Superradiance in Analogue Black Holes

Maurício Richartz (mauricio.richartz@ufabc.edu.br)
Universidade Federal do ABC (UFABC), Santo André, SP, Brasil

(Collaborators: Stefano Liberati, Angus Prain, Silke Weinfurtner)

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Outline

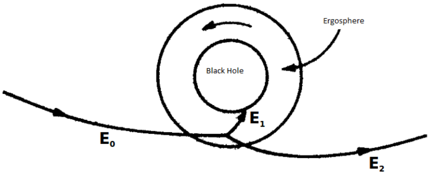
- Superradiance (in general)
- Analogue models of gravity
- Superradiance with gravity waves
  - Simple model
  - Our setup: background and perturbations
  - Numerical results
  - Conclusions

Motivation

- Analogue models provide a theoretical/experimental framework for testing QFTCS.
- The most studied example is Hawking radiation. However we are mostly interested in studying the superradiant scattering of waves by a rotating analogue black hole. Can the amplification be detected in the lab?

Superradiance: the basics

- Superradiance is an important phenomenon for black holes, but was originally proposed in a different context. It is the wave analogue of the Penrose process.



Penrose Process

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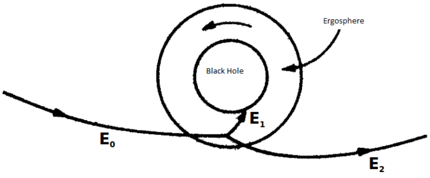
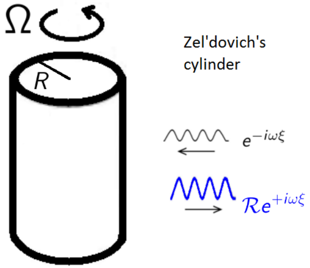


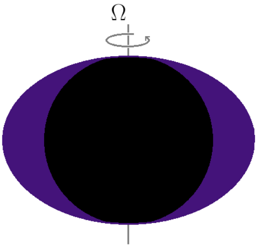
Penrose Process

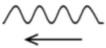


Superradiance

Superradiance: the basics

- Consider a field $\psi = f(x)e^{-i\omega t + im\phi}$ (axisymmetry) which satisfies $f'' + V(x)f = 0$, where $V(x) \to \omega^2$ when $x \to \infty$.

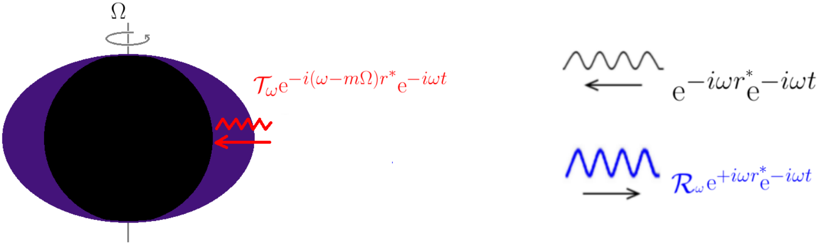




$e^{-i\omega r^*} e^{-i\omega t}$

Superradiance: the basics

BC at $x = x_0$ (x_0 could be the event horizon or the radius of Zeldovich's cylinder) + conservation of "energy" $\Rightarrow$ Superradiance if $\omega < m\Omega$ [MR,Weinfurtner,Penner,Unruh(2009)].



$$|\mathcal{R}_\omega|^2 = 1 - \frac{\omega - m\Omega}{\omega} * (\text{positive quantity})$$

Superradiance: more details for Kerr BHs

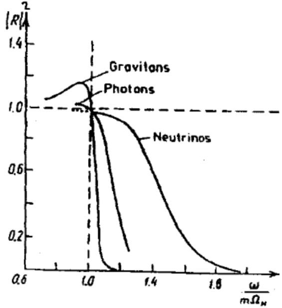
- KG eqn: $\nabla^\mu \nabla_\mu \psi = 0$ and *ansatz* $\psi = R(r)S(\theta)e^{im\phi - i\omega t} \Rightarrow$

$$\text{Teukolsky eqn:} \qquad \Delta \frac{d}{dr}\left(\Delta \frac{dR}{dr}\right) + V(r)R = 0$$

- Scattering process depends on three wave parameters (ℓ, m, $M\omega$) and on one background parameter a/M.

