PONTIFICIA UNIVERSIDAD CATOLICA DE VALPARAISO

Modified Gravity I

Radouane Gannouji
PUCV, Chile

UFES, December 14, 2015

844 Sitzung der physikalisch-mathematischen Klasse vom 25. November 1915

Die Feldgleichungen der Gravitation.

Von A. Einstein.

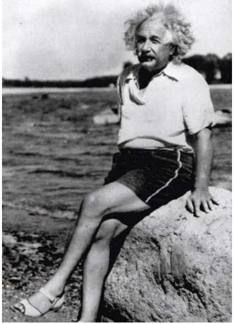
In zwei vor kurzem erschienenen Mitteilungen[1] habe ich gezeigt, wie man zu Feldgleichungen der Gravitation gelangen kann, die dem Postulat allgemeiner Relativität entsprechen, d. h. die in ihrer allgemeinen Fassung beliebigen Substitutionen der Raumzeitvariabeln gegenüber kovariant sind.

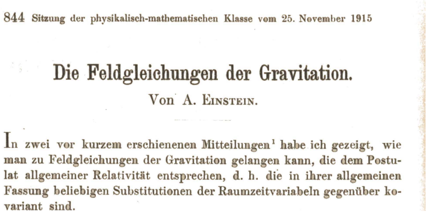
844 Sitzung der physikalisch-mathematischen Klasse vom 25. November 1915

Die Feldgleichungen der Gravitation.

Von A. Einstein.

In zwei vor kurzem erschienenen Mitteilungen[1] habe ich gezeigt, wie man zu Feldgleichungen der Gravitation gelangen kann, die dem Postulat allgemeiner Relativität entsprechen, d. h. die in ihrer allgemeinen Fassung beliebigen Substitutionen der Raumzeitvariabeln gegenüber kovariant sind.



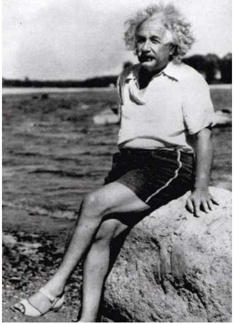


844 Sitzung der physikalisch-mathematischen Klasse vom 25. November 1915

Die Feldgleichungen der Gravitation.

Von A. Einstein.

In zwei vor kurzem erschienenen Mitteilungen[1] habe ich gezeigt, wie man zu Feldgleichungen der Gravitation gelangen kann, die dem Postulat allgemeiner Relativität entsprechen, d. h. die in ihrer allgemeinen Fassung beliebigen Substitutionen der Raumzeitvariabeln gegenüber kovariant sind.



$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$$

- Weak Equivalence Principle is tested through torsion balance experiments (Eöt-Wash) by measuring the fractional difference of the acceleration of different bodies of different composition $\eta < 2 \times 10^{-13}$ (*Wagner et al. 2012*)
- the deflection angle of stars due to the Sun $\theta = (0.99992 \pm 0.00023)\theta_{GR}$ while $\theta_{GR} = 1.75''$ is the prediction in GR (*Shapiro et al. 2004*)
- the time delation due to the Sun's gravitational field measured by Cassini satellite $\Delta t = (1.00001 \pm 0.00001)\Delta t_{GR}$ (*Bertotti et al. 2003*)
- The doppler tracking measurements of the Cassini spacecraft gives for the PPN parameter $|\gamma - 1| < (2.1 \pm 2.3) \times 10^{-5}$
- Lunar laser ranging demonstrates that G is not changing significantly $|\dot{G}/G| < 9 \times 10^{-13}\ \mathrm{yr}^{-1}$
- No Nordtvedt effect (violation of the strong Equivalence Principle) so far $\eta_N \equiv 4\beta - \gamma - 3 < (4.4 \pm 4.5) \times 10^{-4}$
- No evidence of a fifth force, e.g. for a fifth force with a coupling $O(1)$ there is no fifth force down to a distance of $\lambda = 56\ \mu\mathrm{m}$ (*Kapner et al. 2006*)

- Weak Equivalence Principle is tested through torsion balance experiments (Eöt-Wash) by measuring the fractional difference of the acceleration of different bodies of different composition $\eta < 2 \times 10^{-13}$ (*Wagner et al. 2012*)
- the deflection angle of stars due to the Sun $\theta = (0.99992 \pm 0.00023)\theta_{GR}$ while $\theta_{GR} = 1.75''$ is the prediction in GR (*Shapiro et al. 2004*)
- the time delation due to the Sun's gravitational field measured by Cassini satellite $\Delta t = (1.00001 \pm 0.00001)\Delta t_{GR}$ (*Bertotti et al. 2003*)
- The doppler tracking measurements of the Cassini spacecraft gives for the PPN parameter $|\gamma - 1| < (2.1 \pm 2.3) \times 10^{-5}$
- Lunar laser ranging demonstrates that G is not changing significantly $|\dot{G}/G| < 9 \times 10^{-13}\ \mathrm{yr}^{-1}$
- No Nordtvedt effect (violation of the strong Equivalence Principle) so far $\eta_N \equiv 4\beta - \gamma - 3 < (4.4 \pm 4.5) \times 10^{-4}$
- No evidence of a fifth force, e.g. for a fifth force with a coupling $O(1)$ there is no fifth force down to a distance of $\lambda = 56\ \mu$m (*Kapner et al. 2006*)

- Weak Equivalence Principle is tested through torsion balance experiments (Eöt-Wash) by measuring the fractional difference of the acceleration of different bodies of different composition $\eta < 2 \times 10^{-13}$ (*Wagner et al. 2012*)
- the deflection angle of stars due to the Sun $\theta = (0.99992 \pm 0.00023)\theta_{GR}$ while $\theta_{GR} = 1.75''$ is the prediction in GR (*Shapiro et al. 2004*)
- the time delation due to the Sun's gravitational field measured by Cassini satellite $\Delta t = (1.00001 \pm 0.00001)\Delta t_{GR}$ (*Bertotti et al. 2003*)
- The doppler tracking measurements of the Cassini spacecraft gives for the PPN parameter $|\gamma - 1| < (2.1 \pm 2.3) \times 10^{-5}$
- Lunar laser ranging demonstrates that G is not changing significantly $|\dot{G}/G| < 9 \times 10^{-13}\ \text{yr}^{-1}$
- No Nordtvedt effect (violation of the strong Equivalence Principle) so far $\eta_N \equiv 4\beta - \gamma - 3 < (4.4 \pm 4.5) \times 10^{-4}$
- No evidence of a fifth force, e.g. for a fifth force with a coupling $O(1)$ there is no fifth force down to a distance of $\lambda = 56\ \mu\text{m}$ (*Kapner et al. 2006*)

