Cosmology in the holographic braneworld

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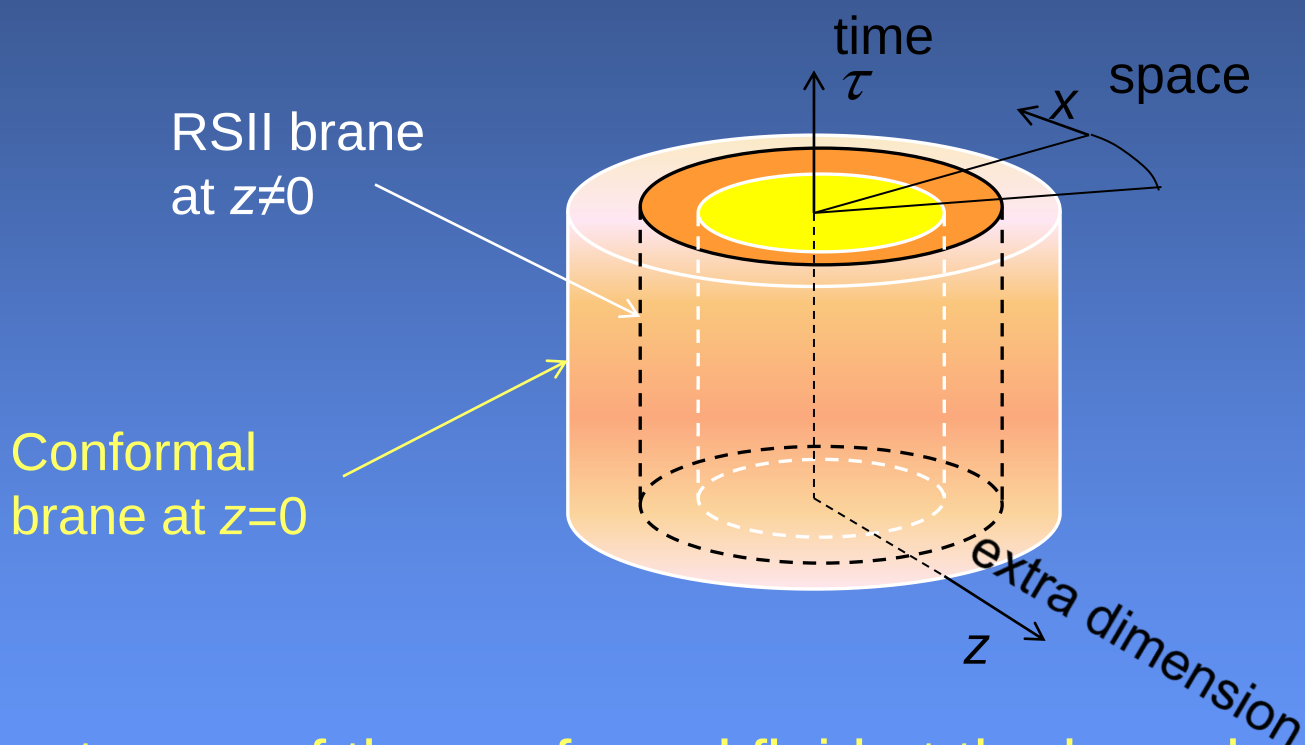
Basic idea

Holographic braneworld is a 3-brane located at the boundary of the asymptotic AdS_5. The cosmology is governed by matter on the brane in addition to the boundary CFT



As the stress tensor of the conformal fluid at the boundary is determined by the geometry of the bulk we expect a deviation from the standard FRW cosmology on the brane.

