PERTURBAÇÕES VETORIAIS EM MODELOS COM RICOCHETE

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I WORKSHOP PPGCOSMO,

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Em homenagem a

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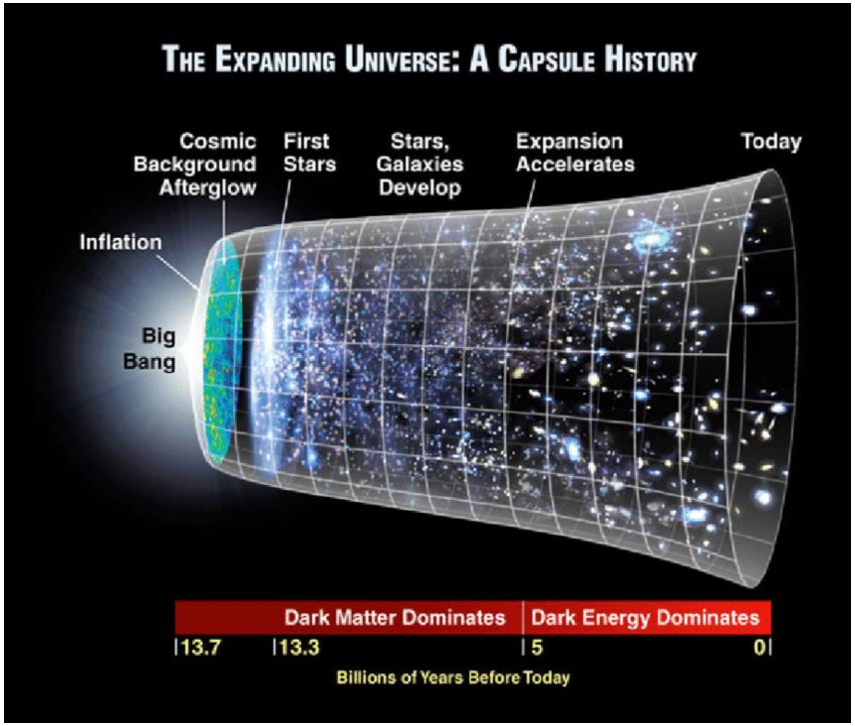
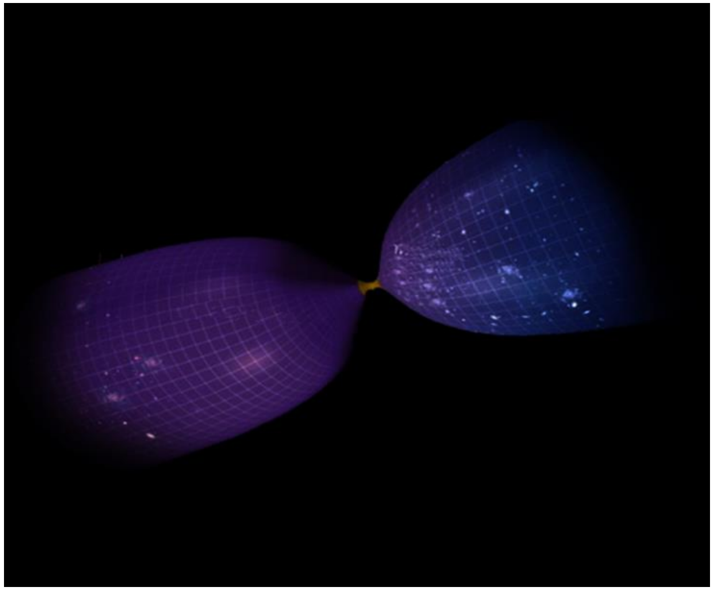
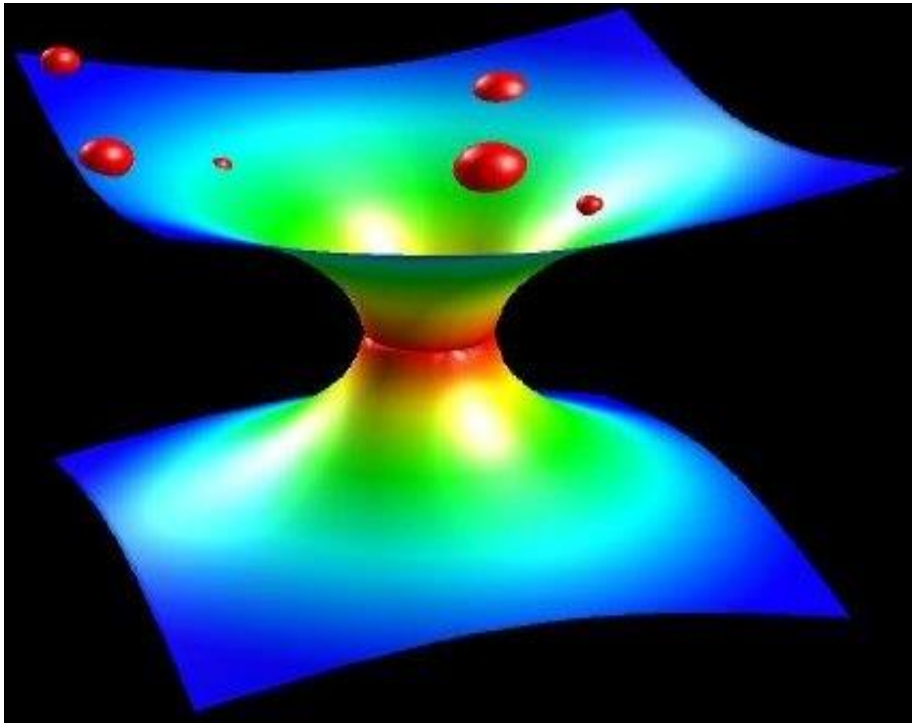




