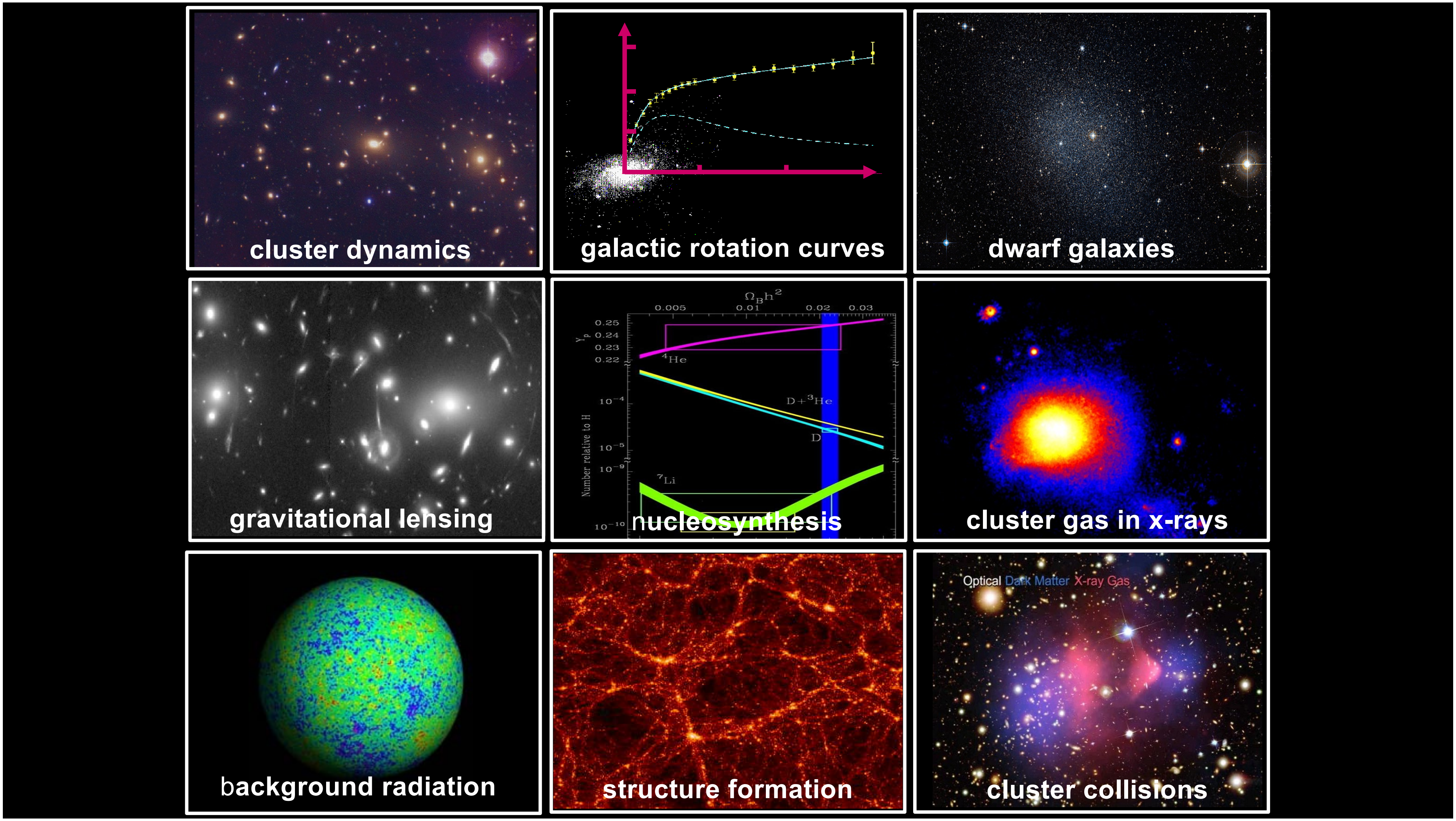
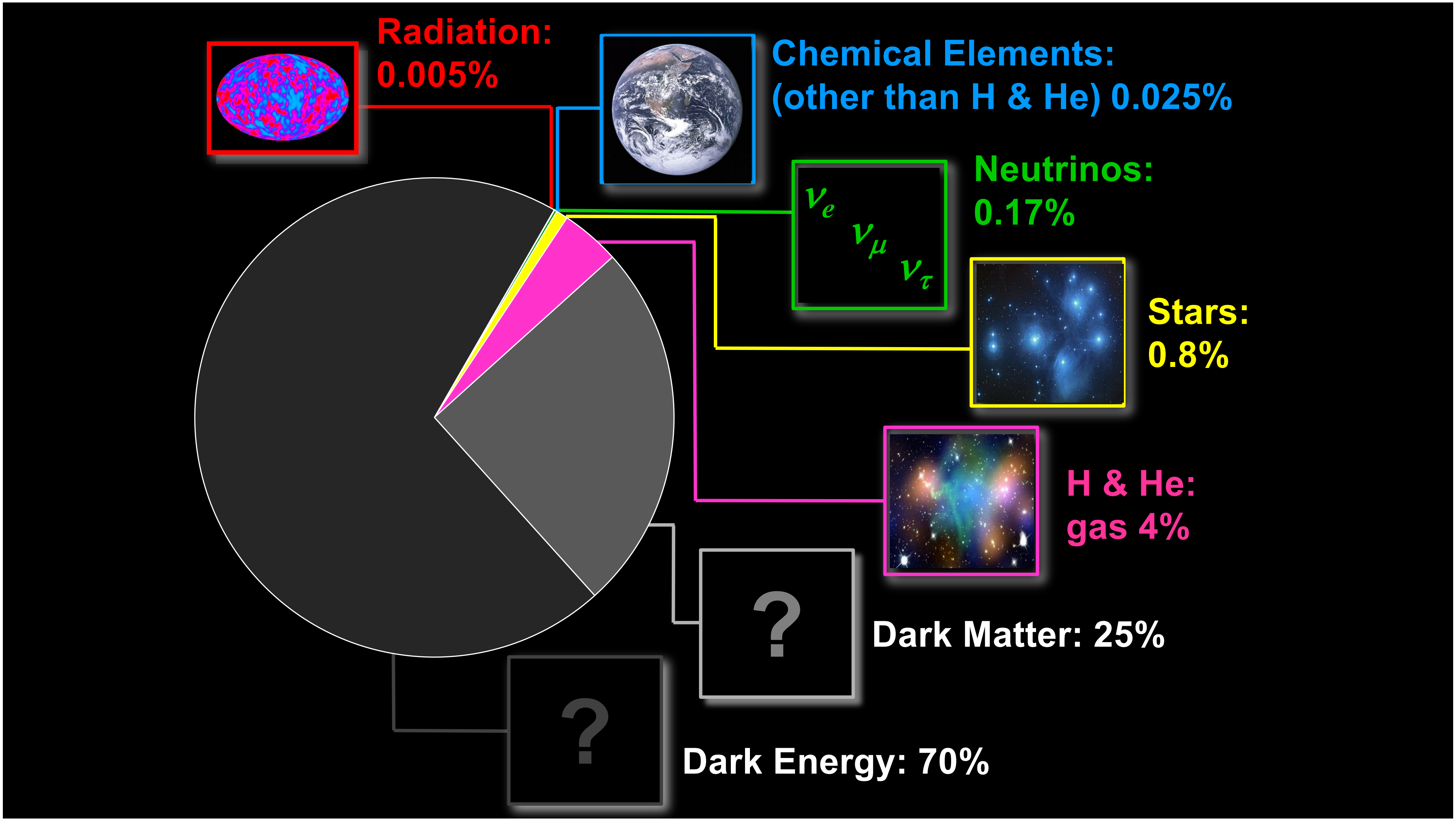
Gravitational Origin of Dark Matter

Rocky Kolb

9/19/2022

VI José Plínio Baptista
School on Cosmology
Friedmann and the
LCDM model


cluster dynamics
galactic rotation curves
dwarf galaxies
gravitational lensing
$\Omega_B h^2$
0.005
0.01
0.02
0.03
0.25
0.24
0.23
0.22
Y_p
^{4}He
10^{-4}
10^{-5}
10^{-9}
10^{-10}
D+^{3}He
D
^{7}Li
Number relative to H
nucleosynthesis
cluster gas in x-rays
background radiation
structure formation
Optical Dark Matter X-ray Gas
cluster collisions



Radiation:
0.005%
Chemical Elements:
(other than H & He) 0.025%
Neutrinos:
0.17%
ν_e ν_μ ν_τ
Stars:
0.8%
H & He:
gas 4%
?
Dark Matter: 25%
?
Dark Energy: 70%

For 40 Years, Leading Candidate*
"Weak"-Scale Cold Thermal Relic

- Mass: GeV – TeV
- "Weak-scale" interaction strength with SM
- No self-interactions
- Produced by "freeze-out" from primordial plasma. COLD dark matter.
- "Detectable" by direct detection, indirect detection, decay products, production at colliders
- Just BSM

But not (convincingly) seen

- In Direct detection (but DAMA/LIBRA)
- In Indirect Detection (but galactic-center excess)
- In Decay (but $3.5\ \mathrm{keV}$ γ-ray line)
- In Colliders no BSM signal (but μ_{g-2}, m_W)

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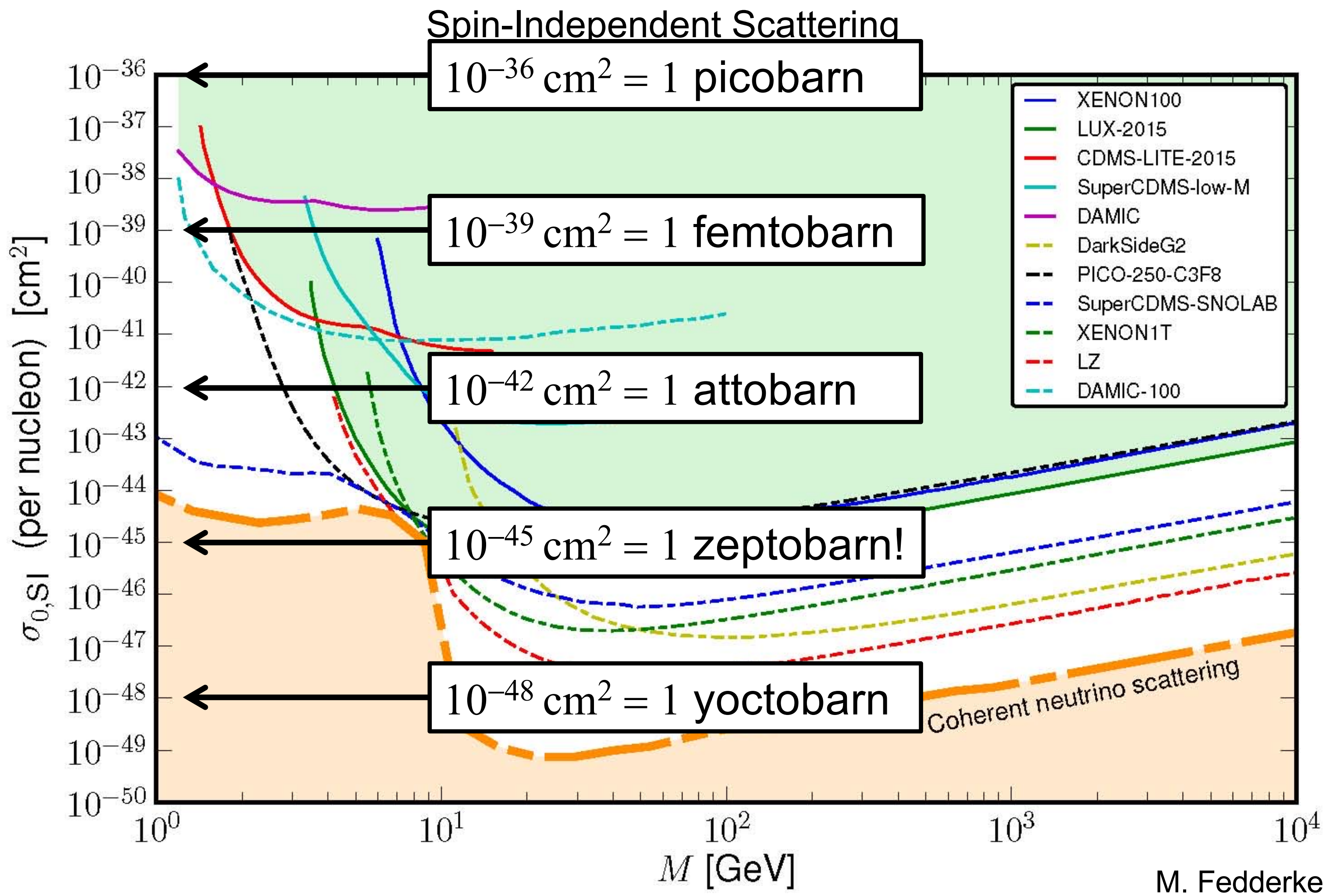
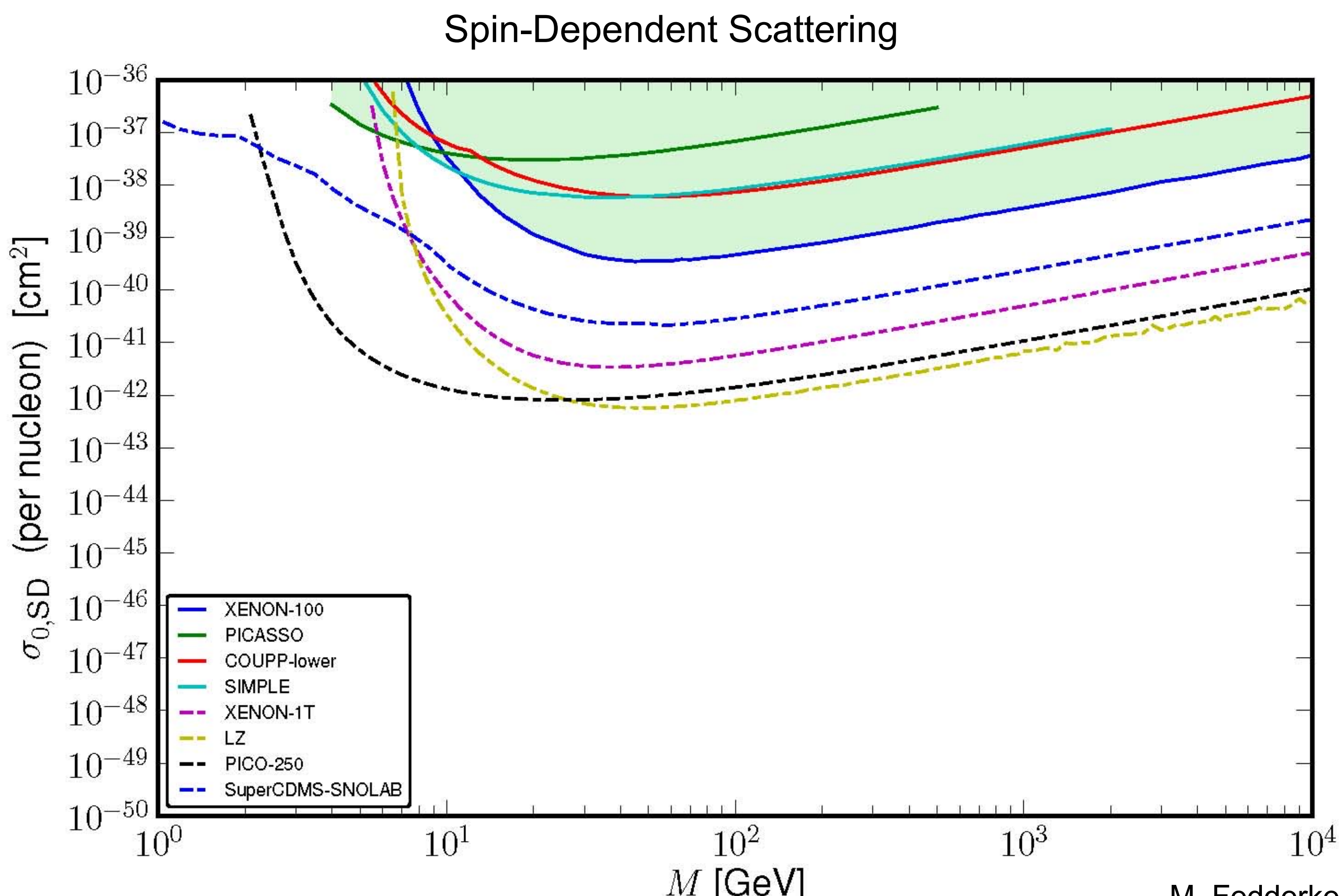
along with axions

Direct detection



Direct detection



M. Fedderke

For 40 Years, Leading Candidate
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What if DM interacts only gravitationally?