Based on:

N.B., *Randall-Sundrum versus holographic cosmology*, Phys. Rev. D 93, (2016) arXiv:1511.07323
N.B. et al., *Tachyon inflation in the holographic braneworld*, JCAP08(2019)034, arXiv:1809.0721,
N. R. Bertini, N.B., D. C. Rodrigues, *Cosmological perturbations in the Holographic braneworld* , work in preparation

and related earlier works

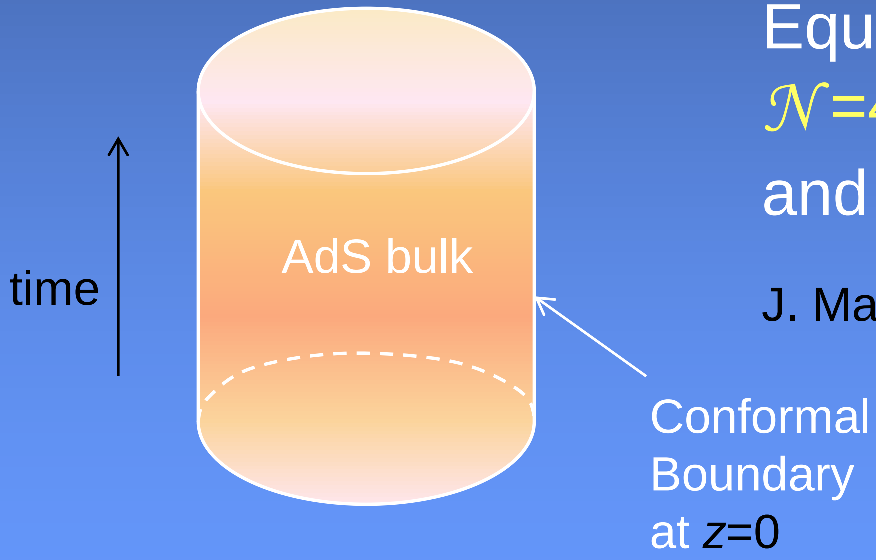
P.S. Apostolopoulos, G. Siopsis and N. Tetradis, Phys. Rev. Lett. **102** (2009), arXiv:0809.3505
P. Brax and R. Peschanski, Acta Phys. Polon. **B 41** (2010) arXiv:1006.3054

Outline

1. AdS/CFT
2. Holographic field equations
3. Holographic cosmology
4. Conclusions and outlook

AdS/CFT

Anti de Sitter/Conformal Field Theory (AdS/CFT) correspondence is a holographic duality between gravity in d+1-dim space-time and quantum CFT on the d-dim boundary. Original formulation stems from string theory:



Equivalence of 3+1-dim
$\mathcal{N}$=4 Supersymmetric YM Theory
and string theory in $AdS_5 \times S_5$

J. Maldacena, Adv. Theor. Math. Phys. **2** (1998)

Consider an action in the asymptotically AdS$_5$ bulk

$$S = \frac{1}{8\pi G_5} \int d^5x \sqrt{-G} \left(-\frac{R^{(5)}}{2} - \Lambda_5 \right)$$

Given induced metric $h_{\mu\nu}$ on the AdS$_5$ boundary the geometry is completely determined by the field equations obtained from the variation principle

$$\frac{\delta S}{\delta G_{\mu\nu}} = 0$$

yielding a solution $G_{\mu\nu}[h]$. Using this we define a functional

$$S[h] = S^{\text{shell}}\left[G_{\mu\nu}[h]\right]$$

where $S^{\text{shell}}\left[G_{\mu\nu}[h]\right]$ is the on shell bulk action

AdS/CFT conjecture: the action $S[h]$ can be identified with the generating functional of a conformal field theory (CFT) on the boundary.

The induced metric $h_{\mu\nu}$ serves as the source for the **energy-momentum tensor** of the dual CFT so that its vacuum expectation value is obtained from the on shell classical action

$$\left\langle T_{\mu\nu}^{\mathrm{CFT}} \right\rangle = \frac{1}{2\sqrt{-h}} \frac{\delta S}{\delta h^{\mu\nu}}$$

Holographic field equations

The on-shell action is IR divergent and must be regularized and renormalized. The asymptotically AdS metric in the Fefferman-Graham form is

$$ds^2_{(5)} = G_{ab}dx^a dx^b = \frac{\ell^2}{z^2}(g_{\mu\nu}dx^\mu dx^\nu - dz^2)$$

where the length scale ℓ is the AdS curvature radius. Near z=0 the metric can be expanded as