- Maximum amplification occurs for $a/M \approx 1$.

Can we see superradiance in real life?



- Real black holes - angular momentum is limited: $a \leq M$
- Press and Teukolsky (1974)
  - Scalar waves: $0,3\%$
  - Electromag. waves: $4,4\%$
  - Grav. waves: 138%

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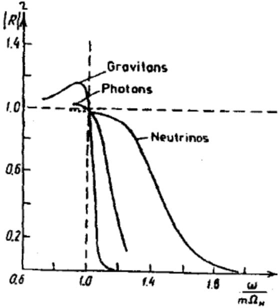

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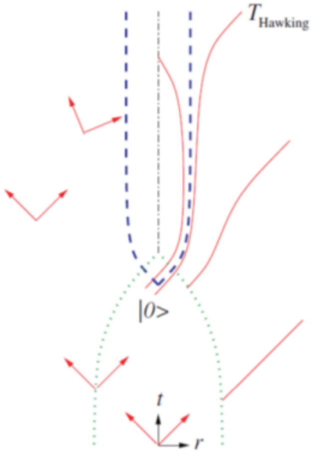
- Scattering of electromagnetic waves by a rotating cylinder (Zel'dovich (1971); Bekenstein and Schiffer (1998)).

Undetectable amplification (because speed of light $c \gg \Omega R_0$)

What about analogue models of gravity?

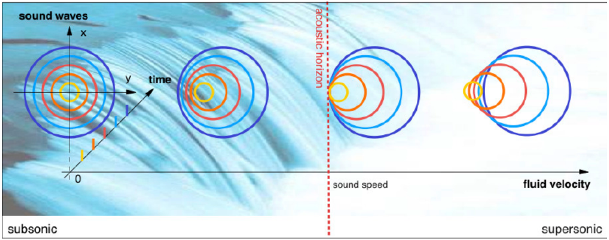
Analogue models of gravity: the basics



- In 1974, Hawking showed that black holes are not completely "black"; they emit the so-called Hawking radiation (HR).
- Transplanckian problem: the outgoing HR is originated in modes which have arbitrarily small wavelengths.
- Ideas of Unruh and Jacobson, based on analogue models, suggest that HR is indeed real.

Ref.: R. Schutzhold (CQG, 2008)

Analogue models: the basics



Ref.: Barcelo, Liberati, Visser (Liv. Rev. Relativity, 2011)

- (i) Barotropic and inviscid fluid; (ii) $\nabla \times \vec{v} = 0$, i.e. $\vec{v} = \nabla\psi$
- Write down the linear perturbations of the continuity + Euler (Bernoulli) eqns: $\rho \to \rho + \delta\rho$, $P \to P + \delta P$, $\psi \to \psi + \delta\psi$
- Field perturbations $\delta\psi$ satisfy a KG equation in a curved spaced time: $\nabla^\mu \nabla_\mu \delta\psi = 0$.

Superradiance with sound waves

- Background: $\vec{v} = -\frac{A}{r}\hat{r} + \frac{B}{r}\hat{\phi}$, ρ and P constants.
- Horizon ($|v_r| = c$) at $r = A/c$.
- Ergosphere ($|\vec{v}| = c$) at $r = \sqrt{A^2 + B^2}/c$.
- Klein–Gordon equation $\left[\text{ansatz } \delta\psi = \sqrt{r}H(r)e^{im\phi - i\omega t}\right]$:

$$\frac{d^2 H}{d\bar{r}_*^2} + \left[\left(\bar{\omega} - \frac{m\bar{B}}{\bar{r}^2}\right)^2 - V(\bar{r}, m)\right] H = 0,$$

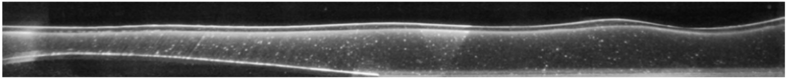
  which depends on only one background parameter ($\bar{B} = B/A$).
- Superradiant scattering is possible if $\omega < m\Omega$, where $\Omega = Bc/A^2$. [Basak, Majumdar (2002,2003), Berti, Cardoso, Lemos (2004)].