- Weak Equivalence Principle is tested through torsion balance experiments (Eöt-Wash) by measuring the fractional difference of the acceleration of different bodies of different composition $\eta < 2 \times 10^{-13}$ (*Wagner et al. 2012*)
- the deflection angle of stars due to the Sun $\theta = (0.99992 \pm 0.00023)\theta_{GR}$ while $\theta_{GR} = 1.75''$ is the prediction in GR (*Shapiro et al. 2004*)
- the time delation due to the Sun's gravitational field measured by Cassini satellite $\Delta t = (1.00001 \pm 0.00001)\Delta t_{GR}$ (*Bertotti et al. 2003*)
- The doppler tracking measurements of the Cassini spacecraft gives for the PPN parameter $|\gamma - 1| < (2.1 \pm 2.3) \times 10^{-5}$
- Lunar laser ranging demonstrates that G is not changing significantly $|\dot{G}/G| < 9 \times 10^{-13}$ yr^{-1}
- No Nordtvedt effect (violation of the strong Equivalence Principle) so far $\eta_N \equiv 4\beta - \gamma - 3 < (4.4 \pm 4.5) \times 10^{-4}$
- No evidence of a fifth force, e.g. for a fifth force with a coupling $O(1)$ there is no fifth force down to a distance of $\lambda = 56\ \mu$m (*Kapner et al. 2006*)

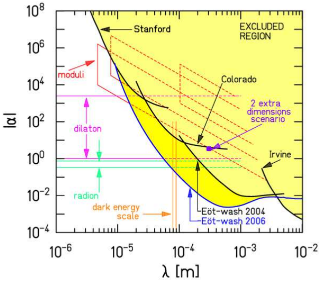
- Weak Equivalence Principle is tested through torsion balance experiments (Eöt-Wash) by measuring the fractional difference of the acceleration of different bodies of different composition $\eta < 2 \times 10^{-13}$ (*Wagner et al. 2012*)
- the deflection angle of stars due to the Sun $\theta = (0.99992 \pm 0.00023)\theta_{GR}$ while $\theta_{GR} = 1.75''$ is the prediction in GR (*Shapiro et al. 2004*)
- the time delation due to the Sun's gravitational field measured by Cassini satellite $\Delta t = (1.00001 \pm 0.00001)\Delta t_{GR}$ (*Bertotti et al. 2003*)
- The doppler tracking measurements of the Cassini spacecraft gives for the PPN parameter $|\gamma - 1| < (2.1 \pm 2.3) \times 10^{-5}$
- Lunar laser ranging demonstrates that G is not changing significantly $|\dot{G}/G| < 9 \times 10^{-13}$ yr^{-1}
- No Nordtvedt effect (violation of the strong Equivalence Principle) so far $\eta_N \equiv 4\beta - \gamma - 3 < (4.4 \pm 4.5) \times 10^{-4}$
- No evidence of a fifth force, e.g. for a fifth force with a coupling $O(1)$ there is no fifth force down to a distance of $\lambda = 56\ \mu$m (*Kapner et al. 2006*)

- Weak Equivalence Principle is tested through torsion balance experiments (Eöt-Wash) by measuring the fractional difference of the acceleration of different bodies of different composition $\eta < 2 \times 10^{-13}$ (*Wagner et al. 2012*)
- the deflection angle of stars due to the Sun $\theta = (0.99992 \pm 0.00023)\theta_{GR}$ while $\theta_{GR} = 1.75''$ is the prediction in GR (*Shapiro et al. 2004*)
- the time delation due to the Sun's gravitational field measured by Cassini satellite $\Delta t = (1.00001 \pm 0.00001)\Delta t_{GR}$ (*Bertotti et al. 2003*)
- The doppler tracking measurements of the Cassini spacecraft gives for the PPN parameter $|\gamma - 1| < (2.1 \pm 2.3) \times 10^{-5}$
- Lunar laser ranging demonstrates that G is not changing significantly $|\dot{G}/G| < 9 \times 10^{-13}\ \mathrm{yr}^{-1}$
- No Nordtvedt effect (violation of the strong Equivalence Principle) so far $\eta_N \equiv 4\beta - \gamma - 3 < (4.4 \pm 4.5) \times 10^{-4}$
- No evidence of a fifth force, e.g. for a fifth force with a coupling $O(1)$ there is no fifth force down to a distance of $\lambda = 56\ \mu$m (*Kapner et al. 2006*)

- Weak Equivalence Principle is tested through torsion balance experiments (Eöt-Wash) by measuring the fractional difference of the acceleration of different bodies of different composition $\eta < 2 \times 10^{-13}$ (*Wagner et al. 2012*)
- the deflection angle of stars due to the Sun $\theta = (0.99992 \pm 0.00023)\theta_{GR}$ while $\theta_{GR} = 1.75''$ is the prediction in GR (*Shapiro et al. 2004*)
- the time delation due to the Sun's gravitational field measured by Cassini satellite $\Delta t = (1.00001 \pm 0.00001)\Delta t_{GR}$ (*Bertotti et al. 2003*)
- The doppler tracking measurements of the Cassini spacecraft gives for the PPN parameter $|\gamma - 1| < (2.1 \pm 2.3) \times 10^{-5}$
- Lunar laser ranging demonstrates that G is not changing significantly $|\dot{G}/G| < 9 \times 10^{-13}\ \mathrm{yr}^{-1}$
- No Nordtvedt effect (violation of the strong Equivalence Principle) so far $\eta_N \equiv 4\beta - \gamma - 3 < (4.4 \pm 4.5) \times 10^{-4}$
- No evidence of a fifth force, e.g. for a fifth force with a coupling $O(1)$ there is no fifth force down to a distance of $\lambda = 56\ \mu$m (*Kapner et al. 2006*)