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + z^2 g^{(2)}_{\mu\nu} + z^4 g^{(4)}_{\mu\nu} + \cdots$$

Explicit expressions for $g^{(2n)}_{\mu\nu}$, in terms of $g^{(0)}_{\mu\nu}$ can be found in

S. de Haro, S.N. Solodukhin, K. Skenderis, Comm. Math. Phys. **217** (2001)

We regularize the action by placing a brane (RSII brane) near the AdS boundary, i.e., at $z = \varepsilon\ell$, $\varepsilon \ll 1$, so that the induced metric on the brane is

$$h_{\mu\nu} = \frac{1}{\varepsilon^2}\left(g^{(0)}_{\mu\nu} + \varepsilon^2\ell^2 g^{(2)}_{\mu\nu} + \cdots\right)$$

The regularized bulk action (RSII action) is then given by

$$S^{\mathrm{reg}}[h] = \frac{1}{8\pi G_5}\int_{z\geq\varepsilon\ell} d^5x\sqrt{-G}\left(-\frac{R^{(5)}}{2} - \Lambda_5\right) + S_{\mathrm{GH}}[h] + S_{\mathrm{br}}[h]$$

$h_{\mu\nu}$ – induced metric on the brane

$S_{\mathrm{GH}}[h]$ – Gibbons-Hawking boundary term

$S_{\mathrm{br}}[h] = -\sigma\int d^4x\sqrt{-h} + \int d^4x\sqrt{-h}\,\mathcal{L}_{\mathrm{matt}}$ – brane action

Next we renormalize the boundary action. The renormalized action is obtained by adding counter-terms and taking the limit $\varepsilon \to 0$

$$S^{\text{reg}}[h] = \lim_{\varepsilon \to 0}(S^{\text{ren}}[h] + S_1[h] + S_2[h] + S_3[h])$$

The necessary counter-terms are

$$S_1[h] = -\frac{6}{16\pi G_5 \ell} \int d^4x \sqrt{-h},$$

$$S_2[h] = -\frac{\ell}{16\pi G_5} \int d^4x \sqrt{-h} \left(-\frac{R[h]}{2} \right),$$

$$S_3[h] = -\frac{\ell^3}{16\pi G_5} \int d^4x \sqrt{-h} \frac{\log \epsilon}{4} \left(R^{\mu\nu}[h] R_{\mu\nu}[h] - \frac{1}{3} R^2[h] \right)$$

S.W. Hawking, T. Hertog, and H.S. Reall, Phys. Rev. D **62** (2000)

Now we demand that the variation with respect to the induced metric $h^{\mu\nu}$ of the regularized on shell bulk action (RSII action) vanishes, i.e.,

$$\delta S^{\mathrm{reg}}[h] = 0$$

This may be expressed as

matter on the brane

$$\delta\left[S^{\mathrm{ren}} - S_3 - \left(\sigma - \frac{3}{8\pi\ell G_5}\right)\int d^4x\sqrt{-h} + \int d^4x\sqrt{-h}\,\mathcal{L}_{\mathrm{matt}} + \frac{\ell}{16\pi G_5}\int d^4x\sqrt{-h}\,\frac{R}{2}\right] = 0.$$

cosmological Constant term

Einstein Hilbert term

AdS/CFT prescription

$$\delta(S^{\mathrm{ren}} - S_3) = \frac{1}{2}\int d^4x\sqrt{-h}\left\langle T^{\mathrm{CFT}}_{\mu\nu}\right\rangle \delta h^{\mu\nu}$$