Superradiance with sound waves

- Can we use this idea to build an experiment?
- Since $c \approx 1500 m/s$, one would need very high fluid velocities in the lab in order to obtain an event horizon.
- Even if this is possible, $v \approx c$ would probably cause the formation of shock waves.
- Is there another analogue model which could be used?.

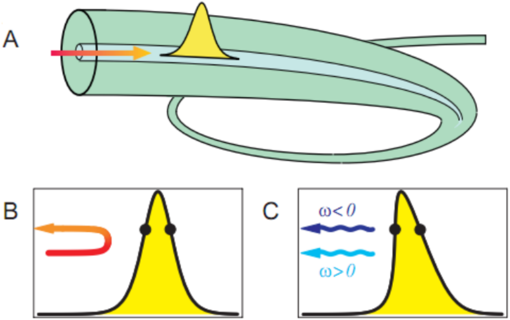
Analogue models in the lab

Surface Waves



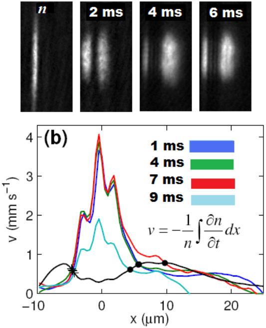
Ref.: Physics in Canada (Unruh, 2010), Weinfurtner et al (PRL, 2011)

Fiber Optics



Ref.: Philbin et. al (Science, 2008)

BEC



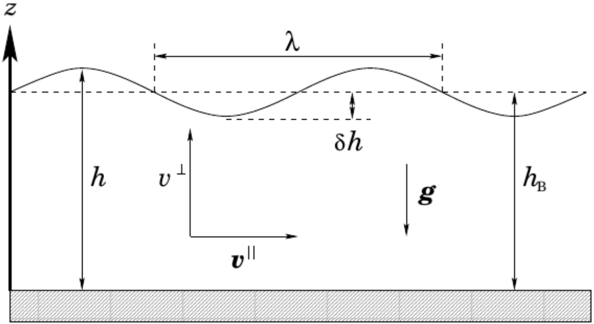
Ref.: Lahav et. al (PRL, 2010)

Superradiance with gravity waves: our model

MR, A. Prain, S. Liberati, S. Weinfurtner (Arxiv:1411.1662)

- The main objectives are:
  - Estimate the superradiant amplification in a realistic setup
  - Construct the setup in the lab and measure the effect (not yet done).
- In order to do the first part we need to:
  - Model the background flow and analyse the propagation of waves on it.
  - Numerical simulations and parameter search to estimate the amplification.

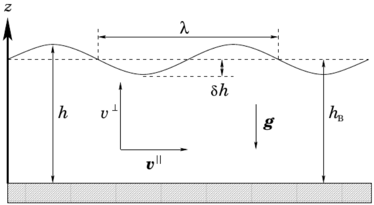
Superradiance with GW



Ref.: Schützhold, Unruh (PRD, 2002)

Gravity waves are waves propagating on the surface of a fluid whose only restoring force is gravity. The formulation as an analogue model was given by Schützhold and Unruh (2002).

