Primordial Black Holes from fully non-Gaussian Curvature Perturbations

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To be released soon. Stay tuned!

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Motivation

Literature

$$\zeta = F(\zeta_G)$$

Power series expansion

$$\zeta = \zeta_G + \frac{3}{5} f_{NL} \zeta_G^2 + \frac{9}{25} g_{NL} \zeta_G^3 + \dots$$

Curvaton

$$\zeta = \log[\chi(\zeta_G)]$$

Ultra slow-roll

$$\zeta = \left(\frac{6}{5} f_{NL}\right)^{-1} \log\left(1 - \frac{6}{5} f_{NL} \zeta_G\right)$$

$+ \dots$

(Ferrante, Franciolini, Iovino and Urbano, 2023)

→ Light spectators with a vanishing mean can produce significant non-gaussian $P(\zeta)$

Motivation

But what is a Primordial Black Hole?

1- Inflationary perturbations in the early universe;

2- Quantum fluctuations during inflation generate overdensities;

3- Density contrast $\delta = \frac{\delta\rho}{\rho}$ → "compaction function" C(r);

4- If $C > C_c$ at horizon entry → PBHs are formed;

Motivation

Why do they matter?

1- Different to astrophysical BHs;

2- Dark matter candidate;

3- Seeds of supermassive black holes;

4- LIGO–Virgo–KAGRA merger events;

5- Constraining early universe physics;

The Curvaton Model

A setup where the superhorizon scale curvature perturbation consists of two uncorrelated parts $\langle \zeta_\varphi \zeta_\chi \rangle = 0$ which we label as the inflaton part ζ_φ and the curvaton part ζ_χ

$$\zeta(\mathbf{k}) = \zeta_\varphi(\mathbf{k}) + \zeta_\chi(\mathbf{k}) \quad (1)$$

We assume that the inflaton part dominates on CMB scales

$$\mathscr{P}_\zeta(k = 0.05\mathrm{Mpc}^{-1}) \simeq \mathscr{P}_{\zeta_\varphi}(k = 0.05\mathrm{Mpc}^{-1}) = 2.1 \times 10^{-9}$$

and that the curvaton part dominates on the small scale k_c relevant for the PBH formation

$$\mathscr{P}_\zeta(k_\mathrm{c}) \simeq \mathscr{P}_{\zeta_\chi}(k_\mathrm{c}) \quad (2)$$

The Curvaton Model

A concrete template for the spectator sector is given by

$$\zeta_\chi(\mathbf{x}) = \alpha \frac{\ln\left[\chi(t_{\text{end}}, \mathbf{x})^2\right] - \langle \chi(t_{\text{end}})^2 \rangle}{\langle \chi(t_{\text{end}})^2 \rangle} \quad (3)$$

where α is a constant and$\chi \equiv \chi(t_{\text{end}})$ denotes the spectator field at the end of inflation.

Such a χ^2 form can arise in the curvaton mechanism (Sasaki,Valiviita, and Wands, 2006):

a weakly coupled scalar field lighter than the Hubble rate during inflation, around the minimum of its potential $V(\chi) = m^2\chi^2/2$, where m is the effective mass of the curvaton.

The Curvaton Model

Fluctuations in the energy density $\delta\rho_\chi$ of a light spectator with a vanishing mean are manifestly non-Gaussian (Herranen, Markkanen and Tranberg, 2014; Markkanen, Rajantie, Stopyra and Tenkanen, 2019) having no leading Gaussian term even for non-interacting with a quadratic potential

The dimensionless density parameter for the curvaton at the decay time is

$$\Omega_\chi = \frac{\rho_{\chi_0}\left(\frac{a_0}{a}\right)^3}{\rho_r\left(\frac{a_0}{a}\right)^4 + \rho_{\chi_0}\left(\frac{a_0}{a}\right)^3} \equiv \frac{\Omega_{\chi_0} x^3}{\Omega_{\chi_0} x^3 + x^4} \quad (4)$$

where we are considering instantaneous decay of the curvaton particles, radiation dominated era for $t_{end} \leqslant t \leqslant t_0$, $x = a_0/a$ and $\Omega_{\chi_0} = \langle \chi_0^2 \rangle / 6M_{Pl}^2$