The variation of the scheme-dependent S_3 combined with $S^{\rm ren}$ yields

$$\frac{2}{\sqrt{-h}}\frac{\delta(S^{\rm ren}-S_3)}{\delta h^{\mu\nu}}=\left\langle T^{\rm CFT}_{\mu\nu}\right\rangle$$

where

$$\langle T^{\rm CFT}_{\mu\nu}\rangle=-\frac{\ell^3}{4\pi G_5}\left\{g^{(4)}_{\mu\nu}-\frac{1}{8}\left[({\rm Tr}g^{(2)})^2-{\rm Tr}(g^{(2)})^2\right]g^{(0)}_{\mu\nu}-\frac{1}{2}(g^{(2)})^2_{\mu\nu}+\frac{1}{4}{\rm Tr}g^{(2)}g^{(2)}_{\mu\nu}\right\}$$

de Haro et al, Comm. Math. Phys. **217** (2001)

$g^{(2n)}_{\mu\nu}$ are the coefficients that appear in the expansion of the bulk metric. They can be expressed in terms of the boundary metric $g^{(0)}_{\mu\nu}$ and associated Ricci tensor. For example

$$g^{(2)}_{\mu\nu}=\frac{1}{2}\left(R_{\mu\nu}-\frac{1}{6}Rg^{(0)}_{\mu\nu}\right)\qquad g^{(4)}_{\mu\nu}=\frac{1}{4}\left(g^{(2)}\right)^2_{\mu\nu}+\tilde{g}_{\mu\nu}$$

where $\tilde{g}_{\mu\nu}$ is traceless and is absent if the bulk metric is conformally flat (e.g., for a pure AdS_5 and an FRW boundary metric)

The variation of the action yields field equations on the boundary

$$R_{\mu\nu} - \frac{1}{2} R g^{(0)}_{\mu\nu} = 8\pi G_{\mathrm{N}} \left(\left\langle T^{\mathrm{CFT}}_{\mu\nu} \right\rangle + T^{\mathrm{matt}}_{\mu\nu} \right)$$

More explicitly

$$R_{\mu\nu} - \frac{1}{2} R g^{(0)}_{\mu\nu} \underbrace{- \frac{\ell^2}{4} \left[\frac{2}{3} R R_{\mu\nu} - R_{\mu\rho} R^{\rho}_{\nu} + \frac{1}{4} \left(2 R_{\alpha\beta} R^{\alpha\beta} - R^2 \right) g^{(0)}_{\mu\nu} \right]}_{\text{modification of Einstein's equations}}$$
$$= 8\pi G_{\mathrm{N}} \left(T^{\mathrm{matt}}_{\mu\nu} + t_{\mu\nu} \right)$$

where $t_{\mu\nu}$ is a traceless part of $T^{\mathrm{CFT}}_{\mu\nu}$ and is absent if the bulk metric is conformally flat. It contributes to the so called dark radiation if there is an AdS Schwarzschild BH in the bulk

Holographic cosmology

We now seek a cosmological solution to the holographic Einstein equations such that the induced metric at the boundary has the FRW form

$$ds^2 = g^{(0)}_{\mu\nu} dx_\mu dx_\nu = dt^2 - a^2(t) d\Omega_\kappa^2$$

From the Einstein equations at the boundary we obtain the holographic Friedmann equation

$$\mathcal{H}^2 - \frac{\ell^2}{4}\mathcal{H}^4 = \frac{8\pi G}{3}\rho + \frac{4\mu\ell^2}{a^4} \qquad \text{where} \quad \mathcal{H}^2 \equiv H^2 + \frac{\kappa}{a^2}$$

quartic deviation

dark radiation

E. Kiritsis, JCAP **0510** (2005); Apostolopoulos et al, Phys. Rev. Lett. **102**, (2009); N.B., Phys. Rev. D 93 (2016), arXiv:1511.07323

The second Friedmann equation can be derived from the energy-momentum conservation