The problem of the standard model: the singularity. To avoid the singularity, one must go beyond GR (Einstein).



Did space-time had a beginning or it always existed?
What are the consequences of a bounce?

Big Bang (Inflation) **Bouncing models**

The singularity problem: a question for the Big Bang, beyond the scope of inflation. Bounce solves, by construction (beyond GR).

Big Bang: puzzles of initial conditions. Not for bounce.

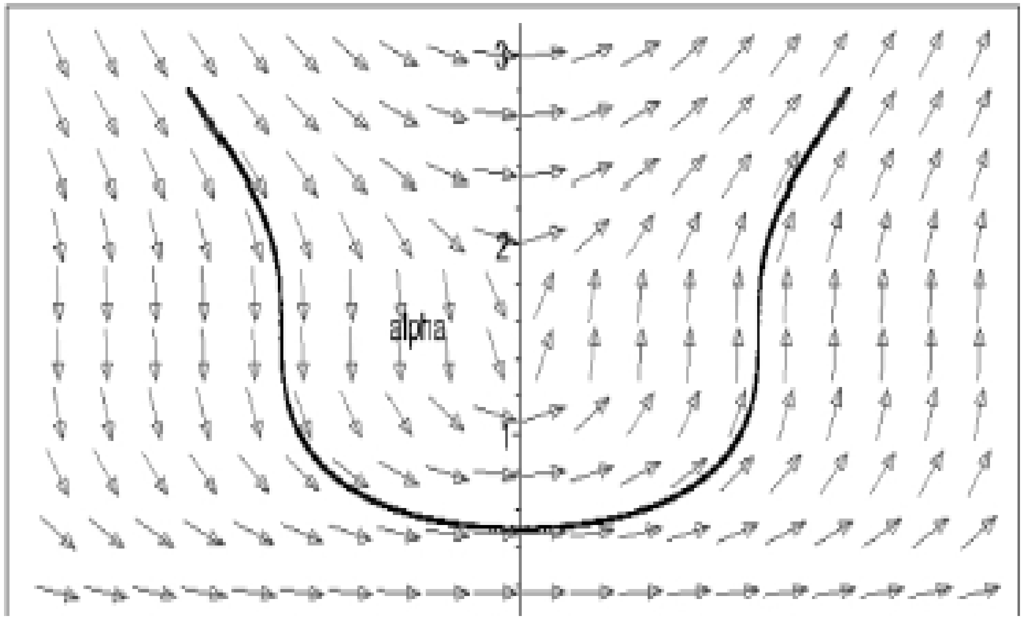
i) Horizon puzzle: it does not apply in the case of a longstanding decelerated contracting phase.

ii) Flatness puzzle: if the contraction phase is much longer then the expansion phase, then the Universe is almost flat because it has not expanded enough!

iii) Origin of perturbations: quantum vacuum fluctuations.