- Laboratory Bounds

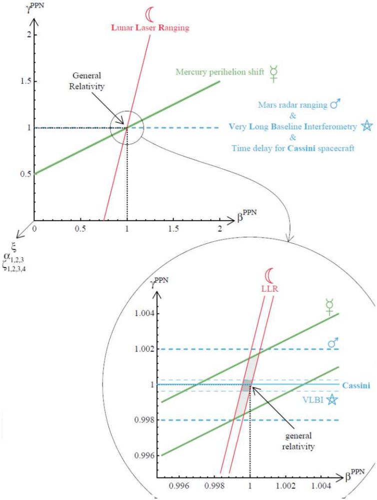
$$V(r) = -G\frac{m_1 m_2}{r}\left[1 + \alpha \exp^{-r/\lambda}\right]$$



$$g_{00} = -1 + 2\frac{GM}{rc^2} - 2\beta^{\mathrm{PPN}}\left(\frac{GM}{rc^2}\right)^2 + \cdots$$

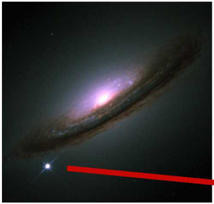
$$g_{ij} = \delta_{ij}\left[1 + 2\gamma^{\mathrm{PPN}}\frac{GM}{rc^2} + \cdots\right]$$

- Solar System Bounds

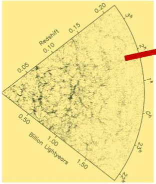


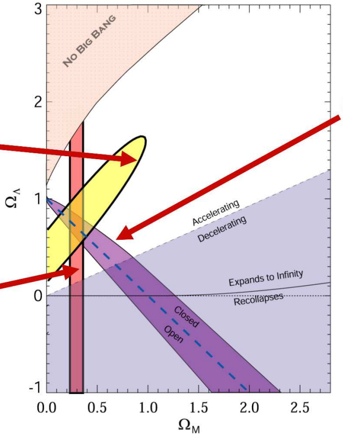
$$\Omega_{DE} = 0.72 \pm 0.015, \qquad -1.11 < w_{DE} = P/\rho < -0.86$$

SNe Ia

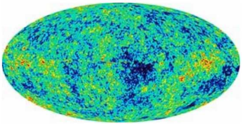


LSS





CMB



- GR is extremely well tested and accurate from laboratory tests to solar system observations with also data from compact objects like pulsars.
- We might also add at larger scales among many observations, the incredible precision of CMB data which are consistent with the model, once a small cosmological constant and an unknown dark matter are added.
- Therefore it seems to be unnecessary and hopeless to modify the theory of gravity.
- But we should nevertheless be open to the possibility that ΛCDM or more generically general relativity is just a first-order approximation of some more fundamental theory.
- In fact, if the theory is perfect at small scales, 95% remains unknown at larger scales (dark energy and dark matter). We are puzzled with various problems

- GR is extremely well tested and accurate from laboratory tests to solar system observations with also data from compact objects like pulsars.
- We might also add at larger scales among many observations, the incredible precision of CMB data which are consistent with the model, once a small cosmological constant and an unknown dark matter are added.
- Therefore it seems to be unnecessary and hopeless to modify the theory of gravity.
- But we should nevertheless be open to the possibility that ΛCDM or more generically general relativity is just a first-order approximation of some more fundamental theory.
- In fact, if the theory is perfect at small scales, 95% remains unknown at larger scales (dark energy and dark matter). We are puzzled with various problems

- GR is extremely well tested and accurate from laboratory tests to solar system observations with also data from compact objects like pulsars.
- We might also add at larger scales among many observations, the incredible precision of CMB data which are consistent with the model, once a small cosmological constant and an unknown dark matter are added.
- Therefore it seems to be unnecessary and hopeless to modify the theory of gravity.
- But we should nevertheless be open to the possibility that ΛCDM or more generically general relativity is just a first-order approximation of some more fundamental theory.
- In fact, if the theory is perfect at small scales, 95% remains unknown at larger scales (dark energy and dark matter). We are puzzled with various problems

- GR is extremely well tested and accurate from laboratory tests to solar system observations with also data from compact objects like pulsars.
- We might also add at larger scales among many observations, the incredible precision of CMB data which are consistent with the model, once a small cosmological constant and an unknown dark matter are added.
- Therefore it seems to be unnecessary and hopeless to modify the theory of gravity.
- But we should nevertheless be open to the possibility that ΛCDM or more generically general relativity is just a first-order approximation of some more fundamental theory.
- In fact, if the theory is perfect at small scales, 95% remains unknown at larger scales (dark energy and dark matter). We are puzzled with various problems

- GR is extremely well tested and accurate from laboratory tests to solar system observations with also data from compact objects like pulsars.
- We might also add at larger scales among many observations, the incredible precision of CMB data which are consistent with the model, once a small cosmological constant and an unknown dark matter are added.
- Therefore it seems to be unnecessary and hopeless to modify the theory of gravity.
- But we should nevertheless be open to the possibility that ΛCDM or more generically general relativity is just a first-order approximation of some more fundamental theory.
- In fact, if the theory is perfect at small scales, 95$\%$ remains unknown at larger scales (dark energy and dark matter). We are puzzled with various problems

The problems

- Cosmological constant problem

It follows from the Equivalence Principle of GR which asserts that all forms of energy curve spacetime

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu}$$

So, even the energy density of the vacuum, which contributes to the cosmological constant, sources the curvature of the spacetime, generically giving it huge contributions

It follows from the Equivalence Principle of GR which asserts that all forms of energy curve spacetime

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

So, even the energy density of the vacuum, which contributes to the cosmological constant, sources the curvature of the spacetime, generically giving it huge contributions

$$\Lambda_{th} = -2 \times 10^8 \mathrm{Gev}^4 + \Lambda_b + \Lambda_{EW} + \Lambda_{QCD}$$

- One promising approaches suggest a modification of GR, in such a way that the graviton no longer universally couples to all sources

It follows from the Equivalence Principle of GR which asserts that all forms of energy curve spacetime