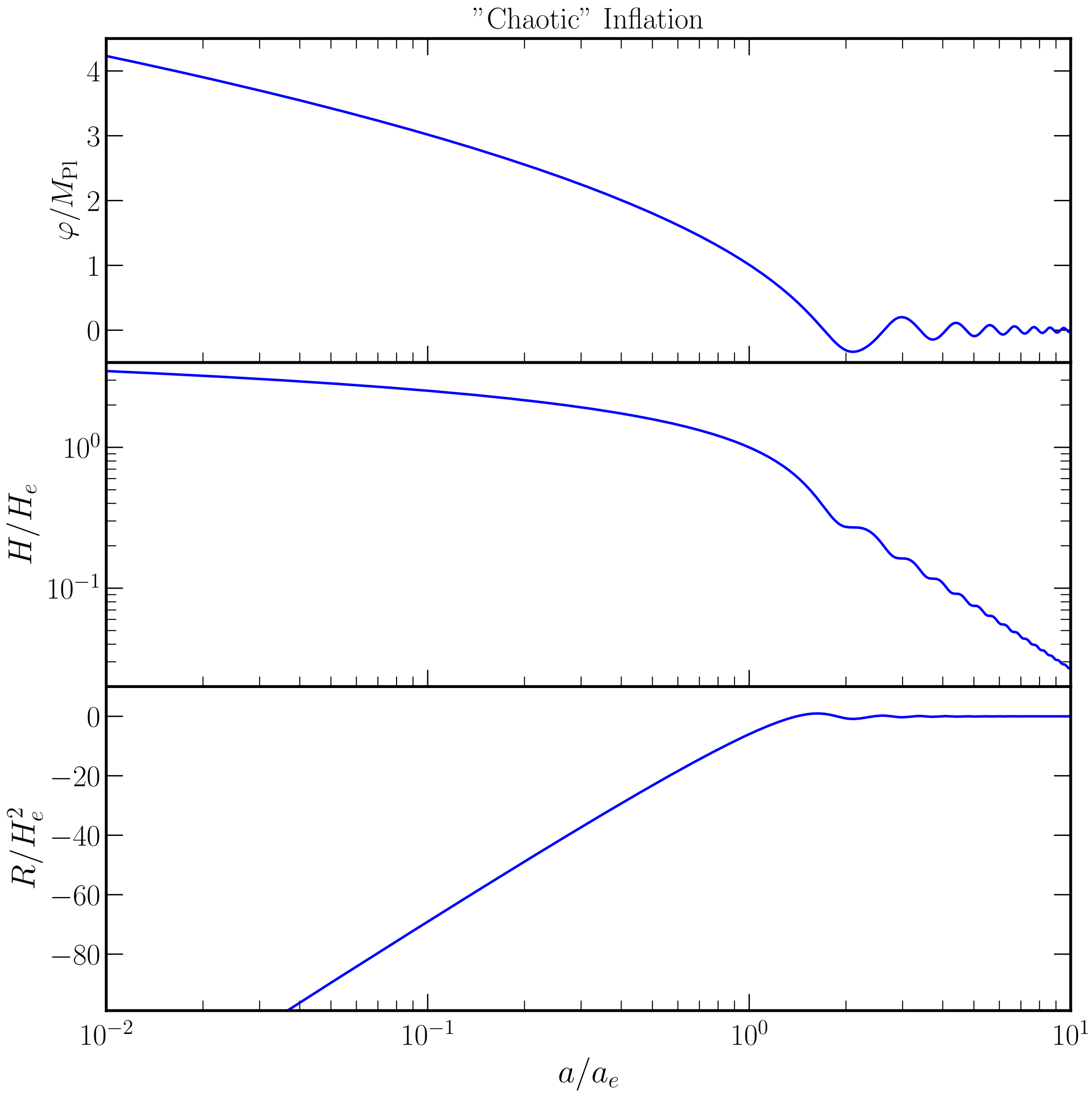
- Gravity must play a prole in its cosmological production
- But gravity weak!

Assume Standard Inflationary Picture

Quasi-de Sitter Phase driven by vacuum energy of inflaton displaced from potential minimum, expansion rate H_e

Matter-dominated phase due to inflaton oscillations about minimum of potential

Inflaton decays and leads to radiation-dominated phase characterized by a reheat temperature T_{RH}



Chaotic Inflation

Potential

$$V(\varphi) = \frac{1}{2} m_\varphi^2 \varphi^2$$

Expansion rate H

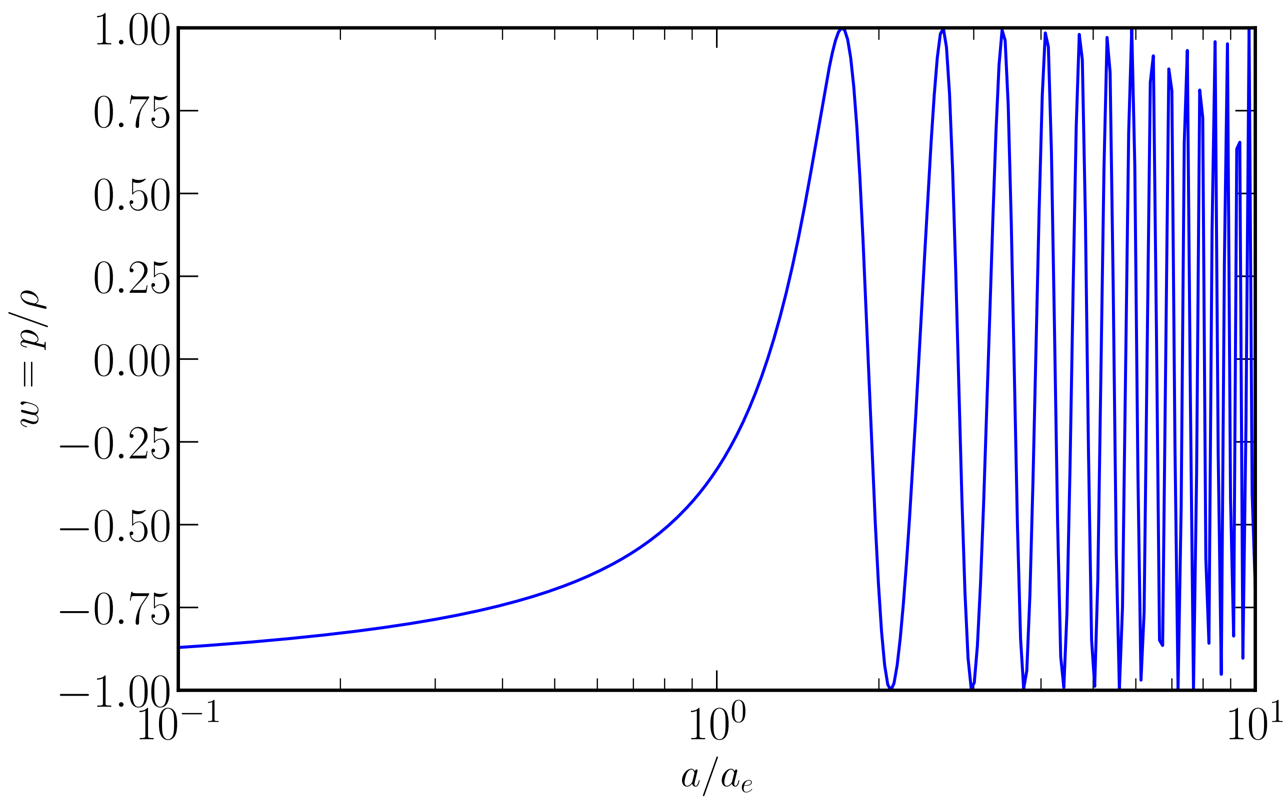
$$3M_{\rm Pl}^2 H^2 = \rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + V(\varphi)$$

$$p_\varphi = \frac{1}{2}\dot{\varphi}^2 - V(\varphi)$$

Ricci Scalar Curvature R

$$M_{\rm Pl}^2 R = -(\rho - 3p)$$

Chaotic Inflation



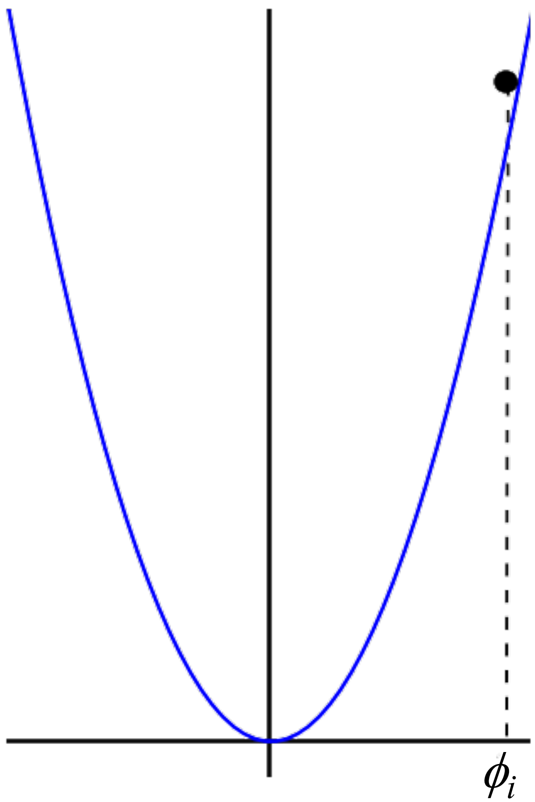
Potential

$$V(\varphi) = \frac{1}{2} m_\varphi \phi^2$$

Equation of state parameter w

Ideas for gravitational particle production

Produce particles through misalignment mechanism



$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0$$

- Scalar field has quantum fluctuations during inflation

$$\Delta\phi = \frac{H}{2\pi}$$

- After inflation field frozen by “Hubble drag” until

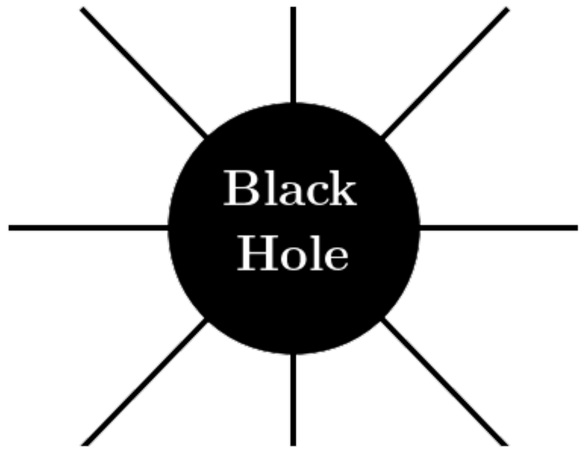
$$H \simeq m_\phi$$

- After which it oscillates with energy density in oscillating field

- cf., axion

Ideas for gravitational particle production

Produce particles via Hawking radiation from primordial black holes
(Hooper, Krnjaic, & McDermott)



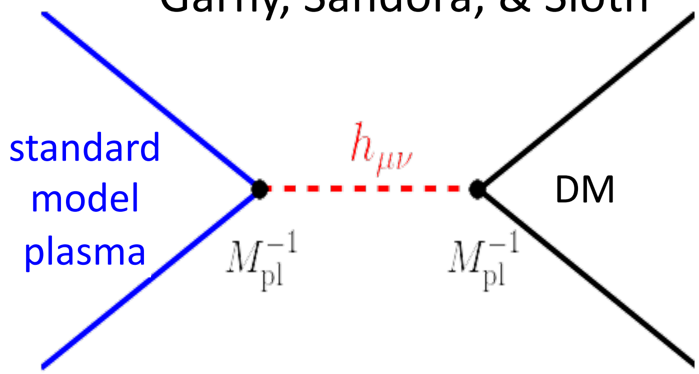
$$\frac{\Omega h^2}{0.12} \approx \left(\frac{10^{11}\mathrm{GeV}}{m}\right)\left(\frac{10^{12}\mathrm{GeV}}{T_i}\right)^3\left(\frac{\epsilon_{\mathrm{BH}}}{10^{-16}}\right)$$

- PBHs of current interest (after first LIGO event)
- Seeds for PBHs from inflation
- Assumes DM mass about $10^{11}\,\mathrm{GeV}$ (WIMPzilla)

Ideas for gravitational particle production

$$\mathcal{L} = M_{\mathrm{Pl}}^{-1} h_{\mu\nu} T^{\mu\nu}$$

Produce particles from SM plasma via
graviton exchange
Garny, Sandora, & Sloth



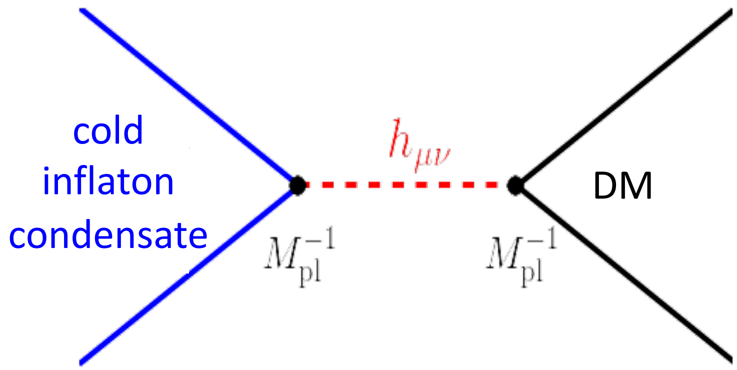
$$\frac{\Omega h^2}{0.12} \approx \left(\frac{\langle\sigma v\rangle}{T^2/M_{\mathrm{Pl}}^4}\right)\left(\frac{m}{10^{13}\mathrm{GeV}}\right)\left(\frac{T_{\mathrm{RH}}}{10^{14}\mathrm{GeV}}\right)^3$$

- Freeze-in
- For DM mass about $10^{13}\,\mathrm{GeV}$ (WIMPzilla)
- Assumes $m < T_{\mathrm{RH}}$