The Curvaton Model

The fully nonlinear relation between the primordial curvature perturbation, ζ, the radiation perturbation, ζ_r, the curvaton perturbation ζ_χ is given by (Sasaki, Valiviita, and Wands, 2006)

$$e^{4\zeta} - \Omega_\chi e^{3\zeta_\chi} e^{\zeta} + \left[\Omega_\chi - 1\right] e^{4\zeta_r} = 0 \quad (5)$$

where we will define a new variable with the mean subtracted $\hat{\zeta}_\chi = \zeta_\chi - \langle\zeta_\chi\rangle$, and also

$$\zeta_\chi = \frac{1}{3}\frac{\ln \chi^2}{\langle\chi^2\rangle}\ , \quad \langle\zeta_\chi\rangle = -\frac{1}{3}\left(\ln 2 - \gamma_E\right)\ , \quad \text{and} \quad \langle\zeta_r\rangle = 0 \quad (6)$$

The Curvaton Model

Since we are interested in the regime of PBHs formation where ζ_χ is large, let us consider the following ansatz

$$\zeta = \zeta_\chi + \frac{1}{3}\ln\Omega_\chi + \delta \quad (7)$$

So we get the asymptotic approximation of curvaton perturbation as

$$\hat{\zeta}(\chi) = \alpha\left[\ln(\chi)^2 - \langle\ln(\chi)^2\rangle\right]\Theta[\chi^2 - \beta^2] \quad (8)$$

where Θ is the Heaviside step function, $\alpha = 1/3$ for the curvaton model, $\chi \equiv \chi(t_{\mathrm{end}})$ denotes the curvaton field at the end of inflation, and $\beta^2 = \langle\chi^2\rangle\left(1 - \Omega_\chi\right)^{3/4} e^{3\zeta_r}/\Omega_\chi$.

This is the template which we will consider in order to estimate the PBH abundance for the curvaton scenario.

The Curvaton Model: a comment

In order to make sure the curvaton contribution does not spoil the cosmological evolution

$\rightarrow$ the spectrum of ζ_χ has restrictions on the space of parameters, indicating that we need a massive curvaton field $m^2/H^2 \geqslant 0.28$ to satisfy $\mathscr{P}_{\zeta_\chi}(0.005\mathrm{Mpc}^{-1}) \sim 10^{-11}$.

So if we have $m^2/H^2 = 0.9$, this translates to the power spectrum at large and small scales, respectively

$$\mathscr{P}_{\zeta_\chi}(0.05\mathrm{Mpc}^{-1}) = 3.8 \times 10^{-11} \quad \text{and} \quad \mathscr{P}_{\zeta_\chi}(10^{13}\mathrm{Mpc}^{-1}) = 2 \times 10^{-5} \qquad (9)$$

Correlators and PDFs

By considering that the high peaks in the density field that form PBHs can be approximated as spherical

$$\chi(t,\mathrm{x}) = \int \frac{d\mathrm{k}}{(2\pi)^3}\left(\hat{a}_{\mathrm{k}}\chi_k + \hat{a}^{\dagger}_{-\mathrm{k}}\chi^{\star}_{-k}\right)e^{i\mathrm{k}\cdot\mathrm{x}}$$

$$= \int \frac{d\mathrm{k}}{(2\pi)^3}\tilde{\chi}_k(t)4\pi\sum_{lm} i^l j_l(kx)Y_{lm}(\hat{x})Y^{\star}_{lm}(\hat{k}) \int \frac{d\mathrm{k}}{(2\pi)^3} j_0(kx)\tilde{\chi}_k = \chi(r) \quad (9)$$

The correlators of the curvaton field $\chi(r)$ and its derivative $\chi'(r)$ are given, in terms of the power spectrum $\mathscr{P}(k)$, by

$$\langle\chi(r)\chi(r)\rangle = \int d\ln(k) j_0^2(kr)\mathscr{P}(k) \quad (10)$$

$$\langle\chi(r)\chi'(r)\rangle = \int d\ln(k)(kr)j_0'(kr)j_0(kr)\mathscr{P}(k) \quad (11)$$

$$\langle\chi'(r)\chi'(r)\rangle = \int d\ln(k)(kr)^2\left[j_0'(kr)\right]^2\mathscr{P}(k) \quad (12)$$

Correlators and PDFs

We assume the lognormal shape for the Gaussian power spectrum

$$\mathscr{P}_G(k) = \frac{A}{\sqrt{2\pi\Delta^2}} \exp\left[-\frac{\ln^2\left(k/k_\star\right)}{2\Delta^2}\right] \quad (13)$$

with amplitude A, width Δ, and peak scale $k_\star$.