→ **INFLATION IS NOT NECESSARY IF THE UNIVERSE DOES NOT HAVE A BEGINNING**

One quantum bounce



Colistete, Fabris, NPN; Phys.Rev. D62, 083507 (2000)

Scalar and tensor perturbations are well behaved around a bounce: what happens with vector perturbations?

Without any viscosity, these perturbations are constrained to change as C/a^2, they are potentially dangerous in the contracting phase, but without a dynamical evolution, the initial conditions are arbitrary. But what happens when viscosity is present?

$$ds^2 = a(\eta)^2\left[d\eta^2 - (\delta_{ij} - \partial_j F_i - \partial_i F_j)dx^i dx^j\right]$$

$$G^0{}_i = -\frac{\nabla^2 \dot{F}_i}{2a^2} \tag{5}$$

$$G^i{}_j = -\left(\frac{\dot{a}^2}{a^4} - \frac{2\ddot{a}}{a^3}\right)\delta^i_j + \left(\frac{\partial^i \ddot{F}_j}{2a^2} + \frac{\partial_j \ddot{F}^i}{2a^2} + \frac{\dot{a}\partial^i \dot{F}_j}{a^3} + \frac{\dot{a}\partial_j \dot{F}^i}{a^3}\right) \tag{6}$$

One needs a non-trivial energy-momentum tensor to give dynamics to the vector perturbations

$$T^{\mu\nu} = (\rho + p)u^\mu u^\nu - p g^{\mu\nu} + \Delta T^{\mu\nu},$$

$$\Delta T^{\mu\nu} = \lambda(\eta)\left[u^{\mu;\nu} + u^{\nu;\mu} - u^\lambda (u^\mu u^\nu)_{;\lambda} - \frac{2}{3}(g^{\mu\nu} - u^\mu u^\nu)\right]$$

λ(η) is the shear viscosity

Defining $$h(\eta) = -kF(\eta), \qquad b^2 = v_t^2/c^2$$

one gets, after some manipulations (Grishchuk, PRD(1993))

$$\ddot{h} + 2\frac{\dot{a}}{a}\dot{h} + k^2 b^2 h = 0$$

Vector shear: σ = h’/k

<σ(k)σ*(k’)> $\propto P_v$**(k)** δ^3**(k-k’)**

P_v**(k)** $\propto l_p^2 \,/\, R_H^2$ **k** $h_k'^2$

We are interested in the VP back-reaction at the bounce

$$ds^2 = a(\eta)^2 \left[d\eta^2 - (\delta_{ij} - \partial_j F_i - \partial_i F_j) dx^i dx^j\right]$$

$$G^i{}_j = -\left(\frac{a'^2}{a^4} - \frac{2a''}{a^3}\right)\delta^i_j + \\ + \left(\frac{\partial^i F''_j}{2a^2} + \frac{\partial_j F^{i\prime\prime}}{2a^2} + \frac{a'\partial^i F'_j}{a^3} + \frac{a'\partial_j F^{i\prime}}{a^3}\right)$$

h(k) = -kF(k) $\quad$ $P_v(\mathbf{k}) \propto l_p^2 \,/\, R_H^2\ \mathbf{k}\ h'^2_k$

$$\langle h\rangle^2 = \frac{8}{\pi}\frac{l_p^2}{R_{H_0}^2}\int_{k_{s,min}}^{k_{s,max}} dk_s k_s^2 |h_{k_s}|^2 \ll 1.$$

$$\frac{4\pi Y^2}{\Omega_r}\frac{l_p^2}{R_H^2}\int_{k_{s,min}}^{k_{s,max}} dk_s k_s P_v(k_s) << 1$$

