

ALTB N-BODY SIMULATIONS

COSMOLOGY BEYOND HOMOGENEITY AND ISOTROPY

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Outline

- The Standard Model of Cosmology
- Tensions / anomalies
- Inhomogeneous Cosmology
- Latest constraints on inhomogeneity
- The large-scale structure on an inhomogeneous background



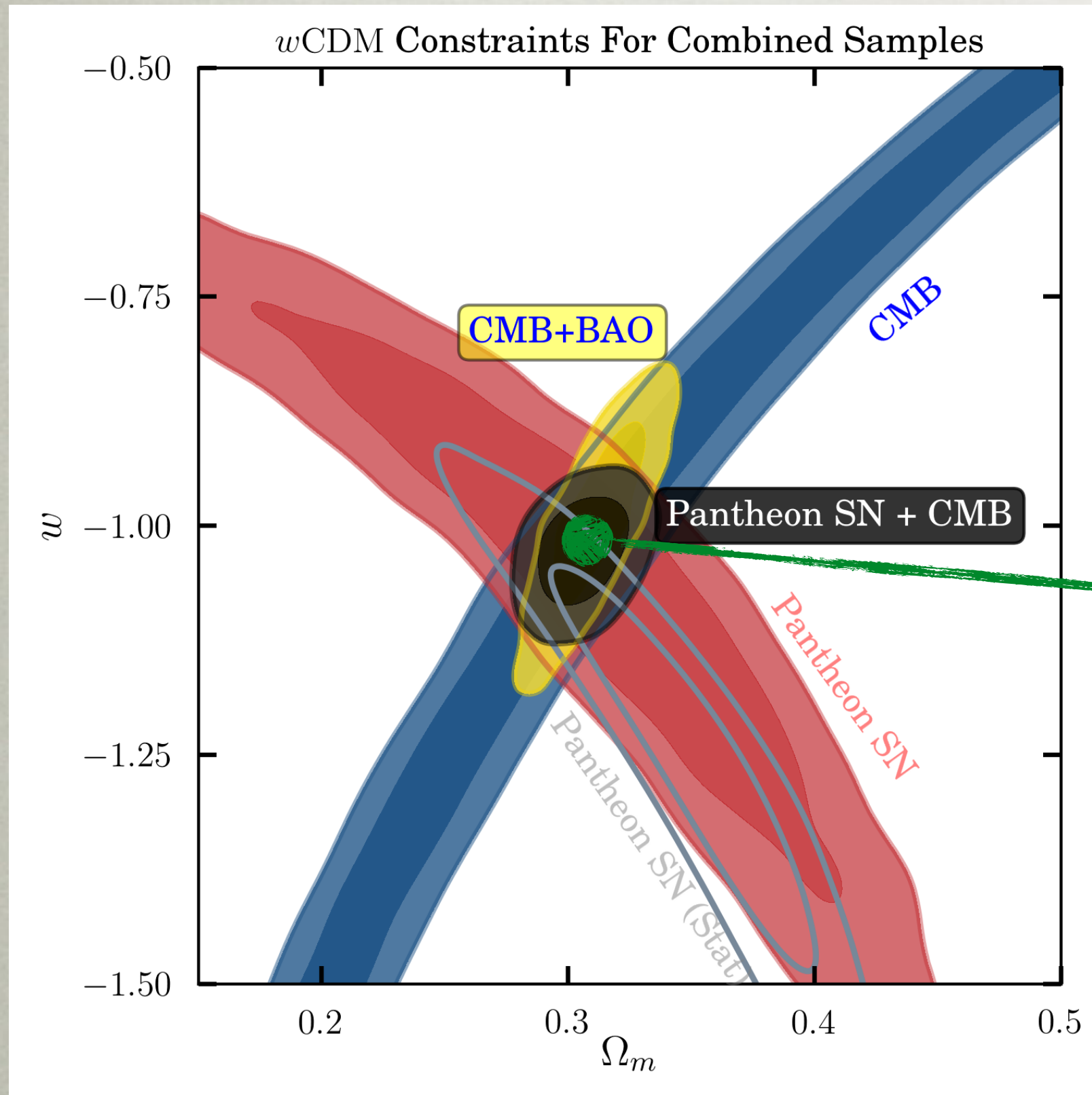
Vom meiste ligen dem biter dz
des in clostern weebrent wart

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ungeleb und geot feneie erwachsen. — und

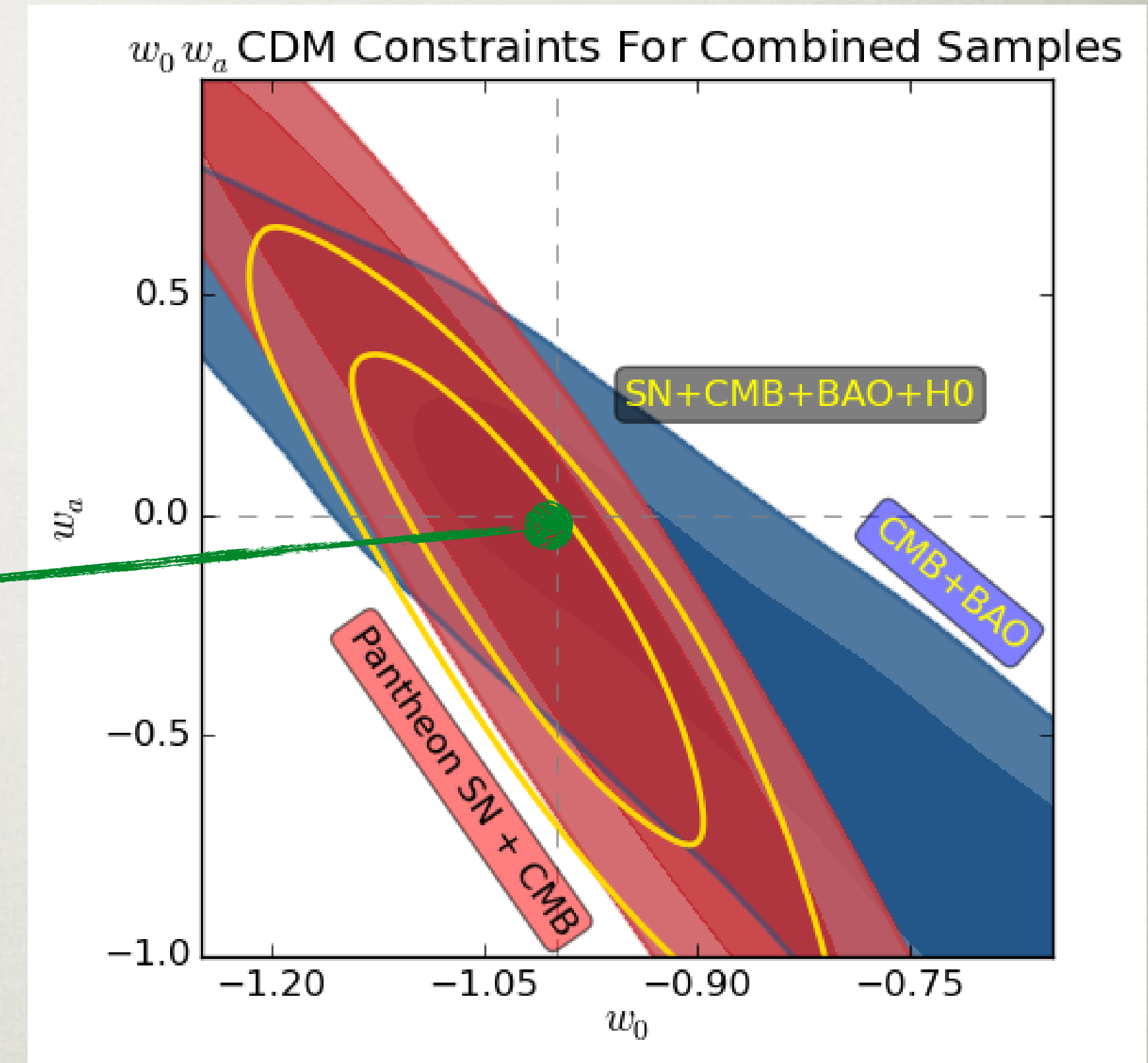
Disclaimer:
heresies ahead!

The standard model of cosmology

Scolnic+ 1710.00845



concordance
model



$$\Omega_M \simeq 0.3$$

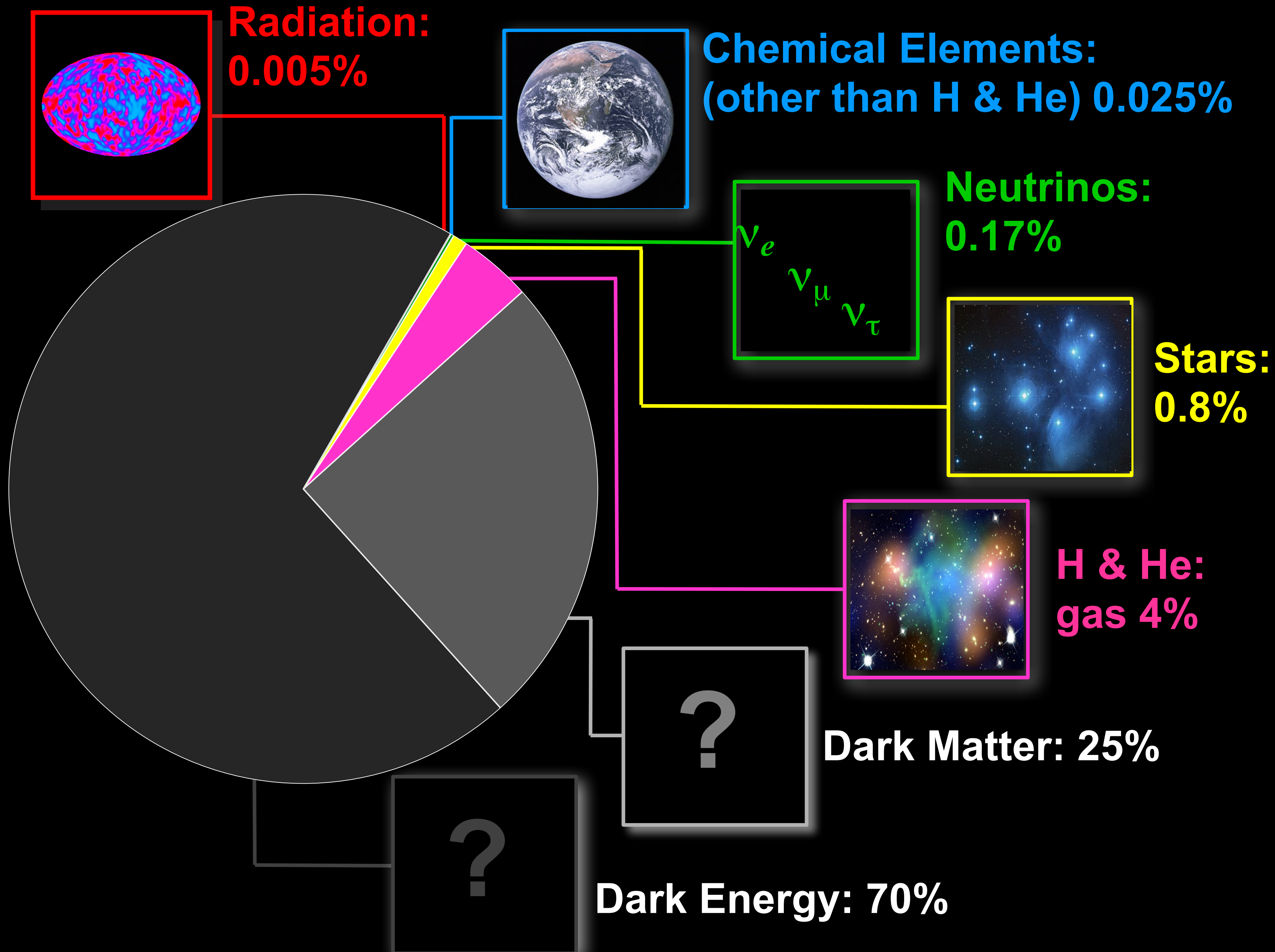
$$\Omega_{\text{DE}} \simeq 0.7$$

$$w_{\text{DE}} \simeq -1$$

strong evidence for
dark energy

2011 Nobel Prize

Λ CDM



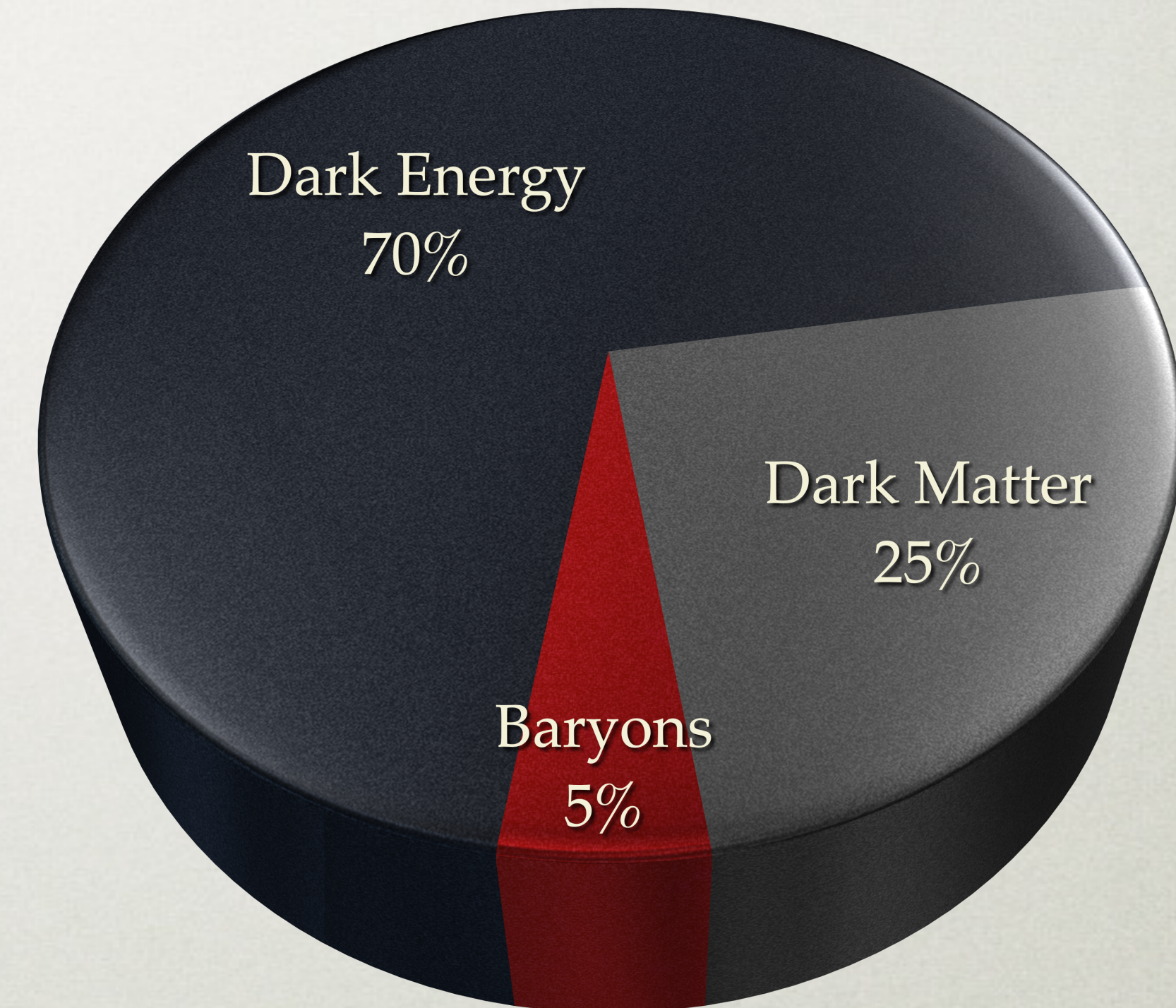
The concordance Λ CDM model

Pros:

- good fit to (a lot of) data
 - simple & consistent
- ➔ well worked-out phenom.
(still working on baryons)

Cons:

- DM??
- Λ ?????
- *bad* fit to some data



Tensions in the Λ CDM model

- CMB anomalies (cold spot, lensing ampl, hemispherical asym, ...)
- large-scale dipole anisotropies (radio, quasars)
- small-scale CDM problems (but also at large scales, see Diego's lectures)
- high- vs low- z σ_8 - Ω_m constraints (halo abundances, weak lensing)
- high- vs low- z H_0 measurements (SH₀ES, ~~strong lensing~~)
- ...

see review
Perivolaropoulos & Skara
2105.05208

Bear in mind the "look-elsewhere effect", which is difficult to assess because there is not yet the "**Journal of Failed Tests**".

Without it, MNRAS, PRD, JCAP are the "**Journals of Outliers**" (successful tests)

These tensions may suffer from "significance inflation".



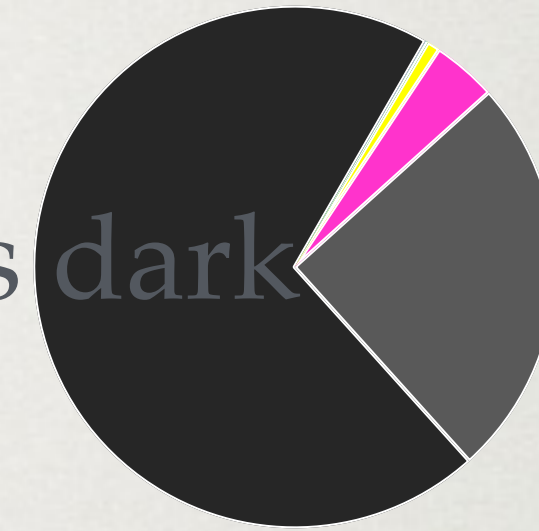
xkcd.com/882

What is inhomogeneous cosmology?

- It is the study of the universe without assuming a priori the FLRW metric
- [Copernican principle valid at larger scales, not at the standard $\approx 100\text{--}200$ Mpc]
- [[it is not the backreaction proposal; questions to Rocky]]

Why inhomogeneous cosmology?

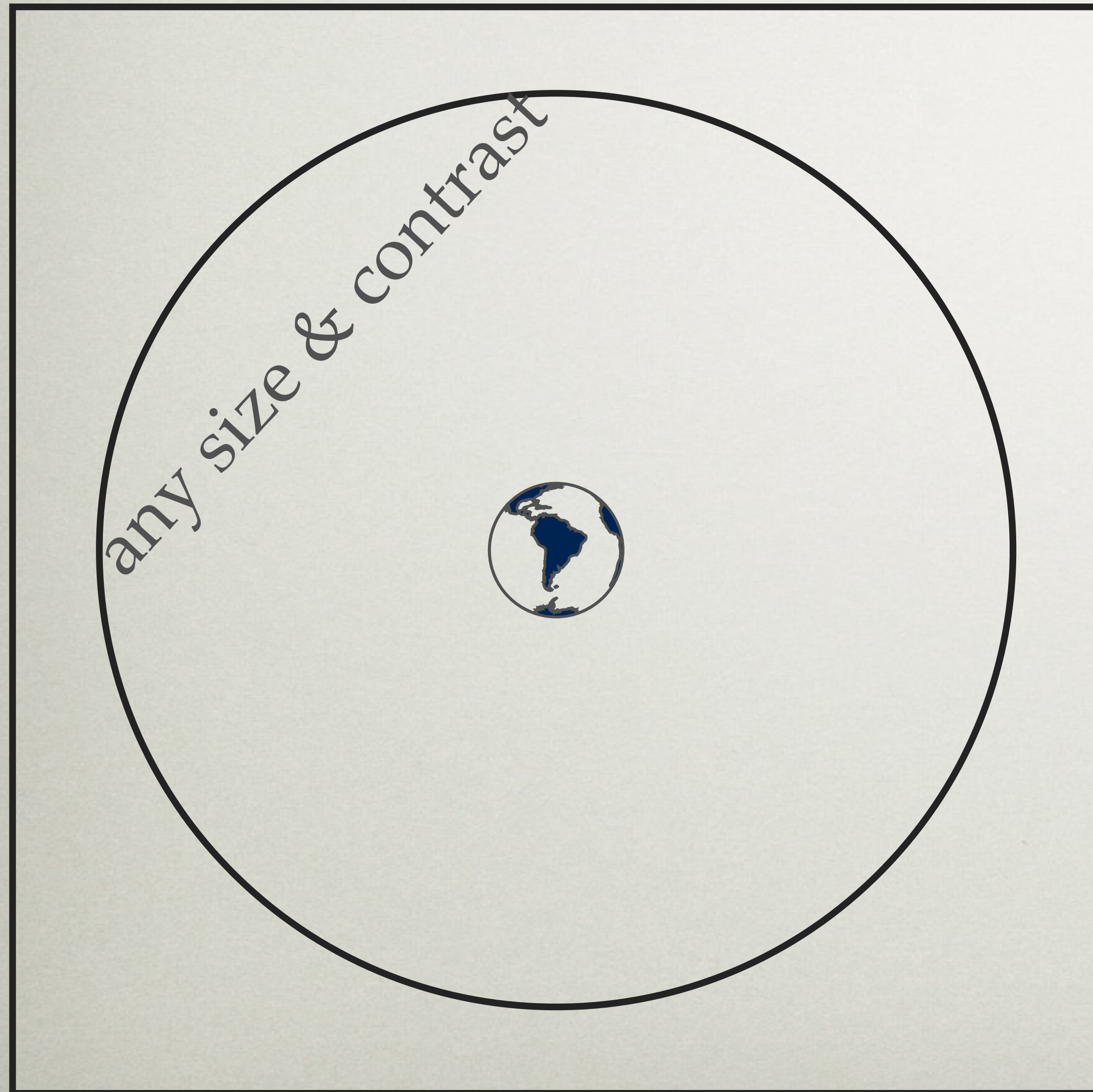
- Test basic assumptions: 95% of the universe content is dark
- Study the universe without assuming homogeneity and isotropy; reconstruct the metric directly from observations
- The universe may feature large-scale inhomogeneities and anisotropies which cannot be explained by the Λ CDM/FLRW paradigm: see the H_0 tension and the comoving dipoles and anisotropy.