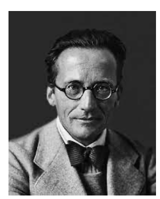
Ideas for gravitational particle production

$$\mathcal{L} = M_{\rm Pl}^{-1} h_{\mu\nu} T^{\mu\nu}$$

Produce particles from inflaton field after quasi-de Sitter era via graviton exchange
Ema, Nakayama, Tang; Mambrini & Olive



- Only works for DM mass < inflaton mass
- DM mass for correct Ωh^2 involved function of several parameters
- “Boltzmann” approach not complete treatment (Kaneta, Lee, Oda)

Boltzmann  $\subset$  Schrödinger $+$  Bolgoliubov

Cosmological Gravitational Particle Production (CGPP)

- A space-dependent (e.g., black holes) or time-dependent (e.g., big bang) gravitational field can create particles from vacuum

    Black Holes – Hawking 1974
    Big Bang – Schrödinger 1939

- Interest here on <u>time-dependent</u> gravitational fields, in particular, the big bang

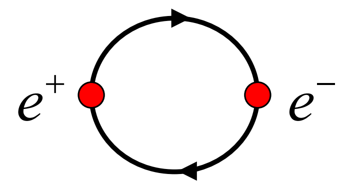
    Inflation: quasi deSitter phase followed by transition to matter-dominated then radiation-dominated phase

- CGPP is an example of QFT in classical gravitation background. Many interesting facets, but …

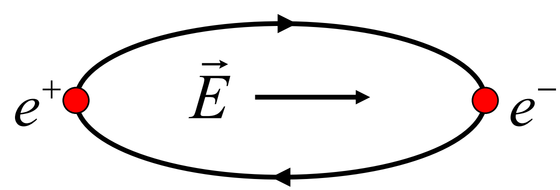
- … my motivation is whether
    - CGPP can be the origin of DARK MATTER (DM), and
    - CGPP can result in cosmological constraints on particle properties

Disturbing the Quantum Vacuum

Electric Field $\longrightarrow$ Particle creation

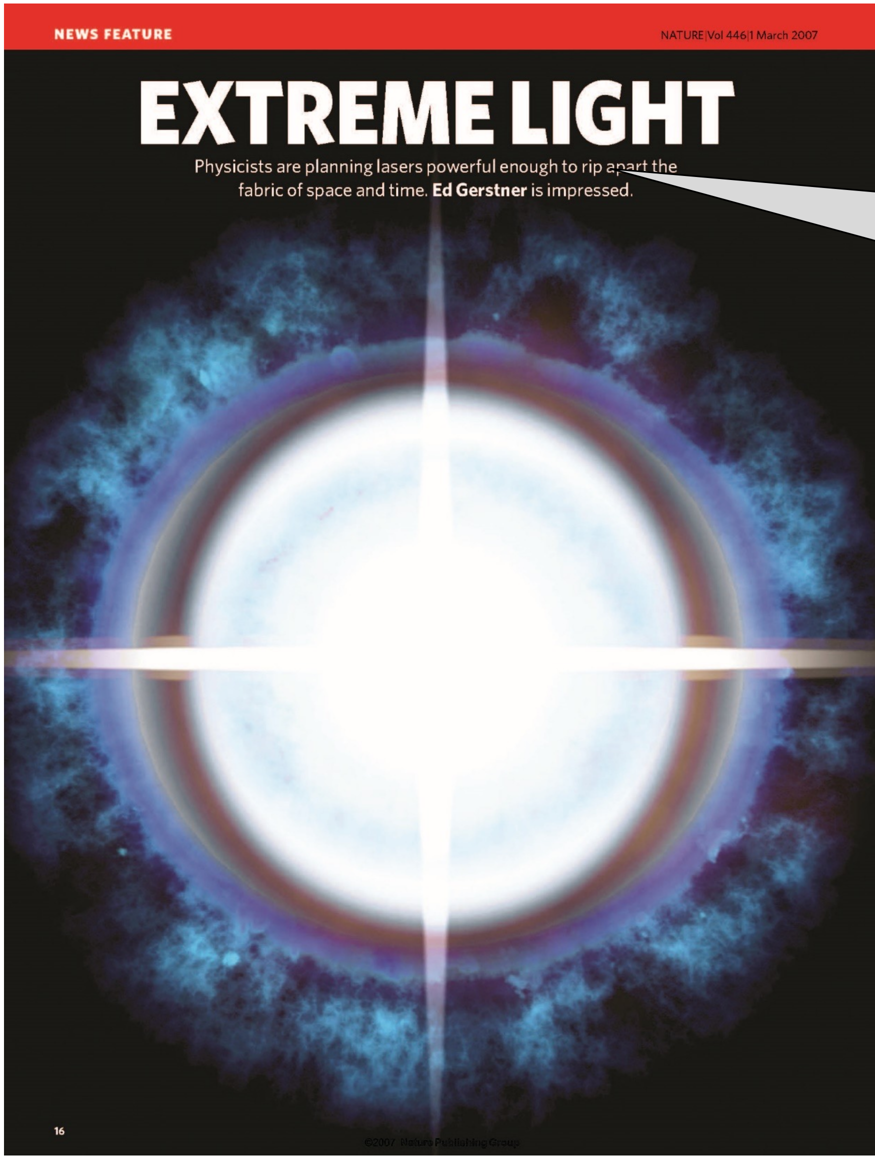


Particle creation if energy gained in acceleration from E-field over a Compton wavelength exceeds the particle's rest mass.



$$\left|\vec{E}_{\mathrm{crit}}\right| = \frac{m_e^2 c^3}{e\hbar} \approx 10^{16}\ \mathrm{V\ cm^{-1}} \qquad \Gamma \propto e^{-\pi E_{\mathrm{crit}}/\left|\vec{E}\right|}$$

Sauter (1931); Heisenberg & Euler (1935); Weisskopf (1936); Schwinger (1951)



NEWS FEATURE NATURE|Vol 446|1 March 2007

EXTREME LIGHT

Physicists are planning lasers powerful enough to rip apart the fabric of space and time. **Ed Gerstner** is impressed.

16

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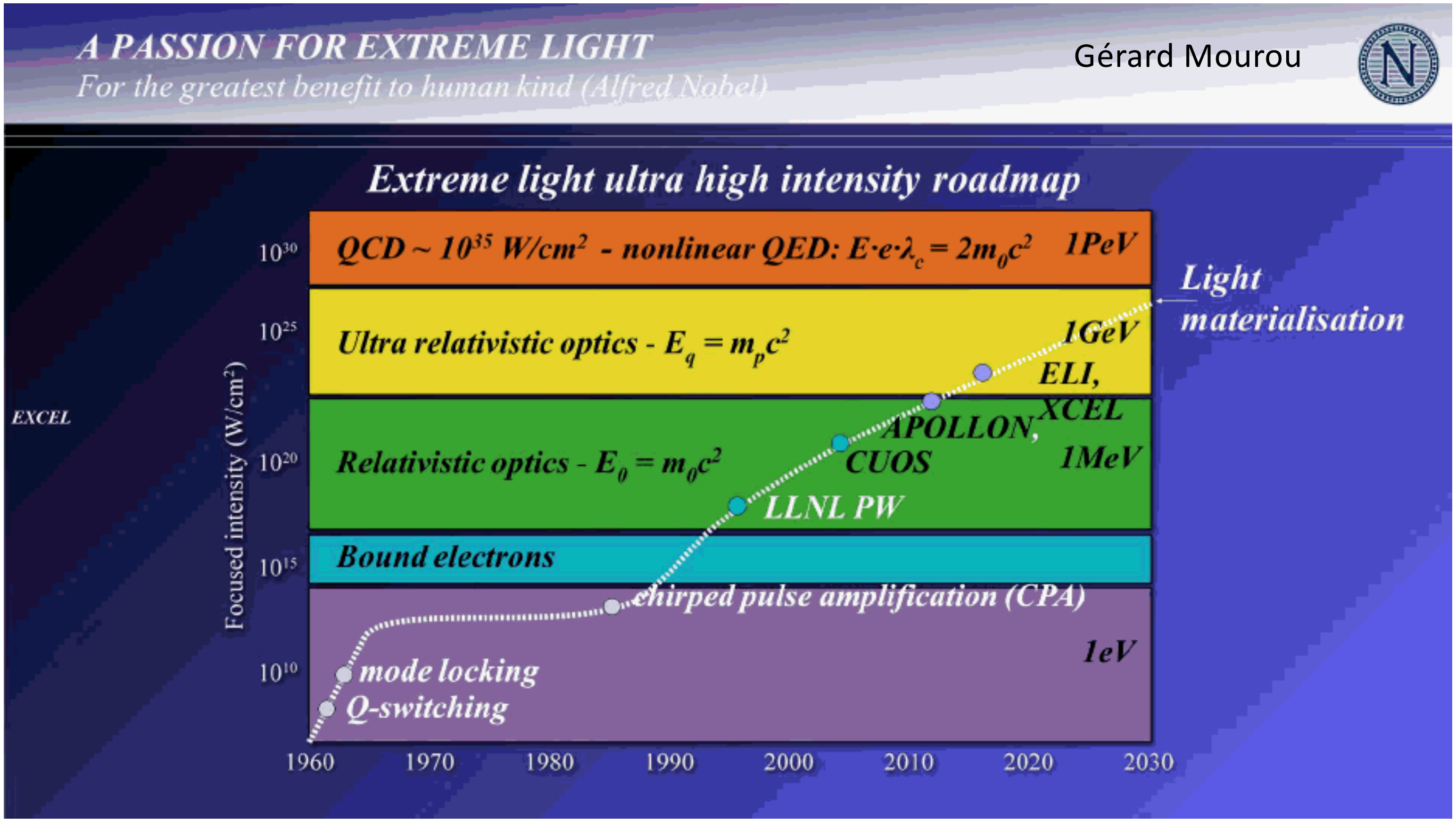
NATURE, Vol 446/1 March 2007

Physicists are planning lasers powerful enough to rip apart the fabric of space and time.

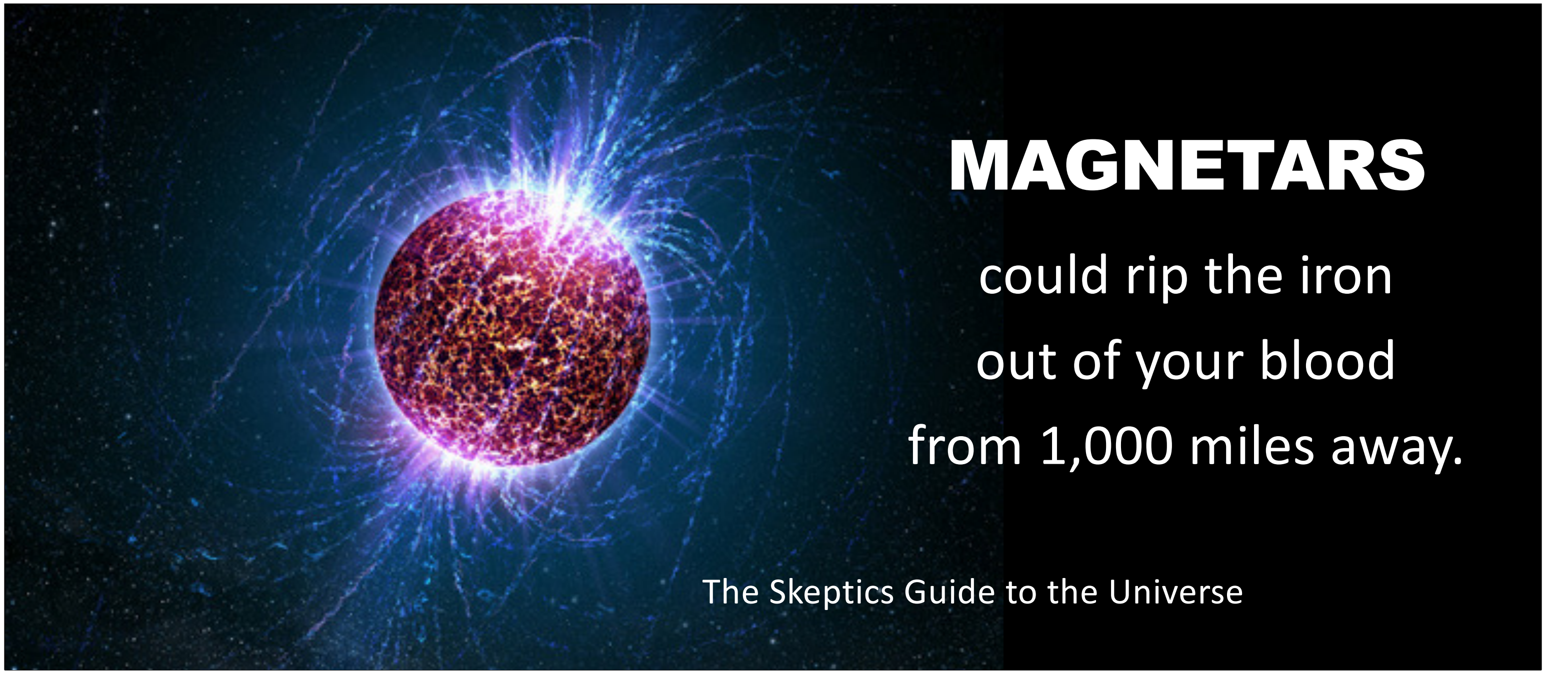
"We're going to change the index of refraction of the vacuum and produce new particles."

Gérard Mourou

Disturbing the Quantum Vacuum



$$I_C \approx \frac{c}{8\pi}\left|\vec{E}_{\mathrm{crit}}\right|^2 \approx 10^{30}\ \mathrm{W\ cm}^2$$



$$\left|\vec{B}_{\mathrm{crit}}\right| = \frac{m_e^2}{e} \approx 5 \times 10^{13}\,\mathrm{G}$$

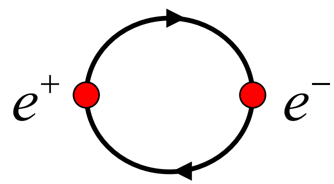
Crab pulsar	3×10^{13} G
Magnetars	$10^{14} - 10^{15}$ G

Strong magnetic fields imply existence of strong electric fields.

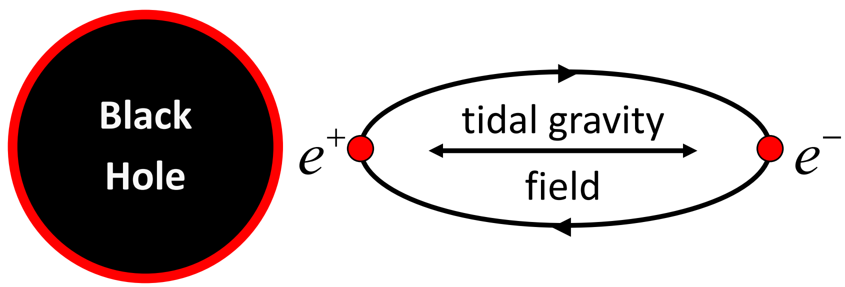
Many unexplained phenomena associated with pulsars, magnetars, etc.

Disturbing the Quantum Vacuum

Tidal gravitational field $\longrightarrow$ Particle creation



Particle creation if energy gained in acceleration from gravity over a Compton wavelength exceeds the particle's rest mass.