$$Y(\eta_s) = \frac{a(\eta_s)}{a_0} = \frac{\Omega_m}{4}\,\eta_s^2 + \sqrt{\frac{1}{x_b^2} + \Omega_r\,\eta_s^2}, \qquad b^2 = v_t^2/c^2 < 1$$

$$L_b = \left.\frac{1}{\sqrt{R}}\right|_{\eta=0} = \frac{a_0 R_H}{x_b^2\sqrt{6\Omega_{r0}}}$$

$$L_b > 10^3 l_p \Rightarrow x_b = \frac{a_0}{a_b} < 10^{31}$$

$$\mu'' + \left(k^2 b^2 - \frac{a''}{a}\right)\mu = 0\,,$$

with $\mu = ah = -akF\,.$

a''/a at the bounce is $x_b^2\Omega_r$ hence $k_{s,max} = \frac{\sqrt{\Omega_r}x_b}{b}$

HAMILTONIAN FOR PERTURBATIONS

$$\mathcal{H} = \frac{\Pi_k^2}{2m} + \frac{m\nu^2 A_k^2}{2} ,$$

$$A_k' = \frac{\Pi_k}{m}; \quad \Pi_k' = -m\nu^2 A_k .$$

VACUUM INITIAL CONDITIONS

$$A_k = \frac{e^{-\mathrm{i}\pi/4}}{\sqrt{2m\nu}} \exp\left[-\mathrm{i}\int_{t_0}^{t} \nu \mathrm{d}t\right] ,$$

$$\Pi_k = -\mathrm{i}e^{-\mathrm{i}\pi/4}\sqrt{\frac{m\nu}{2}} \exp\left[-\mathrm{i}\int_{t_0}^{t} \nu \mathrm{d}t\right]$$

FREQUENCY IRRELEVANT WITH RESPECT TO a''/a

$$\Pi_k(t) = -A_1(k)\int^t m(t_1)\nu^2(t_1)\mathrm{d}t_1 + A_2(k)\left(1 - \int^t m(t_1)\nu^2(t_1)\mathrm{d}t_1 \int^{t_1} \frac{\mathrm{d}t_2}{m(t_2)}\right) + \ldots ,$$