Early-FLRW cosmologies

- Copernican principle valid at larger scales, not at the standard $\approx 100\text{--}200$ Mpc
- Large-scale inhomogeneities with non-standard amplitudes and phases, characterized by bulk flows and coherent perturbations.
A kind of primordial non-Gaussianity (how much room is there given CMB?)
- Early-FLRW cosmologies: standard cosmology endowed with non-standard structure dominated by growing modes. Near-FLRW metric at early times
- Can use existing codes such as Gadget

Early-FLRW cosmologies (cont'd)



spacetime inhom: LTB

justification: 

[Λ LTB model: Λ CDM endowed with a spherical over/underdensity]

observer: at the center

justification:

- if nonlinear, observer is constrained near center by CMB
- if linear, observer's observations averaged over angles

The Λ LTB model

Metric:

$$ds^2 = -dt^2 + \frac{a_{\parallel}^2(t, r)}{1 - k(r)r^2} dr^2 + a_{\perp}^2(t, r) r^2 d\Omega^2$$

$$H_{\perp}(t, r) \equiv \frac{\dot{a}_{\perp}}{a_{\perp}} \qquad H_{\parallel}(t, r) \equiv \frac{\dot{a}_{\parallel}}{a_{\parallel}}$$

Friedmann-like equation:

$$H_{\perp}^2(t, r) = \frac{8\pi G}{3} \rho_m^e(t, r) + \frac{8\pi G}{3} \rho_{\Lambda} - \frac{k(r)}{a_{\perp}^2(t, r)}$$

$$\frac{H_{\perp}^2(t, r)}{H_{\perp 0}^2(r)} = \Omega_{m0}(r) \frac{a_{\perp 0}^3}{a_{\perp}^3} + \Omega_{\Lambda 0}(r) + \Omega_{k0}(r) \frac{a_{\perp 0}^2}{a_{\perp}^2}$$

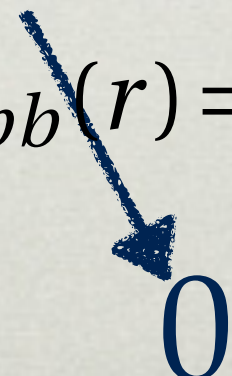
Growing mode:

$$k(r) = k_b + (k_c - k_b) W_3(r/r_b)$$

$$W_n(x) = \begin{cases} e^{-x^n/(1-x)} & \text{for } 0 \leq x < 1 \\ 0 & \text{for } x \geq 1 \end{cases}$$

Decaying mode:

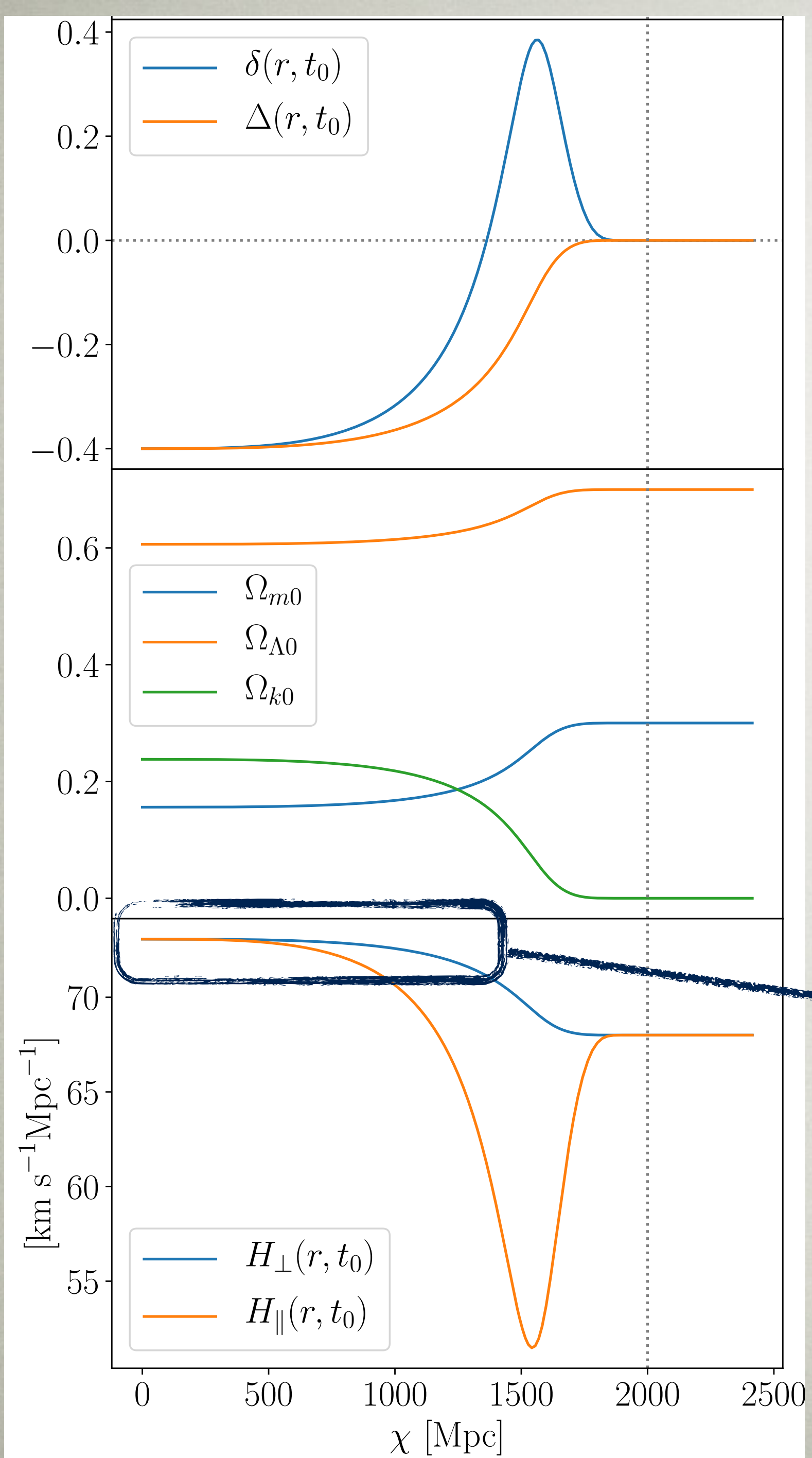
$$t - t_{bb}(r) = \frac{1}{H_{\perp 0}(r)} \int_0^{\frac{a_{\perp}(t, r)}{a_{\perp 0}(r)}} \frac{dx}{\sqrt{\Omega_{m0}(r)/x + \Omega_{\Lambda 0}(r)x^2 + \Omega_{k0}(r)}}$$



Early-FLRW cosmology

Marra et al 2203.04009

The Λ LTB model

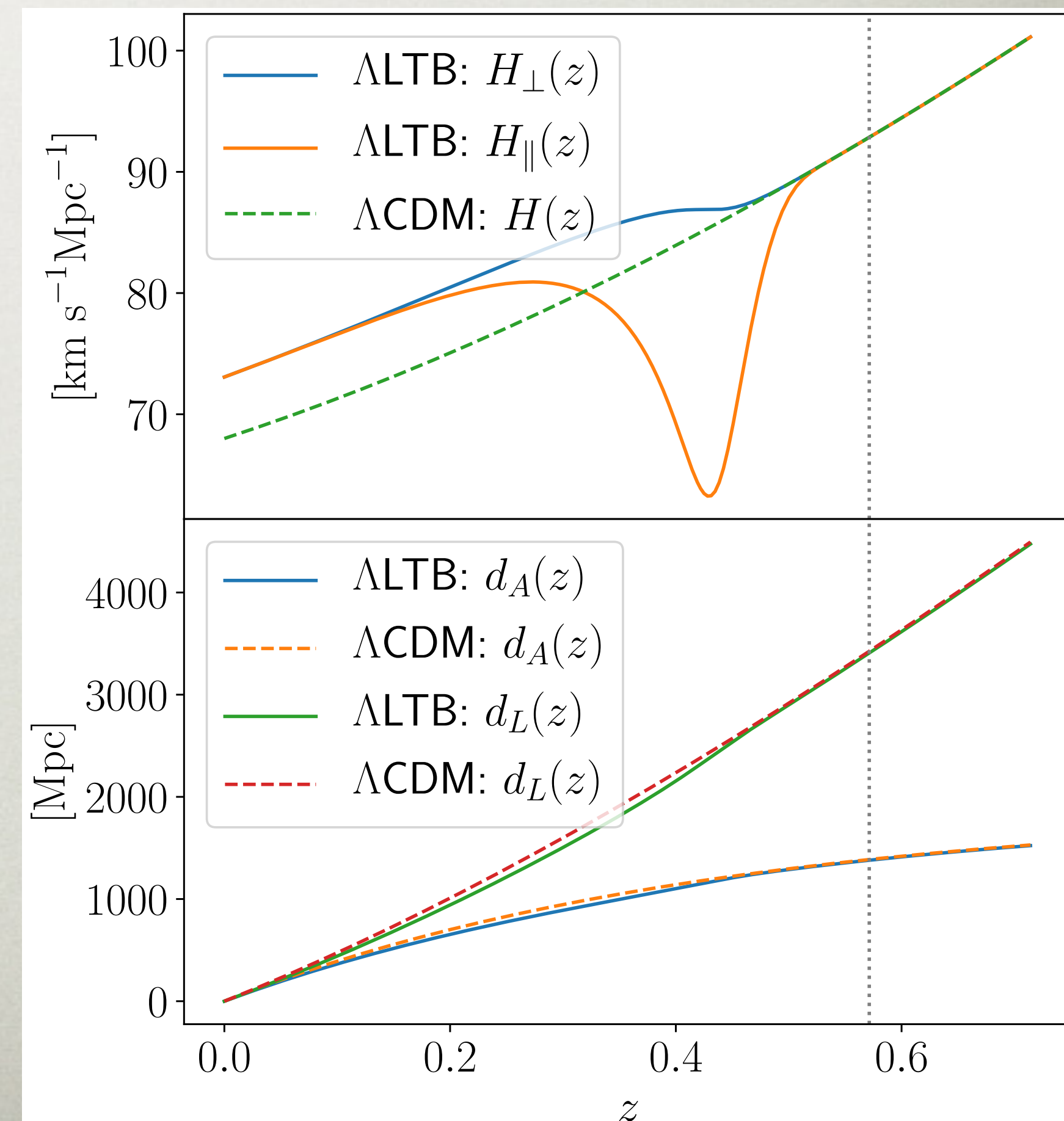


**solution to the
Hubble tension?**

(no void model as
alternative to DE)

$$\Phi_{\text{lin}}(r) = \frac{3}{5} \int_r^{r_b} \frac{E(\bar{r})}{\bar{r}} d\bar{r} \sim E$$

$$E(r) \equiv -\frac{k r^2}{2} = \frac{1}{2} \dot{Y}^2(t, r) - \frac{GF(r)}{Y(t, r)} - \frac{1}{6} \Lambda Y^2(t, r)$$



The Copernican prior

- LTB inhomogeneity parametrized by size (r_b or z_b) and contrast (k_c or δ_0).
- Size and contrast will be constrained by observations.
- What are the “standard” value of size and contrast?

The ones allowed by the standard power spectrum as constrained by the CMB.

The Copernican prior

integrated contrast size of central inhomogeneity
size of LTB patch

$$\mathcal{P}(\delta_0, z_B) \propto \exp \left[-\frac{1}{2} \frac{\delta^2(r_L(\delta_0, z_B), t_0)}{\sigma^2(r_L^{\text{out}}(\delta_0, z_B))} \right]$$

$$\sigma^2(r) = \int_0^\infty \frac{dk}{k} \Delta_{m0}^2(k) \left[3 \frac{j_1(r k)}{r k} \right]^2$$

central contrast

$$P(\delta_0, z_B) = \int dp_i \mathcal{P}(\delta_0, z_B) \mathcal{L}_{\text{CMB}}(p_i, \delta_0, z_B)$$

Observables

- **CMB:** Planck 2018 TT+TE + EE spectrum
- **Type Ia Supernovae:** Pantheon set, $0.01 < z < 2.3$
- **Low-z supernovae:** Pantheon set, $0.023 < z < 0.15$
- **Cosmic chronometers:** 30 measurements in $0 < z < 2$
- **BAO:** 6dFGS, SDSS-MGS, and BOSS-DR12, $0.1 < z < 0.61$
- **Compton y-distortion:** COBE-FIRAS, $y < 1.5 \times 10^{-5}$
- **kinetic Sunyaev–Zeldovich effect:** SPT, $D_{3000}^{\text{kSZ}} = 3.0 \pm 1.0 \mu\text{K}^2$
- **local calibration by SH0ES** on the supernova absolute magnitude

The local prior on M_B

The determination of H_0 by SH0ES is a two-step process:

- (i) Anchors, Cepheids and calibrators are combined to produce a constraint on M_B .
- (ii) Hubble-flow supernovae in the redshift range $0.023 \leq z \leq 0.15$ are used to probe the luminosity distance-redshift relation in order to determine H_0 .

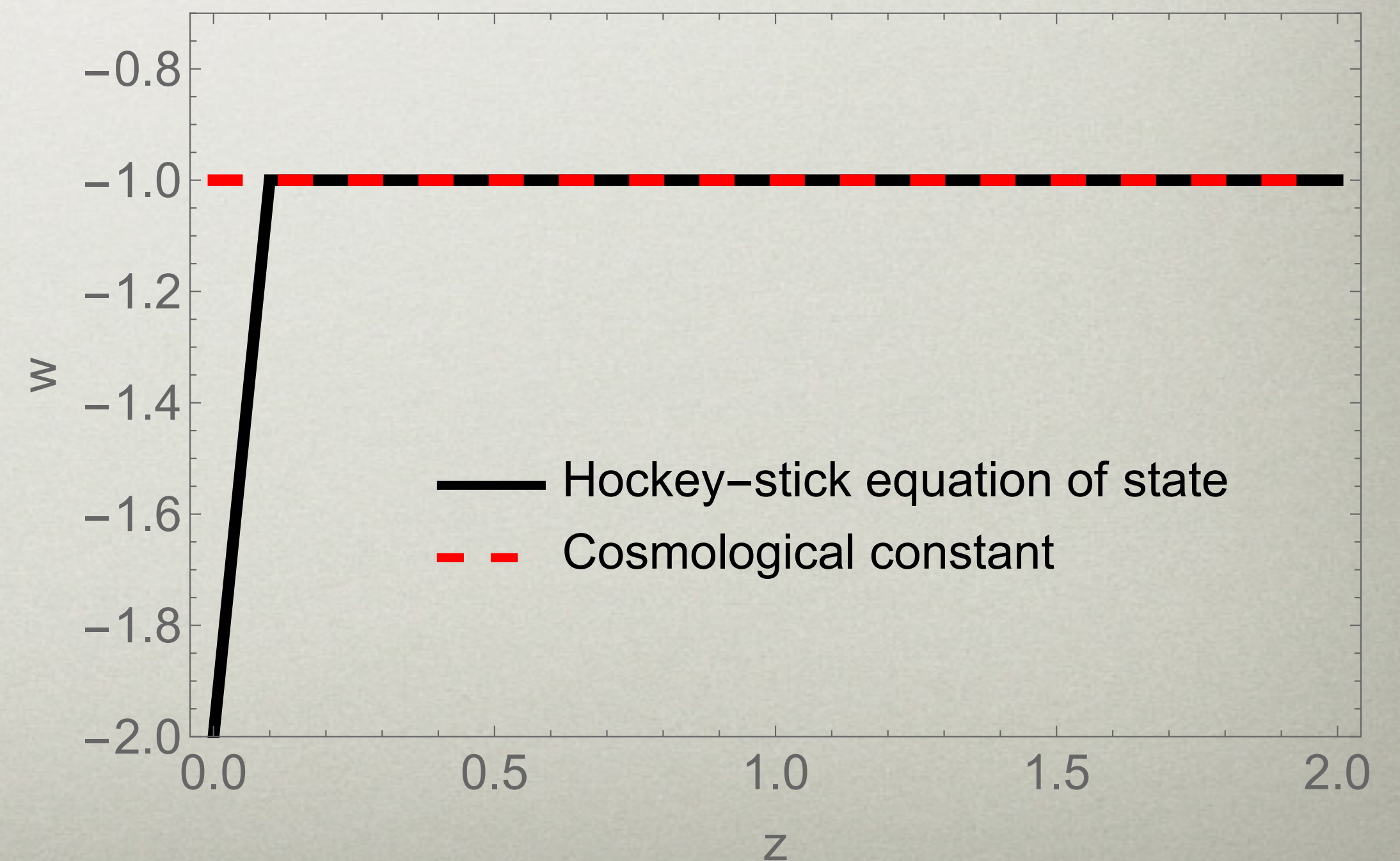
Cosmography with $q_0 = -0.55$ and $j_0 = 1$ is adopted.

The astro-prior on M_B

By skipping the cosmography part and just using the prior on M_B :

- ⦿ one avoids potential double counting of low-redshift supernovae
- ⦿ one avoids cosmography, in particular fixing $q_0 = -0.55$
- ⦿ one includes the fact that M_B is constrained by local calibration

See Camarena & Marra 2101.08641 for its use in the analysis of hockey-stick dark energy



Prior on M_B maintained at:
github.com/valerio-marra/CalPriorSNIa

Module added to Monte Python 3.5

Camarena & Marra 1906.11814, 2101.08641

CalPriorSNIa

Effective calibration prior on the absolute magnitude of Type Ia supernovae

priorMB.nb quickly computes the effective calibration prior on the absolute magnitude M_B of Type Ia supernovae that corresponds to a given determination of H_0 . See Camarena & Marra [arXiv:1906.11814](#) and [arXiv:2101.08641](#) for more details.

Have fun, [David](#) & [Valerio](#)

Calibration priors

The latest H_0 determination by SH0ES is (Riess et al. [arXiv:2012.08534](#)): $H_0 = 73.2 \pm 1.3 \text{ km/s/Mpc}$

The corresponding calibration priors depend on the supernova dataset that one wants to use for the cosmological analysis:

Supercal dataset ([Scolnic et al 2015](#)) ==> $M_B = -19.2421 \pm 0.0375 \text{ mag}$

Pantheon dataset ([Scolnic et al 2018](#)) ==> $M_B = -19.2435 \pm 0.0373 \text{ mag}$

DES-SN3YR dataset ([Brout et al 2018](#)) ==> $M_B = -19.2389 \pm 0.0336 \text{ mag}$