For simplicity, from now on we assume spatial flatness with $\kappa = 0$

The holographic cosmology has interesting properties. Solving the first Friedmann equation as a quadratic equation for H^2 we find

$$H^2 = \frac{2}{\ell^2}(1 \pm \sqrt{1 - 8\pi\ell^2 G\rho / 3})$$

Demanding that this equation reduces to the standard Friedmann equation in the low energy limit, i.e., in the limit when

$$\rho \ll \frac{1}{\ell^2 G}$$

we have to discard the + sign solution. Then, it follows that the physical range of the Hubble rate is between 0 and $\sqrt{2} / \ell$ starting from its maximal value $H_{\max} = \sqrt{2} / \ell$ at an arbitrary initial time t_0. At that time, which may be chosen to be zero, the density and cosmological scale are both finite so the Big-Bang singularity is avoided!

Modified Gauss-Bonnet gravity

Holographic cosmology is related to a modified gravity of the type

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{16\pi G}\left(-R + f(R,\mathcal{G})\right) + \mathcal{L}^{\mathrm{matt}}\right]$$

where $\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ is the Gauss-Bonnet invariant

Models of this type were shown to be ghost free.

D. Comelli, Phys. Rev. D 72, 064018 (2005), gr-qc/0505088.
I. Navarro and K. Van Acoleyen, Phys. Lett. B 622, 1 (2005),gr-qc/0506096.

If one requires that the second Friedmann equation is linear in $\dot{H}$ (cannot be fulfilled in a simple *f*(*R*) modified gravity including the Starobinski model), then *f* turns out to be a function of only one variable *f* = *f*(*J*) where

$$J = \frac{1}{\sqrt{12}}\left(-R + \sqrt{R^2 - 6\mathcal{G}}\right)^{1/2}$$

C. Gao, Phys. Rev. D 86 (2012)

In an FRW cosmology with spatially flat metric one finds $J = \dot{a}/a \equiv H$.
Then the first Friedmann equation takes the form

$$H^2 + \frac{1}{6} f(H) - \frac{1}{6} H \frac{df}{dH} = \frac{8\pi G}{3}\rho$$

The left-hand side is a function of *H* only and takes the holographic form if

$$f(J) = \frac{1}{2}\ell^2 J^4$$

Hence, the holographic cosmology is reproduced in the modified Gauss-Bonnet gravity of the form

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{16\pi G}\left(-R - \frac{\ell^2}{288}\left(\sqrt{R^2 - 6\mathcal{G}} - R\right)^2\right) + \mathcal{L}^{\text{matt}}\right]$$

Cosmological perturbations

To calculate observational parameters we need the scalar and tensor perturbations of the background cosmology.

Scalar perturbations

Assuming a spatially flat background we write the perturbed line element in the Newtonian gauge

$$ds^2 = (1+2\Psi)dt^2 - (1-2\Phi)a^2\delta_{ij}dx^i dx^j$$

The first two Einstein equations at linear order turn out to be very close to the GR expressions

$$\frac{2}{a^2}\left(1-\frac{\ell^2}{2}H^2\right)\left[\nabla^2\Phi - 3a^2H(\dot{\Phi}+H\Psi)\right] = 8\pi G\delta T_0^0$$

$$2\left(1-\frac{\ell^2}{2}H^2\right)\partial_i(\dot{\Phi}+H\Psi) = 8\pi G\delta T_i^0$$

From the third equation

$$-2\left[\ddot{\Phi}+H(\dot{\Psi}+3\dot{\Phi})+\Psi(3H^2+2\dot{H})+\frac{1}{2a^2}\nabla^2(\Psi-\Phi)\right]\delta^i_j+\ell^2\left[H^2\ddot{\Phi}+(3H^3+2H\dot{H})\dot{\Phi}\right.$$
$$\left.+H^3\dot{\Psi}+(3H^4+4H^2\dot{H})\Psi-\frac{1}{6a^2}(3H^2+5\dot{H})\nabla^2\Phi+\frac{1}{6a^2}(3H^2+\dot{H})\nabla^2\Psi\right]\delta^i_j$$
$$-\frac{1}{a^2}\partial^i\partial_j(\Phi-\Psi)+\frac{\ell^2}{2a^2}(H^2-\dot{H})\partial^i\partial_j(\Phi-\Psi)=8\pi G\delta T^i_j$$

for a perfect fluid $\delta T^i_j=-\delta p\delta^i_j$ we infer that the gravitational slip $\eta\equiv\Phi/\Psi=1$ as in GR. Hence we can work in a simple longitudinal gauge with only one scalar.