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}$$

So, even the energy density of the vacuum, which contributes to the cosmological constant, sources the curvature of the spacetime, generically giving it huge contributions

$$\Lambda_{th} = -2 \times 10^{8}\mathrm{Gev}^{4} + \Lambda_{b} + \Lambda_{EW} + \Lambda_{QCD}$$
$$\Lambda_{obs} = 10^{-47}\mathrm{Gev}^{4}$$

- One promising approaches suggest a modification of GR, in such a way that the graviton no longer universally couples to all sources

It follows from the Equivalence Principle of GR which asserts that all forms of energy curve spacetime

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

So, even the energy density of the vacuum, which contributes to the cosmological constant, sources the curvature of the spacetime, generically giving it huge contributions

$$\begin{aligned} \Lambda_{th} &= -2 \times 10^{8} \text{Gev}^{4} + \Lambda_b + \Lambda_{EW} + \Lambda_{QCD} \\ \Lambda_{obs} &= 10^{-47} \text{Gev}^{4} \\ \Lambda_b &= \left(2 \times 10^{8} + 10^{-47}\right) \text{Gev}^{4} \end{aligned}$$

- One promising approaches suggest a modification of GR, in such a way that the graviton no longer universally couples to all sources

It follows from the Equivalence Principle of GR which asserts that all forms of energy curve spacetime

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}$$

So, even the energy density of the vacuum, which contributes to the cosmological constant, sources the curvature of the spacetime, generically giving it huge contributions

$$\begin{aligned}
\Lambda_{th} &= -2 \times 10^{8}\text{Gev}^{4} + \Lambda_{b} + \Lambda_{EW} + \Lambda_{QCD} \\
\Lambda_{obs} &= 10^{-47}\text{Gev}^{4} \\
\Lambda_{b} &= \left(2 \times 10^{8} + 10^{-47}\right) \text{Gev}^{4} \\
\Lambda_{b} &= 200000000,00000000000000000 \\
&\qquad 0000000000000000000000000000001 \text{ Gev}^{4}
\end{aligned}$$

- One promising approaches suggest a modification of GR, in such a way that the graviton no longer universally couples to all sources

It follows from the Equivalence Principle of GR which asserts that all forms of energy curve spacetime

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

So, even the energy density of the vacuum, which contributes to the cosmological constant, sources the curvature of the spacetime, generically giving it huge contributions

$$\begin{aligned}
\Lambda_{th} &= -2 \times 10^8 \text{Gev}^4 + \Lambda_b + \Lambda_{EW} + \Lambda_{QCD} \\
\Lambda_{obs} &= 10^{-47} \text{Gev}^4 \\
\Lambda_b &= \left(2 \times 10^8 + 10^{-47}\right) \text{Gev}^4 \\
\Lambda_b &= 200000000,00000000000000000 \\
&\qquad 000000000000000000000000000001 \text{ Gev}^4
\end{aligned}$$

The "bare" constant should be fine-tuned to $\Lambda_b = \left(2 \times 10^8 + 10^{-47}\right) \text{Gev}^4$

- One promising approaches suggest a modification of GR, in such a way that the graviton no longer universally couples to all sources

It follows from the Equivalence Principle of GR which asserts that all forms of energy curve spacetime

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

So, even the energy density of the vacuum, which contributes to the cosmological constant, sources the curvature of the spacetime, generically giving it huge contributions

$$\begin{aligned} \Lambda_{th} &= -2 \times 10^8 \text{Gev}^4 + \Lambda_b + \Lambda_{EW} + \Lambda_{QCD} \\ \Lambda_{obs} &= 10^{-47} \text{Gev}^4 \\ \Lambda_b &= \left(2 \times 10^8 + 10^{-47}\right) \text{Gev}^4 \\ \Lambda_b &= 200000000,00000000000000000 \\ &\qquad 000000000000000000000000000001 \text{ Gev}^4 \end{aligned}$$

The "bare" constant should be fine-tuned to $\Lambda_b = \left(2 \times 10^8 + 10^{-47}\right)$ Gev^4

- One promising approaches suggest a modification of GR, in such a way that the graviton no longer universally couples to all sources

The problems

- Cosmological constant problem
- Inflation

The inflation scenario has been proposed to solve a certain number of problems faced by the cosmology of the early universe such as

- the flatness problem
- the horizon problem
- the monopole problem

- quantum gravity suggests possible Lorentz violations
- non-commutative spacetimes
- extra-dimensional theories

- All these models propose a solution to the accelerated expansion of the early Universe by modifying the framework of the standard theory of GR, like the Starobinsky model

The inflation scenario has been proposed to solve a certain number of problems faced by the cosmology of the early universe such as

- the flatness problem
- the horizon problem
- the monopole problem

In all these models, additional fields are included such as a scalar field minimally or non-minimally coupled, with or without higher order terms. For example

- quantum gravity suggests possible Lorentz violations
- non-commutative spacetimes
- extra-dimensional theories

- All these models propose a solution to the accelerated expansion of the early Universe by modifying the framework of the standard theory of GR, like the Starobinsky model

The inflation scenario has been proposed to solve a certain number of problems faced by the cosmology of the early universe such as

- the flatness problem
- the horizon problem
- the monopole problem

In all these models, additional fields are included such as a scalar field minimally or non-minimally coupled, with or without higher order terms. For example

- quantum gravity suggests possible Lorentz violations
- non-commutative spacetimes
- extra-dimensional theories

- All these models propose a solution to the accelerated expansion of the early Universe by modifying the framework of the standard theory of GR, like the Starobinsky model

The inflation scenario has been proposed to solve a certain number of problems faced by the cosmology of the early universe such as

- the flatness problem
- the horizon problem
- the monopole problem

In all these models, additional fields are included such as a scalar field minimally or non-minimally coupled, with or without higher order terms. For example

- quantum gravity suggests possible Lorentz violations
- non-commutative spacetimes
- extra-dimensional theories

The latter imply the existence of scalar partners to the graviton, and even more subtle phenomenological effects occur if our 4-dimensional spacetime is a brane embedded in the higher-dimensional bulk.