Statistical analysis

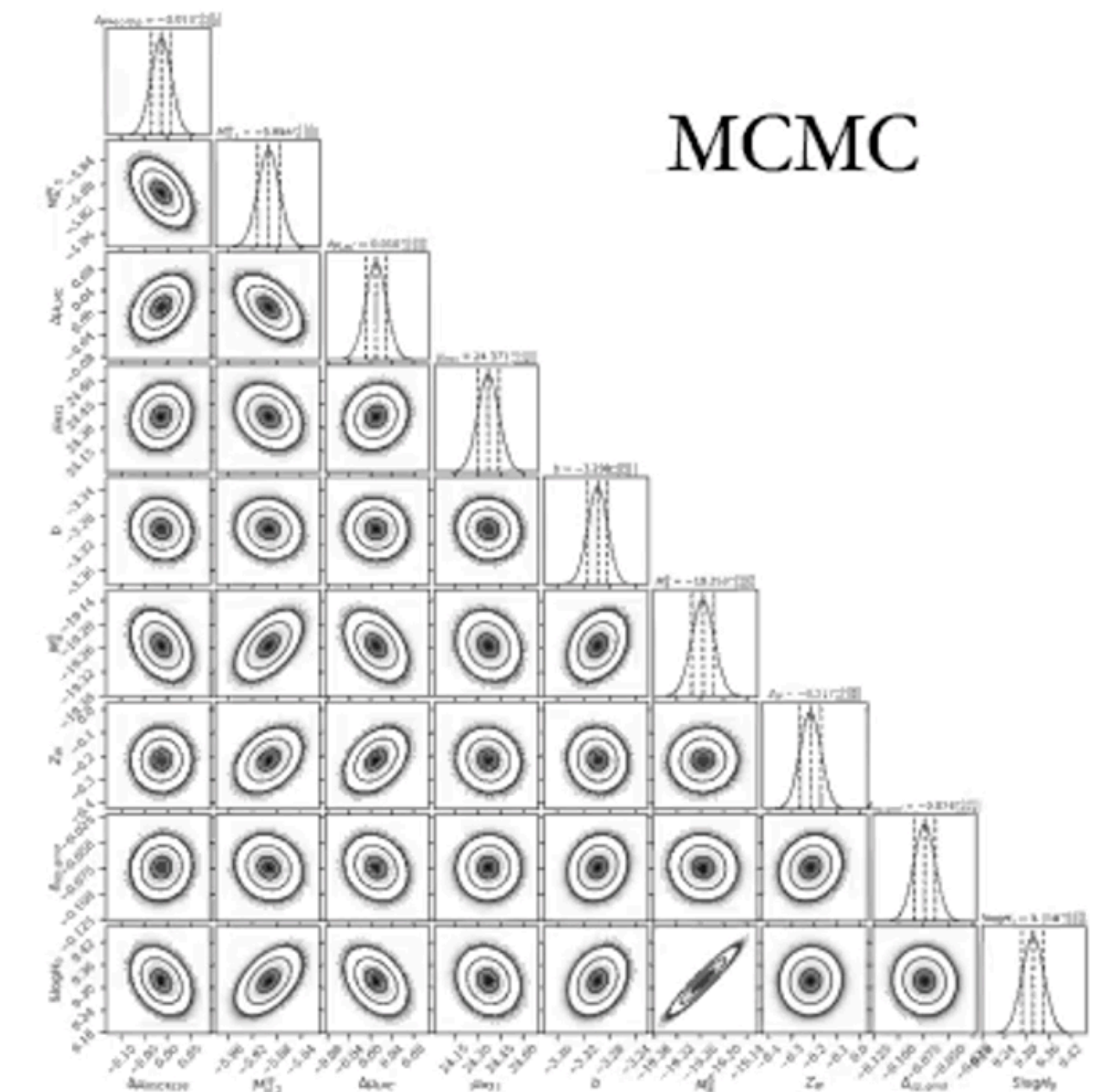
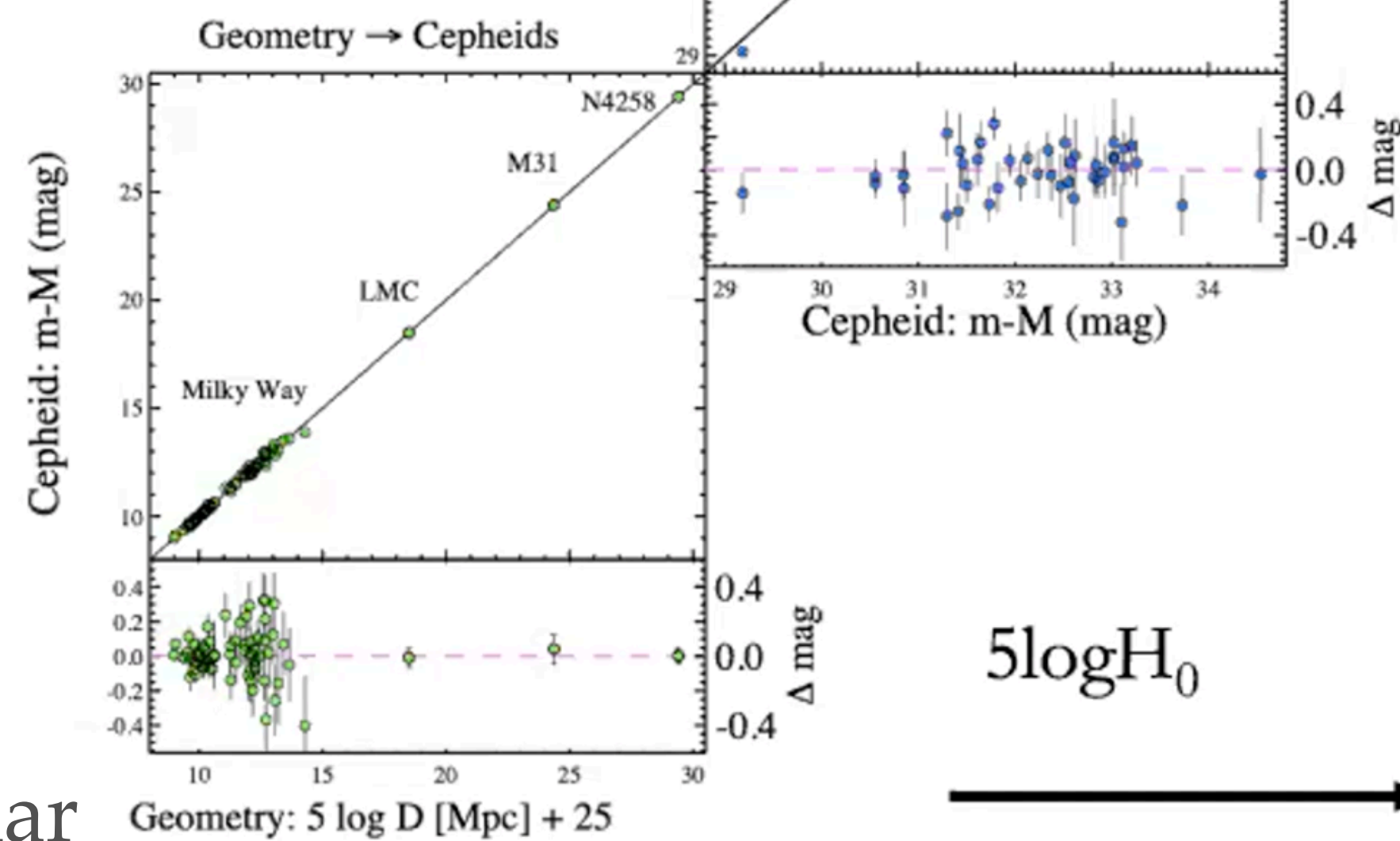
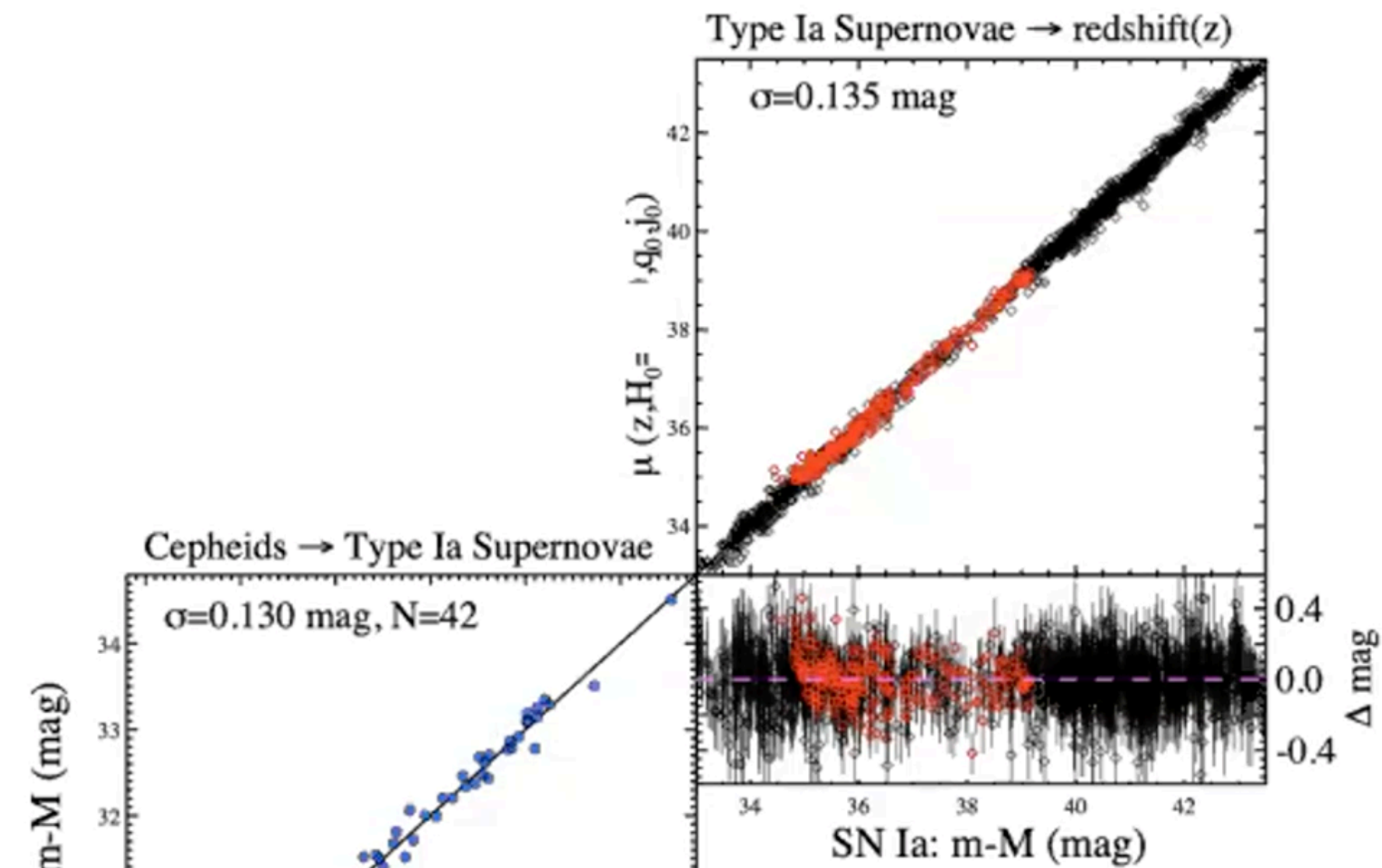
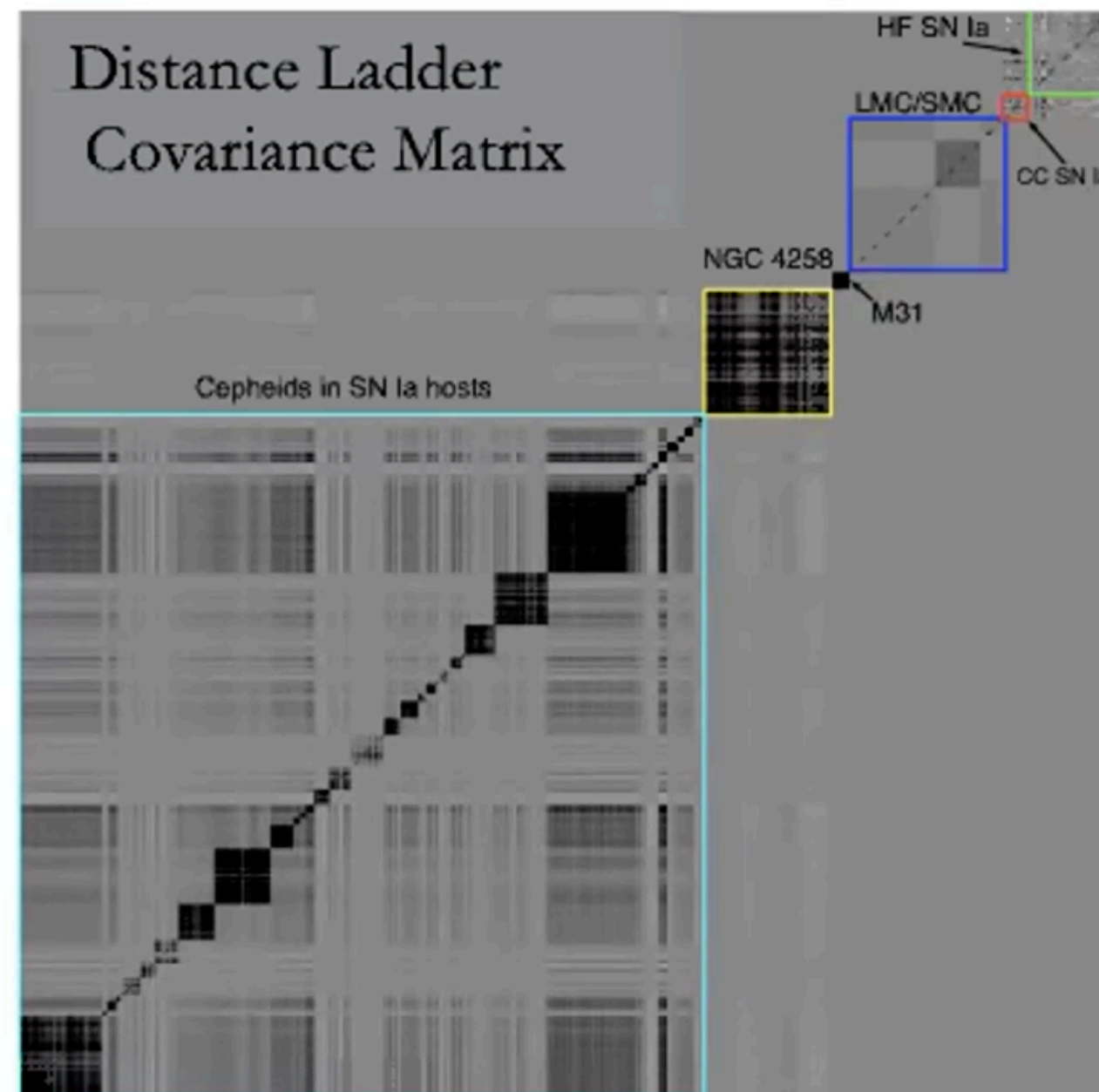
When performing cosmological inference, instead of using the prior on H_0 :

$$\chi^2_{H_0} = \frac{(H_0 - H_0^{\text{SH0ES}})^2}{\sigma_{H_0}^2},$$

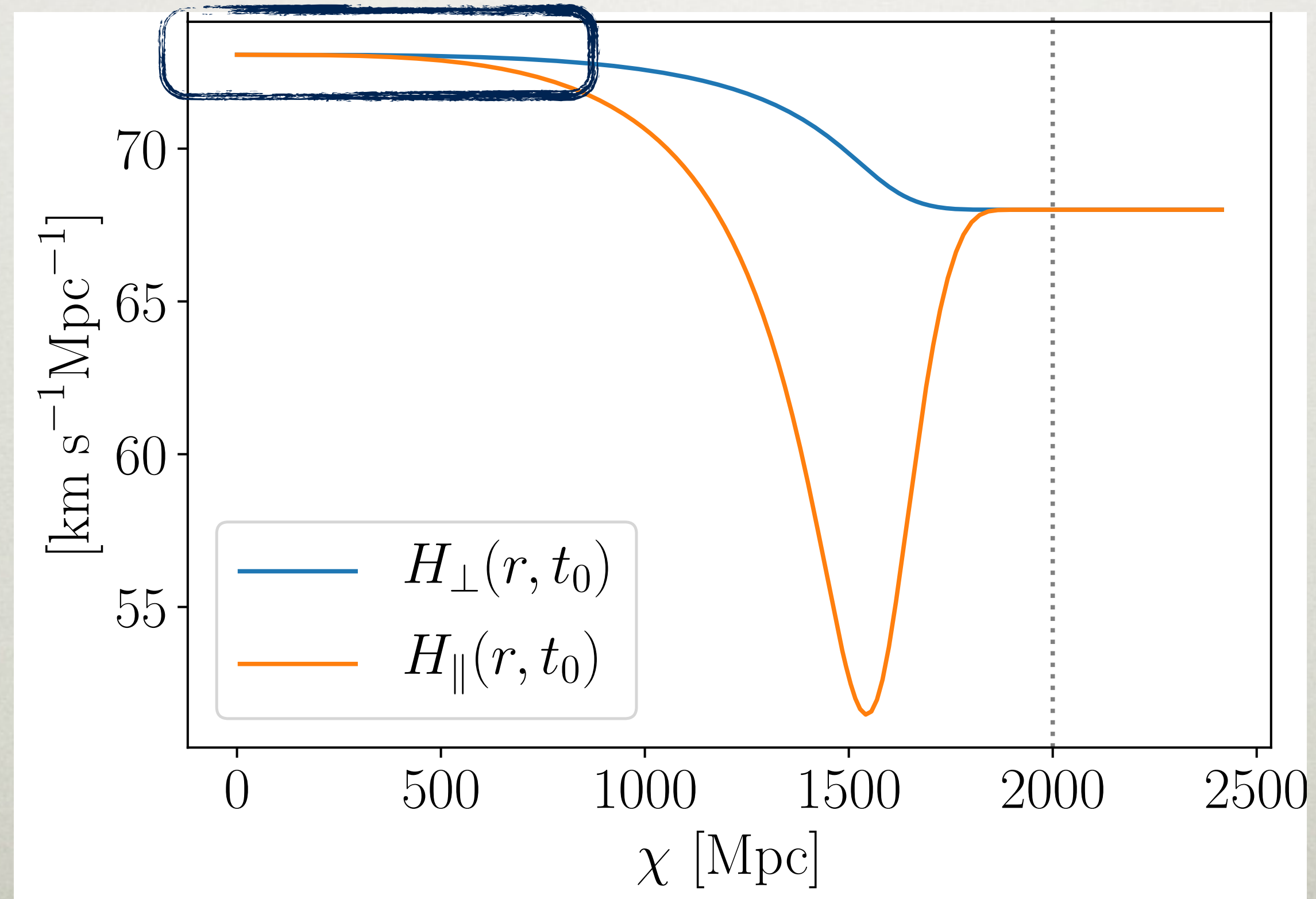
use the prior on M_B :

$$\chi^2_{M_B} = \frac{(M_B - M_B^{\text{SH0ES}})^2}{\sigma_{M_B}^2}.$$

Baseline Fit: 3200 Cepheids, 300 SN, non-diagonal covariance-5 free params

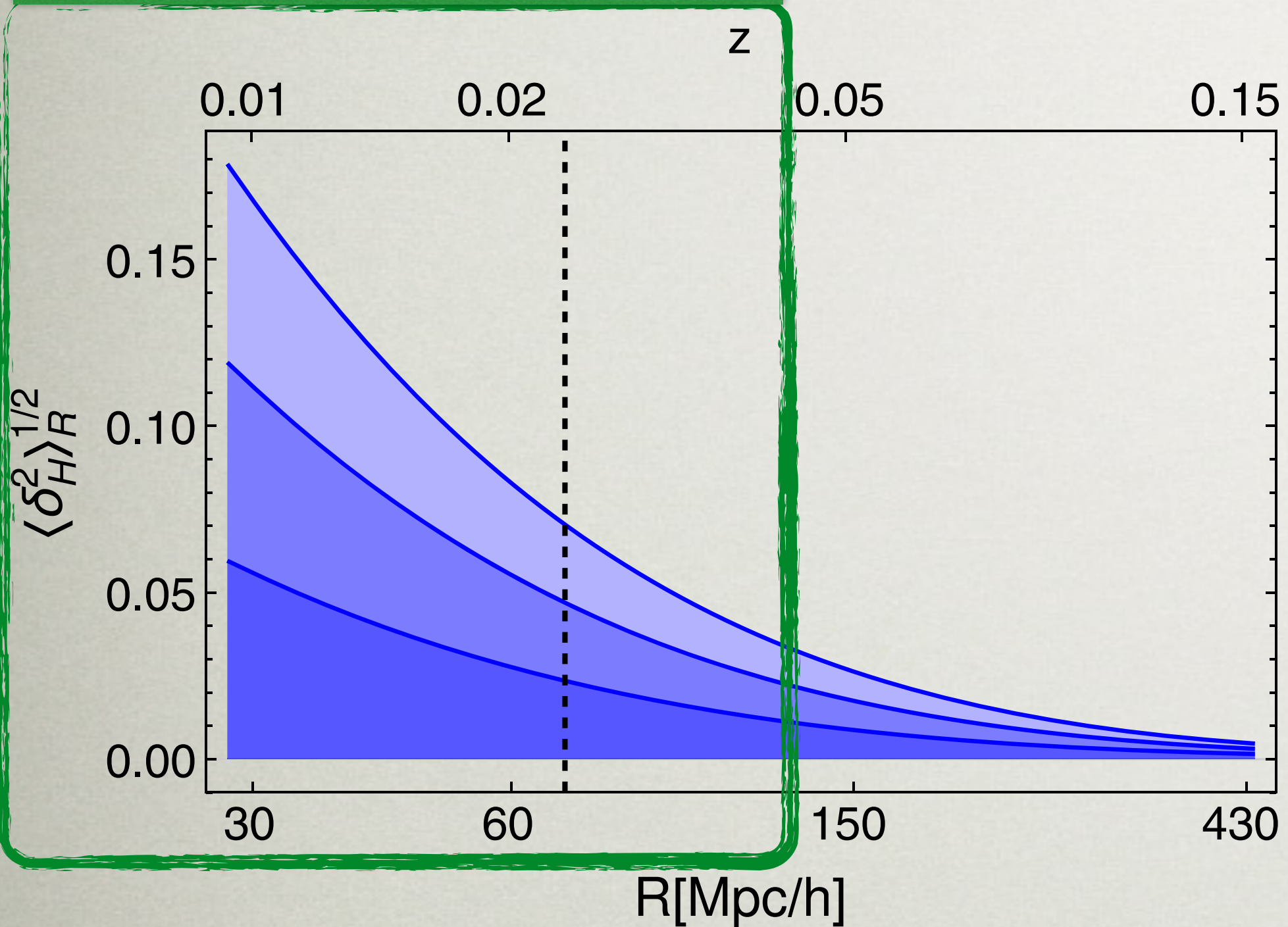


So, can a local void solve the H_0 crisis?