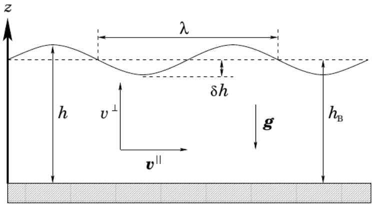
Superradiance with GW



- Similar equations as before: Irrotational flow ($\mathbf{v} = \nabla\psi$) + Continuity + Euler equation.
- However, now the boundary conditions also play a crucial role:

$$\bullet\ \mathbf{v}_z\big|_{z=0} = 0 \qquad \bullet\ \frac{dh}{dt} = v_z\big|_{z=h} \qquad \bullet\ P\big|_{z=h} = 0$$

Superradiance with GW



- Perturb ρ, P, and ψ instead of h, P, and ψ.
- In the shallow water regime ($\lambda \gg h$), the perturbations $\delta\psi$ satisfy a KG eqn, $\nabla_\mu \nabla^\mu \delta\psi_h = 0$, with an effective metric

$$g_{\mu\nu} = \left(\frac{h}{g}\right) \begin{pmatrix} -gh + |\mathbf{v}_{||}|_{z=h}|^2 & -\mathbf{v}_{||}|_{z=h} \\ -\mathbf{v}_{||}|_{z=h} & \mathbf{I}_{2\times 2} \end{pmatrix}, \quad c^2 = gh$$

- Schützhold, Unruh (2002) and Berti, Cardoso, Lemos (2004), basically considered scenarios in which $h = const$.

Superradiance with GW: our model

- Irrotational flow: $\nabla \times \mathbf{v} = 0 \Rightarrow v_\phi = B/r$.
- Cont. eqn + BCs $\Rightarrow v_r = -\frac{Ah_\infty}{rh(r)}$
- Plug v_r and v_ϕ into Bernoulli's eqn:

$$\frac{1}{2}\left(v_r^2 + v_\phi^2\right) + gh = gh_\infty \Rightarrow h^3 + h^2\left[\frac{B^2}{2gr^2} - h_\infty\right] + \frac{A^2 h_\infty^2}{2gr^2} = 0.$$

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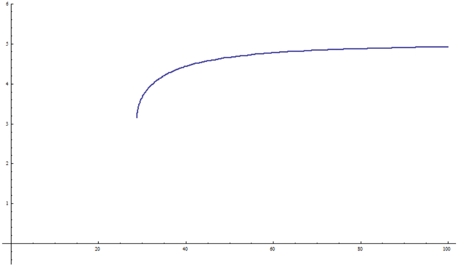


Superradiance with GW: our model

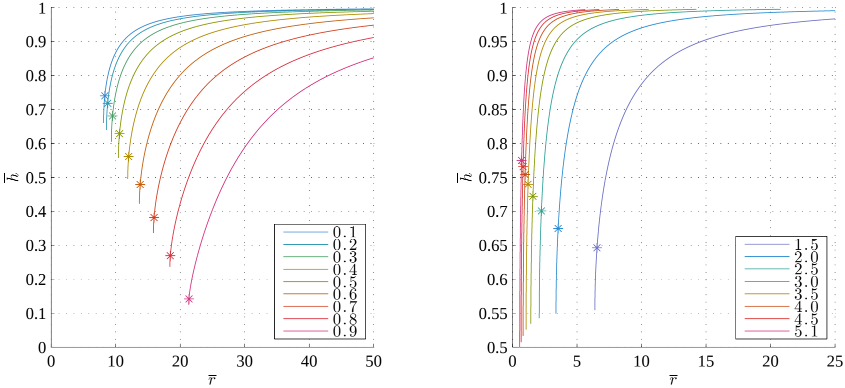
- What about perturbations in our model? They also satisfy a KG eqn in a curved spacetime. The most important difference is, again, the fact that $h(r)$ is not constant and, in particular, g has to be replaced by some function of r: $g \to \tilde{g}(r)$. Therefore, $c = \sqrt{\tilde{g}(r)h(r)}$ instead of $c = \sqrt{gh}$.

- Where is the horizon located in our model?

$$|v_r| = c \Rightarrow \frac{Ah_\infty}{rh} = \sqrt{\tilde{g}h},$$

which can only be solved numerically.