- All these models propose a solution to the accelerated expansion of the early Universe by modifying the framework of the standard theory of GR, like the Starobinsky model

The inflation scenario has been proposed to solve a certain number of problems faced by the cosmology of the early universe such as

- the flatness problem
- the horizon problem
- the monopole problem

In all these models, additional fields are included such as a scalar field minimally or non-minimally coupled, with or without higher order terms. For example

- quantum gravity suggests possible Lorentz violations
- non-commutative spacetimes
- extra-dimensional theories

The latter imply the existence of scalar partners to the graviton, and even more subtle phenomenological effects occur if our 4-dimensional spacetime is a brane embedded in the higher-dimensional bulk.

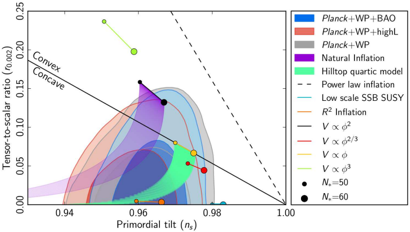
- All these models propose a solution to the accelerated expansion of the early Universe by modifying the framework of the standard theory of GR, like the Starobinsky model

Starobinsky model

- Starobinsky model (Gruber Prize in Cosmology 2013 with V. Mukhanov)

$$S = \int d^4x\sqrt{-g}\left(R + \frac{R^2}{6M^2}\right)$$

- Viable candidate according to last results: Planck



- Many models as Higgs inflation are equivalent to Starobinsky model

(Kehagias, Dizgah, Riotto, 2013)

The problems

- Cosmological constant problem
- Inflation
- Non-renormalizability

- The theory of GR is not perturbatively renormalizable around $g_{\mu\nu} = \eta_{\mu\nu}$
- Also the theory becomes renormalizable if we add higher curvature terms $(\alpha R_{\mu\nu} R^{\mu\nu} + \beta R^2)$ (*Stelle, 1977*)
- But the theory is plagued with ghosts (poles in the propagator with negative residue) $\Pi(p^2) \sim 1/p^2 - 1/(p^2 + m^2)$
- It seems reasonable to consider higher order curvature terms as quantum corrections to the classical theory
- And these ultraviolet corrections to GR might "leak" down to cosmological scales

The problems

- Cosmological constant problem
- Inflation
- Non-renormalizability
- Dark matter

- We have known for a long time that the Universe has a dark component by studying the velocity distribution of galaxies
- But it was later realized that non-luminous matter, dark matter, is needed at all scales from galactic to cosmological
- The standard proposition is to consider additional fields which are predicted by theories beyond the standard model of particle physics such as supersymmetry

The problems

- Cosmological constant problem
- Inflation
- Non-renormalizability
- Dark matter
- Testing GR

If all you have is a hammer, everything looks like a nail

- Other theories give us the opportunity to compare GR with alternatives

- $\Phi = \Psi$ without anisotropic stress (e.g. ΛCDM)
- Alternative theories of gravity violate this condition

- Alternative theories of gravity are also a laboratory on how to test GR

If all you have is a hammer, everything looks like a nail

- Other theories give us the opportunity to compare GR with alternatives

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)dx^2$$

- $\Phi = \Psi$ without anisotropic stress (e.g. ΛCDM)
- Alternative theories of gravity violate this condition

- Alternative theories of gravity are also a laboratory on how to test GR

If all you have is a hammer, everything looks like a nail

- Other theories give us the opportunity to compare GR with alternatives

$$ds^2 = -(1 + 2\Phi)dt^2 + a^2(1 - 2\Psi)dx^2$$

- $\Phi = \Psi$ without anisotropic stress (e.g. ΛCDM)
- Alternative theories of gravity violate this condition
- Alternative theories of gravity are also a laboratory on how to test GR

If all you have is a hammer, everything looks like a nail

- Other theories give us the opportunity to compare GR with alternatives

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)dx^2$$

- $\Phi = \Psi$ without anisotropic stress (e.g. ΛCDM)
- Alternative theories of gravity violate this condition
- Alternative theories of gravity are also a laboratory on how to test GR

If all you have is a hammer, everything looks like a nail

- Other theories give us the opportunity to compare GR with alternatives

$$\mathrm{d}s^2 = -(1+2\Phi)\mathrm{d}t^2 + a^2(1-2\Psi)\mathrm{d}x^2$$

- $\Phi = \Psi$ without anisotropic stress (e.g. ΛCDM)
- Alternative theories of gravity violate this condition

Therefore we might start thinking of experiments to measure with high accuracy these potentials (Φ, Ψ), which is one of the goal of the satellite Euclid which will be launched in 2020

- Alternative theories of gravity are also a laboratory on how to test GR

If all you have is a hammer, everything looks like a nail

- Other theories give us the opportunity to compare GR with alternatives

$$ds^2 = -(1 + 2\Phi)dt^2 + a^2(1 - 2\Psi)dx^2$$

- $\Phi = \Psi$ without anisotropic stress (e.g. ΛCDM)
- Alternative theories of gravity violate this condition

Therefore we might start thinking of experiments to measure with high accuracy these potentials (Φ, Ψ), which is one of the goal of the satellite Euclid which will be launched in 2020

- Alternative theories of gravity are also a laboratory on how to test GR

How to modify the theory of gravity?

- **Lovelock's theorem**

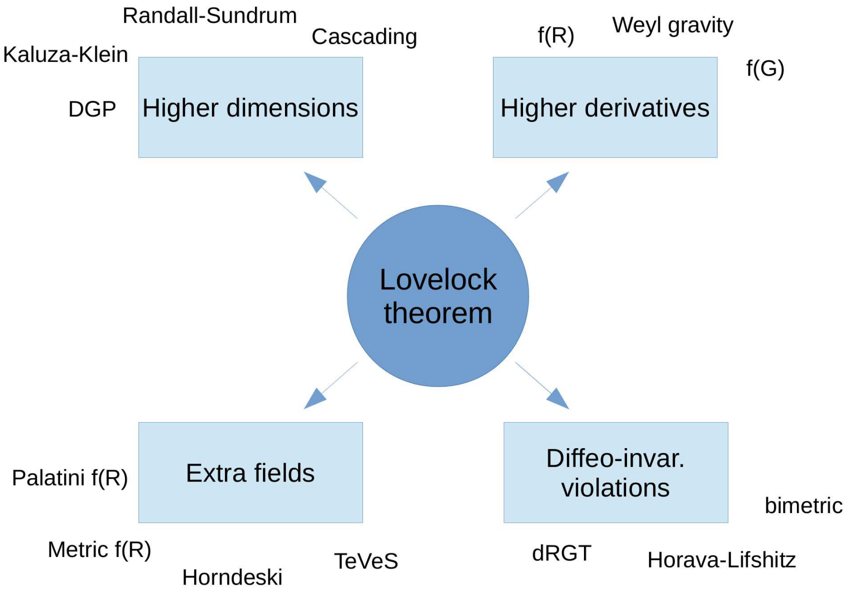
  In 4D the only action constructed solely from the metric $g_{\mu\nu}$ preserving the diffeomorphism and which have field equations involving derivatives of the metric tensor only up to second order is Einstein-Hilbert action

How to modify the theory of gravity?