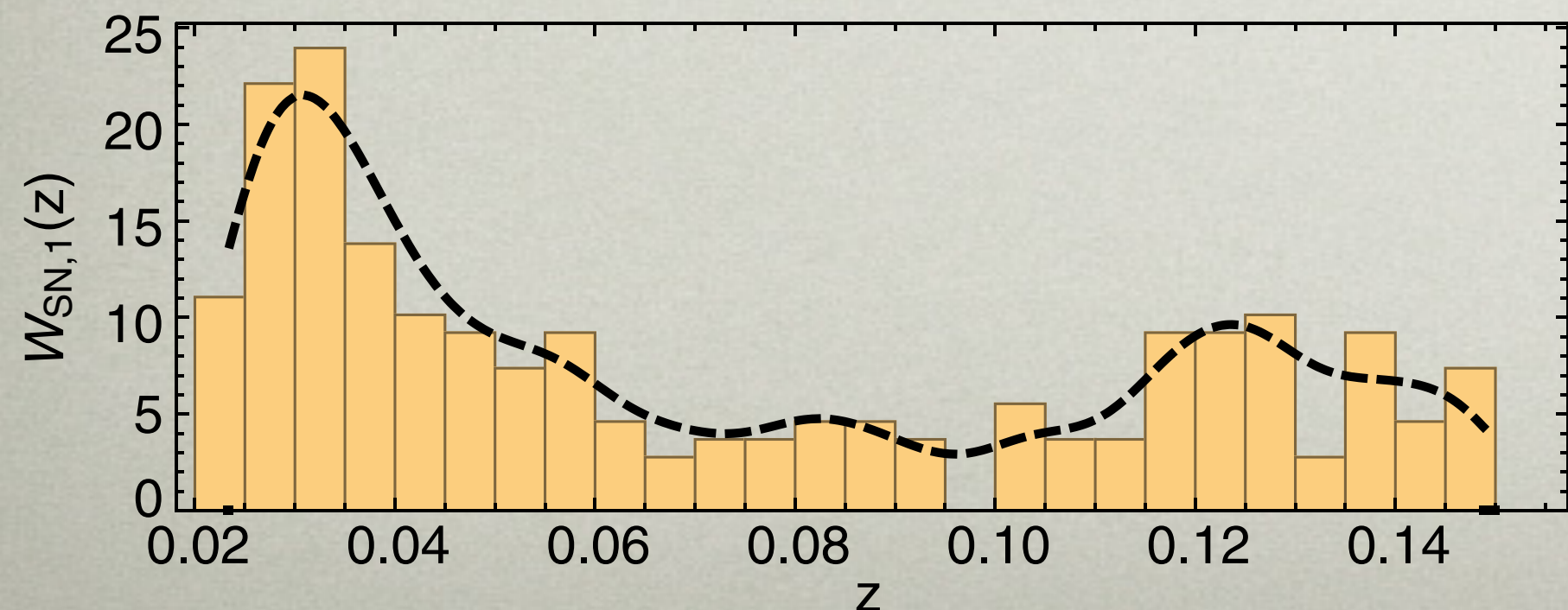
Λ CDM cosmic variance on H_0

SIBELIUS-DARK

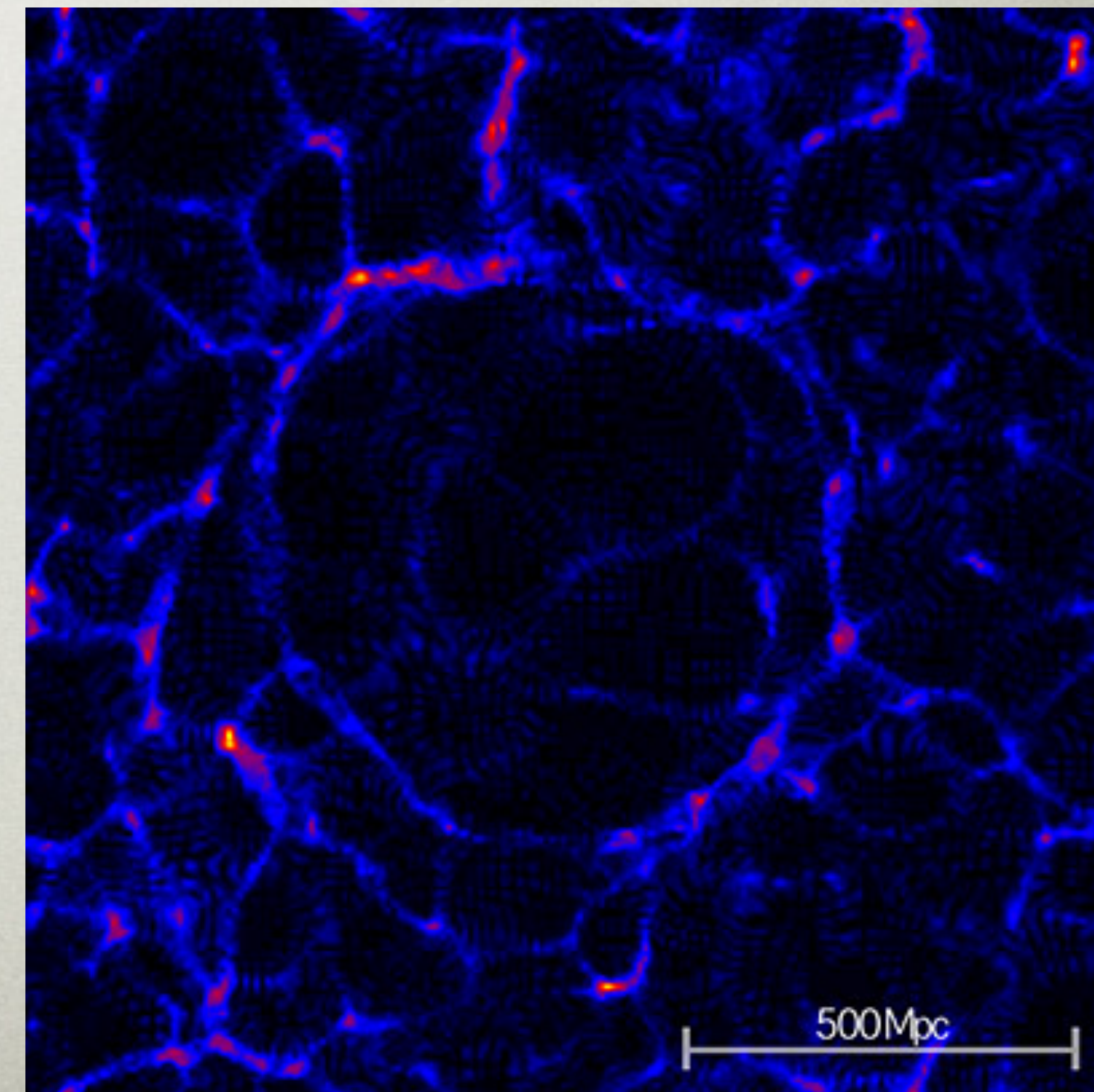


$$\sigma_{\text{cv},1} = H_0^{\text{loc}} \left[\int_{0.0233}^{0.15} dz W_{\text{SN},1}(z) \langle \delta_H^2 \rangle_R \right]^{\frac{1}{2}} \sim 0.5-0.9 \text{ km/s/Mpc}$$

6 km/s/Mpc
necessary!



Marra+ PRL 1303.3121
Camarena&Marra PRD 1805.09900



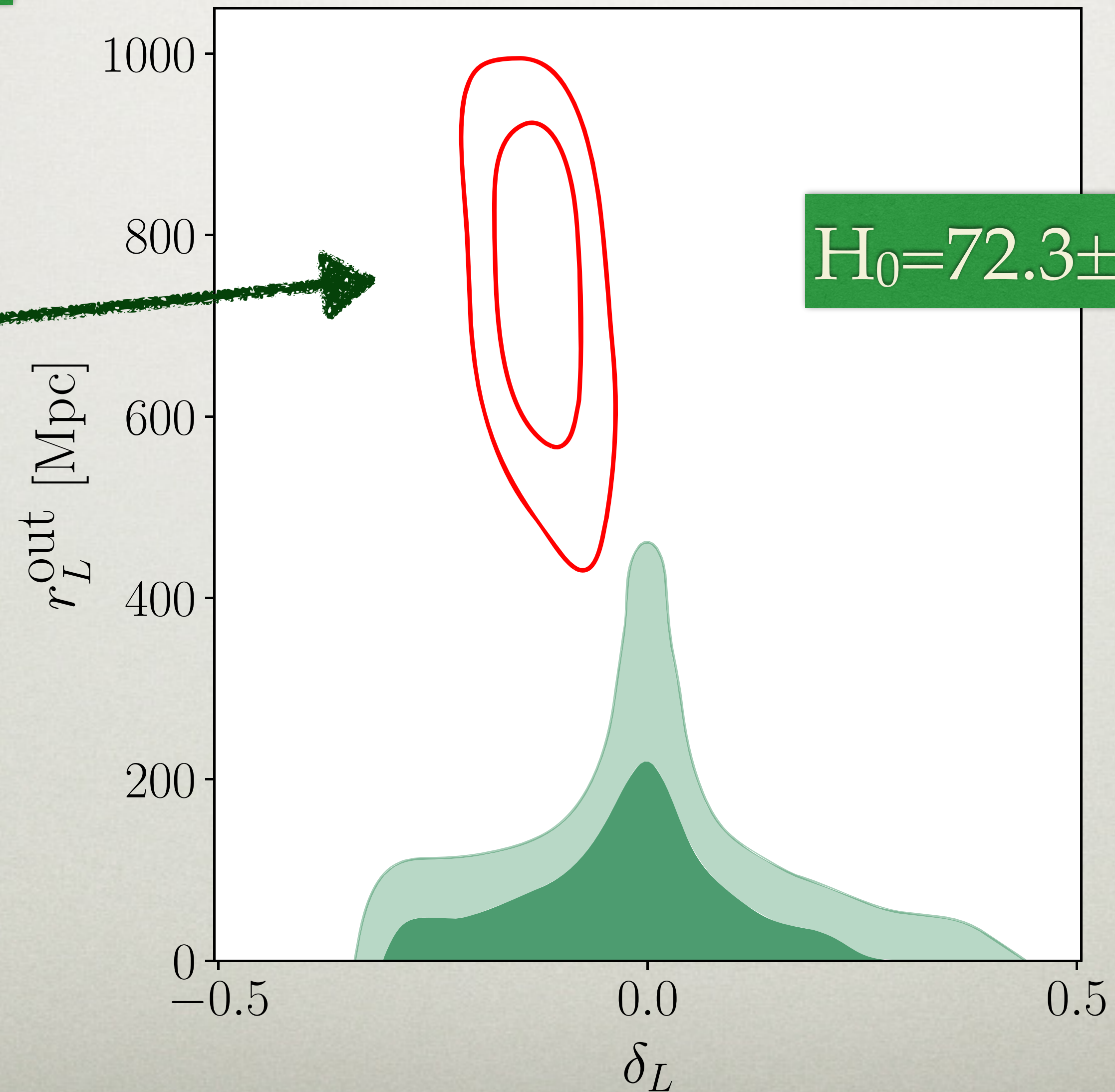
What about an arbitrary local inhomogeneity?

Pantheon SNe
 $0.023 < z < 0.15$

beyond- Λ CDM
underdensity

Copernican Prior + CMB

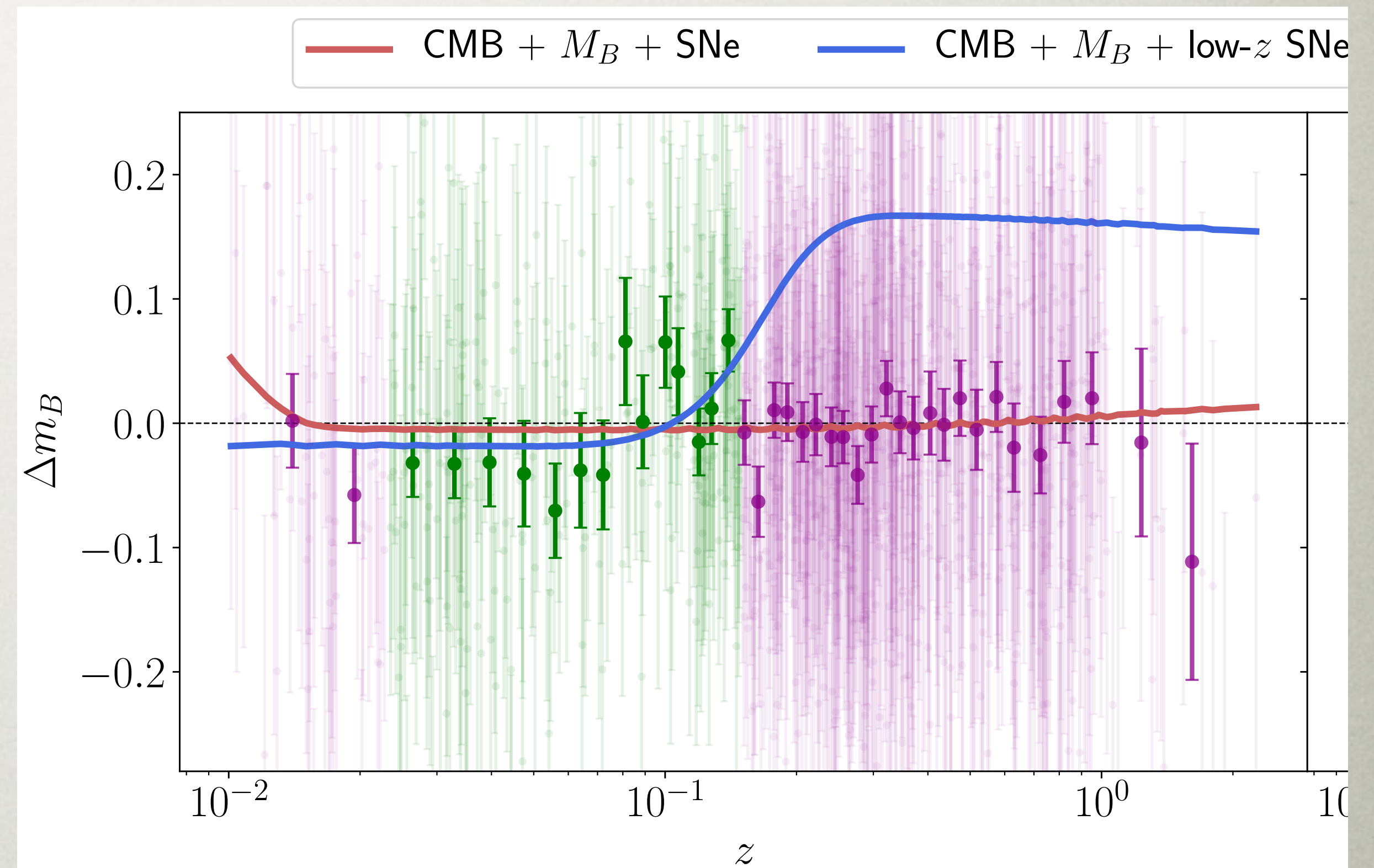
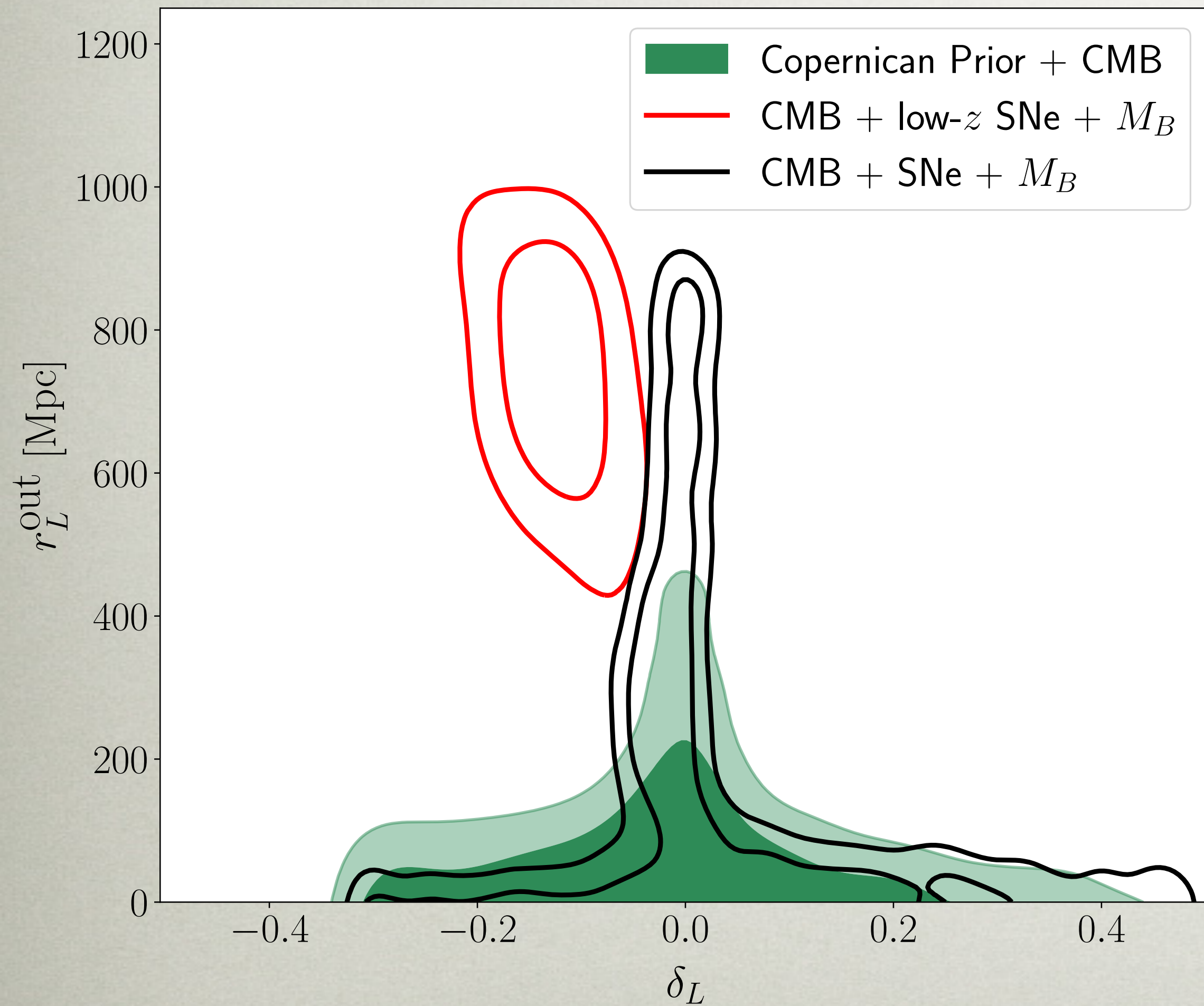
CMB + low- z SNe + M_B



$H_0 = 72.3 \pm 1.1 \text{ km/s/Mpc}$

But...