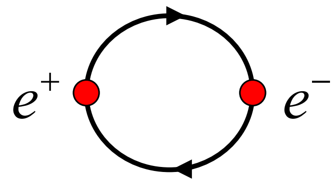


$\mathrm{v} = c$ at Black Hole horizon

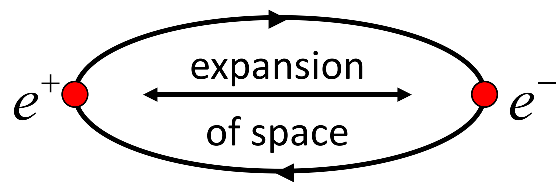
Hawking (1974); Bekenstein (1972)

Disturbing the Quantum Vacuum

Expanding universe $\longrightarrow$ Particle creation



Particle creation if energy gained in acceleration from expansion over a Compton wavelength exceeds the particle's rest mass.



$\mathrm{v} = c$ at Hubble radius

$$H_{\mathrm{crit}} = m \qquad\qquad \Gamma \propto e^{-\pi H_{\mathrm{crit}}/H}$$

Schrödinger's Alarming Phenomenon (1939)

My collaborators on CGPP

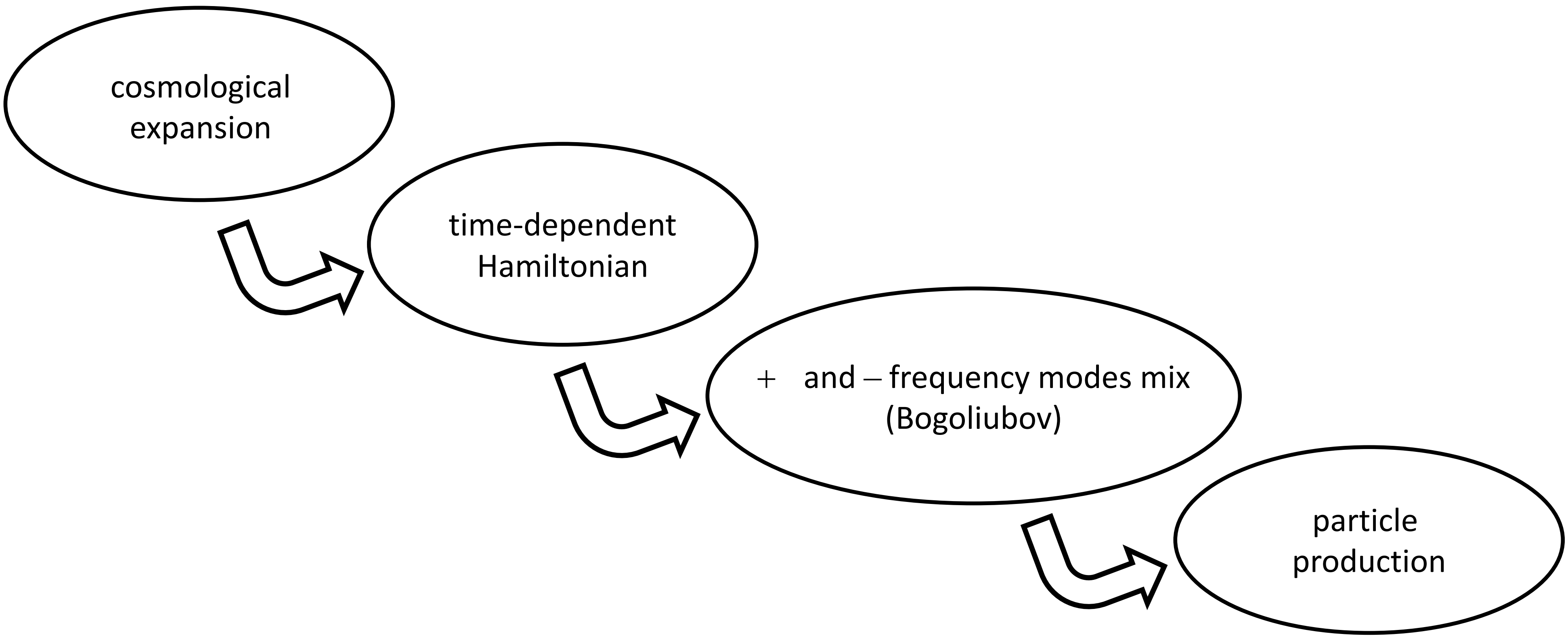
Chung, Kolb, Riotto (1998); Kuzmin & Tkachev (1999)

My collaborators:

Ivone Albuquerque
Edward Basso
Daniel Chung
Patrick Crotty
Michael Fedderke
Gian Giudice
Lam Hui
Siyang Lin
Andrew Long
Evan McDonough
Guillaume Payeur
Toni Riotto
Rachel Rosen
Leo Senatore
Alexi Starobinski
Igor Tkachev
Mark Wyman

Cosmological Gravitational Particle Production (CGPP)

- In Minkowskian QFT, a particle is an IR of the Poincaré group.
- But, expanding universe not Poincaré invariant.
- Notion of a "particle" is approximate.



Scalar field ϕ in Minkowski space

Action:

$$S[\phi(x)] = \int d^4x \left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2\right]$$

Fourier mode decomposition:

$$\widehat{\phi}(t,\boldsymbol{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\left[\widehat{a}_{\mathbf{k}}\chi_{\mathbf{k}}(t)e^{i\mathbf{k}\cdot\mathbf{x}} + \widehat{a}^\dagger_{\mathbf{k}}\chi^*_{\mathbf{k}}(t)e^{-i\mathbf{k}\cdot\mathbf{x}}\right]$$

Mode functions satisfy wave equation and normalization condition:

$$\partial_t^2\chi_k(t) + \omega_k^2\chi_k(t) = 0 \qquad \chi_k\partial_t\chi_k^* - \chi_k^*\partial_t\chi = 0$$

Dispersion relation: $\omega_k^2 = k^2 + m^2$

Vacuum state $|0\rangle$ defined as state where $\widehat{a}_{\mathbf{k}}|0\rangle = 0$

Scalar field ϕ in Minkowski space

Solution to wave equation:

$$\chi_k(t) = \frac{\alpha_k}{\sqrt{2\omega_k}}\, e^{-i\omega_k t} + \frac{\beta_k}{\sqrt{2\omega_k}}\, e^{+i\omega_k t}$$

Bogoliubov coefficients satisfy: $|\alpha_k|^2 - |\beta_k|^2 = 1$

If choose $|\alpha_k| = 0$, will have pure outgoing wave … no mode mixing

Assume Standard Inflationary Picture

Quasi-de Sitter Phase driven by vacuum energy of inflaton displaced from potential minimum, expansion rate H_I roughly constant

Matter-dominated phase due to inflaton oscillations about minimum of potential

Inflaton decays and leads to radiation-dominated phase characterized by a reheat temperature $T_{\rm RH}$

Conformal time $-\infty < \eta < 0$ de Sitter and $0 < \eta < \infty$ matter-dominated → radiation-dominated

Roughly inflation matter-dominated (a_e , H_e are values at end of inflation)

$$a = \frac{a_e}{1-\eta}$$

$$H = H_e$$

$$a = a_e(1 + \tfrac{1}{2}\eta)^2$$

$$H = \frac{H_e}{(1 + \frac{1}{2}\eta)^3}$$

Scalar field ϕ in FRW background

covariant action

$$S[\varphi(x), g_{\mu\nu}(x)] = \int d^4x \sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}m^2\varphi^2 + \frac{1}{2}\xi R\varphi^2\right]$$

Gravity enters the picture

in spatially flat FRW background : $ds^2 = a^2(\eta)[d\eta^2 - d\mathbf{x}^2]$ (η is conformal time)

$$S[\varphi(\eta, \boldsymbol{x})] = \int_{-\infty}^{\infty} d\eta \int d^3\mathbf{x}\left[\frac{1}{2}a^2(\partial_\eta\varphi)^2 - \frac{1}{2}a^2(\nabla\varphi)^2 - \frac{1}{2}a^4m^2\varphi^2 + \frac{1}{2}a^4\xi R\varphi^2\right]$$

field rescaling

$$\phi(\eta, \boldsymbol{x}) = a(\eta)\varphi(\eta, \boldsymbol{x})$$

$aH \to 0$ to zero at $\eta = \pm\infty$

action for canonically-normalized field

$$S[\phi(\eta, \boldsymbol{x})] = \int_{-\infty}^{\infty} d\eta \int d^3\mathbf{x}\left[\frac{1}{2}(\partial_\eta\phi)^2 - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{2}m_{\text{eff}}^2\phi^2 - \frac{1}{2}\partial_\eta(aH\phi^2)\right]$$

time-dependent effective mass

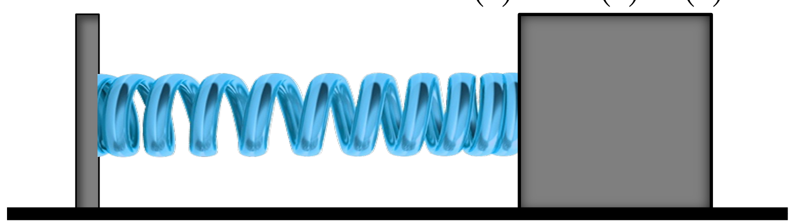
$$m_{\text{eff}}^2(\eta) = a^2(\eta)\left[m^2 + \left(\frac{1}{6} - \xi\right)R(\eta)\right]$$

cosmological expansion ⇒
time-dependent background ⇒
time-dependent Hamiltonian for spectator fields

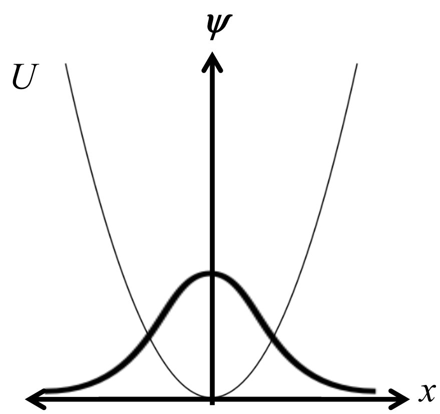
Schrödinger's Alarming Phenomenon

Expansion of the universe causes explicit time dependence in action for "spectator" fields.
Initial ~ de Sitter (early-time) vacuum may not evolve to final ~ Minkowski (late-time) vacuum, but to an excited state populated by particles.

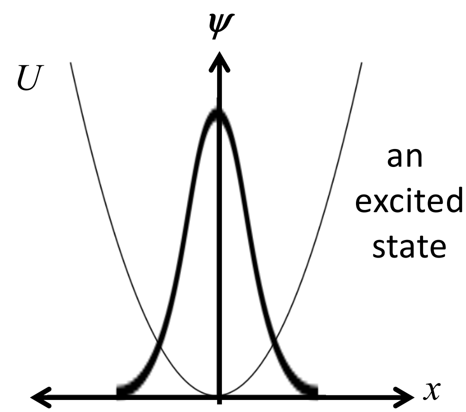
$$\ddot{x}(t) + \omega^2(t)\ x(t) = 0$$



Spring constant varied slowly (adiabatically)



Spring constant varied abruptly (nonadiabatically)



Scalar field ϕ in FRW background

Fourier mode decomposition

$$\widehat{\phi}(\eta, \boldsymbol{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \left[\widehat{a}_{\mathbf{k}} \chi_{\mathbf{k}}(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} + \widehat{a}^\dagger_{\mathbf{k}} \chi^*_{\mathbf{k}}(\eta) e^{-i\mathbf{k}\cdot\mathbf{x}} \right]$$

mode functions satisfy wave equation

$$\partial_\eta^2 \chi_k(\eta) + \omega_k^2(\eta)\chi_k(\eta) = 0$$

but with a time-dependent dispersion relation

$$\omega_k^2(\eta) = |\mathbf{k}|^2 + m_{\text{eff}}^2(\eta)$$

+ & – frequency modes

$$\chi_\pm^{(\text{early})}(\eta) \xrightarrow{\eta\to-\infty} \frac{1}{\sqrt{2k}} e^{\mp ik\eta}$$

$$\chi_\pm^{(\text{late})}(\eta) \xrightarrow{\eta\to+\infty} \frac{1}{\sqrt{2am}} e^{\mp i \int^\eta d\eta' am}$$

leads to mode mixing

$$\begin{pmatrix} \chi_{k+}^{(\text{late})}(\eta) \\ \chi_{k-}^{(\text{late})}(\eta) \end{pmatrix} = \begin{pmatrix} \alpha_k & \beta_k \\ \beta_k^* & \alpha_k^* \end{pmatrix} \begin{pmatrix} \chi_{k+}^{(\text{early})}(\eta) \\ \chi_{k-}^{(\text{early})}(\eta) \end{pmatrix}$$

time-dependent Hamiltonian ⇒ mode mixing
⇒ – frequency modes from + frequency modes

Scalar field ϕ in FRW background

$$m_{\rm eff}^2(\eta) = a^2(\eta)\left[m^2 + \left(\frac{1}{6} - \xi\right)R(\eta)\right]$$

Abrupt changes in $a(\eta)$ leads to nonadiabatic changes in $\omega_k(\eta)$, which *adulterates* positive and negative frequency modes, leading to of particle creation in the expanding universe.

Nonadiabaticity proportional to $\dfrac{\partial_\eta \omega_k}{\omega_k^2}$

Adiabatic deep in quasi-de Sitter phase
Adiabatic at late time after inflation

Nonadiabatic:

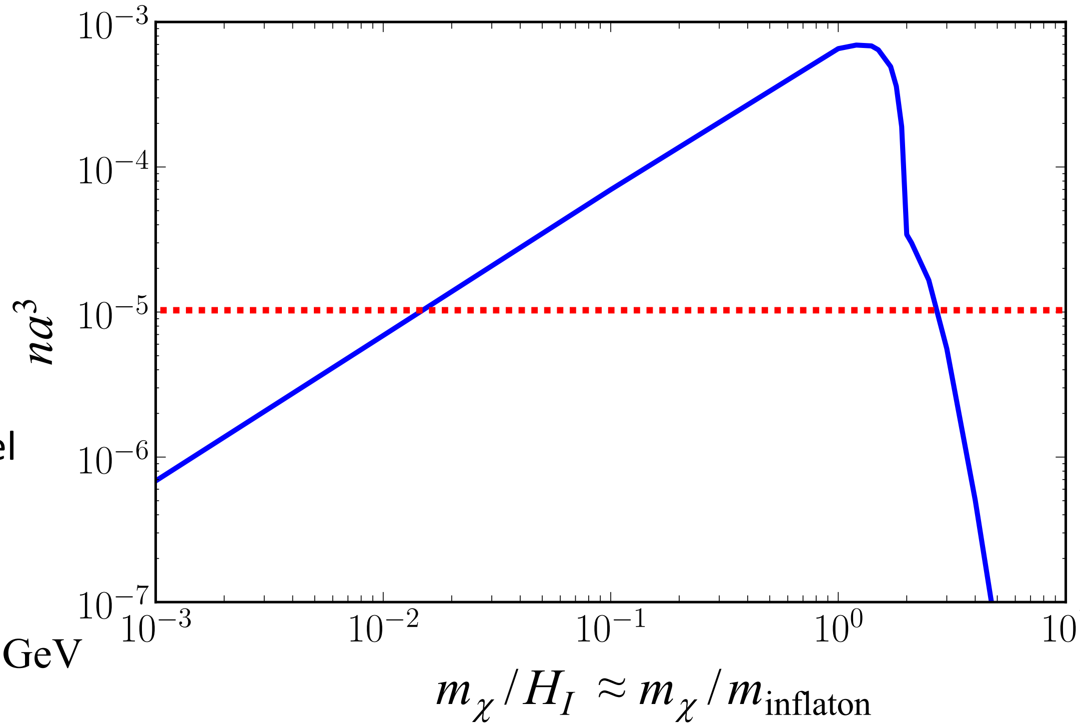
- $\xi = 0$:
  - $k/a \ll H$ when mode exits horizon during inflation. Super-Hubble.
  - $k/a \gg H$ at end of inflation. Sub-Hubble radius.
- $\xi = 1/6$: at end of inflation. Sub-Hubble radius.