The expected value $\langle \ln(\chi)^2 \rangle$ is computed by

$$\langle \ln(\chi)^2 \rangle = \int d\chi P_\chi(\chi) \ln(\chi)^2 = \ln(\sigma_{\chi\chi}^2) - \ln 2 - \gamma_E \quad (14)$$

where the equilibrium probability distribution of the curvaton field $P_\chi(\chi)$ was written as (Starobinsky and Yokoyama, 1994)

$$P_\chi(\chi) = \frac{1}{\sqrt{2\pi\sigma_{\chi\chi}^2}} \exp\left(-\frac{1}{2}\frac{\chi^2}{\sigma_{\chi\chi}^2}\right) \quad (15)$$

Correlators and PDFs

Another ingredient that we will need is

$$\langle \ln(\chi)^2 \rangle' = \frac{(\sigma_{\chi\chi}^2)'}{\sigma_{\chi\chi}^2} = \frac{\int d\ln(k) 2k j_0(kr) j_0'(kr) \mathscr{P}(k)}{\int d\ln(k) j_0^2(kr) \mathscr{P}(k)}$$

$$\equiv 2D' \quad (16)$$

In order to estimate whether a perturbation will collapse to form a PBH, we will use the compaction function C (Young, Byrnes and Sasaki, 2014)

$$C = C_l - \frac{3}{8} C_l^2 \quad (17)$$

where C_l is the linear part of the compaction function.

Correlators and PDFs

C_l is related to the curvature perturbation by

$$C_l = -\frac{4}{3} r\zeta' = -\frac{4}{3} r\alpha \left\{ 2\Theta(\chi^2-\beta^2)\left(\frac{\chi'}{\chi} - D'\right) + \chi' \left(\ln\beta^2 - \langle \ln\chi^2 \rangle\right) \left[-\delta(\chi-\beta) + \delta(\chi+\beta)\right] \right\}$$

$$\equiv \overline{C}_l \quad (18)$$

One can think C_l as the combination of two normally distributed and uncorrelated random variables, χ and χ', and therefore its probability is given by

$$P_{C_l}(C_l) = \int d\chi \int d\chi' P(\chi,\chi')\delta(C_l - \overline{C}_l) \quad (19)$$

where

$$P(\chi,\chi') = \frac{1}{2\pi\sqrt{|\Sigma|}} \exp\left(-\frac{1}{2}\overline{X}^T \Sigma^{-1} \overline{X}\right), \ \overline{X} = \begin{pmatrix} \chi \\ \chi' \end{pmatrix}, \ \overline{X}^T = \begin{pmatrix} \chi & \chi' \end{pmatrix} \text{ and}$$

$$\Sigma = \begin{bmatrix} \langle \chi(r)\chi(r) \rangle & \langle \chi(r)\chi'(r) \rangle \\ \langle \chi(r)\chi'(r) \rangle & \langle \chi'(r)\chi'(r) \rangle \end{bmatrix} = \begin{pmatrix} \sigma^2_{\chi\chi} & \sigma^2_{\chi\chi'} \\ \sigma^2_{\chi\chi'} & \sigma^2_{\chi'\chi'} \end{pmatrix} \quad (20)$$

Correlators and PDFs

By computing the roots of $C_l - \overline{C}_l = 0$, the probability $P_{C_l}(C_l)$ can be split into three regions

$$P_{C_l}(C_l) = \int_{-\infty}^{-\beta} d\chi \int_{-\infty}^{\infty} d\chi' P(\chi,\chi')\delta\left[C_l + \frac{8r\alpha}{3}\left(\frac{\chi'}{\chi} - D'\right)\right]$$

$$+ \int_{\beta}^{\infty} d\chi \int_{-\infty}^{\infty} d\chi' P(\chi,\chi')\delta\left[C_l + \frac{8r\alpha}{3}\left(\frac{\chi'}{\chi} - D'\right)\right] + \delta(C_l)\int_{-\beta}^{\beta} d\chi \int_{-\infty}^{\infty} d\chi' P(\chi,\chi') \quad (21)$$