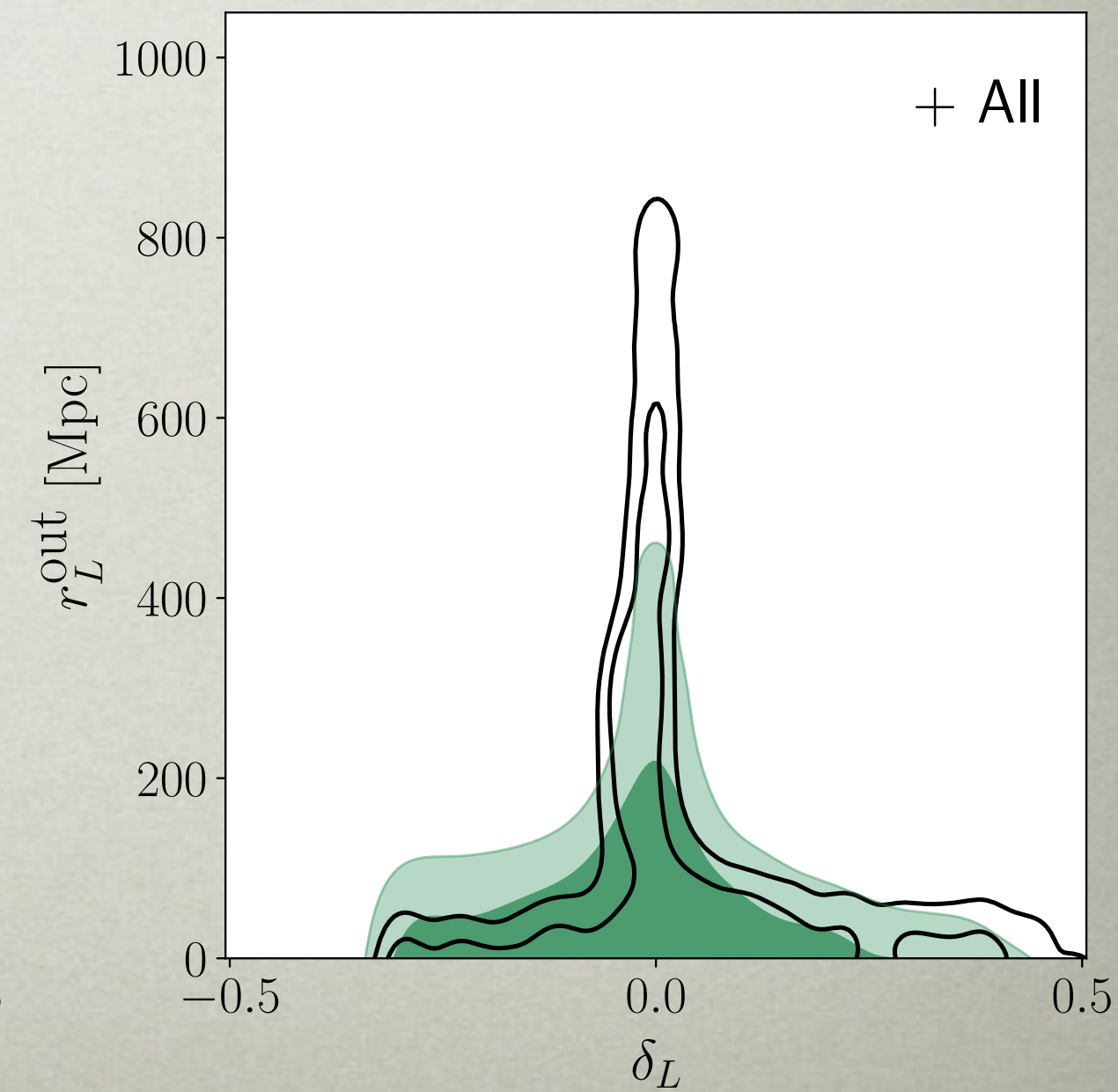
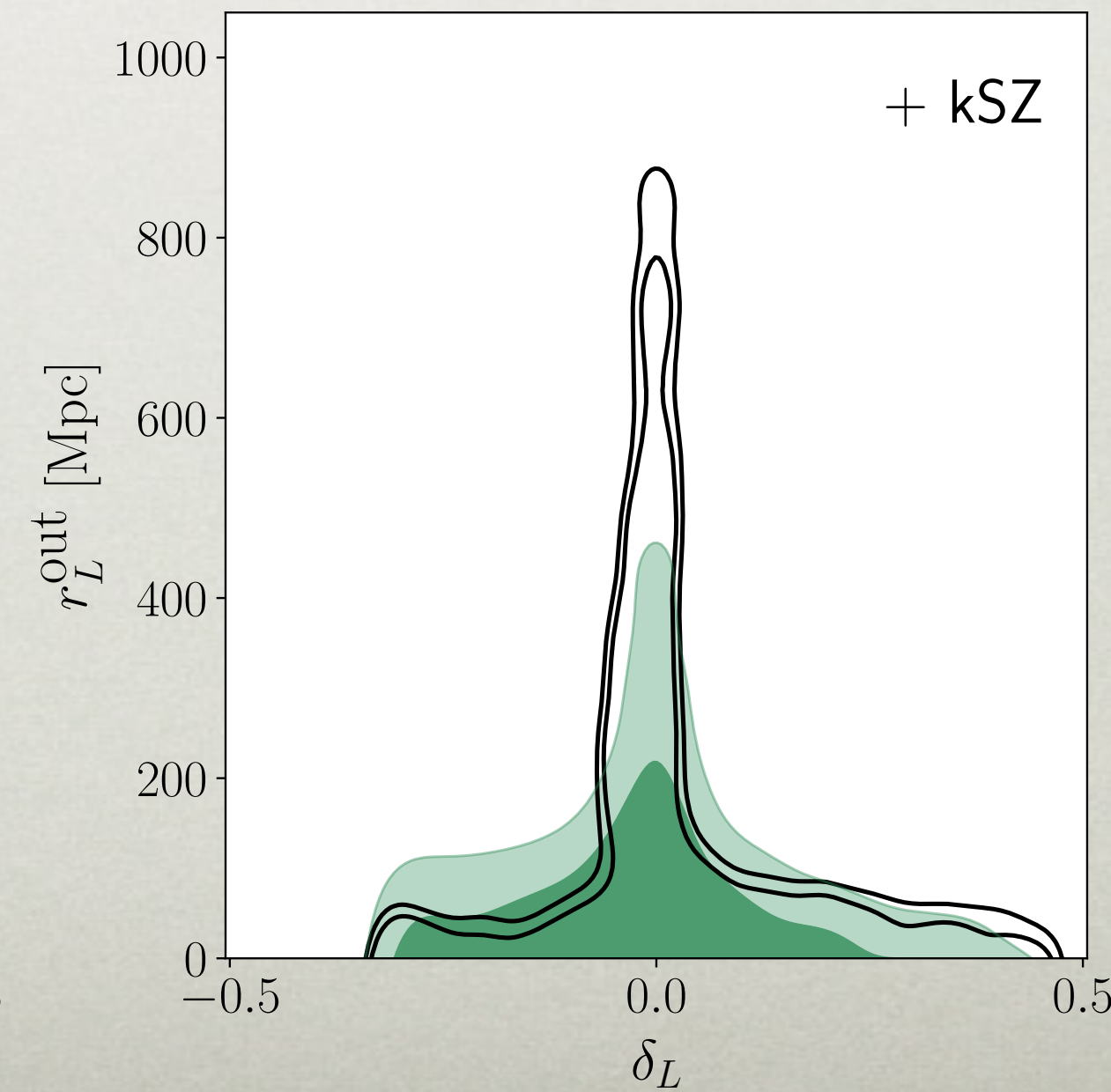
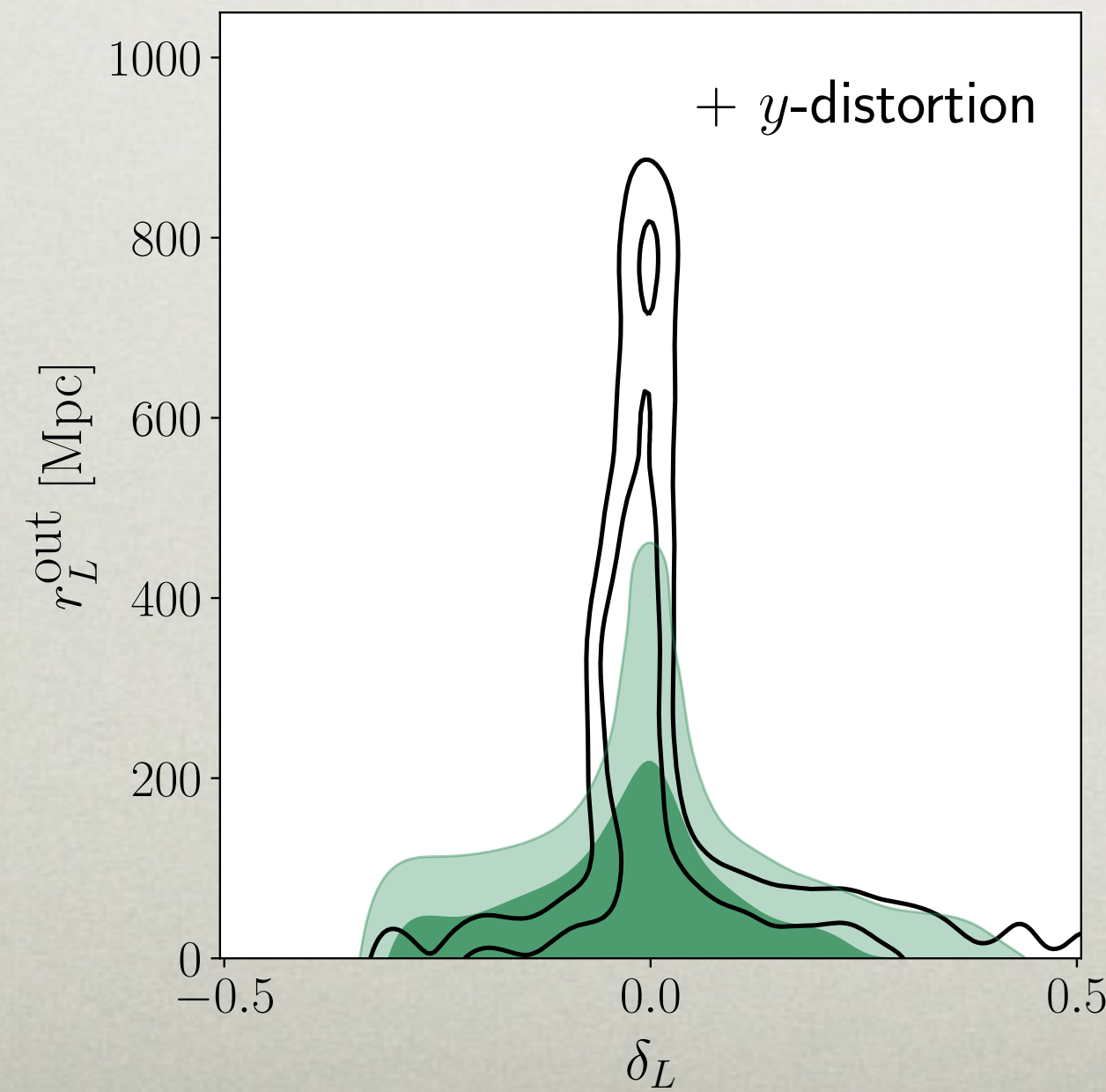
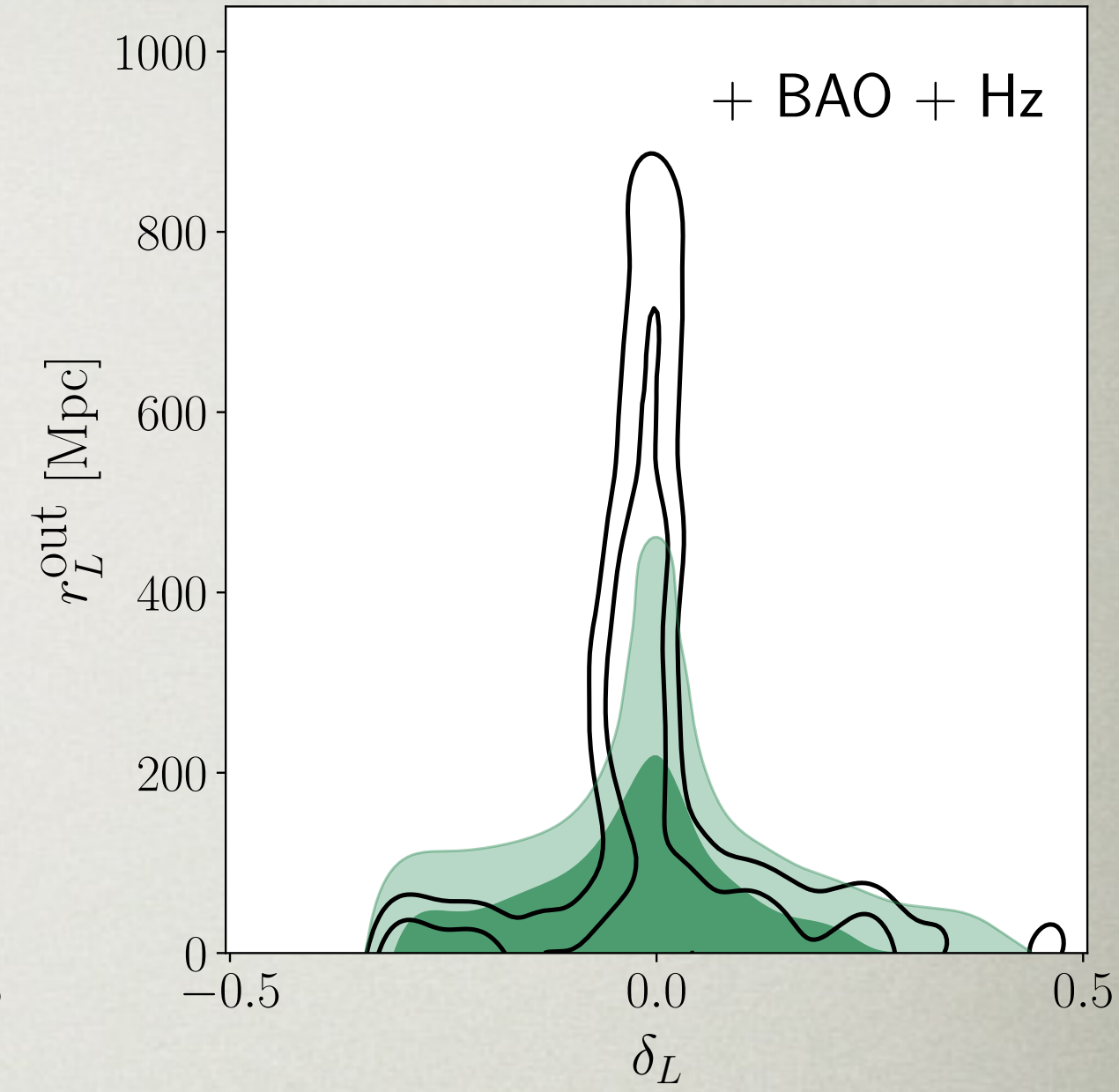
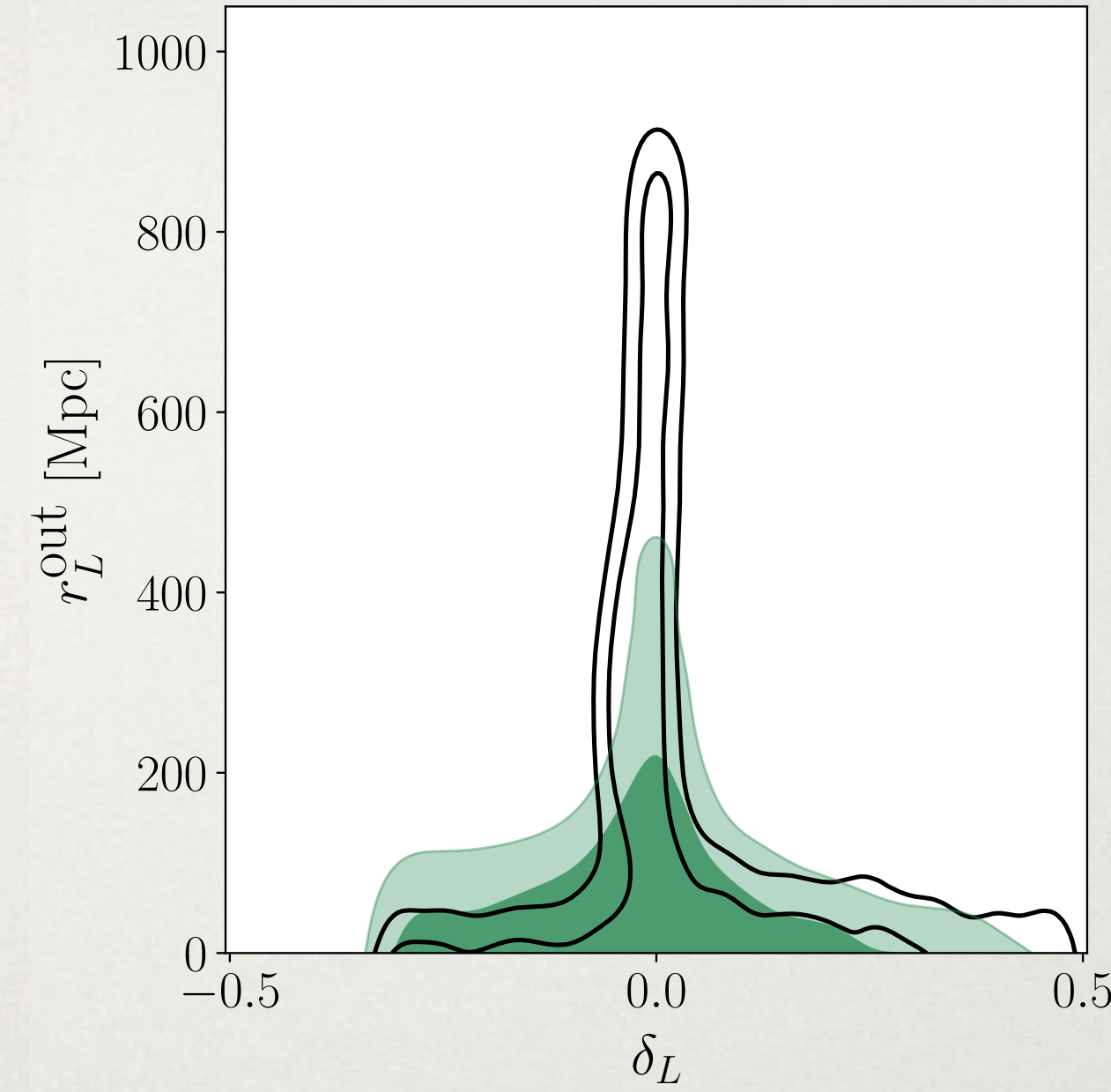
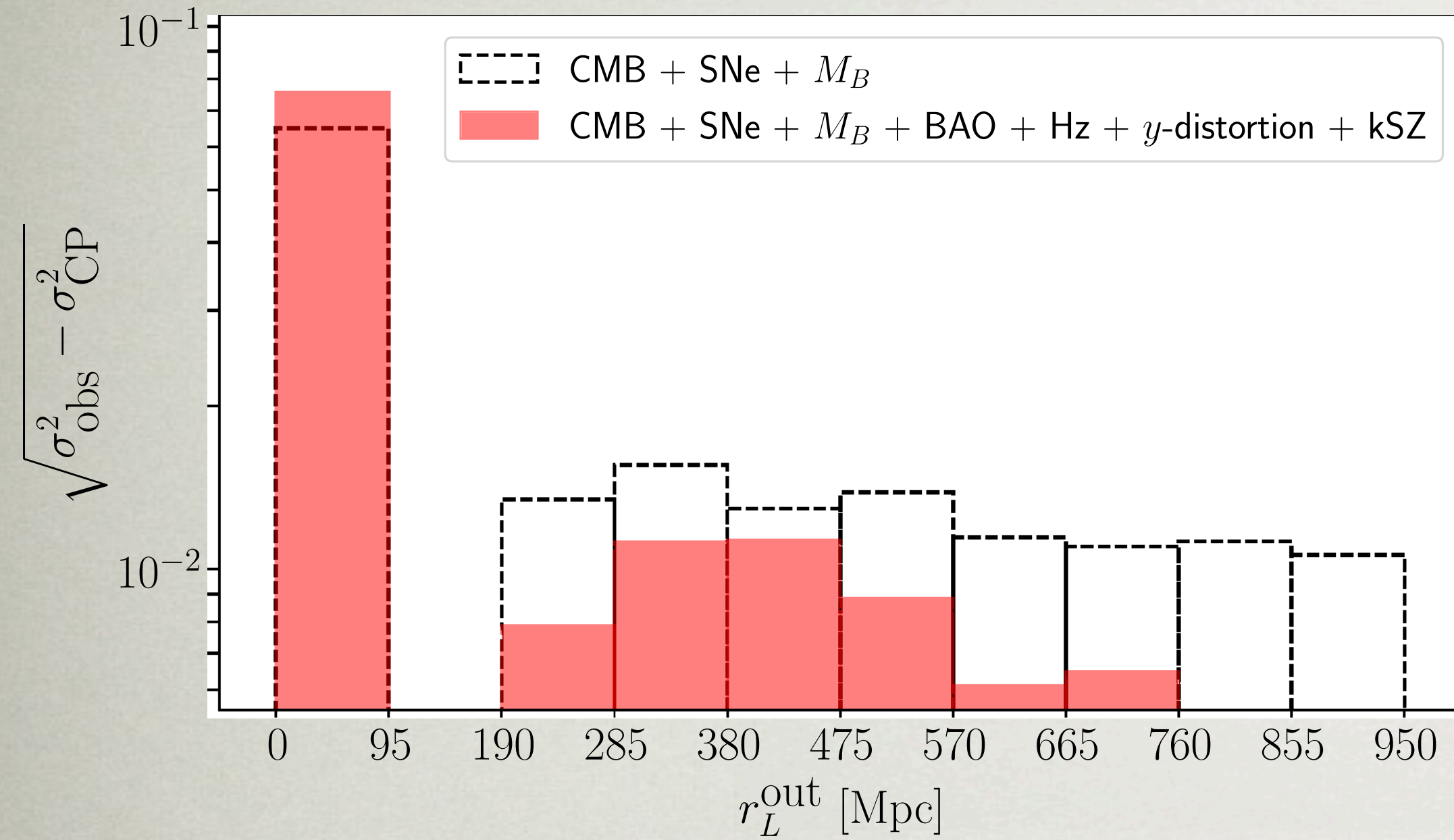
No solution if all supernovae are considered



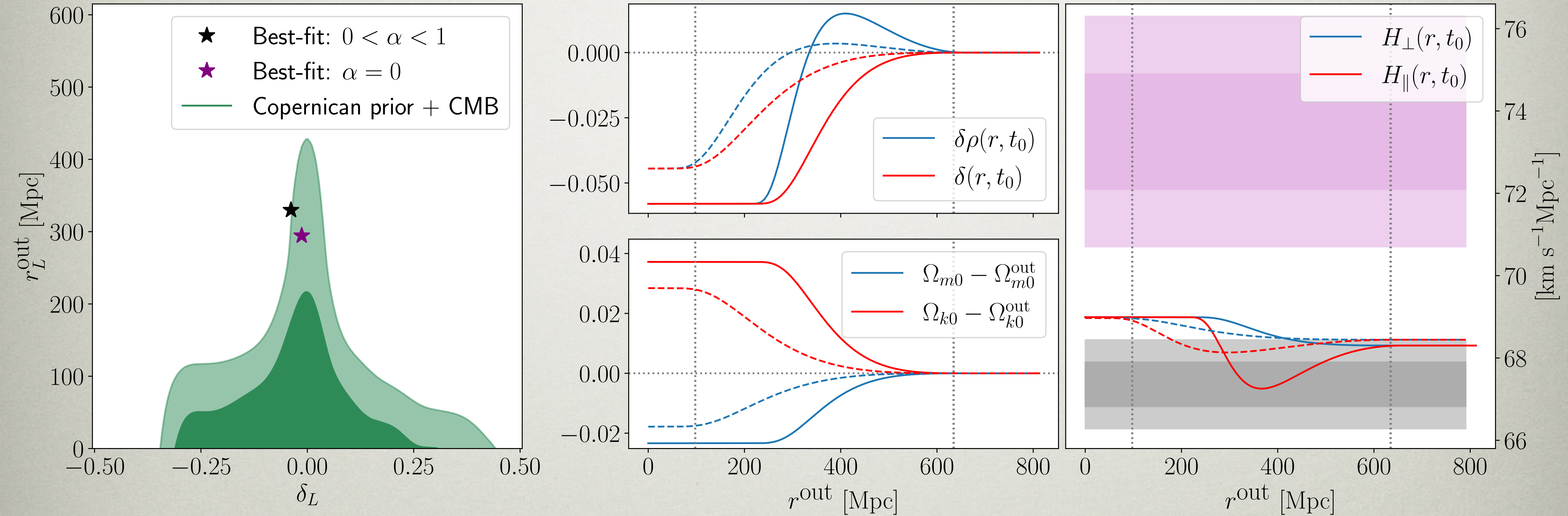
Constraining spherical inhomogeneity

— CMB + SNe + M_B

■ Copernican Prior + CMB



The local structure of the universe



SIBELIUS-DARK also finds a 5% underdensity
McAlpine+ 2202.04099

Beyond background observables

- Tight constraints but simplest setup: no anisotropies, monotonic and rigid profile. Necessary to consider a more general model.
- Constraints from background observables.
- To really probe inhomogeneity we need to also constrain the growth of structure, especially given future surveys.
- The BEHOMO project!

Cosmology **BE**yond **HO**MOgeneity and Isotropy

- Study numerically inhomogeneous cosmology
- LTB, Swiss-cheese, Szekeres (dipole inhom), Bianchi, tilted universe, etc
- Start with early-FLRW cosmologies in their simplest form: Λ LTB
- Archetype of inhomogeneity to explore the effect of spatial gradients
- Project open to the community: valerio-marra.github.io/BEHOMO-project



Why numerical inhomogeneous cosmology?

- Background evolution:

Newtonian codes can reproduce the evolution of the Λ LTB model.

Is this true for more general models? Up to which scale? Do we need GR codes or are Newtonian codes ok?

- Growth of perturbations:

because of mode coupling, (linear) perturbations grow in a not-yet understood way in an inhomogeneous background.

Hic sunt leones.

BEHOMO goal: to study perturbations via the methods of numerical cosmology.