Scalar field ϕ in FRW background

Conformally-coupled scalar
$\xi = 1/6$

$$\frac{\Omega_\chi h^2}{0.12} = \frac{m_\chi}{H_I}\left(\frac{H_I}{10^{12}\mathrm{GeV}}\right)^2\left(\frac{T_{\mathrm{RH}}}{10^9\mathrm{GeV}}\right)\frac{[na^3]}{10^{-5}}$$

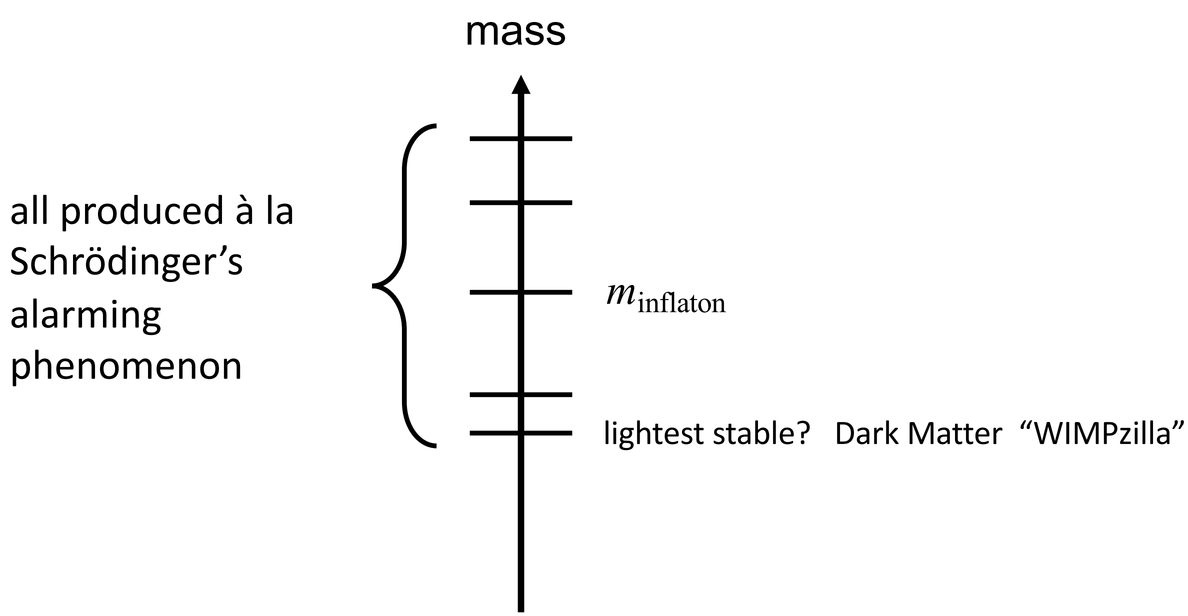
- Calculation assumes particular inflationary model (chaotic, which is ruled out).
- But general picture holds in other models since action occurs around end of inflation.
- We don't know, but $H_I \approx 10^{12}$ GeV and $T_{\mathrm{RH}} \approx 10^9$ GeV are "common."
- If stable and dark matter, $\Omega_\chi h^2 = 0.12 \implies m \approx H_I$. Could have been anything!
- Perhaps inflation scale represents new physics scale, stable particle at that mass scale natural DM candidate.
- WIMPZILLA miracle!



Conformally-coupled scalar WIMPZILLA DM candidate
$m_\chi = \mathcal{O}(m_{\mathrm{inflaton}})$

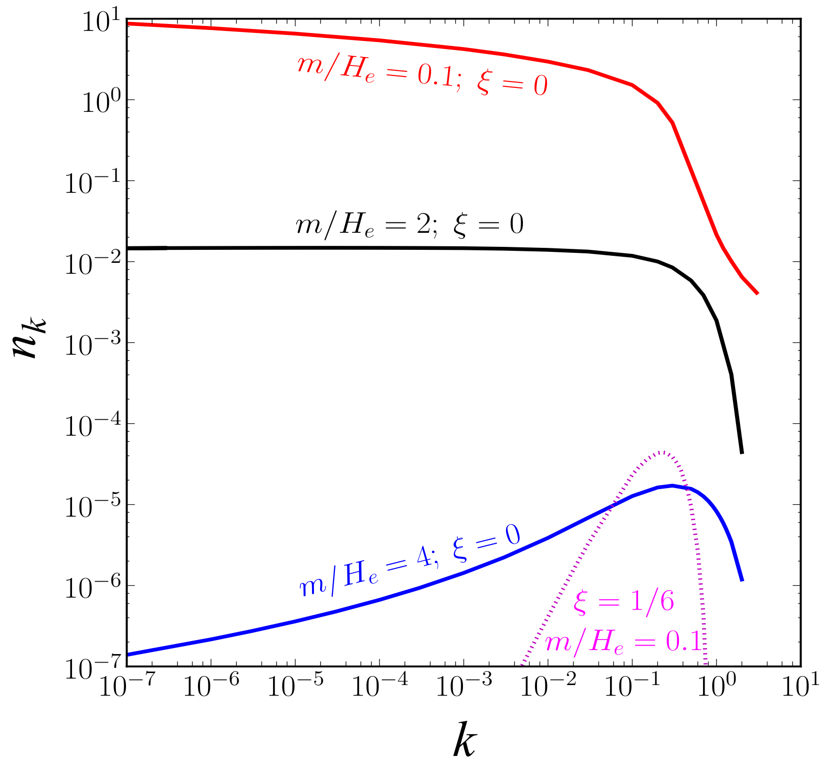
CGPP & Dark Matter

- Inflation indicates a new mass scale
- In most models, $m_{\text{inflaton}} \approx H_{\text{inflation}} \approx 10^{12} - 10^{14}$ GeV?
- $H_{\text{inflation}}$ detectable via primordial gravitational waves in CMB
- (I) expect other particles with mass $\approx m_{\text{inflaton}}$



Scalar field ϕ in FRW background

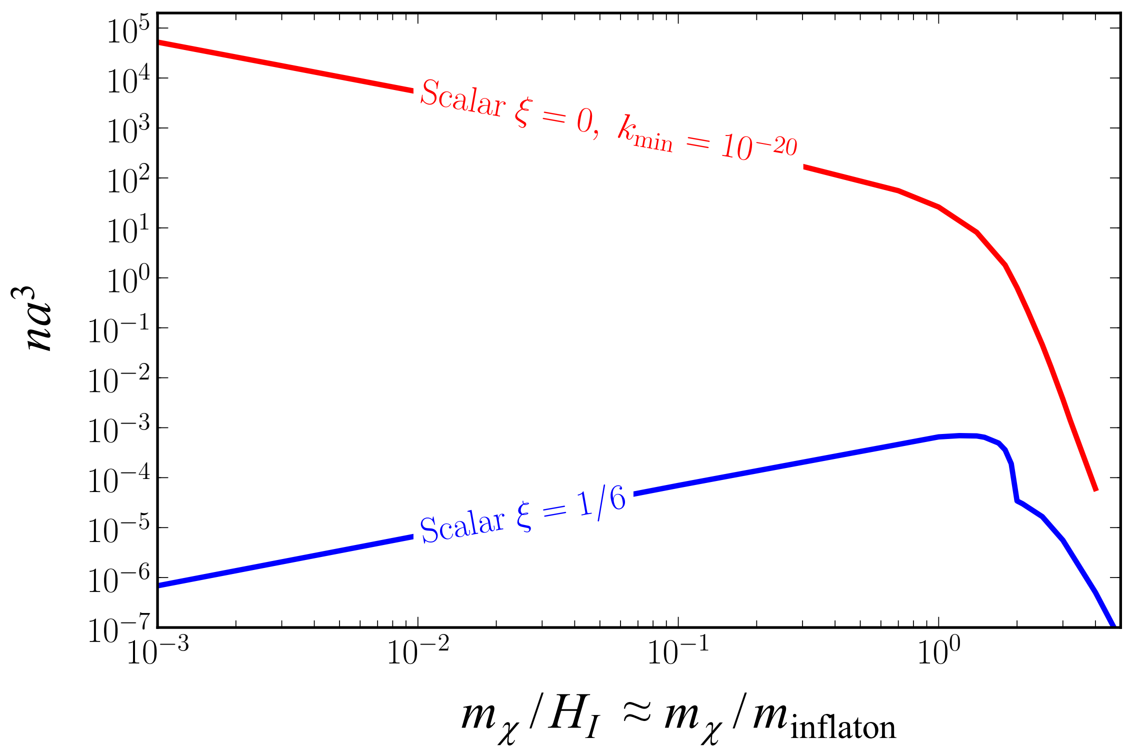
$$[na^3] = \frac{1}{2\pi^2}\int \frac{dk}{k} n_k$$



$\xi = 0$ red for $m/H_I < 2$
blue for $m/H_I > 2$

$\xi = 1/6$ blue for all m/H_I

$$\frac{\Omega_\chi h^2}{0.12} = \frac{m_\chi}{H_I}\left(\frac{H_I}{10^{12}\text{GeV}}\right)^2\left(\frac{T_{\text{RH}}}{10^9\text{GeV}}\right)\frac{[na^3]}{10^{-5}}$$



Minimally-coupled scalar WIMPZILLA not DM candidate unless $m_\chi / H_I \approx m_\chi / m_{\text{inflaton}} \gtrsim$ a few

Scalar field ϕ in FRW background

Red Spectrum Leads to dangerous Isocurvature Fluctuations

If WIMPZILLAS contribute to matter density, two sources of density fluctuations:

Curvature fluctuations from inflation

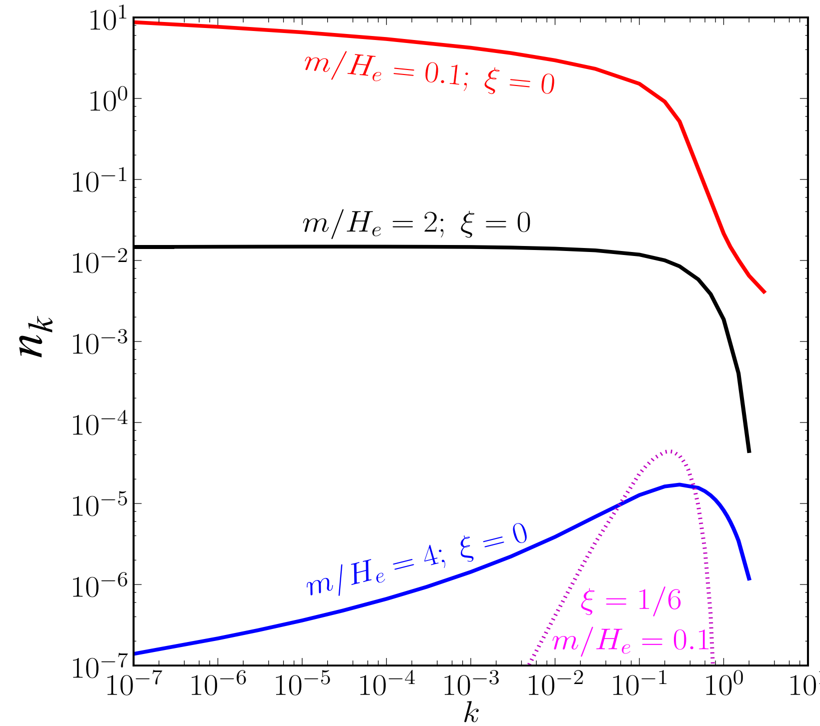
Fluctuations in χ field

They are uncorrelated.

DM density perturbations uncorrelated with baryon and photon perturbations.

For red spectrum, strict CMB limits

Chung, Kolb, Riotto, Senatore for minimally-coupled scalars



Assumes <u>before</u> reheating mode becomes

nonrelativistic $k/a < m$

and sub-Hubble $k/a < H$

Red spectrum survives "early" reheating.

Scalar Fields Not the Only Game in Town

spin-0 (real scalar)

Chung, Kolb, & Riotto (1998); Kuzmin & Tkachev (1998)

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} m^2 \varphi^2 + \frac{1}{2} \xi R \varphi^2$$

spin-$1/2$ (Dirac)

Chung, Kolb, & Riotto (1998); Kuzmin & Tkachev (1998); Chung, Everett, Yoo, & Zhou (2011)

$$\mathcal{L} = \frac{i}{2} \bar{\Psi} \underline{\gamma}^\mu (\nabla_\mu \Psi) - \frac{1}{2} m \bar{\Psi} \Psi + \mathrm{h.c.}$$

spin-1 (de Broglie-Proca)

Dimopoulos (2006) – not for DM; Graham, Mardon, & Rajendran (2016); Ahmed, Grzadkowski, & Socha (2020); Kolb & Long (2020)

$$\mathcal{L} = -\frac{1}{4} g^{\mu\alpha} g^{\nu\beta} F_{\mu\nu} F_{\alpha\beta} + \frac{1}{2} m^2 g^{\mu\nu} A_\mu A_\nu - \frac{1}{2} \xi_1 R g^{\mu\nu} A_\mu A_\nu - \frac{1}{2} \xi_2 R^{\mu\nu} A_\mu A_\nu$$

spin-$3/2$ (Rarita-Schwinger)

Kallosh, Kofman, Linde, & Van Proeyen (1999)
Giudice, Riotto, & Tkachev (1999); Lemoine (1999); Kolb & Long (2020)

$$\mathcal{L} = \frac{i}{4} \bar{\Psi}_\mu \left(\underline{\gamma}^\mu \underline{\gamma}^\rho \underline{\gamma}^\sigma - \underline{\gamma}^\sigma \underline{\gamma}^\rho \underline{\gamma}^\mu \right) (\nabla_\rho \Psi_\sigma) - \frac{1}{2} m \bar{\Psi}_\mu \underline{\gamma}^\mu \underline{\gamma}^\sigma \Psi_\sigma + \mathrm{h.c.}$$

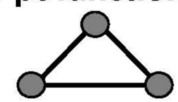
spin-2 (Fierz-Pauli)

DM: Kolb, Liang, Long & Rosen (2022); [see also Babichev, et al (2016)
Bernard, Deffayet, & von Strauss (2015); Mazuet & Volkov (2018)]

$$\mathcal{L} = \frac{1}{2} M_g^2 h_{\mu\nu} \left(\tilde{\mathcal{E}}^{\mu\nu\rho\sigma} + m^2 \mathcal{M}^{\mu\nu\rho\sigma} \right) h_{\rho\sigma}$$