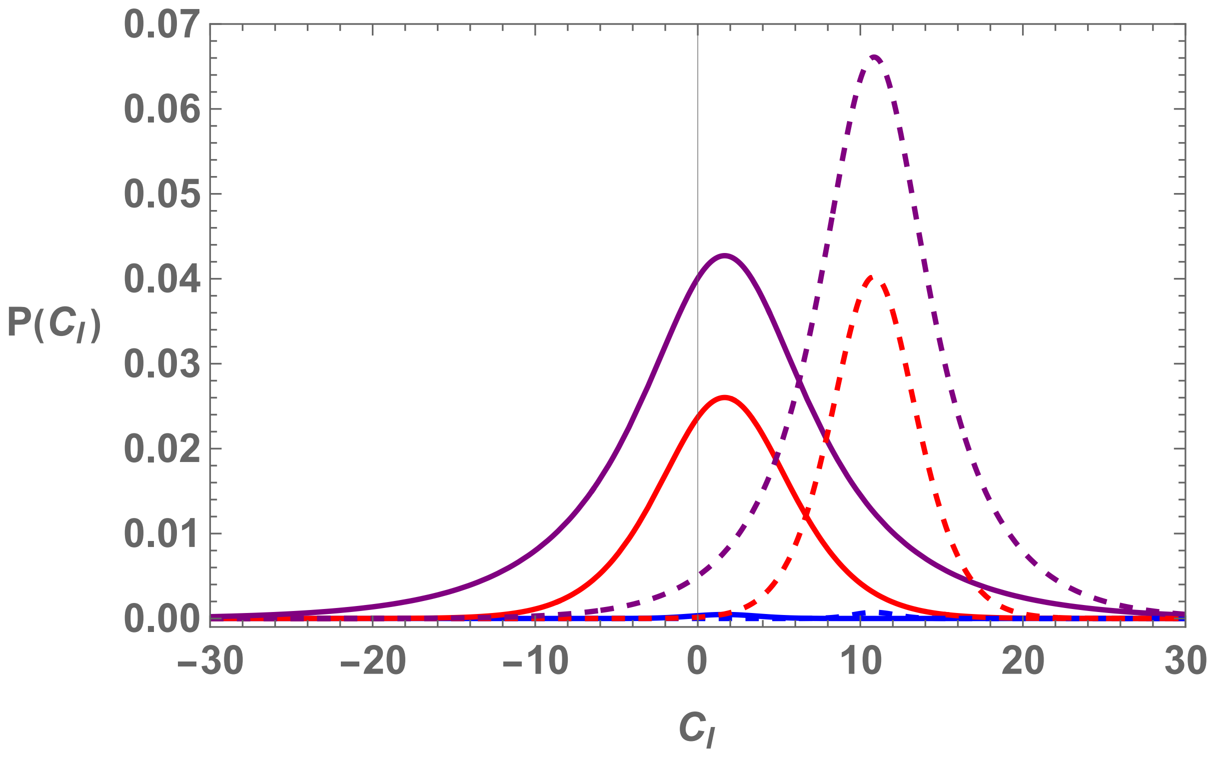
which results in

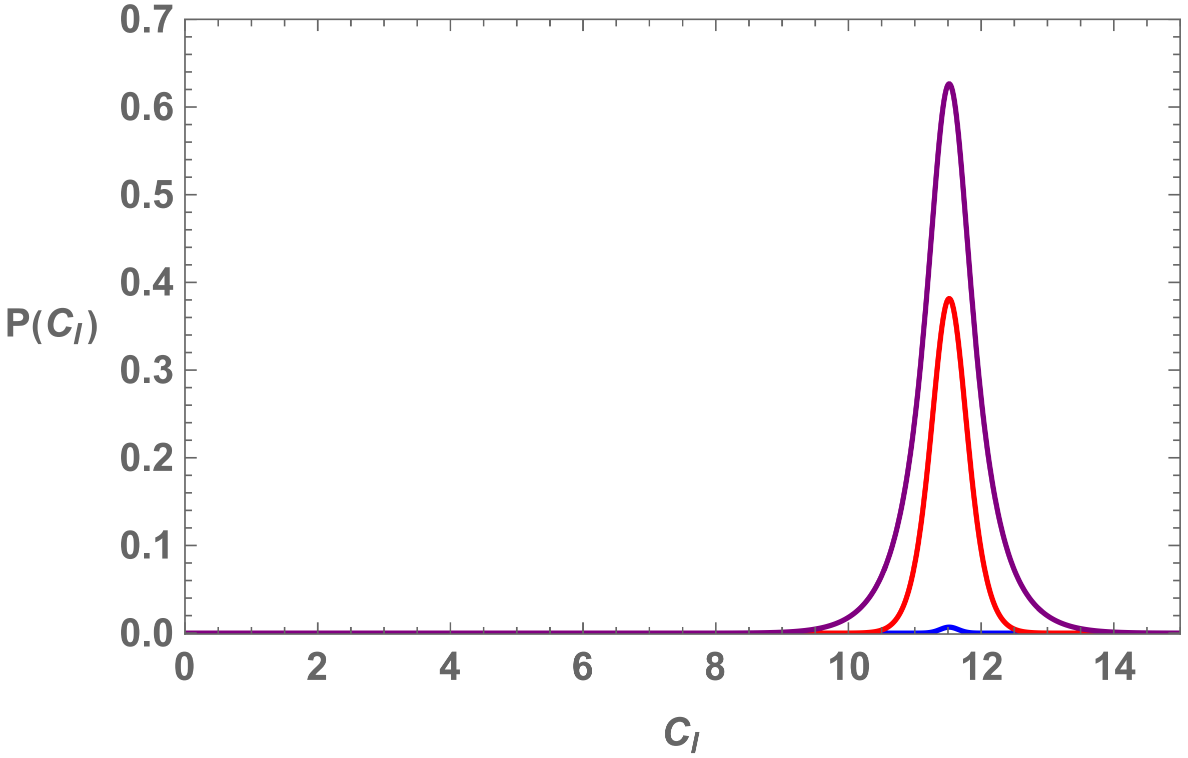
$$P_{C_l}(C_l) = \delta(C_l)\mathrm{Erf}\left(\frac{\beta}{\sqrt{2}\sigma_{\chi\chi}}\right) + \frac{3}{8r|\alpha|\pi\tilde{\sigma}^2\sqrt{|\Sigma|}}\mathrm{Exp}\left(-\tilde{\sigma}^2\beta^2/2\right) \quad (22)$$

where

$$\tilde{\sigma}^2 = \left(\sigma^2_{\chi'\chi'} + 2\tilde{C}_l\sigma^2_{\chi\chi'} + \tilde{C}_l^{\,2}\sigma^2_{\chi\chi}\right)\frac{1}{|\Sigma|} \quad \text{and} \quad \tilde{C}_l = \frac{3}{8r\alpha}C_l - D' \quad (23)$$

Correlators and PDFs





The evolution of the PDF for C_l given by Eq.(22) assuming the lognormal shape for the power spectrum with $\Omega_\chi = 0.1$ (blue), $\Omega_\chi = 0.5$ (red) and $\Omega_\chi = 0.9$ (purple). *Left Panel.* $\Delta = 0.3$ (solid lines) and $\Delta = 0.03$ (dashed lines). *Right Panel*. $\Delta = 0.003$.

Correlators and PDFs

As pointed out in Kitajima, Tada, Yokoyama and Yoo 2021, for a monochromatic power spectrum $\mathscr{P}_\zeta(k) = Ak\delta(k-k_\star)$, we have a singular covariance matrix Σ, i.e.