BEHOMO suite of 68 simulations

	δ_0	r_b (Gpc/h)	L_{box} (Gpc/h)	grav. soft. at $z = 0$ (kpc/h)	PM (# grid el.)	N_{part}	M_{part} ($M_\odot/h$)	$M_{\text{halo}}^{\text{min}}$ ($M_\odot/h$)
Box 1	–	–	0.5	6.4	2048 ³	1024 ³	9.7×10^9	4.8×10^{11}
	...	0.2	"	"	"	"	"	"
Box 2	–	–	1.0	12.8	2048 ³	1024 ³	7.8×10^{10}	3.9×10^{12}
	...	0.4	"	"	"	"	"	"
Box 3	–	–	1.5	9.6	4096 ³	2048 ³	3.3×10^{10}	1.6×10^{12}
	...	0.6	"	"	"	"	"	"
Box 4	–	–	2.0	12.8	4096 ³	2048 ³	7.8×10^{10}	3.9×10^{12}
	...	0.8	"	"	"	"	"	"
Box 5	–	–	3.0	19.2	4096 ³	2048 ³	2.6×10^{11}	1.3×10^{13}
	...	1.2	"	"	"	"	"	"
Box 6	–	–	4.0	22.7	4608 ³	2304 ³	4.4×10^{11}	2.2×10^{13}
	...	1.6	"	"	"	"	"	"

FLRW parameters		value
H_0	68 km/s/Mpc	
Ω_m	0.3	
Ω_k	0	
Perturbation parameters		value
Ω_b	0.048	
$\ln(10^{10} A_s)$	3.0	
n_s	0.97	
τ	0.094	
Y_p	0.25	
N_{eff}	3.046	
Σm_ν	0	
LTB parameters		value
δ_0	[-0.6, 0.6]	
r_b	[500, 4000] Mpc/h	

$\delta_0 = 0, \pm 0.1, \pm 0.15, \pm 0.2, \pm 0.3, \pm 0.45, \pm 0.6$

MSCA at Trieste Obs with Borgani
1.3MCPUh@m100, 170TB
OpenGadget3 + FalconIC

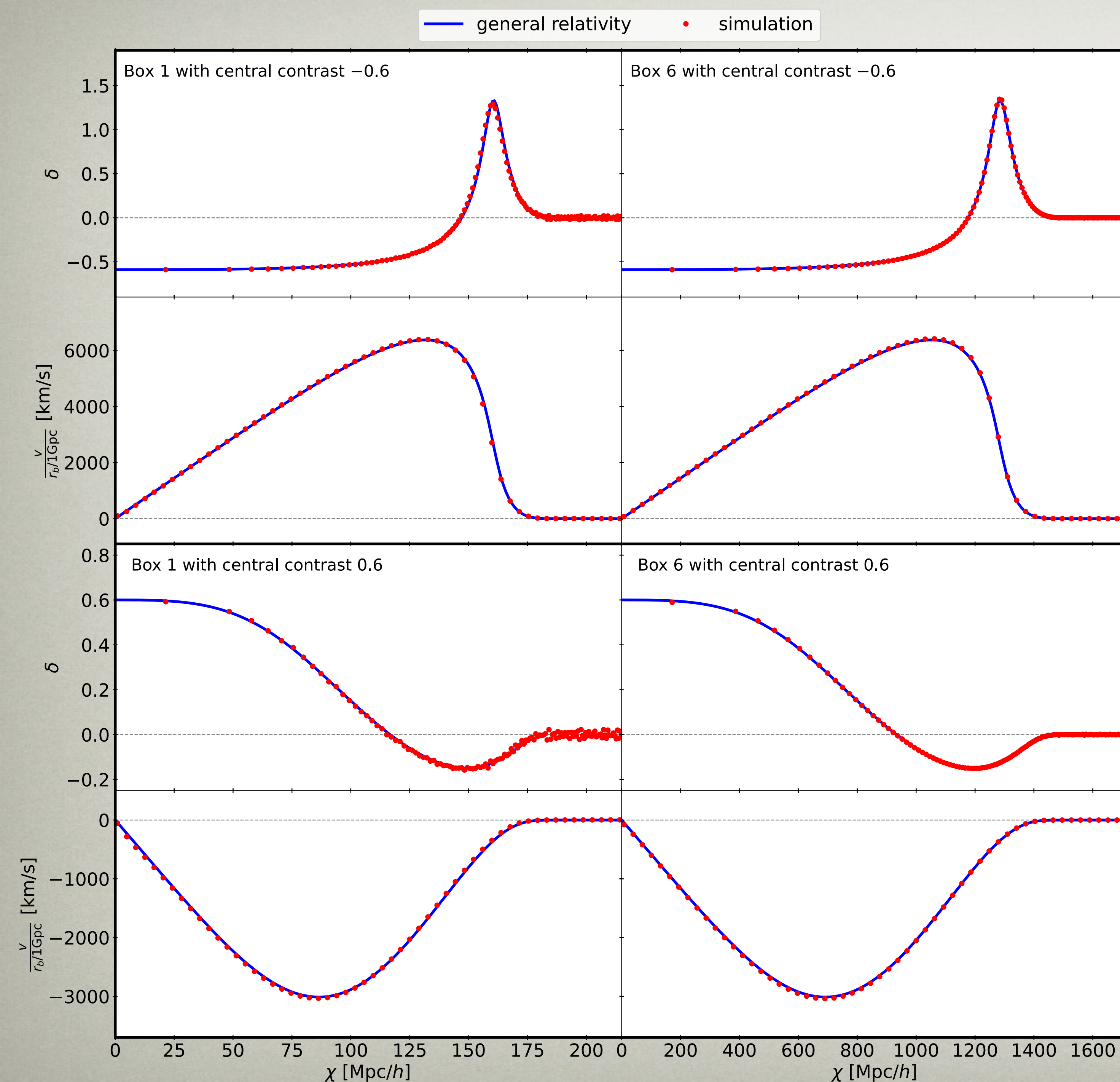
please contact me
if interested in
analyzing the data

Background evolution

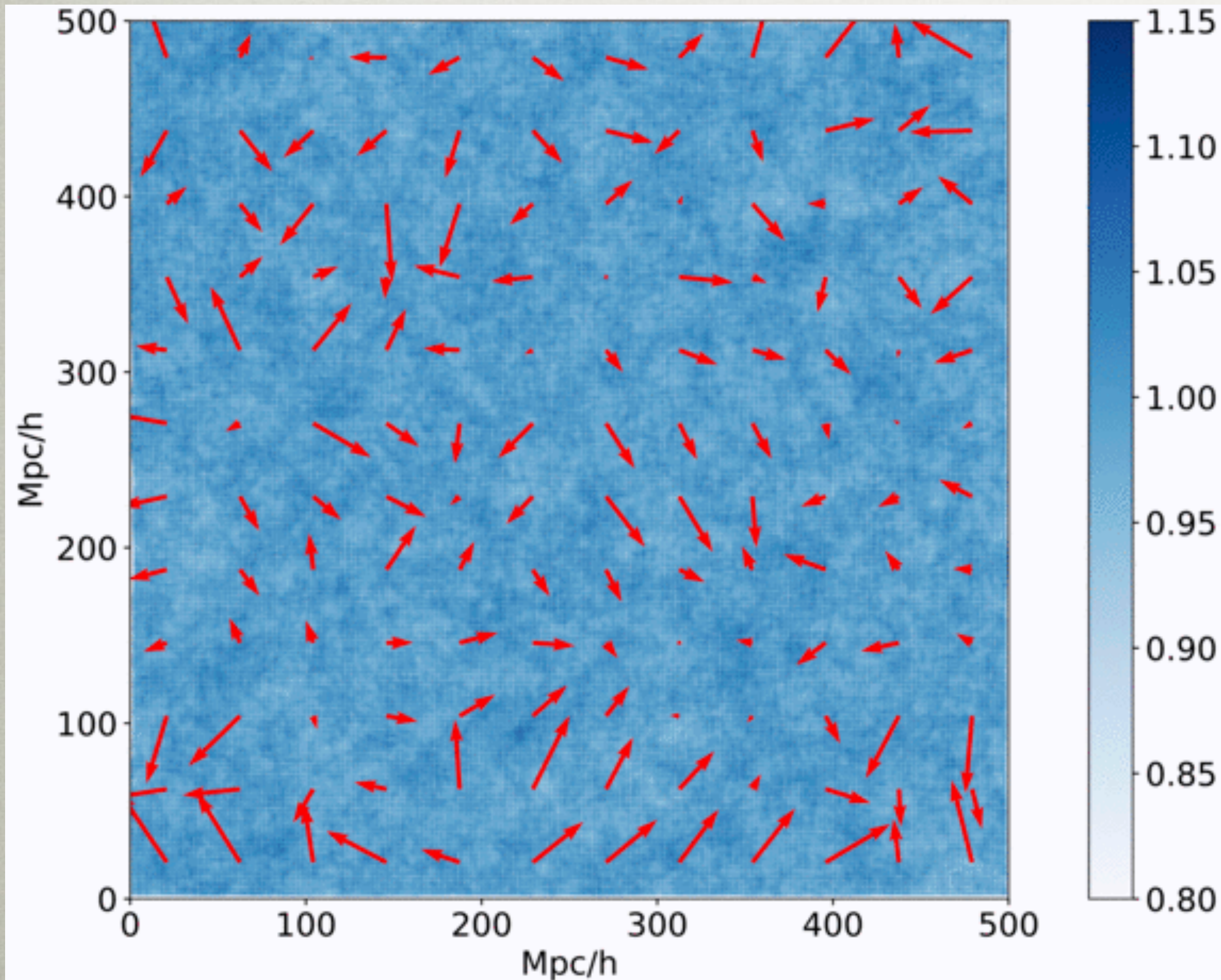
$$A_s \approx 0$$

The inhomogeneous (Newtonian)
N-body simulations perfectly
follow the GR solution.

Thanks to the scale invariance,
their rescaled evolution is the same.



Initial conditions



Special IC from FalconIC
are fed to Gadget:

$$\delta(t_{\text{ini}}, \mathbf{x}) = \delta^{\text{LTB}}(t_{\text{ini}}, \mathbf{x}) + \delta^{\text{gau}}(t_{\text{ini}}, \mathbf{x})$$

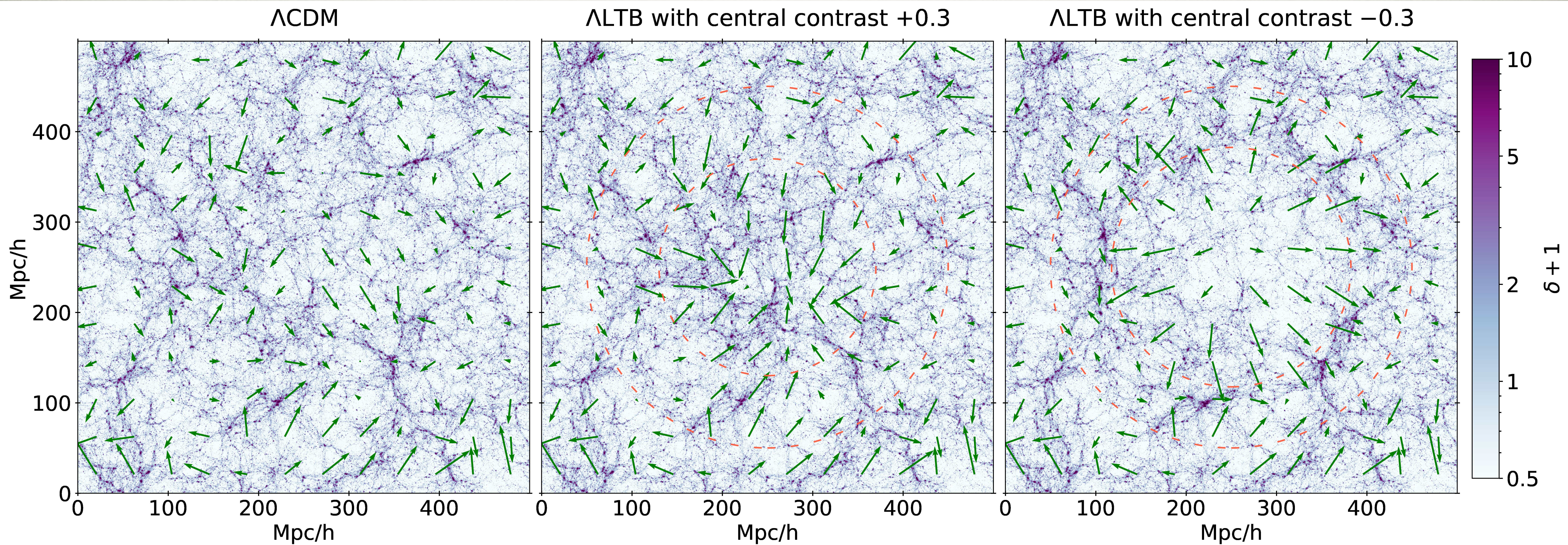
$$\mathbf{x}(t_{\text{ini}}) = \mathbf{q} + \nabla \left(\Phi^{\text{LTB}} + \Phi^{\text{gau}} \right)$$

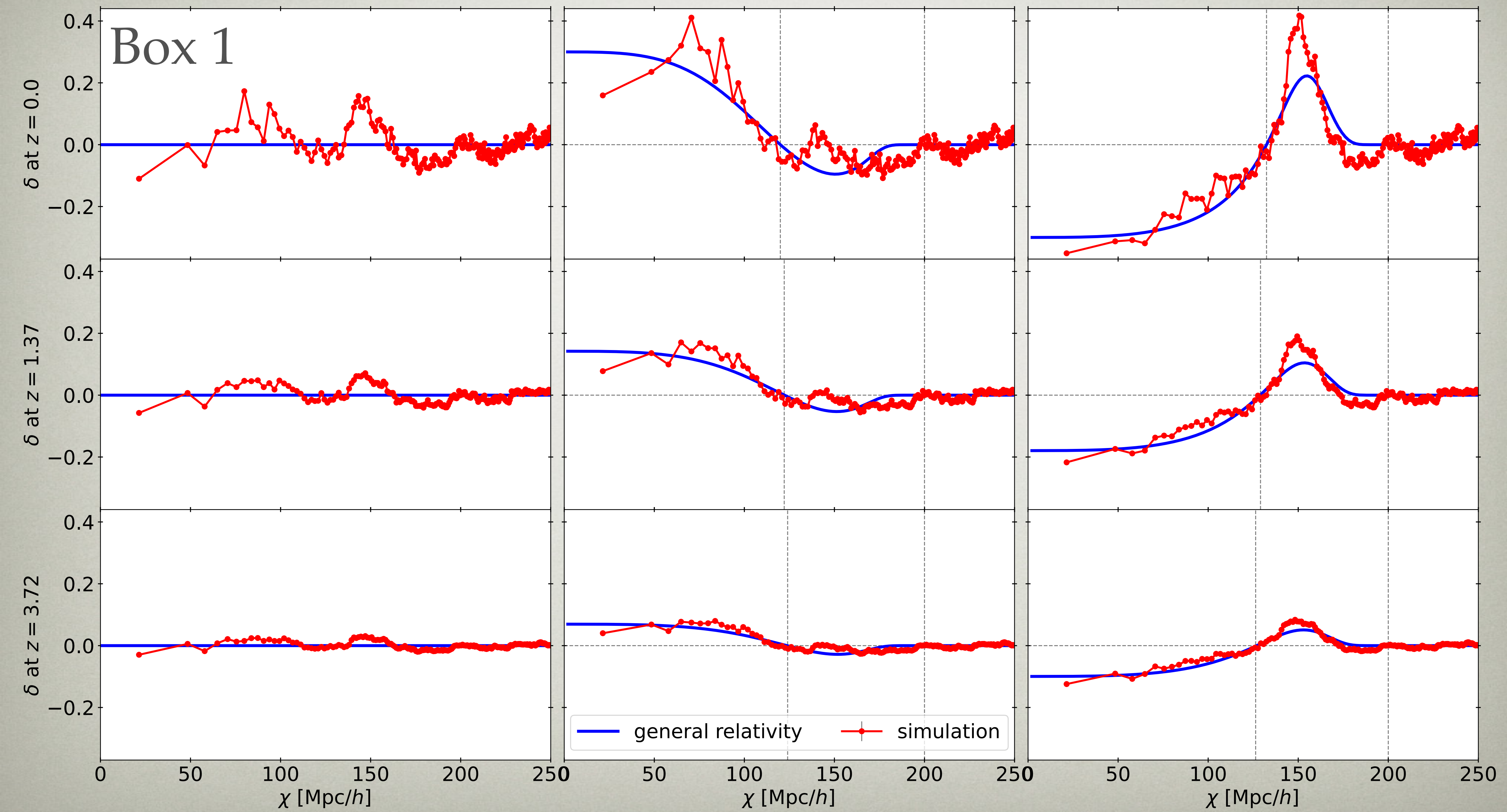
$$\Phi_{\text{lin}}(r) = \frac{3}{5} \int_r^{r_b} \frac{E(\bar{r})}{\bar{r}} d\bar{r} \sim E$$

$$E(r) \equiv -\frac{k r^2}{2} = \frac{1}{2} \dot{Y}^2(t, r) - \frac{GF(r)}{Y(t, r)} - \frac{1}{6} \Lambda Y^2(t, r)$$

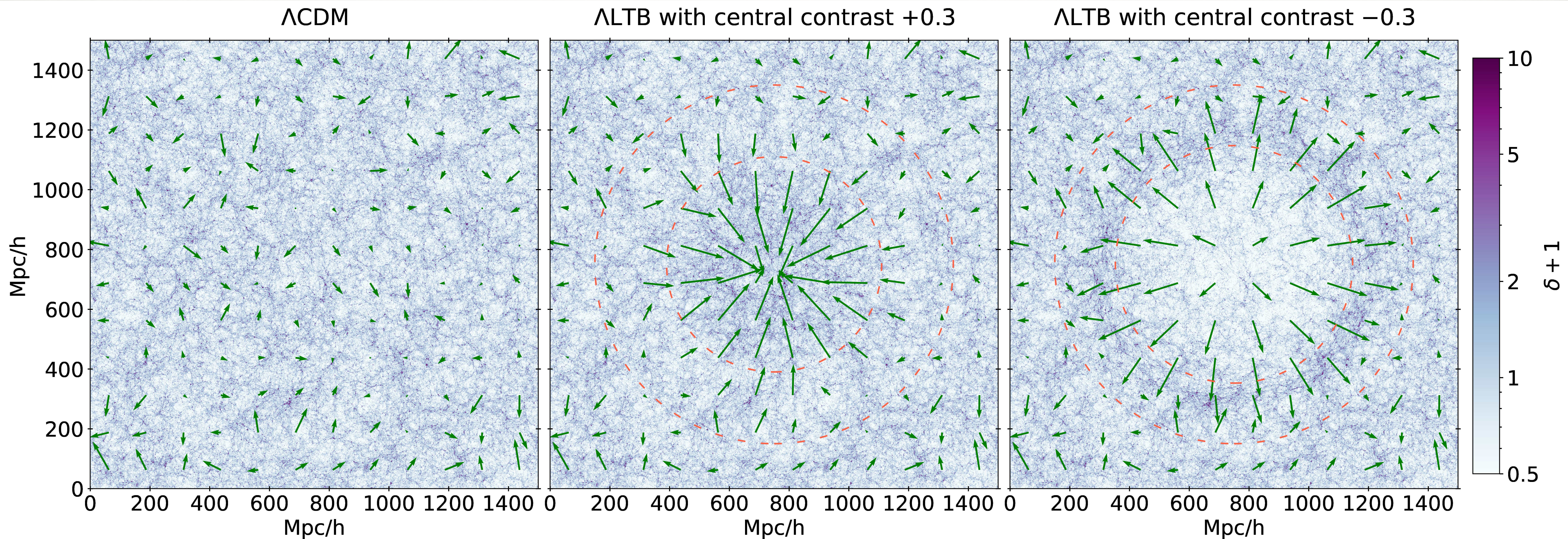
Same Λ CDM seed!