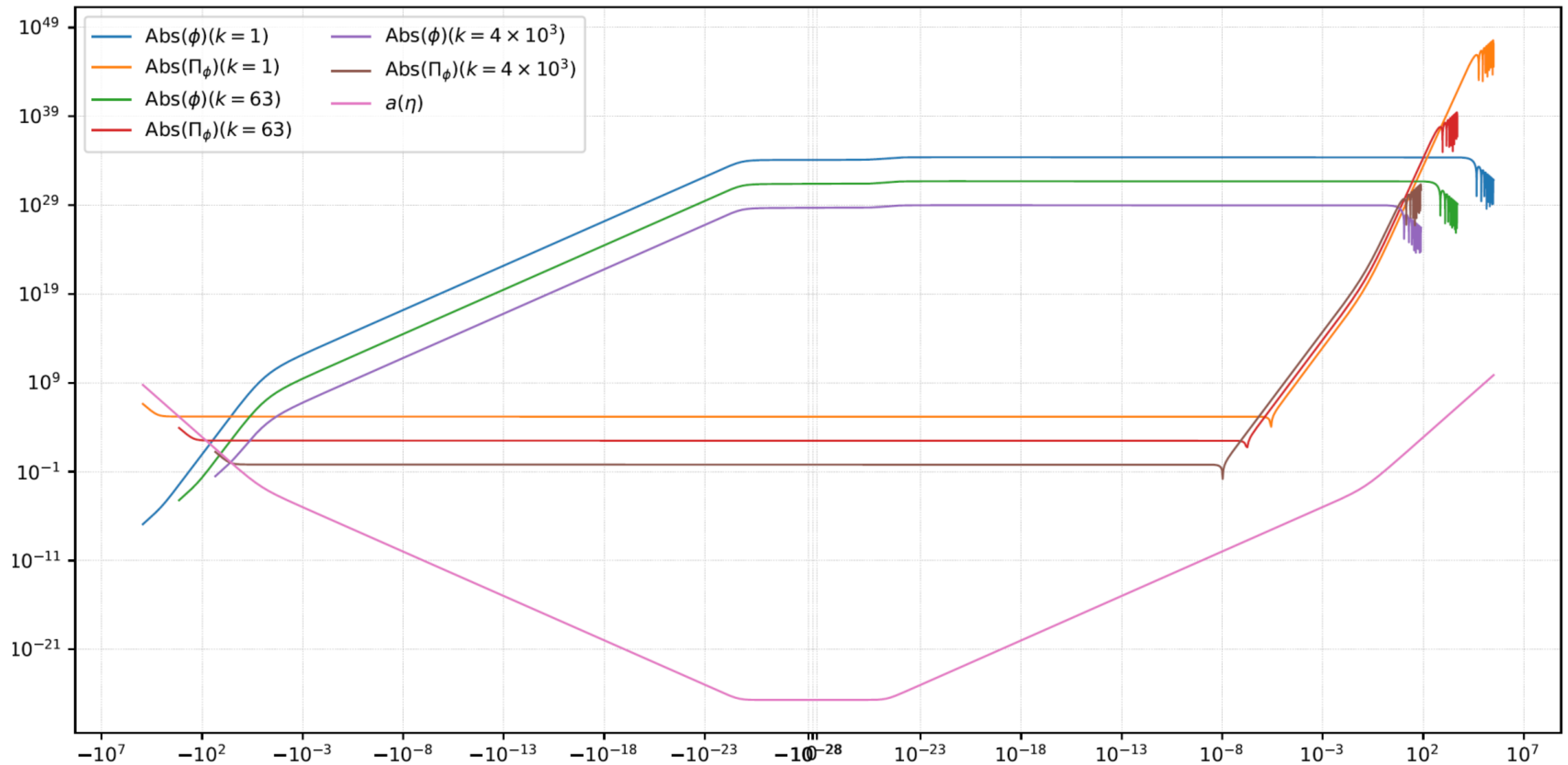
$$A_k(t) = A_1(k)\left(1 - \int^t \frac{\mathrm{d}t_2}{m(t_2)} \int^{t_2} m(t_1)\nu^2(t_1)\mathrm{d}t_1\right) + A_2(k)\left(\int^t \frac{\mathrm{d}t_1}{m(t_1)} - \int^t \frac{\mathrm{d}t_2}{m(t_2)} \int^{t_2} m(t_1)\nu^2(t_1)\mathrm{d}t_1 \int^{t_1} \frac{\mathrm{d}t_3}{m(t_3)}\right) + \ldots ,$$

In our case:

$$\ddot{h} + 2\frac{\dot{a}}{a}\dot{h} + k^2 b^2 h = 0$$

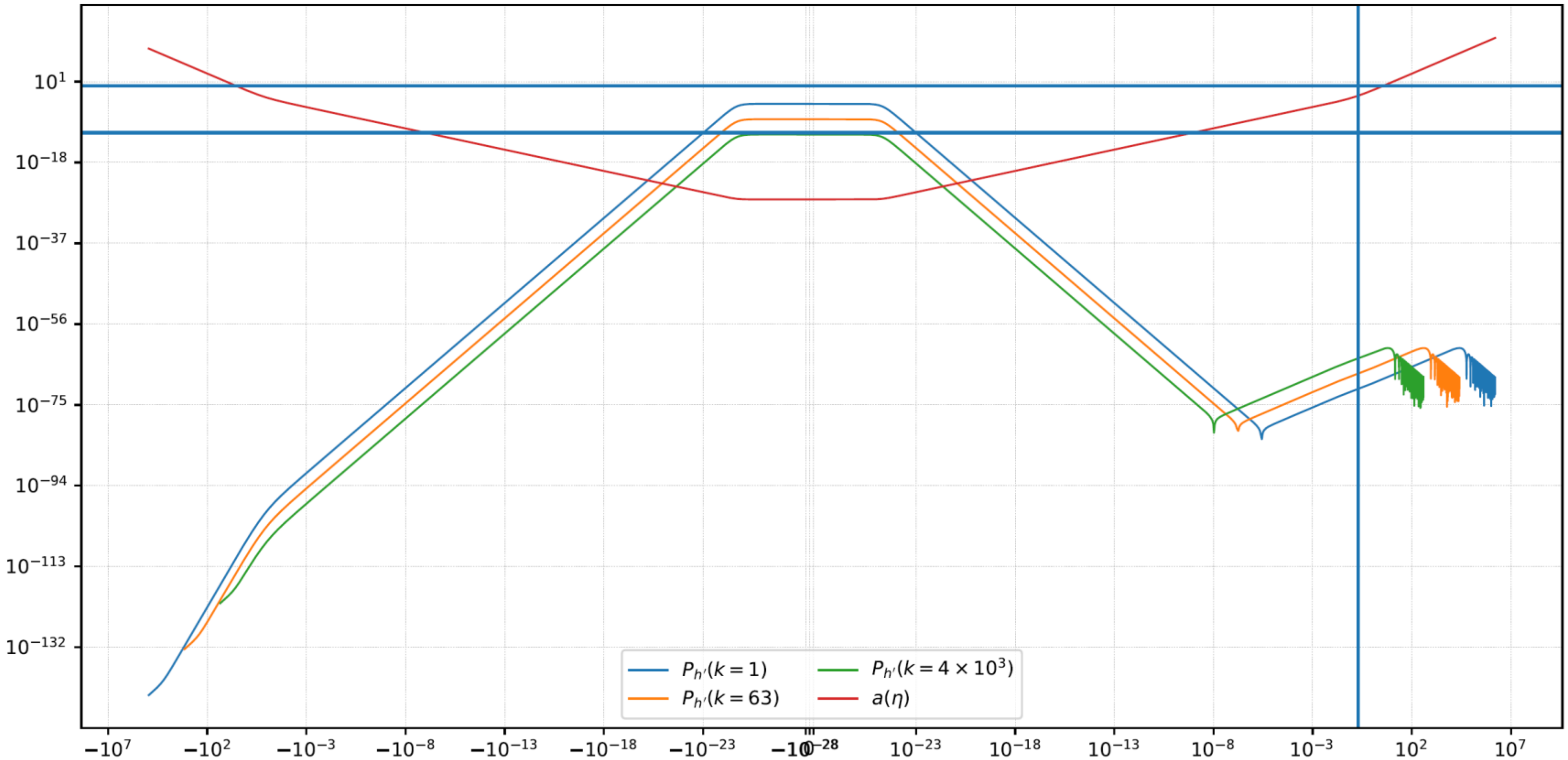
A=h, m = a^2, v = kb

h and π



Abs(ϕ)(k = 1)
Abs(Π_ϕ)(k = 1)
Abs(ϕ)(k = 63)
Abs(Π_ϕ)(k = 63)
Abs(ϕ)(k = 4 × 10^3)
Abs(Π_ϕ)(k = 4 × 10^3)
a(η)
10^49
10^39
10^29
10^19
10^9
10^−1
10^−11
10^−21
−10^7
−10^2
−10^−3
−10^−8
−10^−13
−10^−18
−10^−23
−10^−28
10^−23
10^−18
10^−13
10^−8
10^−3
10^2
10^7

$$P_v(\mathbf{k}) \propto l_p^2 \,/\, R_H^2 \,\mathbf{k}\, h'^2_k = \frac{l_p^2}{R_H^2} k \frac{\pi_k^2}{Y^4}$$



Values of the modes at the bounce, from analytic considerations and numerics:

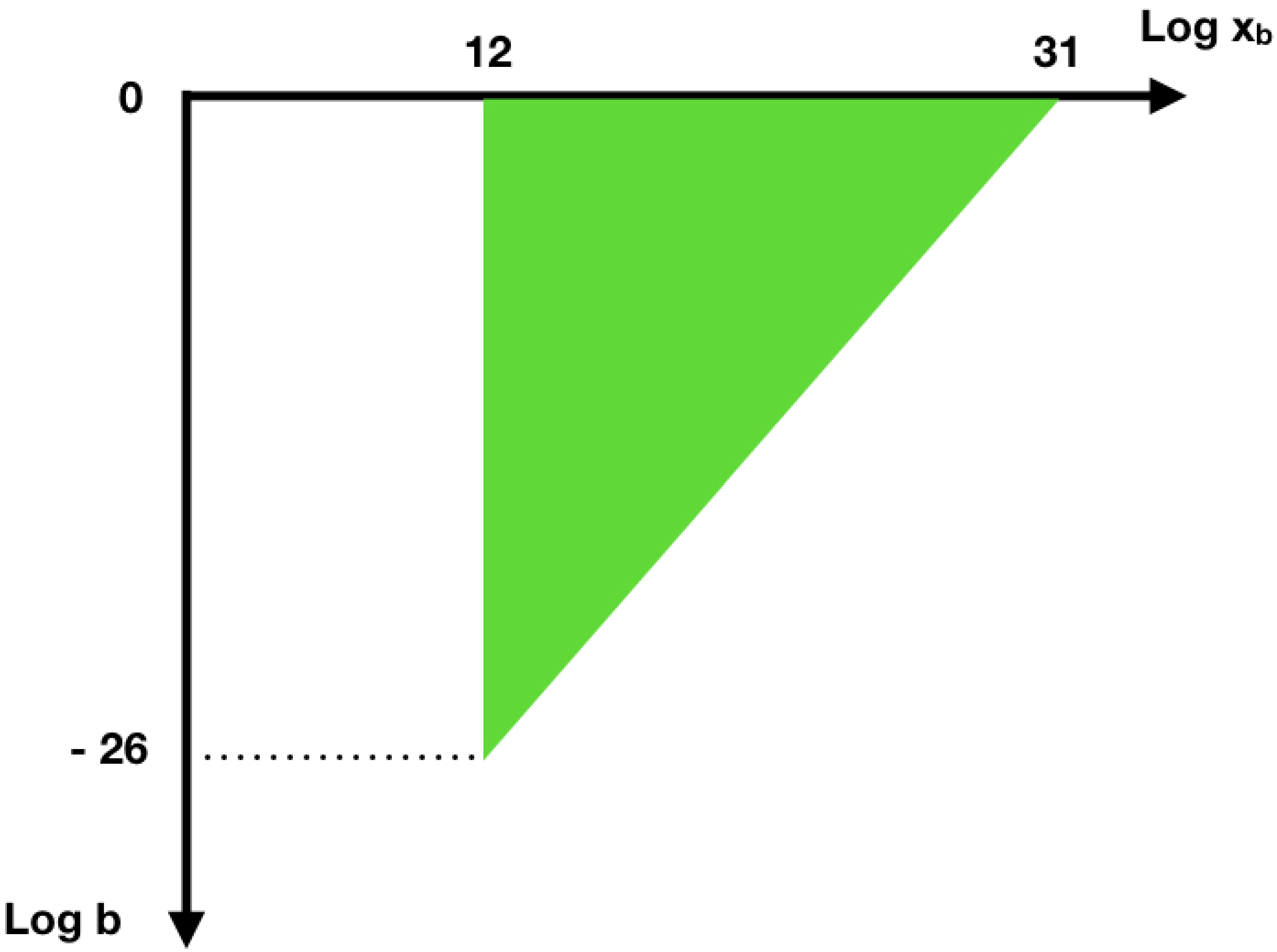
Crossing at matter domination:

$$h = 10^2 \frac{x_b}{(kb)^{3/2}} \;, P_v = 9.66 \times 10^{-124} \frac{x_b^4}{b^3 k^2}$$

Crossing at radiation domination:

$$h = \frac{x_b}{(kb)^{1/2}} \;, P_v = 1.9 \times 10^{-126} \frac{x_b^4}{b}$$

Constraint: $$\frac{x_b^4}{b^3} \ll 10^{126}$$



Conclusion:

- For a huge range of parameters, the bounce is stable against vector perturbations.

- Other bounces and other sources of viscosity should be investigated