$$\sigma^4_{\chi\chi'} = \sigma^2_{\chi\chi}\sigma^2_{\chi'\chi'}$$

which means that the variables χ and χ' are not independent, but can be written as $\chi' = \dfrac{\sigma_{\chi'\chi'}}{\sigma_{\chi\chi}}\chi$. So in that case, the full PDF collapses into a 1-D form given by

$$P_{C_l}(C_l) = \delta(C_l)\mathrm{Erf}\left(\frac{\beta}{\sqrt{2}\sigma_{\chi\chi}}\right) + \delta\left[C_l + \frac{8\alpha r}{3}\left(\frac{\sigma_{\chi'\chi'}}{\sigma_{\chi\chi}} - D'\right)\right]\left[1 - \mathrm{Erf}\left(\frac{\beta}{\sqrt{2}\sigma_{\chi\chi}}\right)\right]$$

It is clear to see that the probability distribution function has infinity value at $C_l = 24.77$ for $\alpha = 1/3$ and $z_\star = 2.74$, but it is zero everywhere else, including the region $0.87 < C_l < 1.33$ relevant for PBHs formation.

Correlators and PDFs

Constraints of $C \to C_l$

Inferior Limit for C

$$C(r) = \frac{2}{R(t,r)} \int_0^{R(r,t)} dR \left[4\pi R(r,t)^2\right] \left(\frac{\rho - \langle\rho\rangle}{\langle\rho\rangle}\right) \quad (24)$$

when

$$\rho = 0 \quad \to \quad C = -1\,, \quad \text{and} \quad C_l = \frac{4}{3}\left(1 - \sqrt{\frac{5}{2}}\right) \quad (25)$$

Superior Limit for C

$$C(r) = C_l - \frac{3}{8}C_l^2 \quad (26)$$

when

$$C = \frac{2}{3} \quad \text{then} \quad C_l = \frac{4}{3} \quad (27)$$

Correlators and PDFs

With the constraints, the PDF is only valid in the regime

$$\frac{4}{3}\left(1-\sqrt{\frac{5}{2}}\right) \leqslant C_l \leqslant \frac{4}{3}$$

and then, the PDF has to be normalised to

$$\hat{P}_{C_l}(C_l) = \frac{1}{\int_{C_{l_{min}}}^{C_{l_{max}}} dC_l P_{C_l}(C_l)} \left[\delta(C_l)\mathrm{Erf}\left(\frac{\beta}{\sqrt{2}\sigma_{\chi\chi}}\right) + \frac{3}{8r|\alpha|\pi\tilde{\sigma}^2\sqrt{|\Sigma|}}\mathrm{Exp}\left(-\tilde{\sigma}^2\beta^2/2\right)\right]$$

Mass distribution of PBHs

The mass spectrum of PBHs is described by the critical collapse relation (Niemeyer and Jedamzik, 1998)

$$m = KM_H \left(C_l - \frac{3}{8} C_l^2 - C_c \right)^{\gamma} \quad (28)$$

where $\gamma = 0.36$ for a radiation dominated fluid, M_H indicates the mass of the cosmological horizon measured at time t_H, $K = 1$ is the coefficient which depends on the particular shape of the perturbations, and $C_c = 0.587$ is the critical amplitude of the compaction function for a monochromatic curvature perturbation power spectrum.

The scale r is chosen such that $k_\star r = 2.74$, which maximises the compaction function for a monochromatic power spectrum (Musco, De Luca, Franciolini and Riotto, 2021)

Mass distribution of PBHs

Then last step to compute the probability of forming a PBH is to calculate the mass fraction