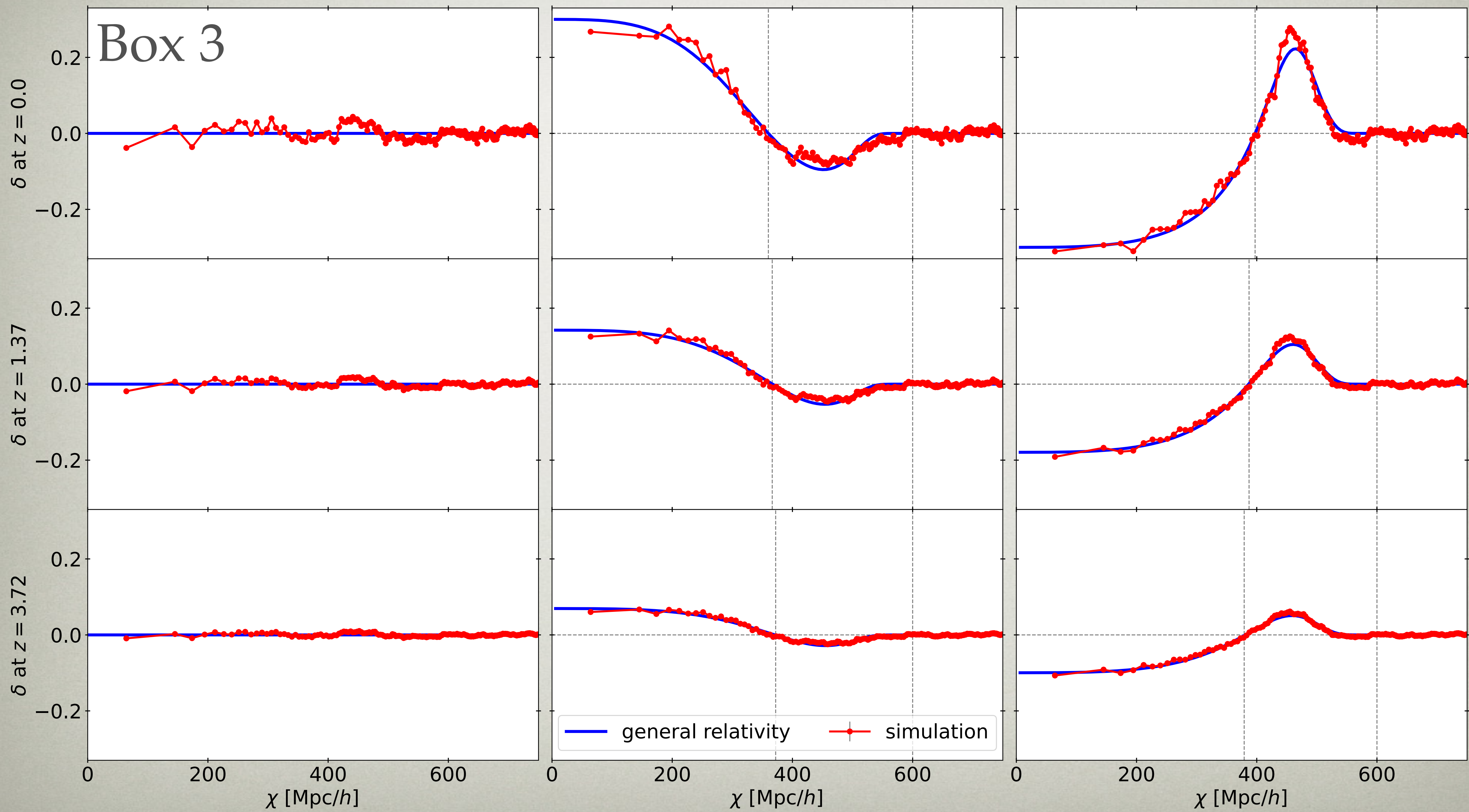
Box 1 of 500 Mpc

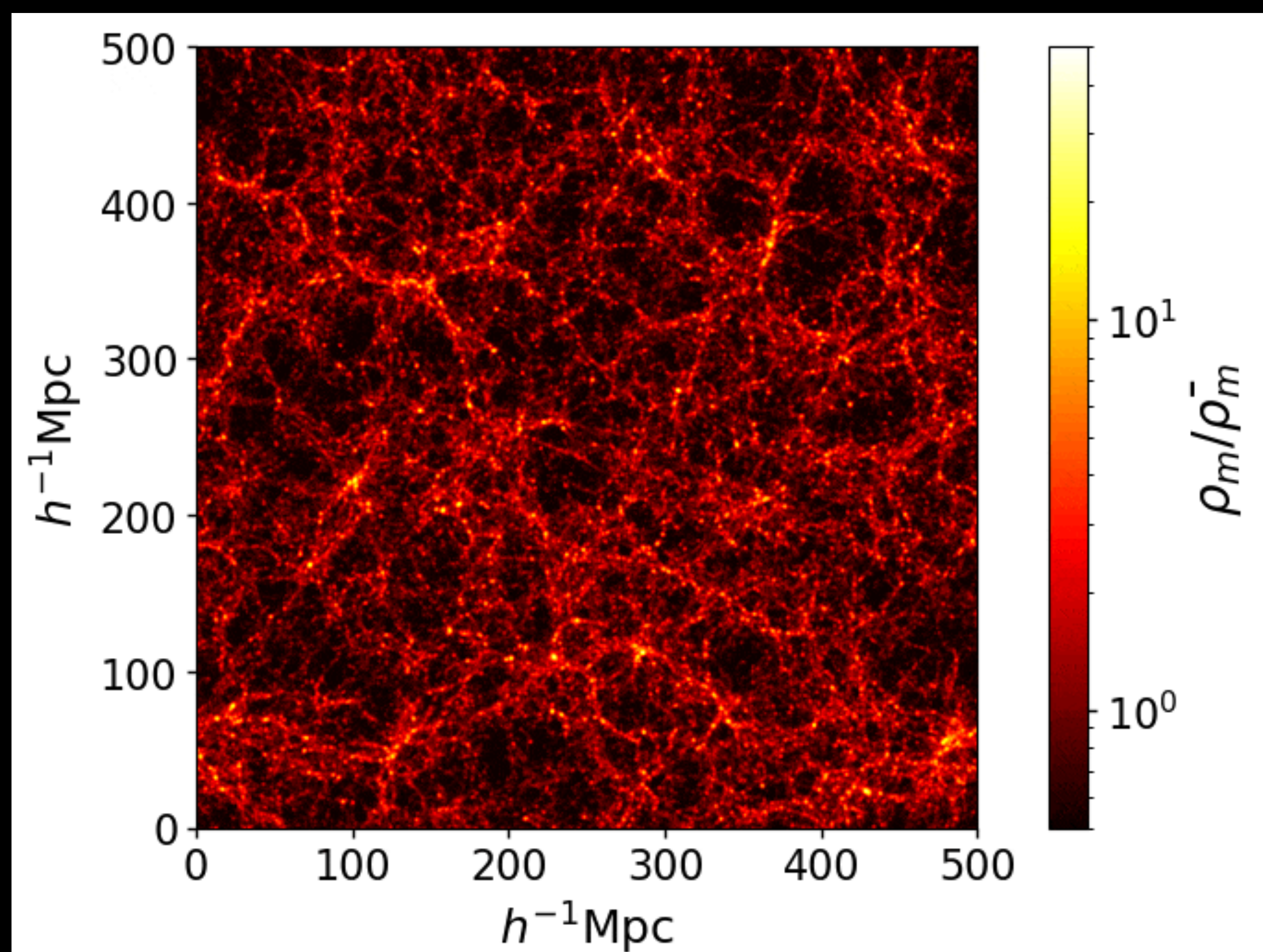
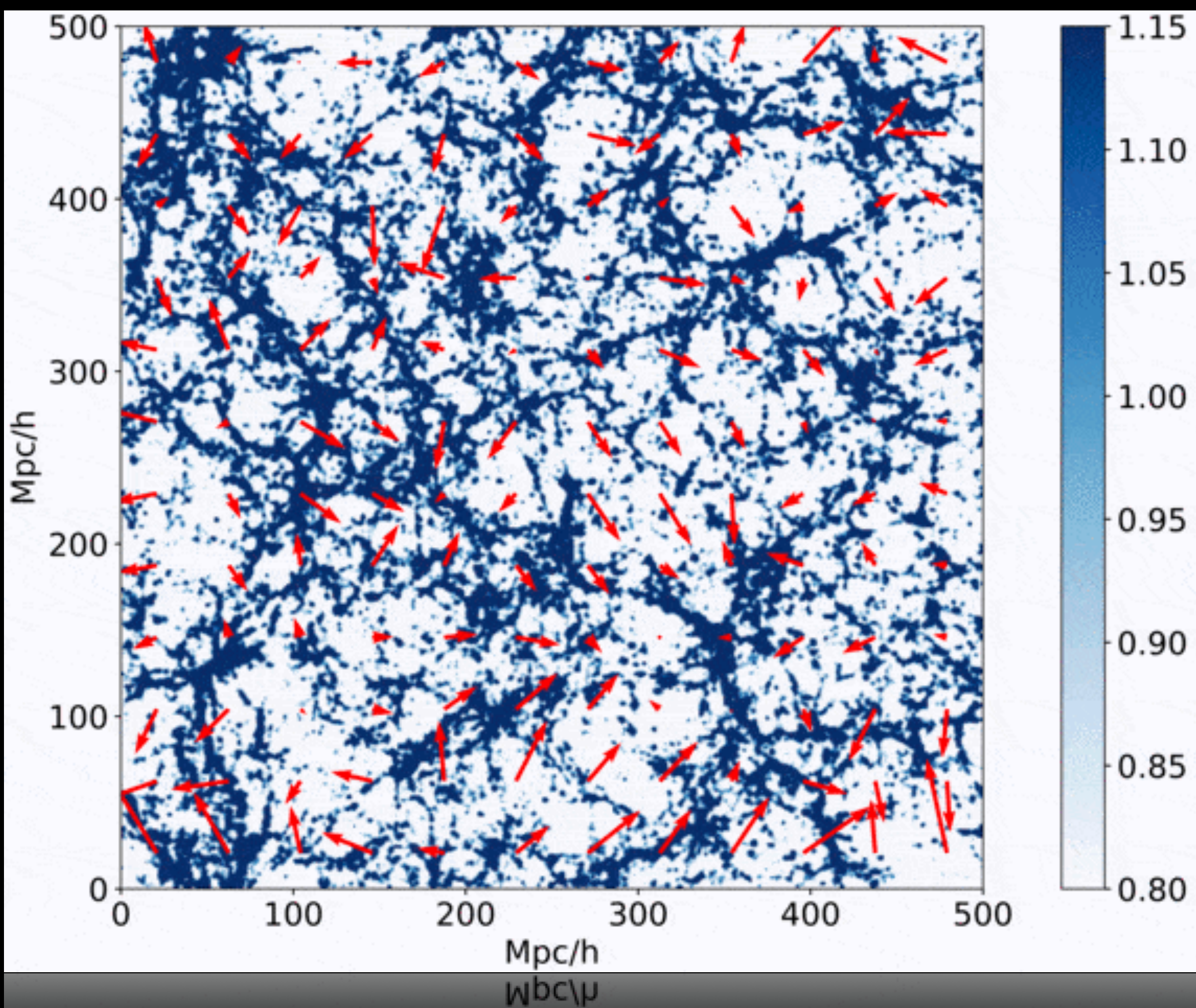




Box 3 of 1500 Mpc





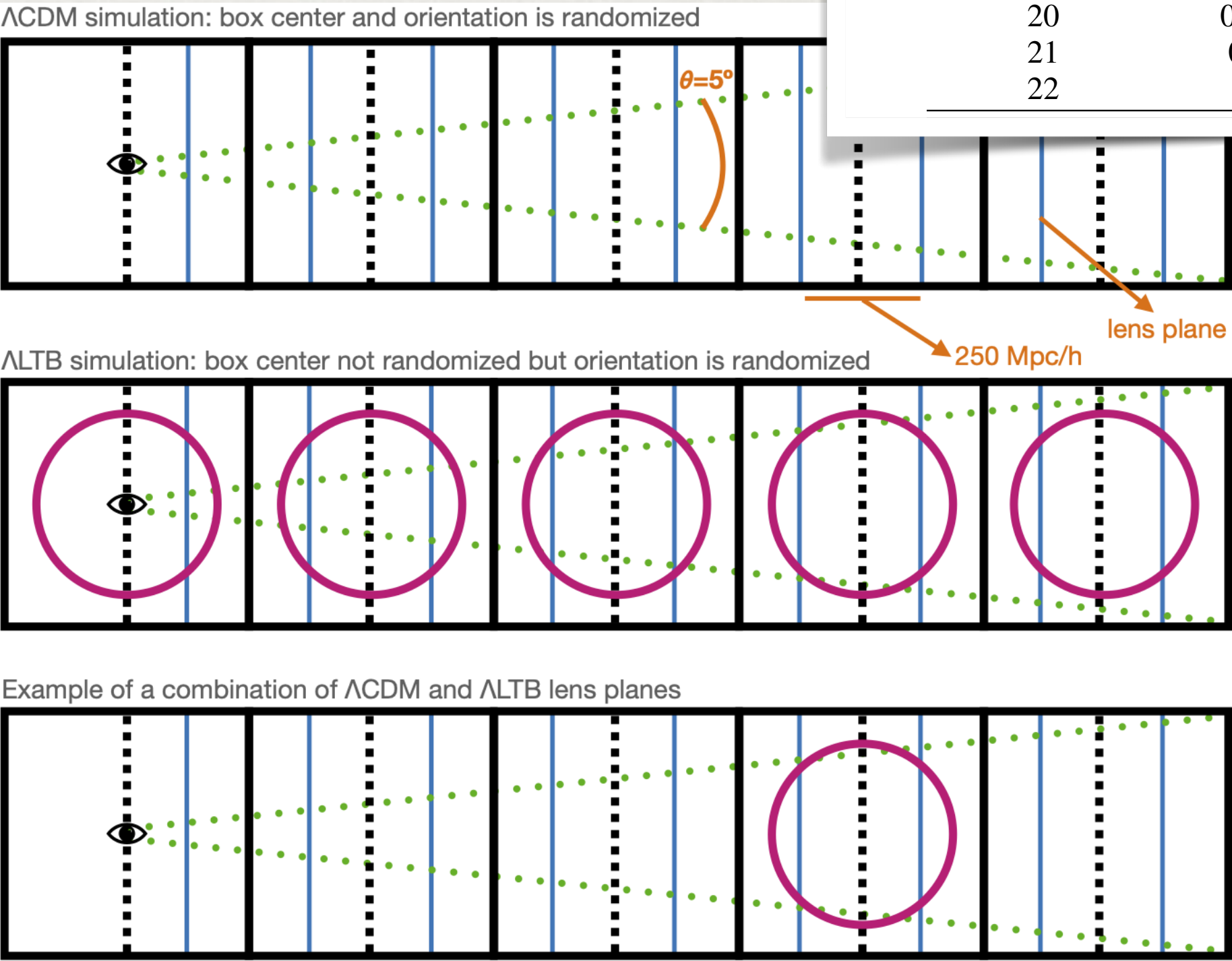


Data products

- Full snapshots
- Halo catalogs from Rockstar:
merger trees, halo parameters, ...
(40 properties)
- Lens planes from SLICER:
10x22 lensing planes of 2048^2
pixels with a $5^\circ \times 5^\circ$ fov up to $z=4.2$
(8.8" of resolution).

Table 3. Snapshots that were saved during the runs for the generation of the halo catalog and lens planes. Only even-numbered snapshots are kept for long-term storage.

snapshot	a	z	d_C (Mpc/h)
0	0.02	49	8360
1	0.191	4.225	5125
2	0.212	3.715	4875
3	0.234	3.275	4625
4	0.257	2.892	4375
5	0.281	2.557	4125
6	0.307	2.261	3875
7	0.333	1.999	3625
8	0.362	1.765	3375
9	0.391	1.555	3125
10	0.423	1.366	2875
11	0.456	1.194	2625
12	0.491	1.037	2375
13	0.528	0.893	2125
14	0.568	0.761	1875
15	0.611	0.638	1625
16	0.656	0.523	1375
17	0.706	0.416	1125
18	0.76	0.315	875
19	0.82	0.22	625
20	0.886	0.129	375
21	0.96	0.042	125
22	1	0	0



Growth of perturbations

At the background level, LTB is 'silent'.

At the perturbation level the first-order **magnetic Weyl tensor couples scalar to tensor perturbation modes**. Four approaches:

1. Full set of coupled equations
- 2. Silent approximation**
- 3. Local-FLRW approximation**
4. Second-order perturbations theory

Growth of silent perturbations

2-scalars in the 1+1+2 decomposition:

$$\rho_m(t, r) = \frac{F'(r)}{4\pi Y^2(t, r) Y'(t, r)}$$

$$\theta(t, r) = H_{\parallel}(t, r) + 2H_{\perp}(t, r),$$

$$\Sigma(t, r) = \frac{2}{3} [H_{\parallel}(t, r) - H_{\perp}(t, r)],$$



$$\mathcal{E}(t, r) = \frac{8\pi G}{3} \rho_m(t, r) - \frac{2GF(r)}{Y^3(t, r)} \equiv \frac{8\pi G}{3} [\rho_m(t, r) - \rho_m^e(t, r)],$$

electric Weyl scalar

$$\delta X_i(t, r, n^\mu) = T_{ij}^{\text{LTB}}(t, r) \delta X_j(t_i, r, n^\mu)$$

$$X_i = \{\rho_m, \theta, \Sigma, \mathcal{E}\}$$

Linear evolution:

$$\delta \dot{\rho}_m = -\theta \delta \rho_m - \rho_m \delta \theta,$$

$$\delta \dot{\theta} = -\frac{2}{3} \theta \delta \theta - 4\pi G \delta \rho_m - 3\Sigma \delta \Sigma,$$

$$\delta \dot{\Sigma} = -\left(\frac{2}{3} \theta + \Sigma\right) \delta \Sigma - \frac{2}{3} \Sigma \delta \theta - \delta \mathcal{E},$$

$$\delta \dot{\mathcal{E}} = -\left(\theta - \frac{3}{2} \Sigma\right) \delta \mathcal{E} - \mathcal{E} \left(\delta \theta - \frac{3}{2} \delta \Sigma\right) - 4\pi G (\rho_m \delta \Sigma + \Sigma \delta \rho_m)$$

initial conditions at the
matter-dominated era:

$$\frac{\delta \rho_m}{\rho_m} = -\frac{18}{5} \frac{D^2}{\theta^2} \mathcal{R},$$

$$\frac{\delta \theta}{\theta} = -\frac{1}{3} \frac{\delta \rho_m}{\rho_m},$$

$$\frac{\delta \Sigma}{\theta} = \frac{6}{5} \frac{1}{\theta^2} \left(\frac{\partial^2}{\partial r^2} - \frac{1}{3} D^2 \right) \mathcal{R},$$

$$\frac{\delta \mathcal{E}}{\theta} = -\frac{1}{2} \delta \Sigma.$$

Local-FLRW approximation

$$\Gamma \equiv \frac{3\Sigma}{2\theta/3} = \frac{H_{\parallel}(t, r) - H_{\perp}(t, r)}{[H_{\parallel}(t, r) + 2H_{\perp}(t, r)]/3} \ll 1$$

$$\delta\dot{\rho}_m = -\theta\delta\rho_m - \rho_m\delta\theta,$$

$$\delta\dot{\theta} = -\frac{2}{3}\theta\delta\theta - 4\pi G\delta\rho_m - 3\Sigma\delta\Sigma,$$

$$\delta\dot{\Sigma} = -\left(\frac{2}{3}\theta + \Sigma\right)\delta\Sigma - \frac{2}{3}\Sigma\delta\theta - \delta\mathcal{E},$$

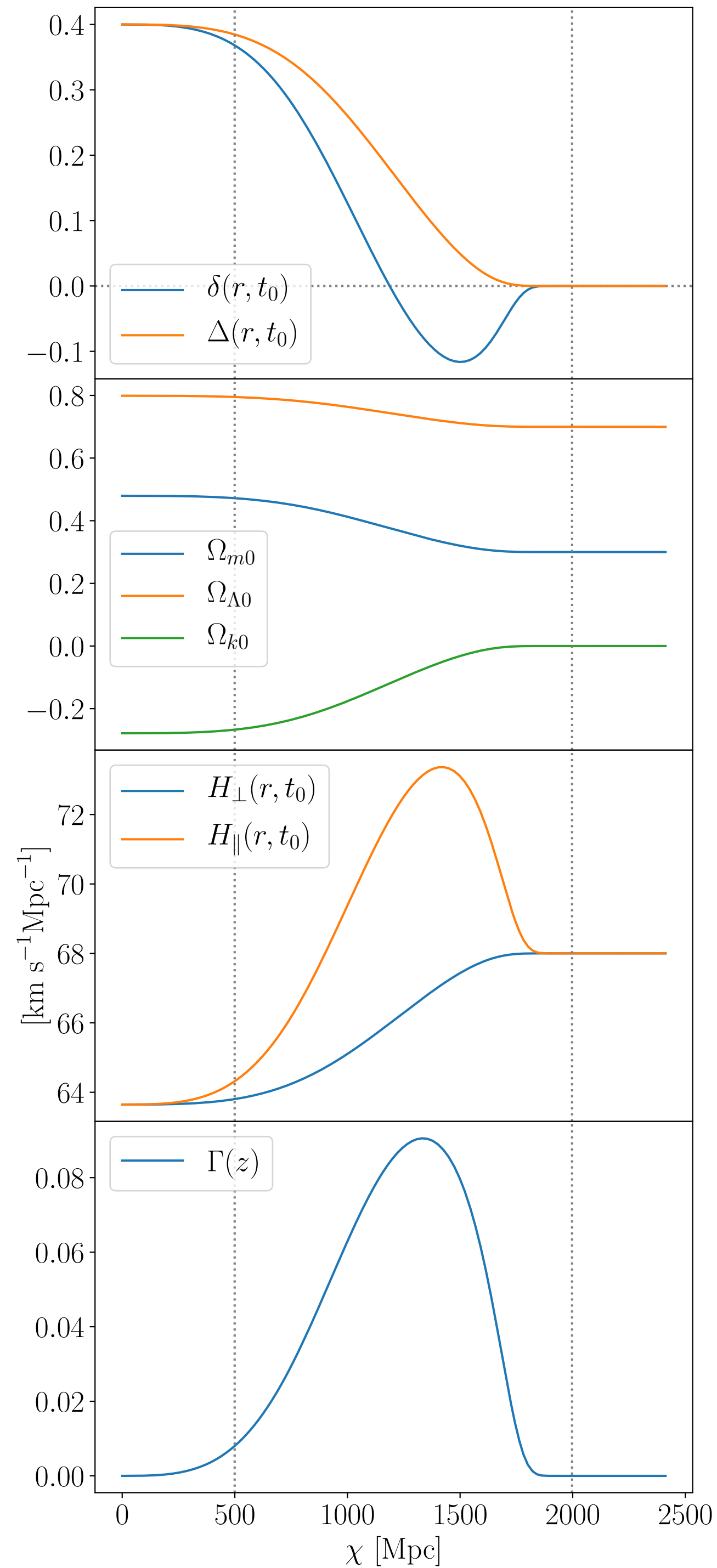
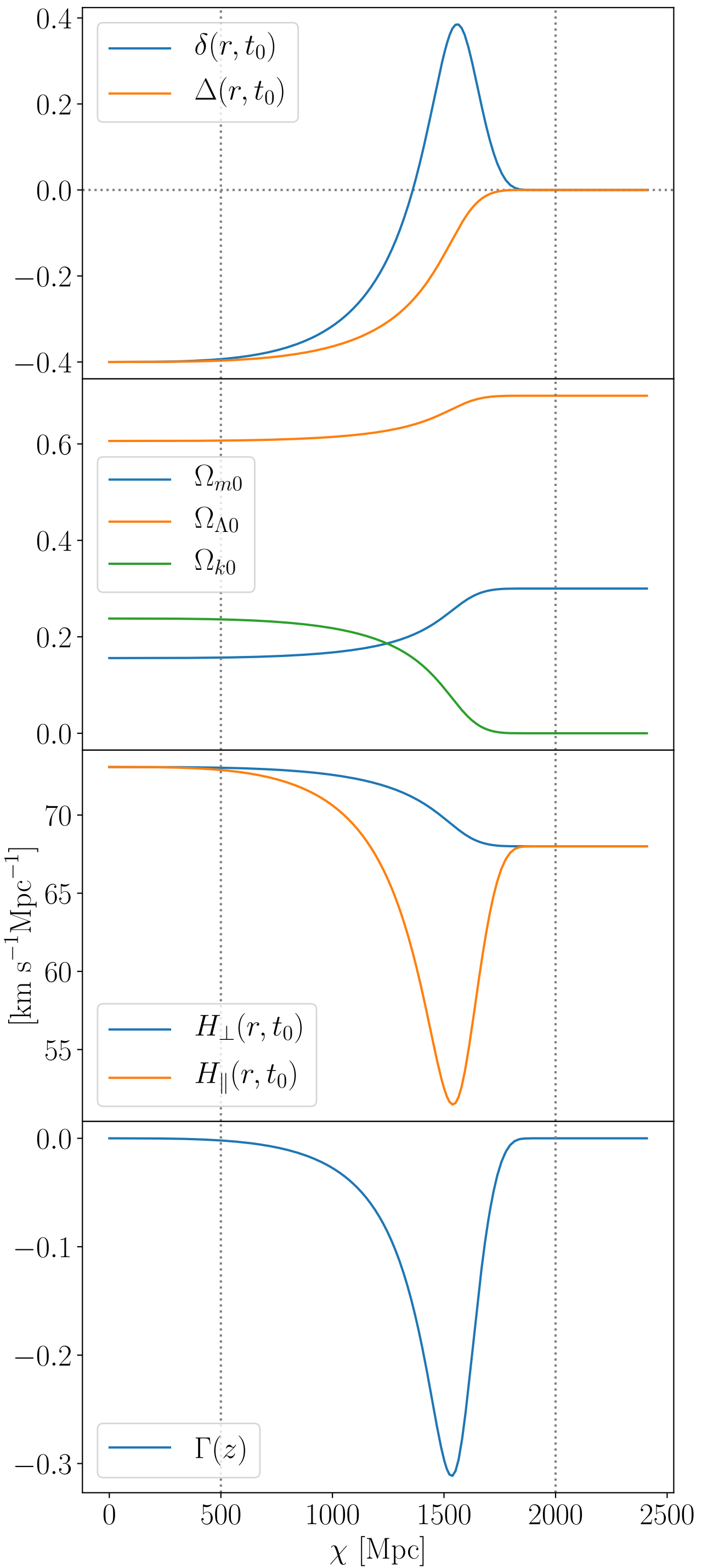
$$\delta\dot{\mathcal{E}} = -\left(\theta - \frac{3}{2}\Sigma\right)\delta\mathcal{E} - \mathcal{E}\left(\delta\theta - \frac{3}{2}\delta\Sigma\right) - 4\pi G(\rho_m\delta\Sigma + \Sigma\delta\rho_m)$$