N.R. Bertini, N.B., D.C. Rodrigues, *Cosmological perturbations in the Holographic braneworld* , work in preparation

From these equations we can work out the power spectrum $\mathcal{P}_S$ and the spectral index n_s-1 for various inflation models, e.g., for a canonical scalar or *k*-essence.

Tensor perturbations

are related to the production of gravitational waves during inflation.
The metric perturbation are defined as

$$ds^2 = dt^2 - a^2(t)\left(\delta_{ij} + h_{ij}\right) dx^i dx^j$$

In the absence of anisotropic stress the tensor perturbations are decoupled from matter and consequently they are only determined by the dynamics of the background metric, so we expect no deviation from the standard results with Einstein's equations at linear order

$$h''_{ij} + 2aH h'_{ij} - \Delta h_{ij} = 0$$

Then, the calculation of the power spectrum $\mathcal{P}_{\mathrm{T}}$ proceeds as usual.

Now we can apply the formalism of J. Garriga and V. F. Mukhanov, Phys. Lett. B 458 (1999), to the perturbations of the field equations.

Preliminary results for a general k-essence $\mathcal{L} = \mathcal{L}(X, \varphi)$, $X = g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu}$

$$\mathcal{P}_{\mathrm{S}} = \frac{GH^2}{\pi c_{\mathrm{s}}\varepsilon_1}\left[1 - 2(1+C)\varepsilon_1 - C\varepsilon_2\right]$$

$$\mathcal{P}_{\mathrm{T}} = \frac{16GH^2}{\pi}\left[1 - 2(1+C)\varepsilon_1\right]$$

$$C = -2 + \ln 2 + \gamma \simeq -0.72$$

exactly as in the standard cosmology. A deviation shows up in the slow roll parameters and sound speed. Typically

$$\varepsilon_2 = \varepsilon_2\big|_{\mathrm{st}} - \frac{2\ell^2 H^2}{\left(2-\ell^2H^2\right)\left(4-\ell^2H^2\right)}\varepsilon_1$$

Deviations from the standard k-essence inflation

$$c_{\mathrm{s}}^2 \equiv \left.\frac{\partial p}{\partial \rho}\right|_s = 1 + \frac{4}{3}\frac{\mathcal{L}\,\mathcal{L}_{,XX}}{\mathcal{L}_{,X}^2}\frac{2-\ell^2H^2}{4-\ell^2H^2}\varepsilon_1 + o(\varepsilon_i)$$