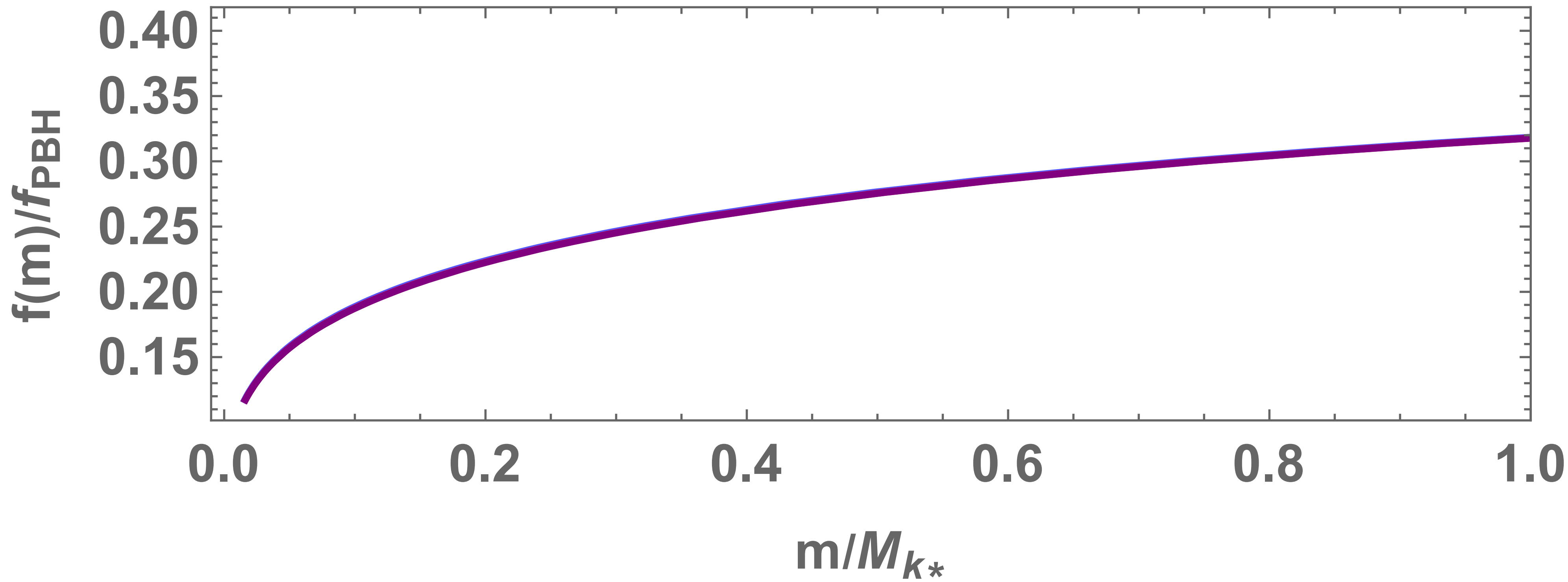
$$f_{PBH} = \int_0^{m_{max}} d(\ln m) f(m) \quad (29)$$

where

$$f(m) = Z \frac{\left(C_l - \frac{3}{8}C_l^2 - C_c\right)^{\gamma+1}}{\gamma\left(1 - \frac{3}{4}C_l\right)} \quad (30)$$