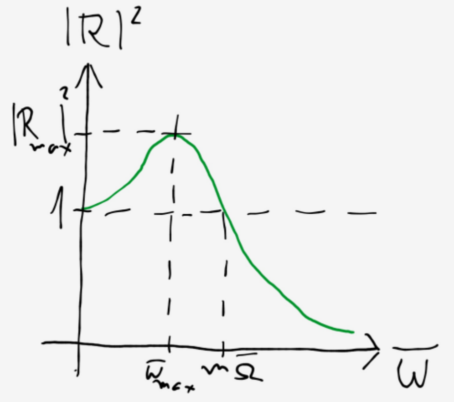
Surface profile / Horizon



Surface profile and location of the horizon (Arxiv:1411.1662)

Superradiance with GW: our model

- By imposing adequate BCs at $r \to r_h$ and $r \to \infty$, one can show that superradiance will occur (actually, it does not depend on the specifics of $h(r)$).
- For a given set of parameters, this is the typical spectrum obtained:



Superradiance with GW: our model

- We would like to do a parameter search to determine the best regime for superradiance that also satisfies our assumptions (namely, $h'^2 << 1$ and linear dispersion).
- To do that, we need first to rescale our equations:

$$\overline{A} = \frac{A}{\sqrt{2gh_\infty^3}}, \quad \overline{B} = \frac{B}{\sqrt{2gh_\infty^3}}, \quad \overline{g} = \tilde{g}/g$$

$$\overline{r} = \frac{r}{h_\infty}, \quad \overline{h} = \frac{h}{h_\infty}, \quad \overline{\omega} = \frac{\omega}{\omega_{disp}}, \quad \omega_{disp} = \sqrt{\frac{g}{h_\infty}}.$$

- In the end, the perturbation eqn for $\delta\psi = R(r)e^{im\phi - i\omega t}$ depends on two background parameters: $\overline{A}$ and $\overline{B}$ (it also depends on $\overline{\omega}$, m and, of course, $\overline{r}$).

Superradiance with gravity waves: our model

Instead of $\overline{A}$ and $\overline{B}$, we would like to use more "natural parameters":

- For Kerr BHs, there is only one parameter: $a/M = 2\Omega r_+ \leq 1$. This suggests that $\Omega r_{\mathrm{H}} = v_\phi|_{r=r_{\mathrm{H}}} = B/r_{\mathrm{H}}$ might be important in our model. Furthermore, because of the Bernoulli equation, we have $v_\phi \leq \sqrt{2gh_\infty}$, and therefore $\overline{v}_\phi(r_{\mathrm{H}}) \leq 1$.
- Another important quantity associated with horizons is the surface gravity:

$$\overline{\kappa}(r_{\mathrm{H}}) = \kappa_{\mathrm{H}}/g = \kappa_{\mathrm{H}} = \frac{1}{2}\frac{d}{dr}\left(c^2 - v_r^2\right)\Big|_{\text{horizon}}$$

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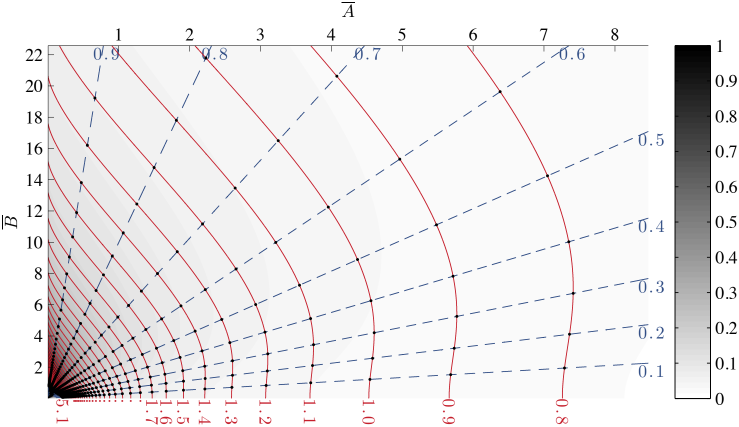
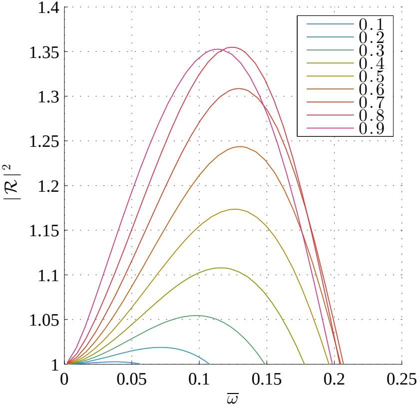
Superradiance with GW: results

