$$

can be rewritten

$$\frac{1}{2}\int \frac{d^4q}{(2\pi)^4}\sum_{JPij} a_{ij}(J^{\mathcal{P}})\,\Psi P_{ij}(J^{\mathcal{P}})\Psi\ ,$$

where $\Psi = (A, h)$, e.g in the A-A sector

$$\frac{1}{2}\int \frac{d^4q}{(2\pi)^4}\sum_{JPij} a_{ij}(J^{\mathcal{P}})\,A^{\lambda\mu\nu} P_{ij}(J^{\mathcal{P}})_{\lambda\mu\nu}{}^{\tau\rho\sigma} A_{\tau\rho\sigma}\ ,$$

Ghost- and tachyon-free MAGs

Antisymmetric MAG:

E. Sezgin and P. van Nieuwenhuizen, Phys. Rev. D21, (1980) 3269

Y.C. Lin, M.P. Hobson and A.N. Lasenby, arXiv:1812.02675

Symmetric MAG:

R.P. and E. Sezgin, Phys. Rev. D101, (2020) 084040

C. Marzo, Phys.Rev.D 105 (2022) 6, 065017

DIY MAG:

A. Baldazzi, O. Melichev and R.P. Ann.Phys. 438 (2022) 168757

- measure zero in theory space
- consistency only checked at level of free propagators

A six-parameter family of symmetric MAGs

Projective invariance, no spin 3^-, only one (massless) spin 2^+

$$
\begin{aligned}
S(g, A) = -\frac{1}{2}\int d^n x\,\sqrt{|g|}\,\Bigg[& -\frac{1}{4}(10c_7 + 3c_{11})F^{\mu\nu\rho\sigma}\Big(F_{\mu\nu\rho\sigma} - 2F_{\mu\rho\nu\sigma}\Big) \\
& +2F^{(13)}_{[\mu\nu]}\Big(c_7 F^{(13)\mu\nu} + c_{11} F^{(14)\mu\nu}\Big) - \frac{2}{3}(c_7 + 4c_{11})\, F^{(14)}_{[\mu\nu]} F^{(14)\mu\nu} \\
& -a_0 F - \frac{1}{16}(15a_0 + 120a_6 + 28a_7 - 22a_8)\, Q^{\rho\mu\nu} Q_{\rho\mu\nu} \\
& +\frac{1}{8}(9a_0 + 40a_6 - 28a_7 - 6a_8)\, Q^{\rho\mu\nu} Q_{\nu\mu\rho} \\
& +a_6\, Q_\mu Q^\mu + a_7\, \widetilde{Q}_\mu \widetilde{Q}^\mu + \frac{1}{72}(a_0 + 8a_6 - 4a_7 + 74a_8)\, Q_\mu \widetilde{Q}^\mu \Bigg]
\end{aligned}
$$

New ghost and tachyon-free theories

No ghosts and tachyons provided

$$a_0 > 0\ , \quad 4\,a_0 + 20\,a_6 - 7\,a_7 + 2\,a_8 < 0\ ,$$

$$(a_0 + 8a_6 - 4a_7 + 2a_8)(89a_0 + 520a_6 - 212a_7 + 82a_8) > 0\ ,$$

$$c_{11} < -2c_7 \ \text{ for } \ c_7 \le 0\ , \quad \text{or} \quad c_{11} < -\frac{10}{3}c_7 \ \text{ for } \ c_7 > 0\ .$$

Contain: one massless spin 2^+ one massive spin 2^-, 1^+ and 1^- each

In Einstein form, contain no R^2 terms. Non-renormalizable.

Probably radiatively unstable.

Spin 2^-

Antisymmetric MAG

$$\mathcal{L} = b_1^{TT}\Big[-\frac{1}{3}\nabla_\mu K_{\nu\rho\sigma}\nabla^\mu K^{\nu\rho\sigma} - \frac{1}{3}\nabla_\mu K_{\nu\rho\sigma}\nabla^\mu K^{\rho\nu\sigma} + \nabla_\mu K^\alpha{}_{\alpha\nu}\nabla^\mu K_\beta{}^{\beta\nu}$$
$$+\frac{1}{3}\nabla_\mu K^\mu{}_{\rho\sigma}\nabla_\nu K^{\nu\rho\sigma} + \frac{2}{3}\nabla_\mu K^\mu{}_{\rho\sigma}\nabla_\nu K^{\rho\nu\sigma} + \frac{2}{3}\nabla_\mu K_\rho{}^\mu{}_\sigma\nabla_\nu K^{\rho\nu\sigma}$$
$$+\frac{1}{3}\nabla_\mu K_\rho{}^\mu{}_\sigma\nabla_\nu K^{\sigma\nu\rho} - \nabla_\mu K^\alpha{}_\alpha{}^\mu\nabla_\nu K^\beta{}_\beta{}^\nu + 2\nabla_\mu K^{\mu\nu}{}_\rho\nabla_\nu K^\alpha{}_\alpha{}^\rho\Big]$$