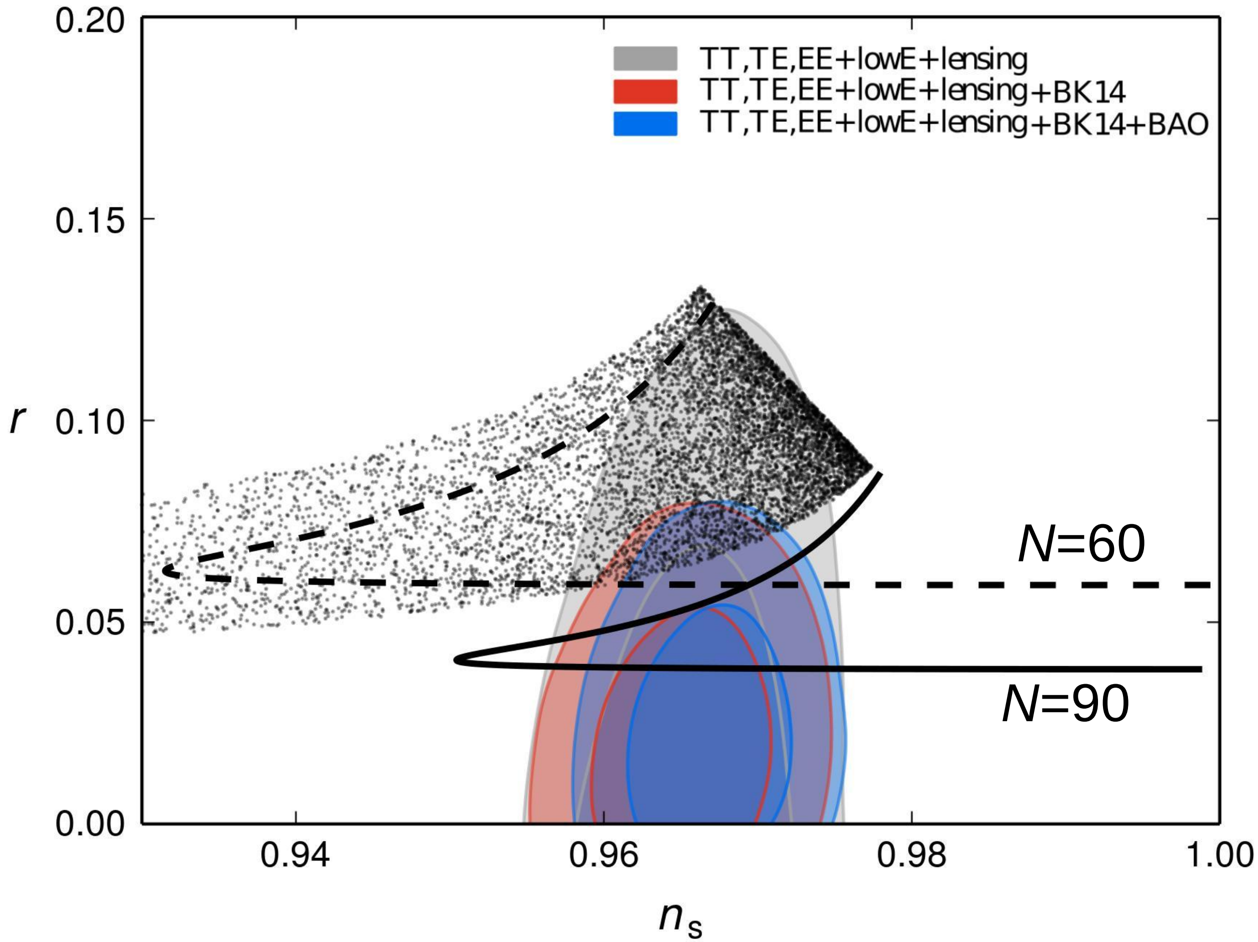
In the slow roll regime ℓH is typically of the order of 1 so the deviation from the standard cosmology can be significant

N.B. et al., *Tachyon inflation in the holographic braneworld*, JCAP08 (2019), arXiv:1809.0721

Conclusions and outlook

- The cosmology on the holographic brane deviates from the standard cosmology by additional terms with dependence on Hubble rate
- The holographic cosmology appears also in a modified Gauss-Bonnet gravity of the specific form
- The perturbations in Newtonian gauge deviate from GR but the gravitational slip remains equal to 1 as in GR
- What remains to be done is to solve the exact equations numerically for various inflation models. Our preliminary estimate for a k-essence inflation model shows deviations in the power spectra with respect to the standard inflation.
- It would be of considerable interest to compare our results with those in the corresponding modified Gauss-Bonnet gravity. This work is in progress.

Muito obrigado



r versus n_s with observational constraints from Planck 2018. The dots represent theoretical predictions obtained numerically for randomly chosen N ranging between 60 and 90 and h_i^2 between 0 to 2. The parameter ω is also varying in view of the functional dependence $N=N(h_i, \omega)$. The black lines represent the analytical results in the slow roll approximation.