- **Lovelock's theorem**

In 4D the only action constructed solely from the metric $g_{\mu\nu}$ preserving the diffeomorphism and which have field equations involving derivatives of the metric tensor only up to second order is Einstein-Hilbert action

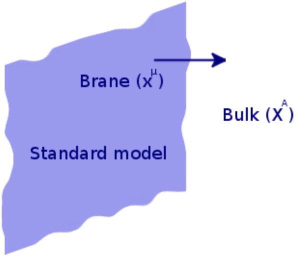
Following these assumptions, GR emerges as an unique theory. But any violation of these axioms gives rise to a different class of modified theories of gravity


Randall-Sundrum
Cascading
Kaluza-Klein
DGP
Higher dimensions
f(R)
Weyl gravity
f(G)
Higher derivatives
Lovelock theorem
Palatini f(R)
Extra fields
Metric f(R)
Horndeski
TeVeS
Diffeo-invar. violations
bimetric
dRGT
Horava-Lifshitz

In this very rich zoo of models, we can find a fundamental common idea

In this very rich zoo of models, we can find a fundamental common idea

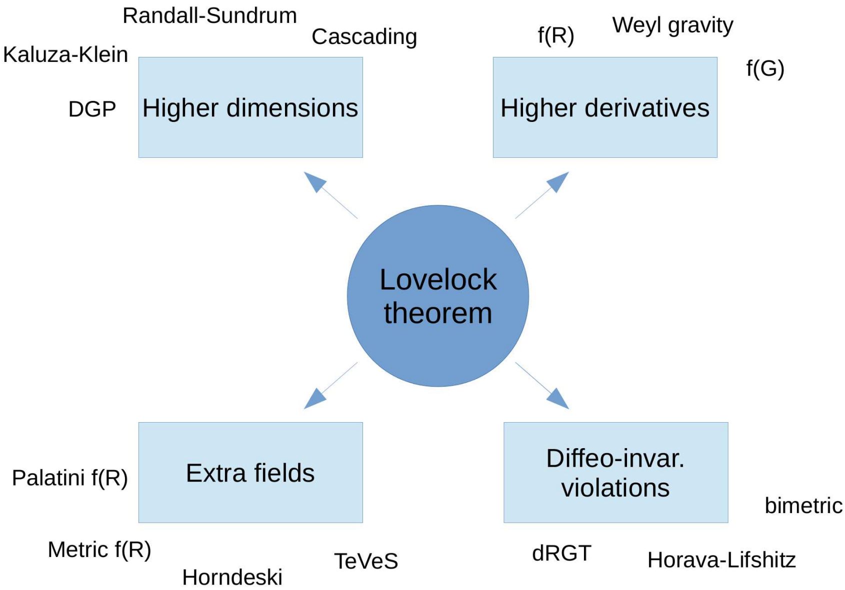
- In higher dimensional theories, the models have more than 2 degrees of freedom. For example, the DGP model has 5 degrees of freedom which can be split into a massless graviton, a massless vector field and a scalar field
- Integrating out the additional dimension gives rise to an effective theory describing the brane bending mode, known as galileon theory
- Higher dimensional models contain often scalar field
- In fact, more generically many theories of high-energy physics, such as string theory and supergravity, predict light gravitationally coupled scalar fields



$$S = M_{(5)}^3 \int \sqrt{-g}\, d^5x\, R + M_{Pl}^2 \int_{\text{brane}} \sqrt{-g}\, d^4x\, R + \int_{\text{brane}} \sqrt{-g}\, d^4x\, \mathscr{L}_{\text{matter}}$$

- First framework where cosmic acceleration is linked to a large distance modification of gravity
- A good description of many DGP key properties is given by

$$S = \int \sqrt{-g}\, d^4x \left[\frac{M_{Pl}^2}{2} R - \frac{1}{2} (\partial\phi)^2 - \frac{1}{\Lambda^3} (\partial\phi)^2 \Box\phi + \phi T \right]$$


Randall-Sundrum
Cascading
Kaluza-Klein
DGP
Higher dimensions
f(R)
Weyl gravity
f(G)
Higher derivatives
Lovelock theorem
Palatini f(R)
Extra fields
Metric f(R)
Horndeski
TeVeS
Diffeo-invar. violations
bimetric
dRGT
Horava-Lifshitz

In this very rich zoo of models, we can find a fundamental common idea

- In higher dimensional theories, the models have more than 2 degrees of freedom. For example, the DGP model has 5 degrees of freedom which can be split into a massless graviton, a massless vector field and a scalar field
- Integrating out the additional dimension gives rise to an effective theory describing the brane bending mode, known as galileon theory
- Higher dimensional models contain often scalar field
- In fact, more generically many theories of high-energy physics, such as string theory and supergravity, predict light gravitationally coupled scalar fields
- In higher derivative theories such as $f(R)$ in the metric formalism, we have also an additional degree of freedom, the scalaron which appears explicitly in the Einstein frame of the theory

$$S_1 = \int d^4x\sqrt{-g}f(R) \Leftrightarrow S_2 = \int d^4x\sqrt{-g}\left(f(A) + F(A)(R - A)\right)$$

$$\text{with } F = \frac{df}{dR}$$

$$S_1 = \int d^4x\sqrt{-g}f(R) \Leftrightarrow S_2 = \int d^4x\sqrt{-g}\left(f(A) + F(A)(R-A)\right)$$