$\overline{v}_\phi(r_{\mathrm{H}})$ and $\overline{\kappa}(r_{\mathrm{H}})$ can be related to $\overline{A}$ and $\overline{B}$ only numerically:

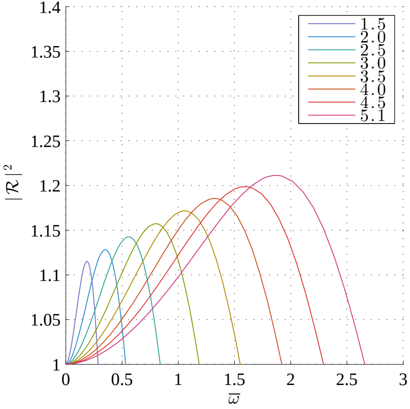


Lines of constant κ (solid red) and lines of constant v_ϕ (dashed blue).

Superradiance with GW: results



Amplification spectra for fixed $\overline{\kappa}(r_h)$



Amplification spectra for fixed $\overline{v}_\phi(r_h)$

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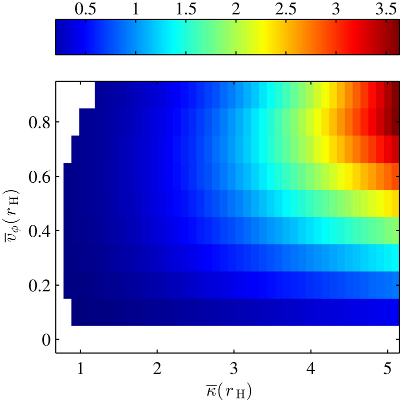
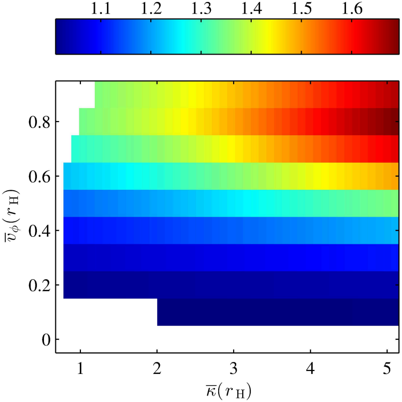
Superradiance with GW: results



Freq. of max. amplification $\overline{\omega}_{\max}$



Maximum amplification $|\mathcal{R}|^2_{\max}$

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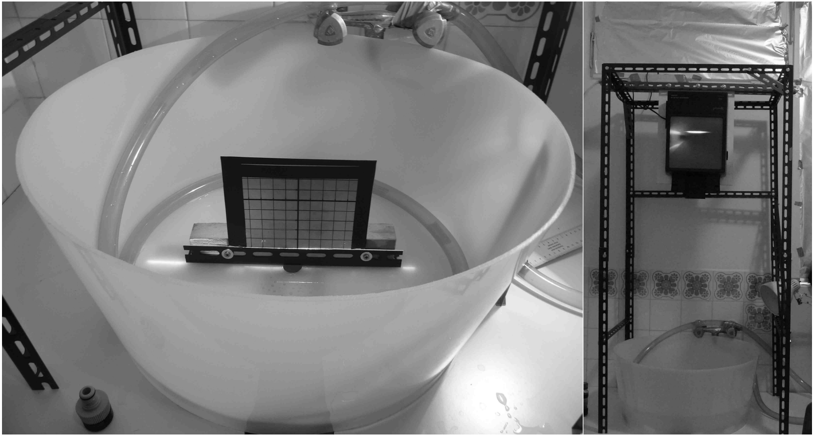
Superradiance with GW: conclusions

- There is a region of the parameter space where the maximum amplification is ≈ 1.4 and where our basic assumptions are still valid.
- This should be further investigated, with even more realistic models and with real experiments.
- There might be other ways (indirect) to detect superradiance in the lab (e.g. ergosphere instability or “black hole bomb” instability).