The Fourier transform of the linearized Lagrangian is

$$\mathcal{L} \;=\; -\frac{1}{2}K\cdot q^2\left[P_{22}\left(2^-\right) - P_{33}\left(1^-\right)\right]\cdot K$$

Spin 3^-

General MAG

$$
\begin{aligned}
\mathcal{L} \;=\; & -2b_1^{QQ}\big[\tfrac{1}{4}\nabla_\mu Q_{\alpha\beta\gamma}\nabla^\mu Q^{\alpha\beta\gamma} + \tfrac{1}{2}\nabla_\mu Q_{\alpha\beta\gamma}\nabla^\mu Q^{\beta\alpha\gamma} \\
& -\nabla_\mu \mathrm{tr}_{(12)}Q_\alpha \nabla^\mu \mathrm{tr}_{(12)}Q^\alpha - \nabla_\mu \mathrm{tr}_{(12)}Q_\alpha \nabla^\mu \mathrm{tr}_{(23)}Q^\alpha - \tfrac{1}{4}\nabla_\mu \mathrm{tr}_{(23)}Q_\alpha \nabla^\mu \mathrm{tr}_{(23)}Q^\alpha \\
& -\tfrac{1}{4}\mathrm{div}_{(1)}Q_{\alpha\beta}\mathrm{div}_{(1)}Q^{\alpha\beta} - \mathrm{div}_{(1)}Q_{\alpha\beta}\mathrm{div}_{(2)}Q^{\alpha\beta} - \mathrm{div}_{(2)}Q_{(\alpha\beta)}\mathrm{div}_{(2)}Q^{(\alpha\beta)} \\
& -\tfrac{1}{8}\mathrm{trdiv}_{(1)}Q_{\alpha\beta}\mathrm{trdiv}_{(1)}Q^{\alpha\beta} - \tfrac{1}{2}\mathrm{trdiv}_{(1)}Q_{\alpha\beta}\mathrm{trdiv}_{(2)}Q^{\alpha\beta} - \tfrac{1}{2}\mathrm{trdiv}_{(2)}Q_{\alpha\beta}\mathrm{trdiv}_{(2)}Q^{\alpha\beta} \\
& +\mathrm{div}_{(12)}Q_\alpha \mathrm{tr}_{(23)}Q^\alpha + \tfrac{1}{2}\mathrm{div}_{(23)}Q_\alpha \mathrm{tr}_{(23)}Q^\alpha \\
& +2\mathrm{div}_{(12)}Q_\alpha \mathrm{tr}_{(12)}Q^\alpha + \mathrm{div}_{(23)}Q_\alpha \mathrm{tr}_{(12)}Q^\alpha\big]\ ,
\end{aligned}
$$

where $\mathrm{div}_{(1)}Q_{\alpha\beta} = \nabla^\lambda Q_{\lambda\alpha\beta}$ etc.. when linearized, propagates only a massless spin-3 state.

Indeed setting $S_{\alpha\beta\gamma} = Q_{(\alpha\beta\gamma)}$, and $b_1^{QQ} = 1/3$ it becomes the Fronsdal Lagrangian

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\Big[- q^2 S_{\alpha\beta\gamma} S^{\alpha\beta\gamma} + 3q^2 \mathrm{tr}_{(12)} S_\alpha \mathrm{tr}_{(12)} S^\alpha \\ &\quad + 3\mathrm{div}_{(1)} S_{\alpha\beta} \mathrm{div}_{(1)} S^{\alpha\beta} + \frac{3}{2}(\mathrm{trdiv}_{(1)} S)^2 - 6\mathrm{div}_{(12)} S_\alpha \mathrm{tr}_{(12)} S^\alpha \Big]\end{aligned}$$

The “higher spin symmetry” $\delta S_{\alpha\beta\gamma} = \partial_{(\alpha}\Lambda_{\beta\gamma)}$ is an accidental symmetry.
Massive case under investigation
(restrictions as an EFT - see e.g. B. Bellazzini, F. Riva, J. Serra and F. Sgarlata, JHEP **10** (2019), 189 [arXiv:1903.08664 [hep-th]])

Extension to the UV

In Landau-Ginzburg theory and in the SM the Goldstone bosons are embedded in a linear representation.
One more field is needed.