$$\text{with } F = \frac{df}{dR}$$

Variation with respect to A leads to the equation

$$S_1 = \int d^4x\sqrt{-g}f(R) \Leftrightarrow S_2 = \int d^4x\sqrt{-g}\left(f(A) + F(A)(R-A)\right)$$

$$\text{with } F = \frac{df}{dR}$$

Variation with respect to A leads to the equation

$$F'(A)(R-A) = 0 \;\Rightarrow\; A = R \quad \text{if} \quad F'(A) \neq 0$$

$$S_1 = \int d^4x \sqrt{-g} f(R) \Leftrightarrow S_2 = \int d^4x \sqrt{-g} \left(f(A) + F(A)(R - A) \right)$$

$$\text{with } F = \frac{df}{dR}$$

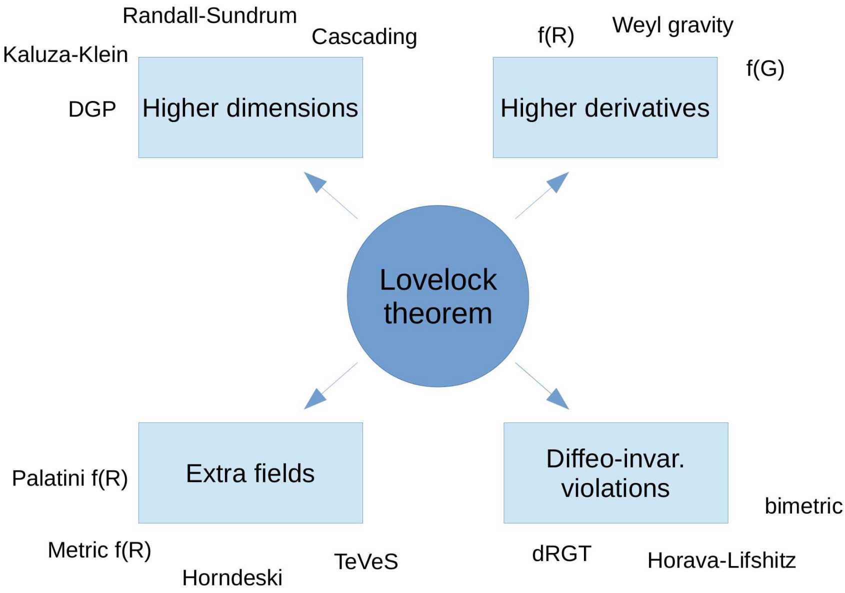
Variation with respect to A leads to the equation

$$F'(A)(R - A) = 0 \;\Rightarrow\; A = R \quad \text{if} \quad F'(A) \neq 0$$

$$F(A) = \phi, \text{ and } V(\phi) = A(\phi)\phi - f\left[A(\phi)\right]$$

$$\Downarrow$$

$$S_2 = \int d^4x \sqrt{-g} \left(\phi R - V(\phi) \right)$$



Randall-Sundrum
Cascading
Kaluza-Klein
DGP
Higher dimensions
f(R)
Weyl gravity
f(G)
Higher derivatives
Lovelock theorem
Palatini f(R)
Extra fields
Metric f(R)
Horndeski
TeVeS
Diffeo-invar. violations
bimetric
dRGT
Horava-Lifshitz

In this very rich zoo of models, we can find a fundamental common idea

- In higher dimensional theories, the models have more than 2 degrees of freedom. For example, the DGP model has 5 degrees of freedom which can be split into a massless graviton, a massless vector field and a scalar field
- Integrating out the additional dimension gives rise to an effective theory describing the brane bending mode, known as galileon theory
- Higher dimensional models contain often scalar field
- In fact, more generically many theories of high-energy physics, such as string theory and supergravity, predict light gravitationally coupled scalar fields
- In higher derivative theories such as $f(R)$ in the metric formalism, we have also an additional degree of freedom, the scalaron which appears explicitly in the Einstein frame of the theory
- In the case we break the diffeomorphism invariance such as dRGT model, we have additional degrees of freedom. And like for extra dimensions, the very interesting aspects of the model are encoded in the scalar degree of freedom which become transparent in the so-called decoupling limit

Therefore we see that even if the models appear very different, they can be unified in a single structure

Therefore we see that even if the models appear very different, they can be unified in a single structure

They all have in common an additional degree of freedom, a **scalar field**

$$\Phi$$

Quintessence

$$S = \int \sqrt{-g} d^4x \left[R - \frac{1}{2}(\partial\phi)^2 - V(\phi) \right]$$

For a homogeneous profile $\phi = \phi(t)$, the scalar field acts like a perfect fluid with equation of state $w = P/\rho$

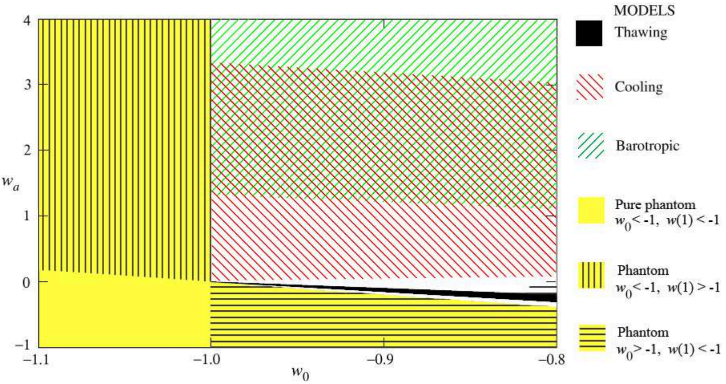
$$w = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}$$

ϕ is a source to the observed accelerated expansion if $w \simeq -1 \Rightarrow \dot{\phi}^2 \ll V(\phi)$

- Tracker solution (e.g. with the Ratra-Peebles potential $V = M^{4+n}/\phi^n$)
- Phantom ($w < -1$)
- Thawing/freezing models
- Dark energy and inflation
- Black holes (scalar hair)