Experimental efforts



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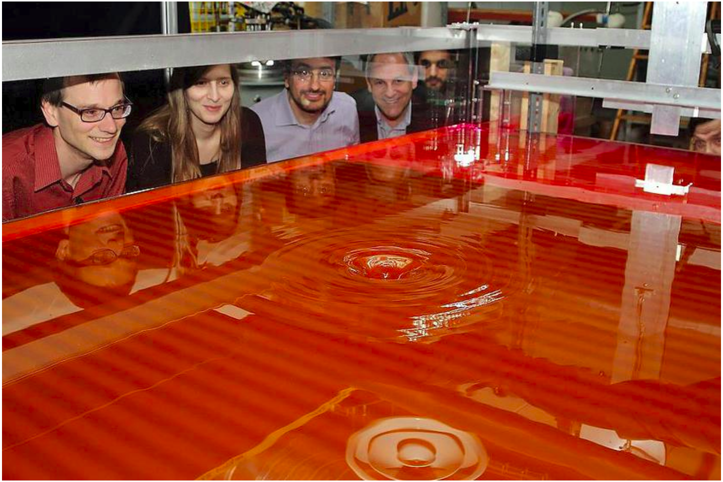

Ref.: `www.gravitylaboratory.com`

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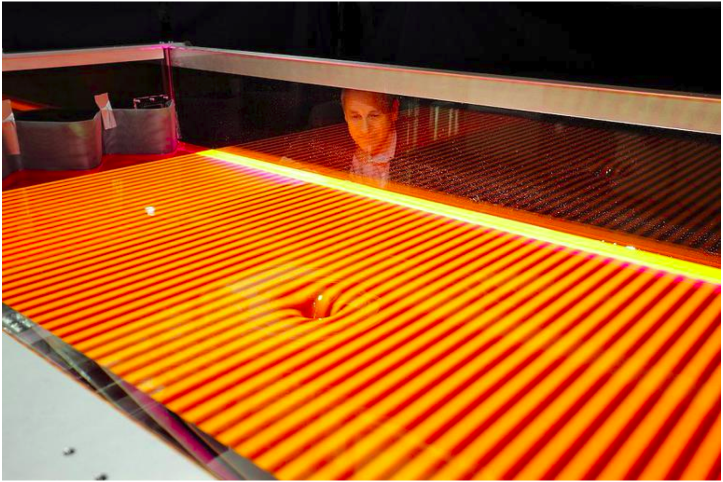

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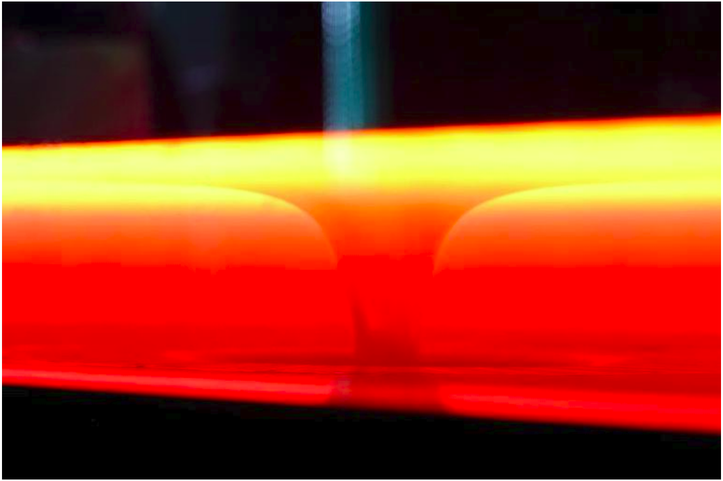

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