Long-term Program

Complexity

* related to but not cosmological collider program	1-pt function 	2-pt function 	3-pt function* 
Observable	**Dark Matter**	**Isocurvature Fluctuations**	**CMB Non-Gaussianities**
Massive scalar field (conformal)	Chung, Kolb, & Riotto (98) Kuzmin & Tkachev (99)	Expected to be very small	Chung & Yoo
Massive scalar field (minimal)	Kuzmin & Tkachev (99)	Chung, Kolb, Riotto & Senatore	
Massive Dirac field	Chung, Kolb & Riotto (98, 99)	Similar to conformal scalar	Similar to conformal scalar?
Proca-de Broglie field	Massive: Kolb & Long Light: Graham, Mardon & Rajendran		
Massive Rarita-Schwinger field	Kolb, McDonough, Long		
Massive Fierz-Pauli field	Kolb, Ling, Long & Rosen (in progress)		

Complexity

Dirac field ψ in FRW background

Dirac Equation in FRW:

$$i\partial_\eta \begin{pmatrix} u_A(\eta) \\ u_B(\eta) \end{pmatrix} = \begin{pmatrix} a(\eta)m & k \\ k & -a(\eta)m \end{pmatrix} \begin{pmatrix} u_A(\eta) \\ u_B(\eta) \end{pmatrix}$$

Dispersion relation same as conformally-coupled scalar

$$\omega_k^2(\eta) = k^2 + m^2 a^2(\eta)$$

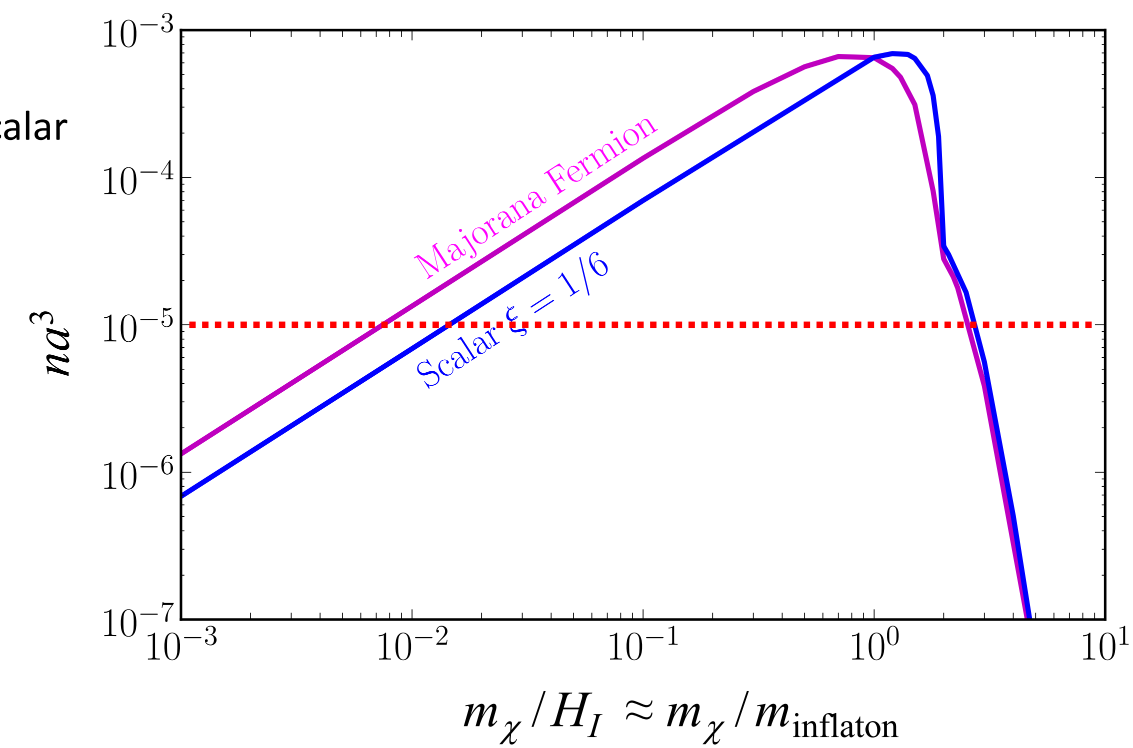
Adiabaticity parameter k/m × conformal scalar

$$A_k = \frac{a^2 H m k}{(k^2 + a^2 m^2)^{3/2}}$$

Blue spectrum: no isocurvature issues

Dirac WIMPZILLA DM candidate for $m_\chi = \mathcal{O}(m_{\text{inflaton}})$

$$\frac{\Omega_\chi h^2}{0.12} = \frac{m_\chi}{H_I}\left(\frac{H_I}{10^{12}\text{GeV}}\right)^2\left(\frac{T_{\text{RH}}}{10^9\text{GeV}}\right)\frac{[na^3]}{10^{-5}}$$



Fields with Spin $> 1/2$

For bosons, $\omega_k(\eta)$ tells all:

$$\omega_k^2(\eta) = \begin{cases} k^2 + a^2(\eta)m^2 + (\frac{1}{6} - \xi)a^2(\eta)R(\eta) & s = 0 \\ k^2 + a^2(\eta)m^2 \quad \text{Like conformally-coupled scalar: in massless limit no production} & s = 1 \quad \lambda = \pm 1 \\ k^2 + a^2(\eta)m^2 + \dfrac{1}{6}\dfrac{k^2 a^2(\eta)R(\eta)}{k^2 + a^2(\eta)m^2} + 3\dfrac{k^2 a^4(\eta)H^2(\eta)m^2}{(k^2 + a^2(\eta)m^2)^2} \quad \text{Interesting (i.e., complicated)} & s = 1 \quad \lambda = 0 \\ k^2 + a^2(\eta)m^2 + \frac{1}{6}a^2(\eta)R(\eta) \quad \text{Like minimally-coupled scalar; graviton in massless limit} & s = 2 \quad \lambda = \pm 2 \\ k^2 + a^2(\eta)m^2 + \frac{1}{6}\dfrac{a^2(\eta)(2k^2 + a^2(\eta)m^2)R(\eta)}{k^2 + a^2(\eta)m^2} - \dfrac{a^2(\eta)k^2(2k^2 - a^2(\eta)m^2)H^2(\eta)}{(k^2 + a^2(\eta)m^2)^2} & s = 2 \quad \lambda = \pm 1 \\ \text{way, way too long to show} & s = 2 \quad \lambda = 0 \end{cases}$$

de Broglie—Proca field A_μ in FRW background

Graham, Mardon, & Rajendran (2016); Ahmed, Grzadkowski, & Socha (2020); Kolb & Long (2020)

Dispersion relation for transverse modes, $\omega_k^2 = k^2 + a^2(\eta)m^2$, same as conformally-coupled scalar.

Dispersion relation for longitudinal modes, $\omega_k^2 = k^2 + a^2(\eta)m^2 + \frac{1}{6}\frac{k^2a^2(\eta)R(\eta)}{k^2+a^2(\eta)m^2} + 3\frac{k^2a^4(\eta)H^2(\eta)m^2}{(k^2+a^2(\eta)m^2)^2}$, "interesting."

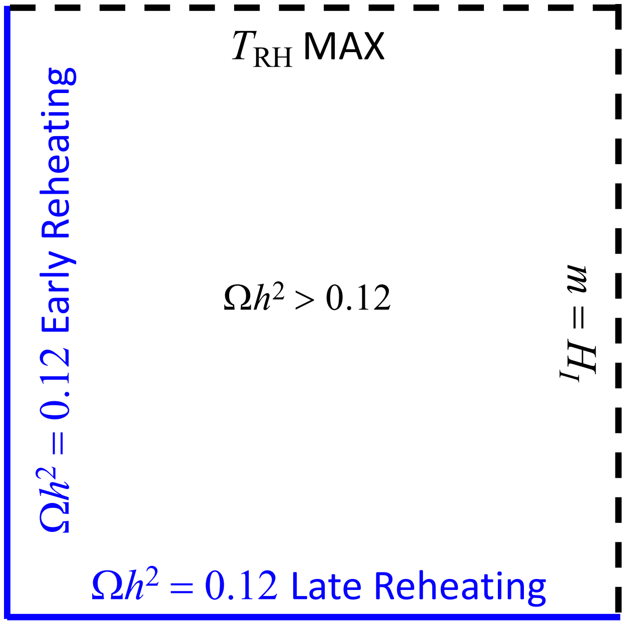
- power spectrum is blue-tilted at low k
- negligible power at CMB scales
- no problem with isocurvature even for m << H_I

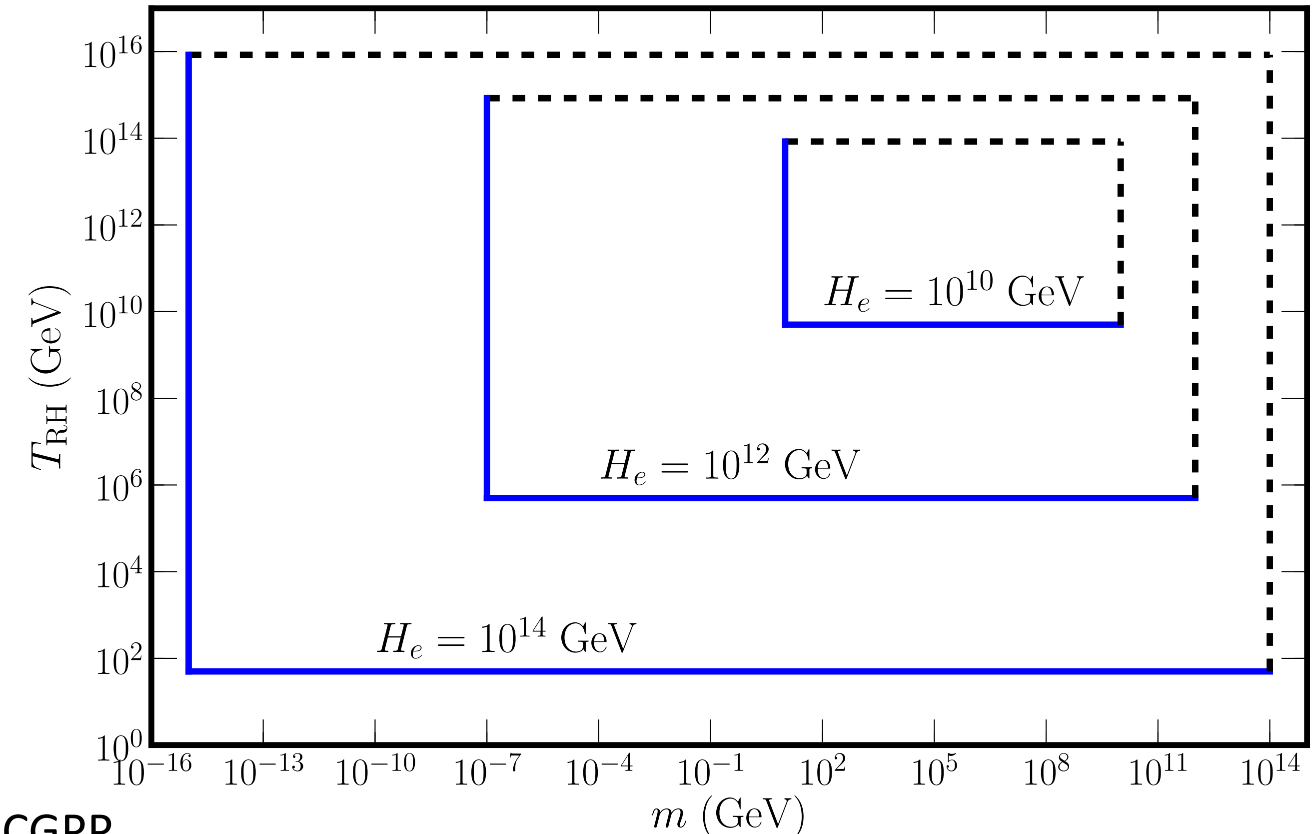
Late Reheating $$\frac{\Omega h^2}{0.12} = \left(\frac{H_I}{10^{11}\mathrm{GeV}}\right)^2 \left(\frac{T_{\mathrm{RH}}}{5\times 10^7\mathrm{GeV}}\right) \qquad \left(T_{\mathrm{RH}} < 8.4\times 10^8 \left(\frac{m}{\mathrm{GeV}}\right)^{1/2}\mathrm{GeV}\right)$$

Early Reheating $$\frac{\Omega h^2}{0.12} = \left(\frac{m}{10^{-6}\mathrm{eV}}\right)^{1/2} \left(\frac{H_I}{10^{14}\mathrm{GeV}}\right)^2 \qquad \left(T_{\mathrm{RH}} > 8.4\times 10^8 \left(\frac{m}{\mathrm{GeV}}\right)^{1/2}\mathrm{GeV}\right)$$

Ωh^2 depends on m, H_I, and T_{RH}

de Broglie—Proca field A_μ in FRW background





Kolb & Long (2020)

Very light (μeV) DM from CGPP

or

Very massive (10^{14} GeV) DM from CGPP

Fierz-Pauli field $f_{\mu\nu}$ in FRW background

- It was believed that massive gravity theories had ghostly 6th degree of freedom at the nonlinear level until de Rahm, Gabadadze & Trolly (dRGT 2011)
- Hassan & Rosen (2012) showed how to construct ghost-free "bimetric" theories with correct number of propagating d.o.f.
- We (with Ling, Long, and Rosen) are examining two massive spin-2 theories à la Hassan & Rosen

Minimal coupling

$$S[g,f,\Phi] = \int d^4x \left[m_g^2 \sqrt{-g}\, R(g) + m_f^2 \sqrt{-f}\, R(f) - 2m^4 \sqrt{-g}\, V(\mathbb{X};\beta_n) + \sqrt{-g}\, \mathcal{L}_m(g,\Phi) + \sqrt{-f}\, \mathcal{L}_m(f,\Phi) \right]$$

$$\mathbb{X}^\mu{}_\nu = \sqrt{g^{\mu\lambda} f_{\lambda\nu}}$$

Non-minimal coupling

$$S[g,f,\Phi] = \int d^4x \left[m_g^2 \sqrt{-g}\, R(g) + m_f^2 \sqrt{-f}\, R(f) - 2m^4 \sqrt{-g}\, V(\mathbb{X};\beta_n) + \sqrt{-g^{\mathrm{eff}}}\, \mathcal{L}_m(g^{\mathrm{eff}},\Phi) \right]$$

$$g^{\mathrm{eff}}_{\mu\nu} = a^2 g_{\mu\nu} + 2ab(g\mathbb{X}) + b^2 f_{\mu\nu}$$