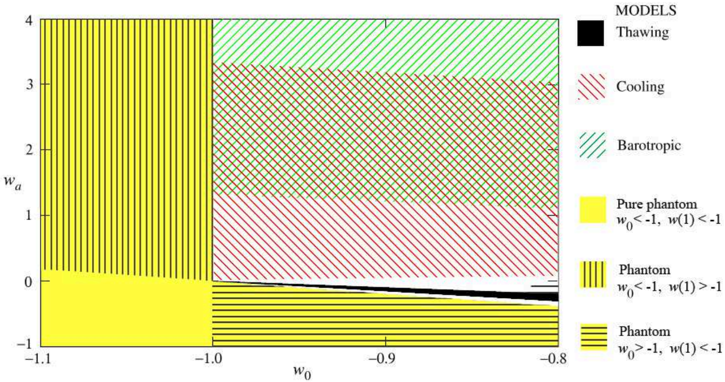
- Tracker solution (e.g. with the Ratra-Peebles potential $V = M^{4+n}/\phi^n$)
- Phantom ($w < -1$)
- Thawing/freezing models
- Dark energy and inflation
- Black holes (scalar hair)

- Tracker solution (e.g. with the Ratra-Peebles potential $V = M^{4+n}/\phi^n$)
- Phantom ($w < -1$)
- Thawing/freezing models



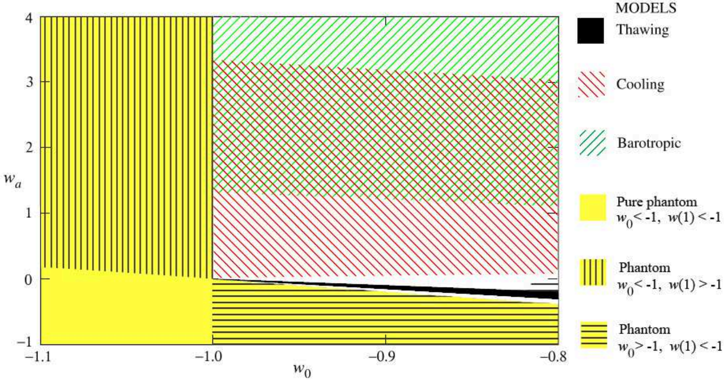
- Dark energy and inflation
- Black holes (scalar hair)

- Tracker solution (e.g. with the Ratra-Peebles potential $V = M^{4+n}/\phi^n$)
- Phantom ($w < -1$)
- Thawing/freezing models



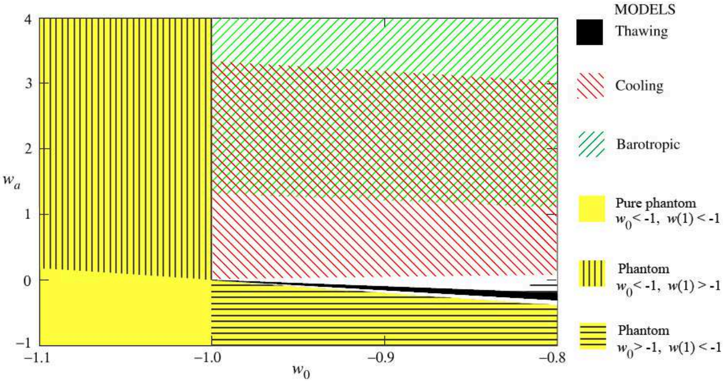
- Dark energy and inflation
- Black holes (scalar hair)

- Tracker solution (e.g. with the Ratra-Peebles potential $V = M^{4+n}/\phi^n$)
- Phantom ($w < -1$)
- Thawing/freezing models



- Dark energy and inflation
- Black holes (scalar hair)

- Tracker solution (e.g. with the Ratra-Peebles potential $V = M^{4+n}/\phi^n$)
- Phantom ($w < -1$)
- Thawing/freezing models



- Dark energy and inflation
- Black holes (scalar hair)

Not solving the CCP, same fine-tuning

$$S = \int \sqrt{-g} d^4x \left[R - \frac{1}{2} Z^{\mu\nu}(\phi, \partial\phi, \cdots) \partial_\mu \phi \partial_\nu \phi - V(\phi) + \phi L_m \right]$$

$$S = \int \sqrt{-g} d^4x \Big[R - \frac{1}{2} Z^{\mu\nu}(\phi, \partial\phi, \cdots) \partial_\mu \phi \partial_\nu \phi - V(\phi) + \phi L_m \Big]$$

$$S = \int \sqrt{-g}\, d^4x \left[R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{\Lambda^3}(\partial\phi)^2 \Box\phi + \phi L_m \right]$$

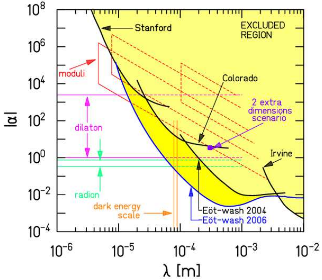
$$Z^{\mu\nu} = g^{\mu\nu}(1 + \frac{2}{\Lambda^3}\Box\phi)$$

$$S=\int\sqrt{-g}d^4x\Big[R-\frac{1}{2}Z^{\mu\nu}(\phi,\partial\phi,\cdots)\partial_\mu\phi\partial_\nu\phi-V(\phi)+\phi L_m\Big]$$

The coupling to matter produces a fifth force

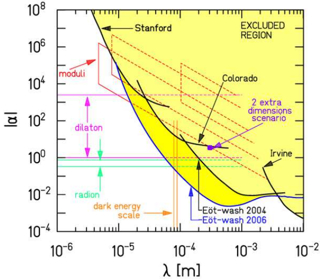
$$S = \int \sqrt{-g}d^4x\left[R - \frac{1}{2}Z^{\mu\nu}(\phi, \partial\phi, \cdots)\partial_\mu\phi\partial_\nu\phi - V(\phi) + \phi L_m\right]$$

The coupling to matter produces a fifth force



$$S = \int \sqrt{-g} d^4x \Big[R - \frac{1}{2} Z^{\mu\nu}(\phi, \partial\phi, \cdots)\partial_\mu \phi \partial_\nu \phi - V(\phi) + \phi L_m \Big]$$

The coupling to matter produces a fifth force



Solar System and lab experiments rule out any long range gravitationally coupled fifth force

So the challenge is to modify gravity only at large (cosmic) distances, while keeping it unaltered at short (Solar System) distances