In gravity no new field needed. Simplest linear realization would keep the same fields but relax constraints.

In linear realization, unbroken phase can be realized,
but difficult to write actions that make sense in both phases.

Some quantum calculations in MAG

The RG for Palatini/Holst action
J.-E. Daum, M. Reuter, Phys.Lett.B 710 (2012) 215-218, e-Print: 1012.4280 [hep-th]

The RG for the general dimension-2 Lagrangian
C. Pagani and R.P. Class.Quant.Grav. 32 (2015) 19, 195019, e-Print: 1506.02882 [gr-qc]

The RG for non-integrable Weyl theory
C. Pagani and R.P. Class.Quant.Grav. 31 (2014) 115005, e-Print: 1312.7767 [hep-th]

All non-universal results.

Very recently

The beta functions generated by the Lagrangian

$$c_1 L_1^{FF} + a_1 L_1^{TT}$$

with

$$L_1^{FF} = F_{abcd} F^{abcd} \; , \qquad L_1^{TT} = T_{abc} T^{abc}$$

have been calculated by
O. Melichev, PhD thesis, SISSA 2023

A technical point

Both in YM theory and in GR, calculation of divergences greatly simplified by choosing Feynman-Lorentz, resp. Feynman De Donder gauge (α=1).

If we use metric formalism, no freedom to eliminate nonminimal terms. In antisymmetric MAG must use tetrad gauge.

Not yet clear whether this can work for any MAG action.

Log divergences in Einstein form

$$
\begin{aligned}
-\frac{1}{2}\frac{1}{(4\pi)^2}\int d^4x\sqrt{g}\Big[&\frac{239}{60}H_1^{RR}+\frac{103}{30}H_2^{RR}-\frac{7}{4}H_3^{RR}\\
&+\frac{299}{24}H_3^{RT}-\frac{13}{12}H_5^{RT}\\
&+\frac{9}{4}H_1^{TT}+2H_2^{TT}-2H_4^{TT}+\frac{7}{6}H_5^{TT}\\
&-\frac{35}{24}H_6^{TT}-\frac{15}{4}H_7^{TT}+3H_8^{TT}+\frac{7}{6}H_9^{TT}\Big]
\end{aligned}
$$

Log divergences in Cartan form

$$
\begin{aligned}
-\frac{1}{2}\frac{1}{(4\pi)^2}\int d^4x\sqrt{g}\Big[&\frac{76}{15}L_1^{FF}+\frac{107}{60}L_3^{FF}-\frac{86}{15}L_4^{FtF}\\
&+\frac{137}{30}L_7^{FF}-\frac{17}{15}L_8^{FF}-\frac{7}{4}L_{16}^{FF}\\
&-\frac{91}{10}L_1^{FT}+\frac{13}{3}L_8^{FT}-\frac{93}{10}L_9^{FT}-\frac{39}{5}L_{13}^{FT}\\
&+\frac{19}{30}L_1^{TT}-\frac{1}{5}L_2^{TT}-\frac{47}{30}L_3^{TT}+\frac{32}{15}L_5^{TT}\Big]
\end{aligned}
$$

Lessons

A single dimension-4 term in the action generates all the others.

Some consequences for quantum MAG:

Poincaré gauge theory is not renormalizable, not even on shell renormalizable
General antisymmetric MAG is renormalizable (did not calculate the T^3 and T^4 terms)

A couple of terms can be removed by linear field redefinitions.
With Palatini term:
all these terms can be absorbed by field redefinitions

Summary for part 3

- the EFT of gravitons may be extended to MAG treated as an EFT;
- if all new dof's have Planck masses, they are indistinguishable;
- gravitational Higgs phenomenon explains why we have the Levi-Civita connection at low energy
  furthermore; it is a prelude to a possible unification of forces;
- deeper questions: why is the metric nondegenerate?
  This is the analog of the question: what drives electroweak symmetry breaking?
  In the symmetric phase there is no metric
  For this we need a genuine quantum theory of spacetime.