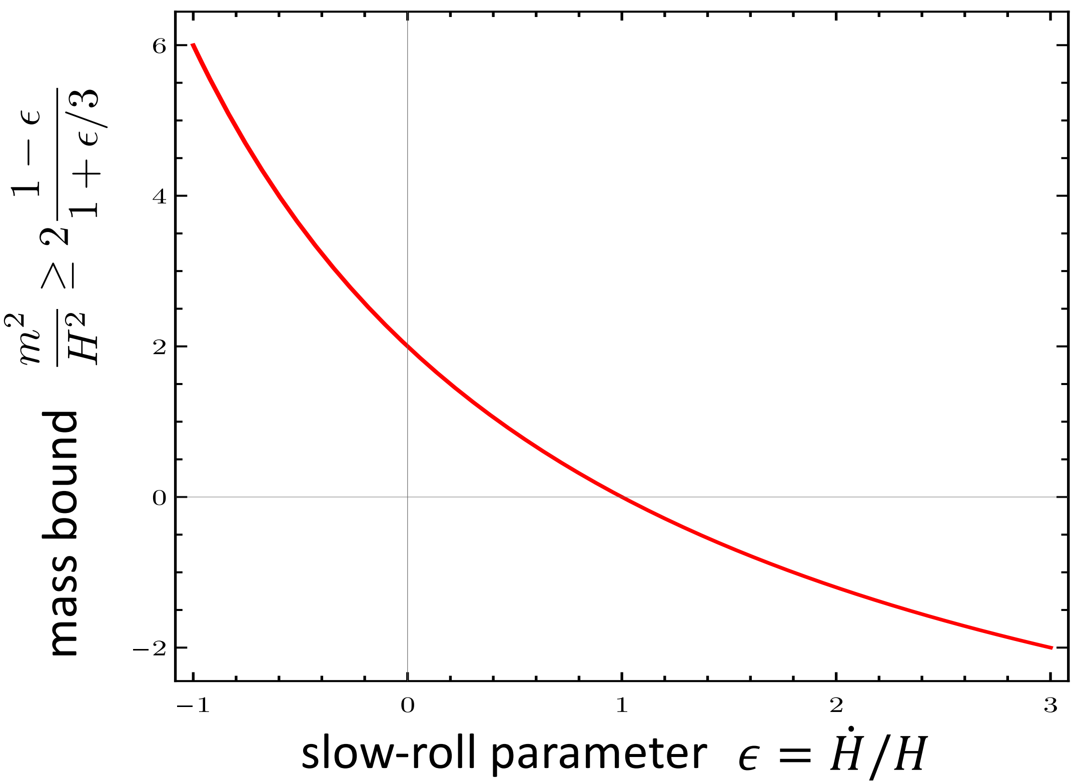
- Looking for WIMPZILLAS, but interesting things along the way

Fierz—Pauli field $f_{\mu\nu}$ in FRW background

- In 1987 Higuchi demonstrated that for fields of spin two or greater in de Sitter space, there are ghosts unless

$$m > 2H$$

- We (with Ling, Long, and Rosen) generalize the Higuchi bound to FRW:



$$m^2 \geq 2H^2 \, \frac{H^2 + \dot{H}}{H^2 - \dot{H}/3} = 2H^2 \frac{1-\epsilon}{1+\epsilon/3}$$

$$\epsilon = \begin{cases} 0 & \text{dS} \\ -3/2 & \text{MD} \\ -2 & \text{RD} \end{cases}$$

Cosmological Limit on the mass of massive spin-2 field!

Rarita—Schwinger field ψ_μ in FRW background

"Dirac" Equation in FRW:

$$i\partial_\eta \begin{pmatrix} u_A(\eta) \\ u_B(\eta) \end{pmatrix} = \begin{pmatrix} a(\eta)m & k \\ k & -a(\eta)m \end{pmatrix} \begin{pmatrix} u_A(\eta) \\ u_B(\eta) \end{pmatrix}$$

$s = 3/2$; $\lambda = \pm\, 3/2$ (same as $s = 1/2$)

$$i\partial_\eta \begin{pmatrix} u_A(\eta) \\ u_B(\eta) \end{pmatrix} = \begin{pmatrix} a(\eta)m & (C_A + iC_B)k \\ (C_A - iC_B)k & -a(\eta)m \end{pmatrix} \begin{pmatrix} u_A(\eta) \\ u_B(\eta) \end{pmatrix}$$

nonzero for gravitino

$s = 3/2$; $\lambda = \pm\, 1/2$

C_A & C_B function of (H, m, R, $\partial_\eta m$)

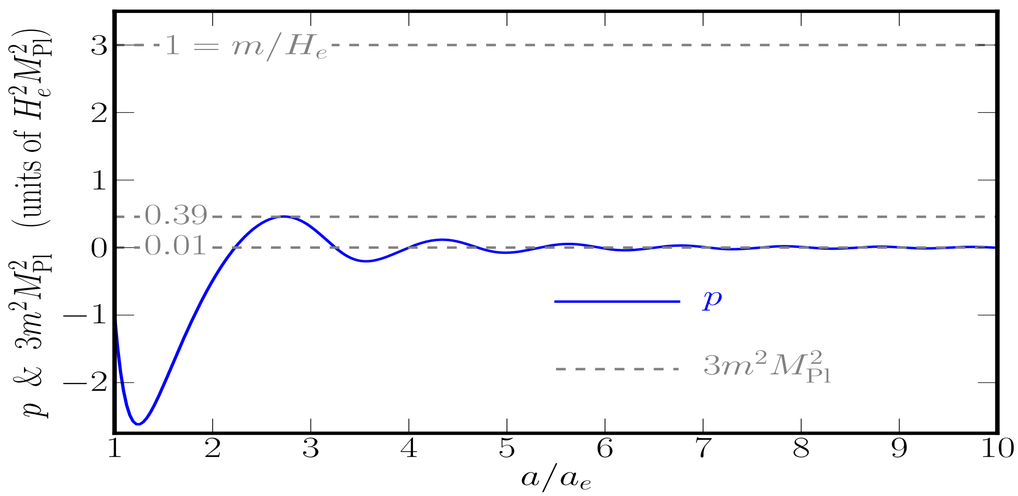
$C_A{}^2 + C_B{}^2 = c_s{}^2 =$ sound speed

New feature: $c_s = \dfrac{\left|p(\eta) - 3m^2 M_{\rm Pl}^2\right|}{\rho(\eta) + 3m^2 M_{\rm Pl}^2}$ time-dependent effective sound speed!

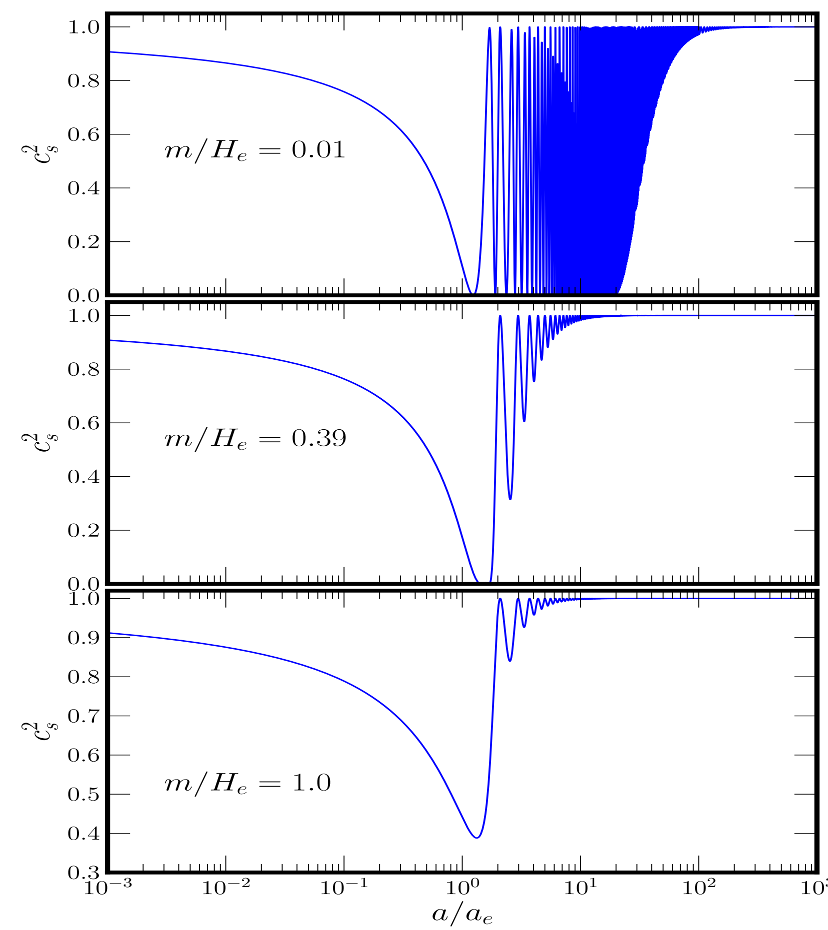
Can vanish when $p = 3m^2 M_{\rm Pl}^2$!!

Rarita—Schwinger field ψ_μ in FRW background

$$c_s = \frac{\left|p(\eta) - 3m^2 M_{\rm Pl}^2\right|}{\rho(\eta) + 3m^2 M_{\rm Pl}^2}$$



vanishing sound speed



Sound speed will vanish (perhaps many times) if $m < 0.39\ H_e$
(assumes harmonic potential after inflation)

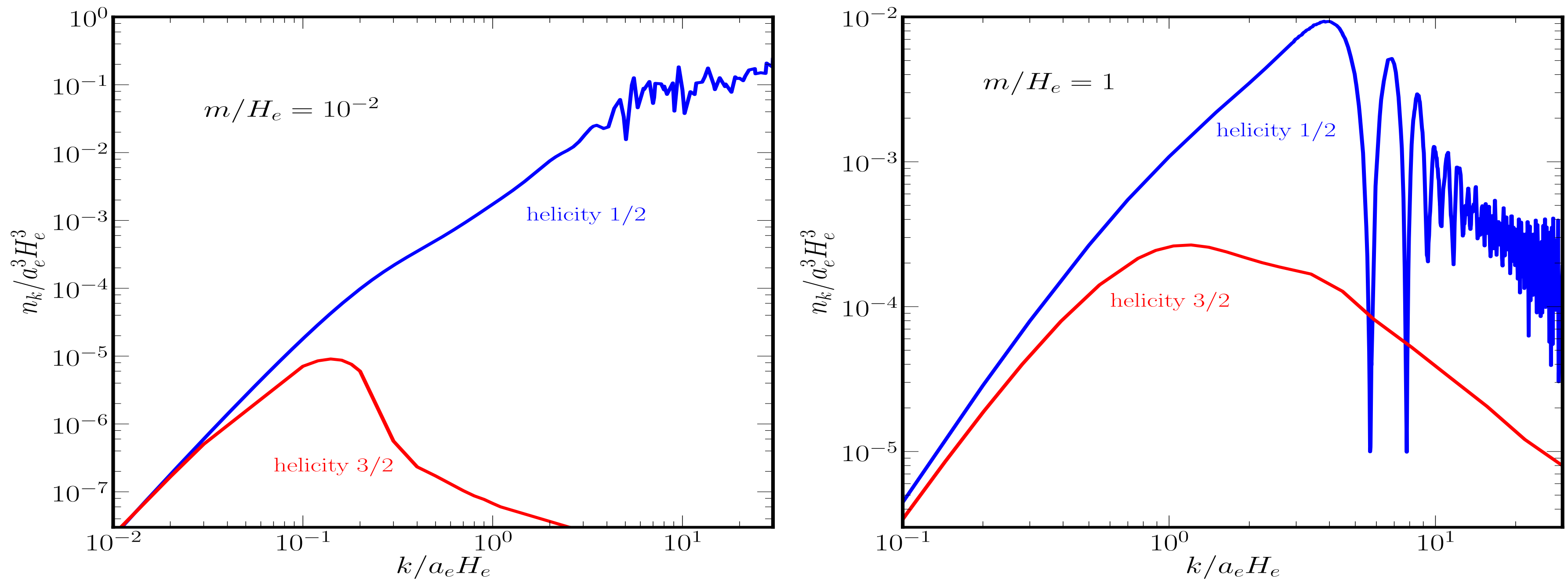
Rarita—Schwinger field ψ_μ in FRW background

Dispersion relation is $\omega_k^2(\eta) = c_s^2 k^2 + a^2(\eta) m^2$

Usual case: $c_s^2 = 1 \Rightarrow \omega_k(\eta) = k$ and constant for $k \Rightarrow \infty$

CGPP depends on changing $\omega_k(\eta)$, so no production of high-k modes!

If $c_s^2 = 0$: as $k \Rightarrow \infty$, $\omega_k(\eta)$ is independent of k, production of high-k modes unsuppressed!



Rarita—Schwinger field ψ_μ in FRW background

Supergravity employs spin-3/2 field (gravitino, inflation, …), the superpartner to graviton.

Catastrophic production of gravitinos dependent on model.

For models with a single chiral superfield gravitino mass is time dependent ($\partial_\eta m \neq 0$).

$c_s = 1$ at all times $\Longrightarrow$ no catastrophic production

For models with multiple chiral superfields (most modern models)

c_s depends on relative orientation of inflaton direction & susy breaking

$c_s = 0$ in models with a nilpotent superfield and orthogonal constraint KKLT

mixing between the goldstino & inflatino may avoid the catastrophe (explicit calculation needed)

Dudas, Garcia, Mambrini, Olive, Peloso, & Verner (2021); Antoniadis, Benakli, & Ke (2021)

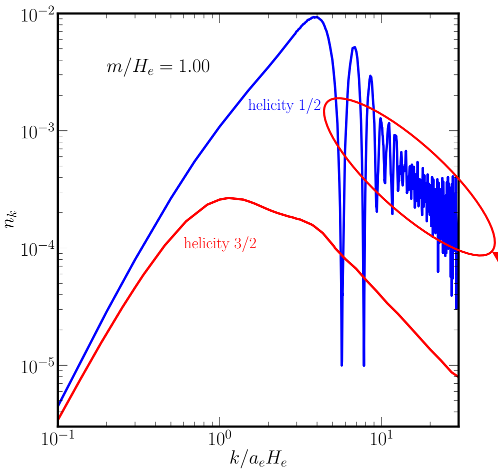
Models with $c_s = 0$ are in a SWAMPLAND! Kolb, Long, & McDonough (2021)

CGPP may provide constraints on SUGRA model building.