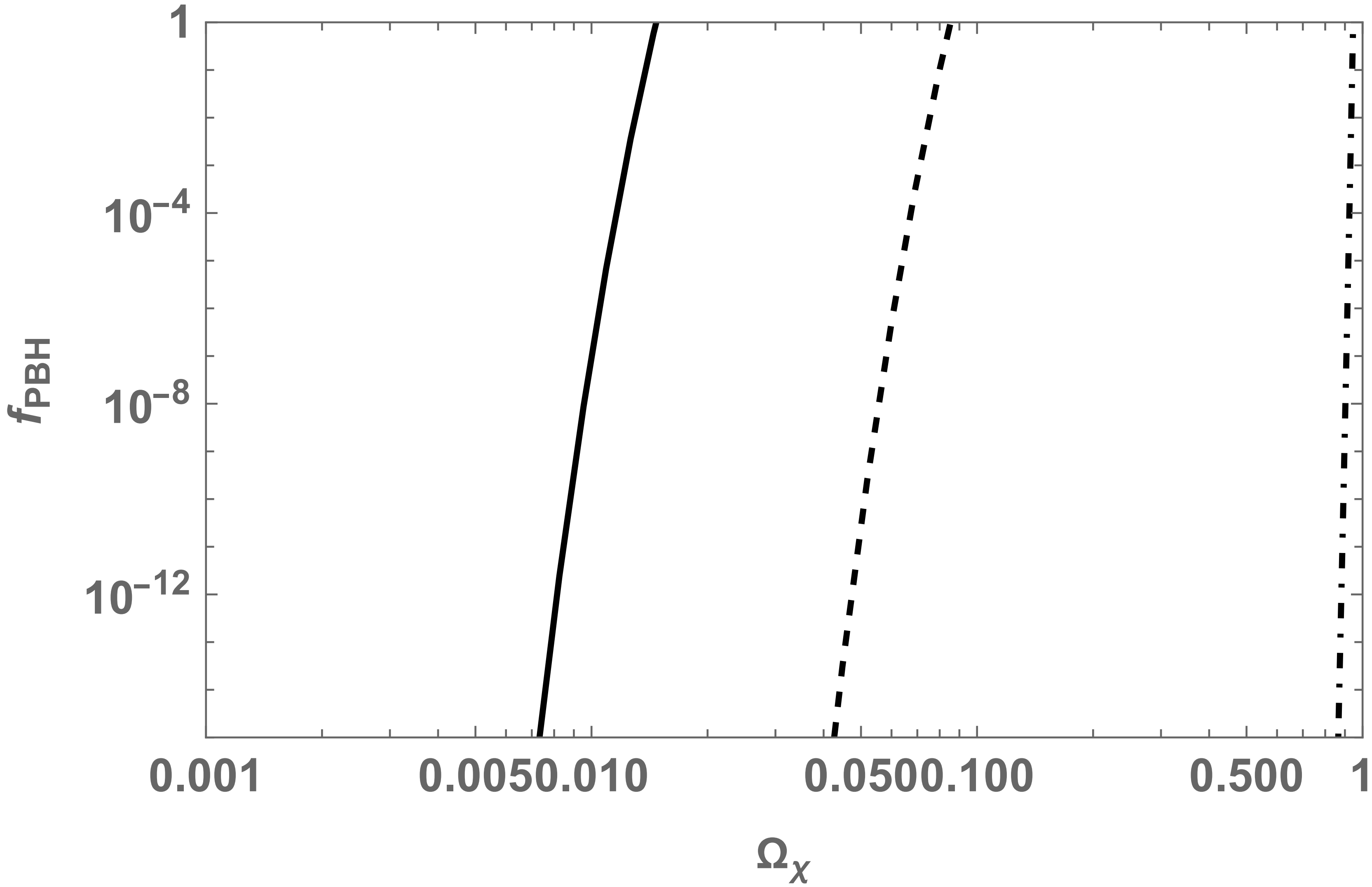
The horizon mass M_H evaluated at the peak scale $k_\star = 1.56 \times 10^{13}\mathrm{Mpc}^{-1}$ in the power spectrum is $M_{k_\star}$, and for these values $Z = 6.88 \times 10^{16}$. Also m_{max} corresponds to $C_l = 4/3$.

Mass distribution of PBHs



The evolution of the mass distribution $f(m)$, normalised by the total dark matter fraction f_{PBH} assuming the lognormal shape for the power spectrum with $\Delta = 0.3$ and $z_\star = 2.74$. The chosen values for the energy density of the curvaton are $\Omega_\chi = 0.1$ (blue), $\Omega_\chi = 0.5$ (red) and $\Omega_\chi = 0.9$ (purple).

Mass distribution of PBHs



The total dark matter fraction f_{PBH} as function of Ω_χ for $\Delta = 0.3$ (solid line), $\Delta = 0.03$ (dashed line), and $\Delta = 0.001$ (dot-dashed line).

Final Considerations

1- Non-Gaussianity can greatly enhance PBH formation;

2- If the curvaton mass is reasonably heavy compared to the Hubble scale, $m^2/H^2 \geqslant 0.28$, then it will have a blue spectrum at small scales without spoiling the large scale structure formation;

3- For the curvaton model with $\langle \chi \rangle = 0$, there is a considerable probability to form PBHs (1.11×10^{-2}) if the curvaton doesn't decay too late, $\Omega_\chi \propto 0.013$ for $\Delta = 0.3$;

4- Our model doesn't depend explicitly on the amplitude of the power spectrum A, nor the the radial coordinate r. The PDF will depend only on the $z_\star$, which is associated to the peak scale by $z_\star = rk_\star$, and the width Δ;

5-By decreasing the width of the lognormal power spectrum, it is possible to see that the condition to form PBHs becomes more narrow, for example, for $\Omega_\chi = 0.9$ and $\Delta = 0.001$, the mass fraction is $f_{PBH} \sim 10^{-179}$;

Final Considerations

The limit $f_{PBH} \leqslant 1$ put a tight constraint on the maximum value of Ω_χ!

Muito obrigada!