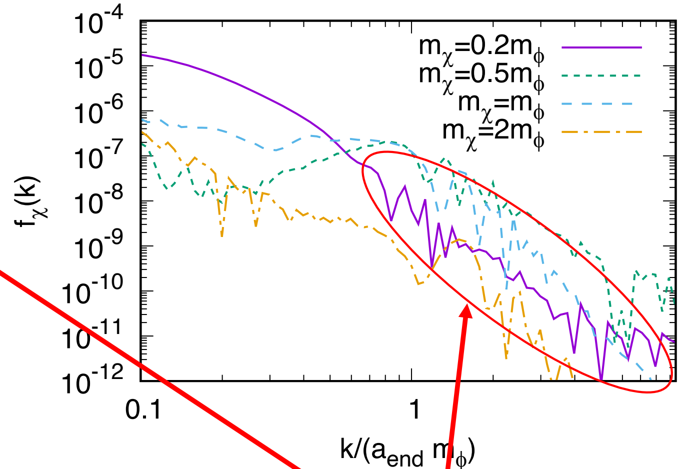
Quantum interference in gravitational particle production

(Basso, Chung, Kolb, Long)

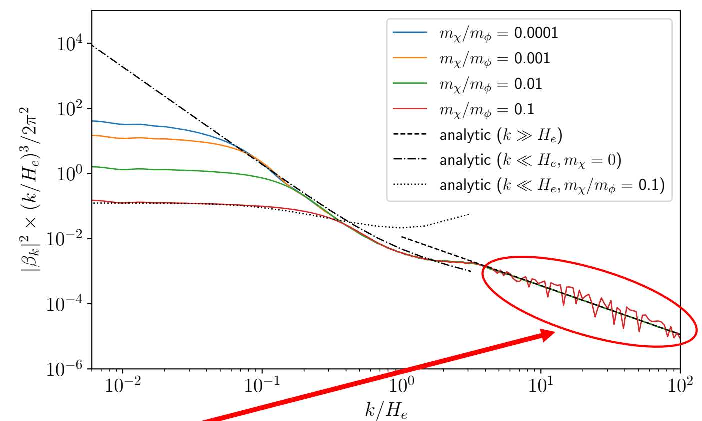
Catastrophic Production of Slow Gravitinos
Kolb, Long, Mcdonough



Production of Purely Gravitational Dark Matter
Ema, Nakayama, Tang

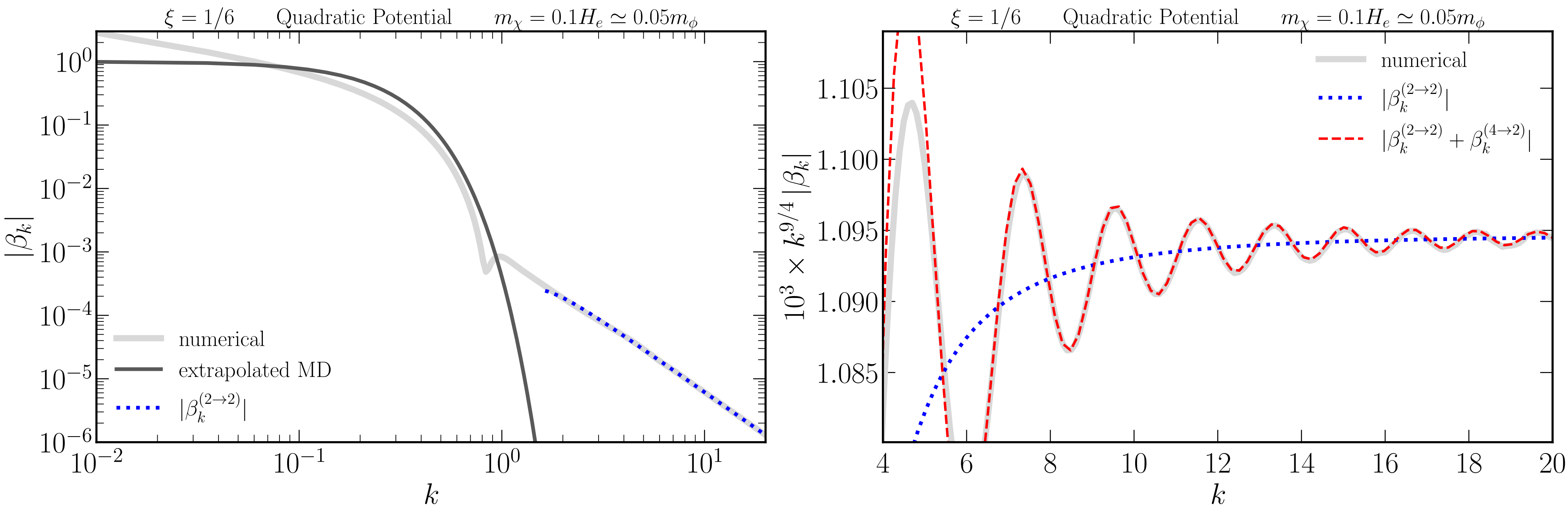


Boltzmann or Bogoliubov?
Kaneta, Mook, Oda



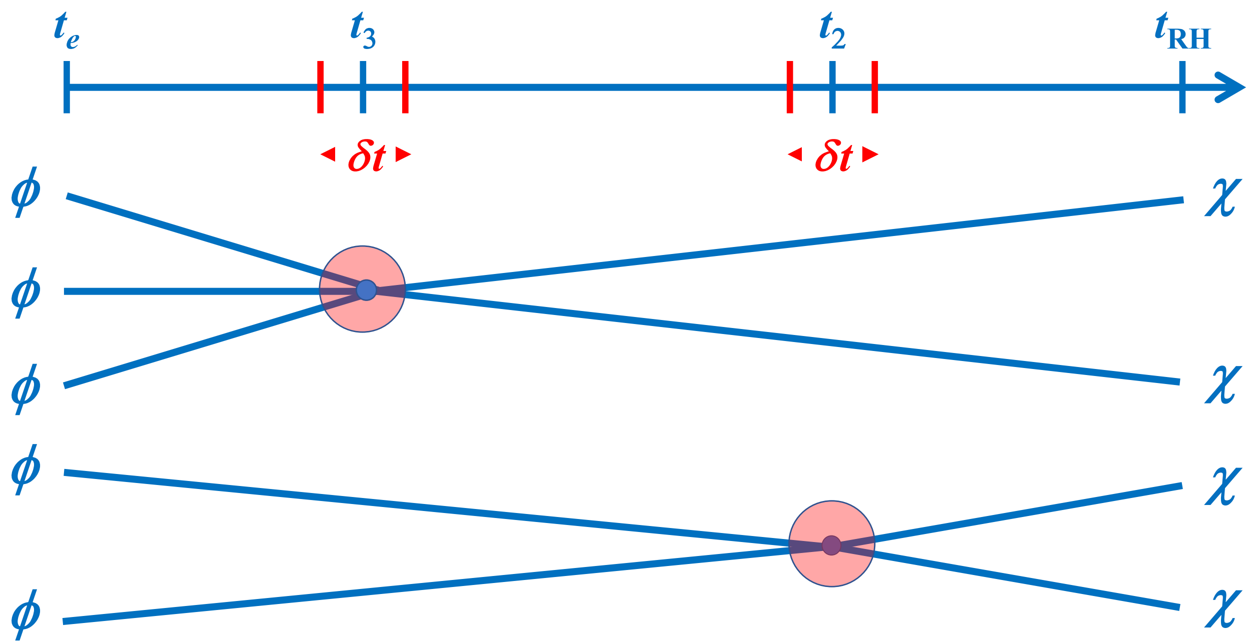
WTF? (Why These Features?) also, power-law decrease instead of exponential

We argue that these features are due to the quantum interference of coherent scattering reactions. We find analytic formulae for the particle production amplitude for a conformally-coupled scalar field, including an interference effect in the kinematic region where the production can be interpreted as inflaton scattering into scalar final states via graviton exchange.

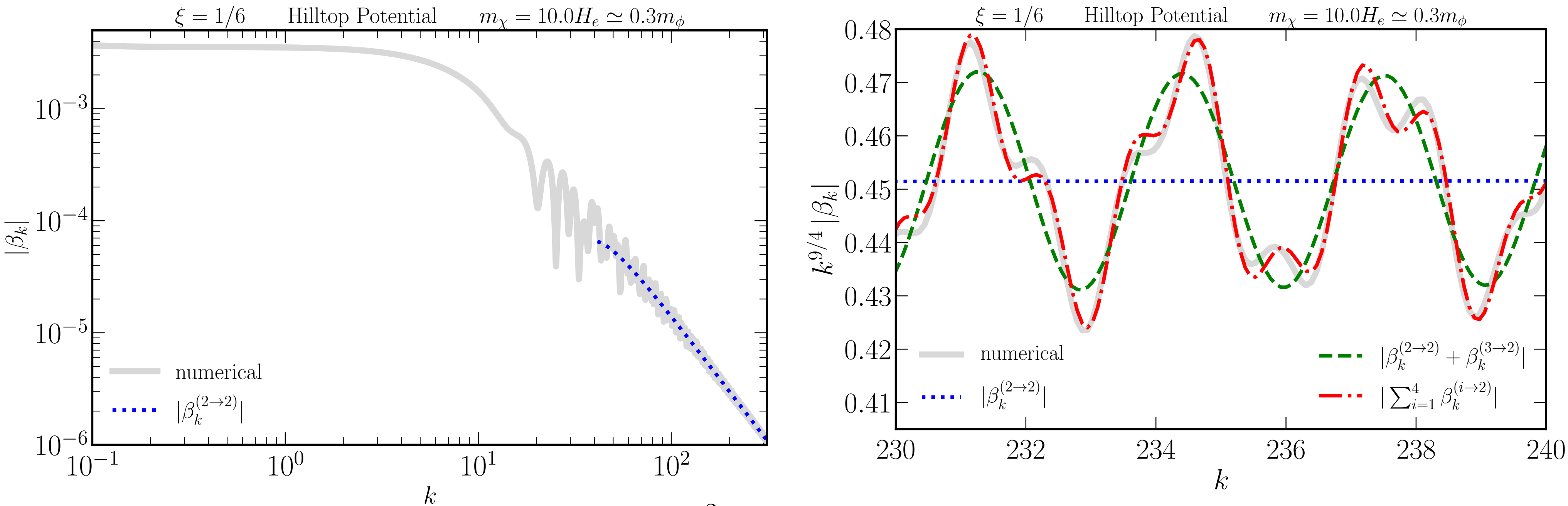


- Extrapolated MD (matter dominated-no inflaton oscillations) $|\beta_k| \propto \exp(-k^{3/2})$ for $k > 1$
- Numerical (quadratic inflaton potential with inflaton oscillations) $|\beta_k| \propto k^{-9/4}$ for $k > 1$
- Power-law behavior can be understood as $\phi + \phi \to \chi + \chi$ via a classical Boltzmann approach
- But, $\phi + \phi \to \chi + \chi$ via a classical Boltzmann approach cannot explain oscillations
- Oscillations due to quantum interference $|\, c_1 \langle \chi\chi | U | \phi\phi \rangle + c_2 \langle \chi\chi | U | \phi\phi\phi\phi \rangle \,|^2$

Quantum interference in gravitational particle production



- Initial macroscopic inflaton scattering state can be viewed as cold coherent superposition of $n\phi$ states
- Bogoliubov treatment allows processes that can be interpreted as $\left| \, c_1\langle\chi\chi|U|\phi\phi\rangle + c_2\langle\chi\chi|U|\phi\phi\phi\phi\rangle \, \right|^2$



- Hilltop inflaton potential $V \sim \left(1 - \frac{\phi^6}{v^6}\right)^2$ effective cubic term
- Quantum interference much more pronounced

A Question

What to make of a QFT that is “perfectly” reasonable in Minkowski,

but

Pathological in FLRW?
(examples: Fierz-Pauli, spin-2 with small mass)

Finally, Summary: GPP can make DM & constrain BSM physics!

Dark matter might have only gravitational interactions (that's all we really "know")

If so, dark matter must have a gravitational origin.

Cosmological Gravitational Particle Production through Schrödinger's alarming phenomenon promising.

Scalars:

Conformally-coupled: promising DM candidate if $m \approx H_I$ (WIMPZILLA miracle).

Minimally-coupled: not promising DM candidate.

Late reheating: Ωh^2 much too large unless $m \gtrsim$ few H_I. Early reheating: Ωh^2 much too large unless $m \gtrsim$ few H_I.

Isocurvature constraints unless $m \gtrsim$ few H_I.

Dirac fermions:

Similar to conformally-coupled scalars: promising DM candidate if $m \approx H_I$ (WIMPZILLA miracle).

de Broglie—Proca vectors:

DM candidate could be very light ($\mu\mathrm{eV}$) or very massive (H_I)

Rarita-Schwinger fermions:

Catastrophic production if c_s vanishes. Implications for models of supergravity.

Fierz-Pauli tensors:

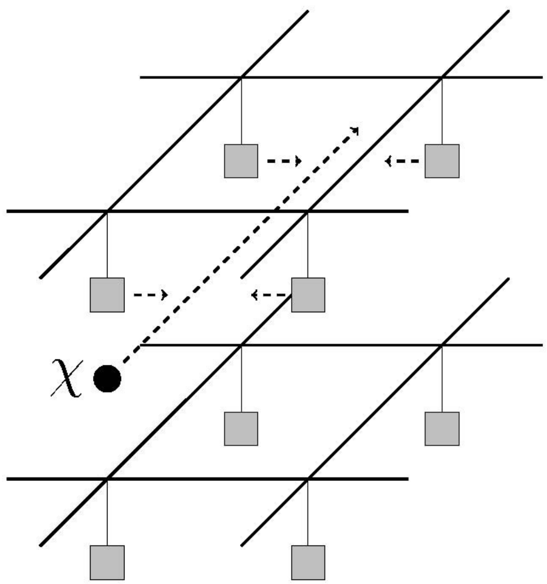
FRW-generalization of the Higuchi bound; DM relic abundance in progress.

Spin greater than 2: Alexander, Jenks, McDonough

Detect WIMPzillas with only gravitational coupling?

“Gravitational Direct Detection of Dark Matter”
Carney, Ghosh, Krnjaic, Taylor arXiv: 1903.00492

$$\mathrm{SNR}^2 = 10^4 \left(\frac{M_\chi}{1\ \mathrm{mg}}\right)^2 \left(\frac{M_D}{1\ \mathrm{mg}}\right)^2 \left(\frac{1\ \mathrm{mm}}{d}\right)^4$$



Meter-scale detector

Billion microgram to milligram sensors

Lattice spacing millimeter to centimeter

Detect DM of mass greater than Planck mass

How about 10^{-6} Planck mass?

Coming soon to a Reviews of Modern Physics Near You

"Cosmological Gravitational Particle Production and its Implications for the Origin of Dark Matter"

With Andrew Long 2023

Gravitational Origin of Dark Matter

Rocky Kolb

9/19/2022

VI José Plínio Baptista School on Cosmology
Friedmann and the LCDM model

Some Notation

FLRW model $$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2\right]$$

My convention: scale factor a has dimension of length. An observer "at rest" has constant (r, θ, ϕ) "comoving coordinates"

In early universe $k = 0$

Define conformal time $$a d\eta = dt$$

Expansion rate $$H = \frac{\dot{a}}{a} = \frac{a'}{a^2}$$ prime indicates derivative wrt conformal time

Gravitational Particle Production

"Outstanding importance"?

Schrödinger 1939: "Generally speaking this is a phenomenon of outstanding importance. With particles it would mean the production or annihilation of matter, merely by the expansion." [why would that be of outstanding importance?]

Forgotten in 40s, 50s, 60s (by Schrödinger also).

"Great Cosmological Significance"?

Leonard Parker Thesis 1966. In 1968 paper: "…for the early stages of a Friedmann expansion it [particle creation] may well be of great cosmological significance, especially since it seems inescapable if one accepts quantum field theory and general relativity." [no speculation as to the "great cosmological significance"]

Zel'dovich 1970s proposed an application: explains homogeneity and isotropy of the universe.

Gravitational Particle Production

Other interest in CGPP in the 1970s (mostly regarded as a curiosity).

US:	Parker, Ford, Fulling, Allen, Friedman, Wald, …
Soviet Union:	Zel'dovich, Starobinski, Grishchuk, Grib, Mostepanenko, Lukash, … (CGPP in the CCCP)
UK:	Bunch, Davies, Birrell, Hawking, …

Great cosmological significance in the 1980s (inflation):

Mukhanov & Chibisov

Gravitational Particle Production

Great cosmological significance in the 1980s (inflation):

Mukhanov & Chibisov

Gravitational Particle Production universal*

Could there be more?

Gravitational Particle Production not a large effect (cf. curvature perturbations ≈ 10^{-5})

What else could be observable?

Dark matter

CMB Isocurvature perturbations

CMB Nongaussianities

* So long as Weyl conformal symmetry violated.