$$\ddot{\delta}(t, r) + 2H_{\text{loc}}(t, r)\dot{\delta}(t, r) - \frac{3}{2}H_{\text{loc}}^2(t, r)\Omega_m^{\text{loc}}(t, r)\delta(t, r) = 0$$

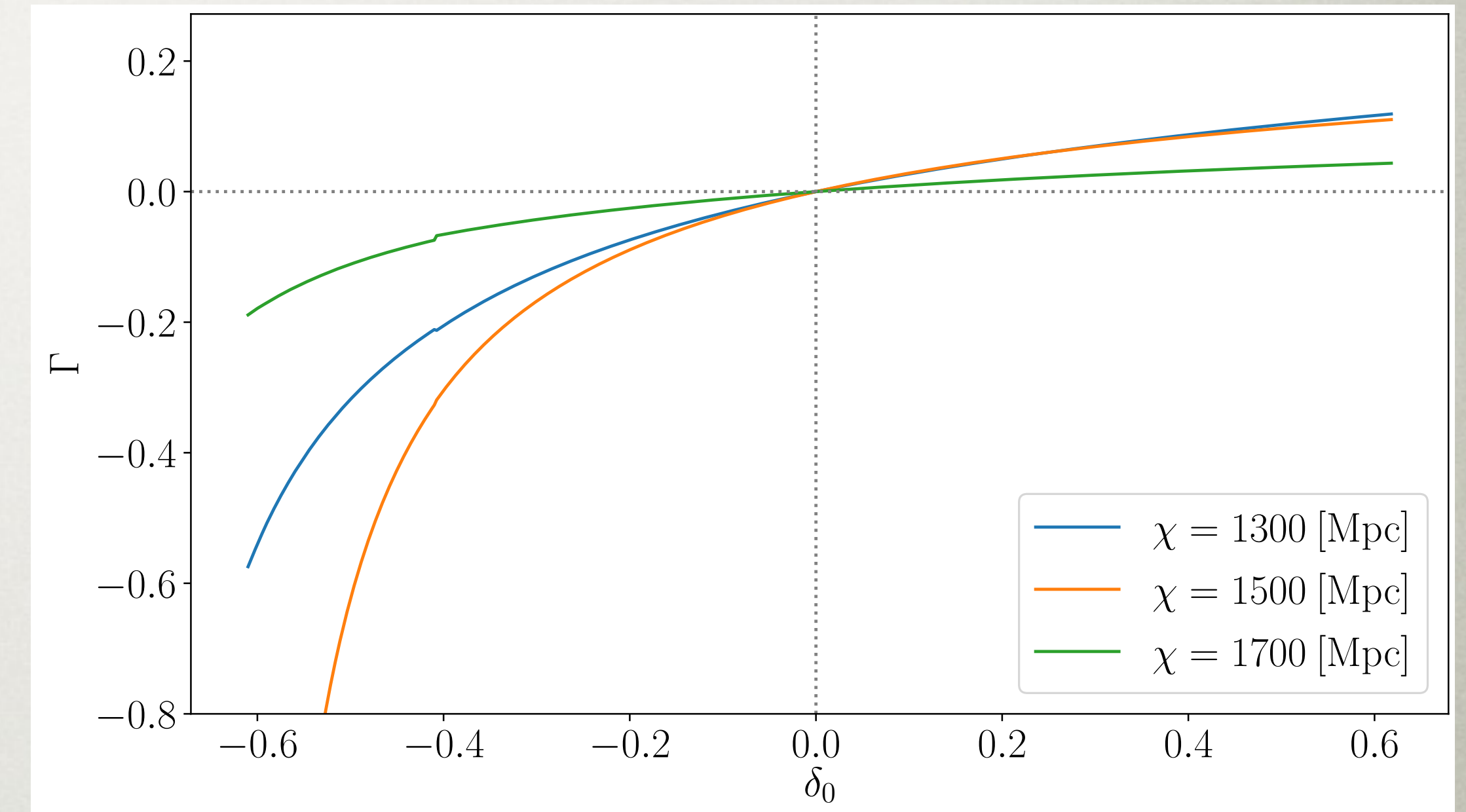
$$\delta(t_i, r) = a_{\text{loc}}(t_i, r), \quad \dot{\delta}(t_i, r) = \dot{a}_{\text{loc}}(t_i, r)$$

$$H_{\text{loc}}(t, r) \equiv \frac{\theta(t, r)}{3},$$

$$\Omega_m^{\text{loc}}(t, r) \equiv \frac{8\pi G\rho_m(t, r)}{3H_{\text{loc}}^2(t, r)}.$$



Local-FLRW approximation



$$\Gamma \equiv \frac{3\Sigma}{2\theta/3} = \frac{H_{\parallel}(t, r) - H_{\perp}(t, r)}{[H_{\parallel}(t, r) + 2H_{\perp}(t, r)]/3}$$

Clustering: model

FLRW

$$C_\ell^{\text{exact}}(t, \chi) = 4\pi \int \frac{dk}{k} \Delta^2(k, t) F_{\chi, \ell}^2(k),$$

$$F_{\chi, \ell}(k) = \int d\bar{\chi} F_\chi(\bar{\chi}) j_\ell(k\bar{\chi}),$$

$$F_\chi(\bar{\chi}) = \begin{cases} 1/\Delta\chi & \text{for } \chi_1 < \bar{\chi} < \chi_2 \\ 0 & \text{otherwise} \end{cases}$$

$$\Delta_{\text{lin}}^2(k, t) \equiv \frac{k^3}{2\pi^2} P_{\text{lin}}(k, z) = \delta_{H0}^2 \left(\frac{k}{a_0 H_0} \right)^{3+n_s} T^2(k) D^2(t),$$

$$\delta_{H0}^2 = \frac{4}{25} \left(\frac{\delta(t_0)}{\Omega_{m,0}} \right)^2 \left(\frac{a_0 H_0}{k_p} \right)^{n_s-1} A_s,$$

LTB

$$C_\ell(t, \chi) = \frac{16\pi}{25} \int_0^\infty \frac{dk}{k} T^2(k) \Delta_{\mathcal{R}}^2(k) T_\ell^2(t, \chi, k)$$

local-FLRW approx

silent approx

$$T_\ell(t, \chi, k) = \int d\bar{\chi} F_\chi(\bar{\chi}) T_{\text{LTB}, \rho}(t, \bar{\chi}, k, \ell)$$

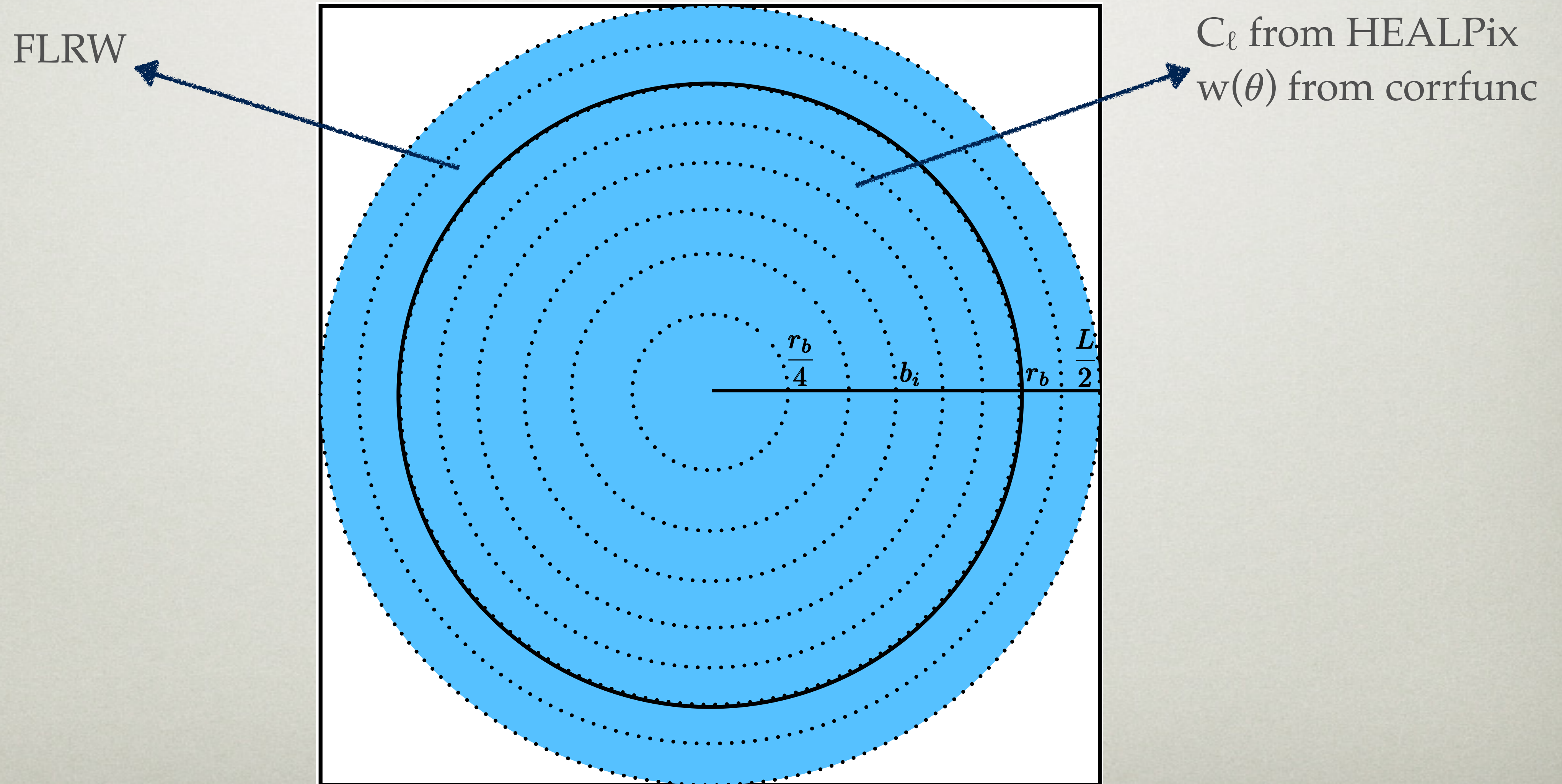
$$= \frac{c^2 k^2}{a_i^2 H_i^2} \left[T_{\rho\rho}^{\text{LTB}}(t, r(t, \chi)) F_{\chi, \ell}(k) - T_{\rho\Sigma}^{\text{LTB}}(t, r(t, \chi)) A_{\chi, \ell}(k) \right],$$

where

$$A_{\chi, \ell}(k) = \int d\bar{\chi} F_\chi(\bar{\chi}) \left[\frac{2}{k\bar{\chi}} j_{\ell+1}(k\bar{\chi}) + \left(\frac{\ell(\ell-1)}{k^2 \bar{\chi}^2} - \frac{2}{3} \right) j_\ell(k\bar{\chi}) \right]$$

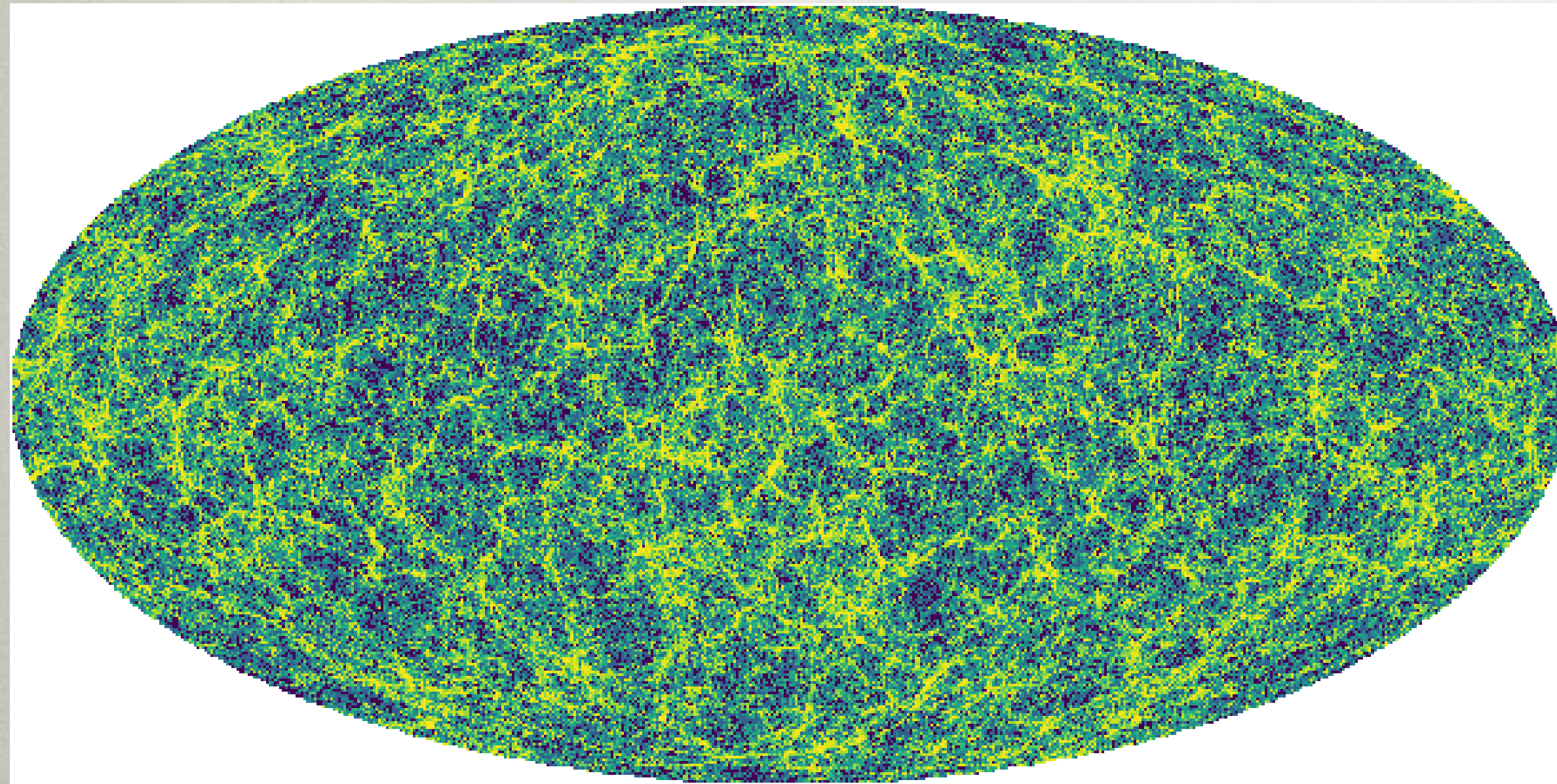
$$C_\ell^{\text{LTB}}(t, \chi) = C_\ell(t, \chi) \frac{\delta_m^2(t, r(t, \chi))}{\delta_m^2(t)}$$

Clustering: simulation

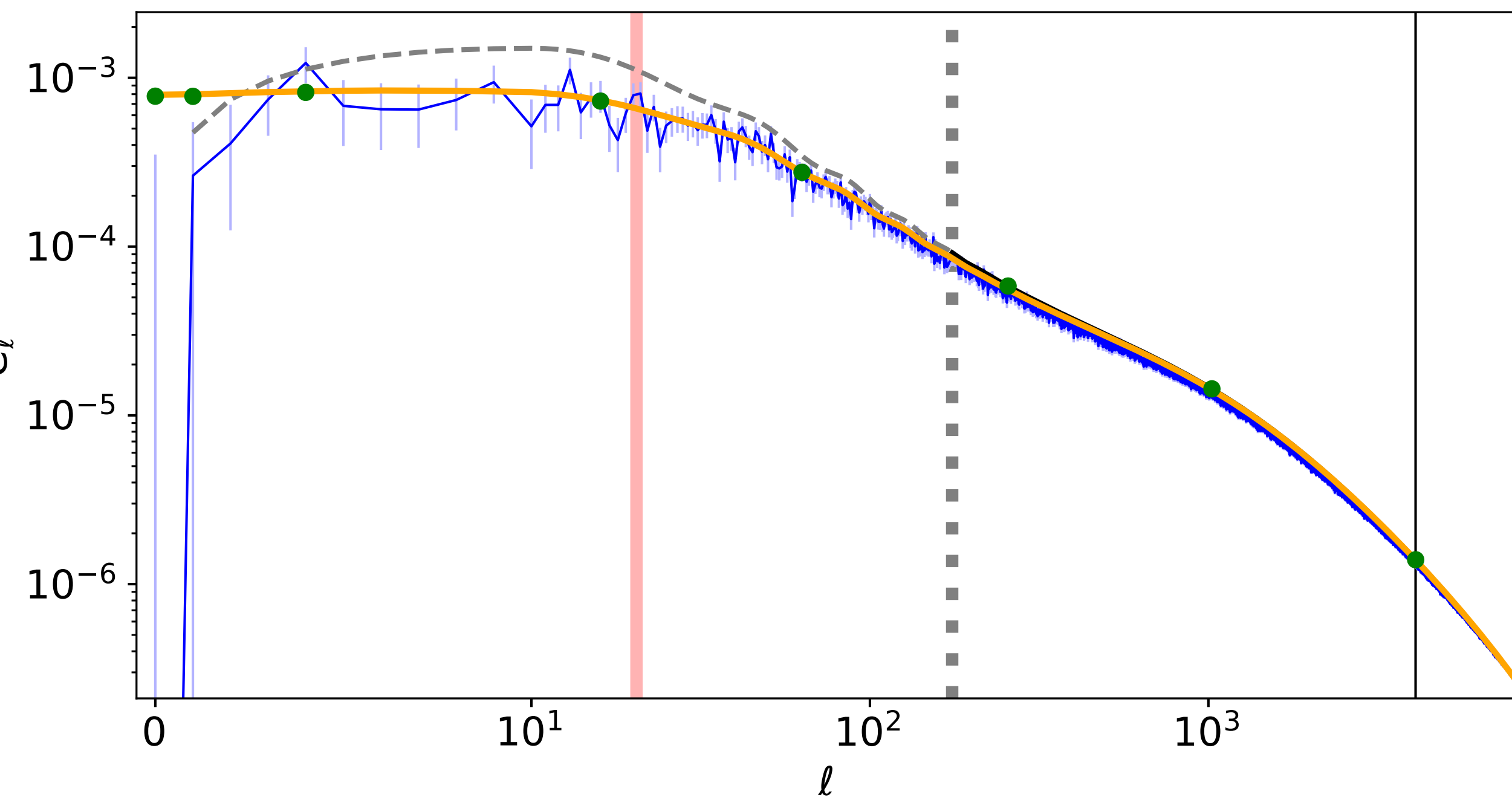
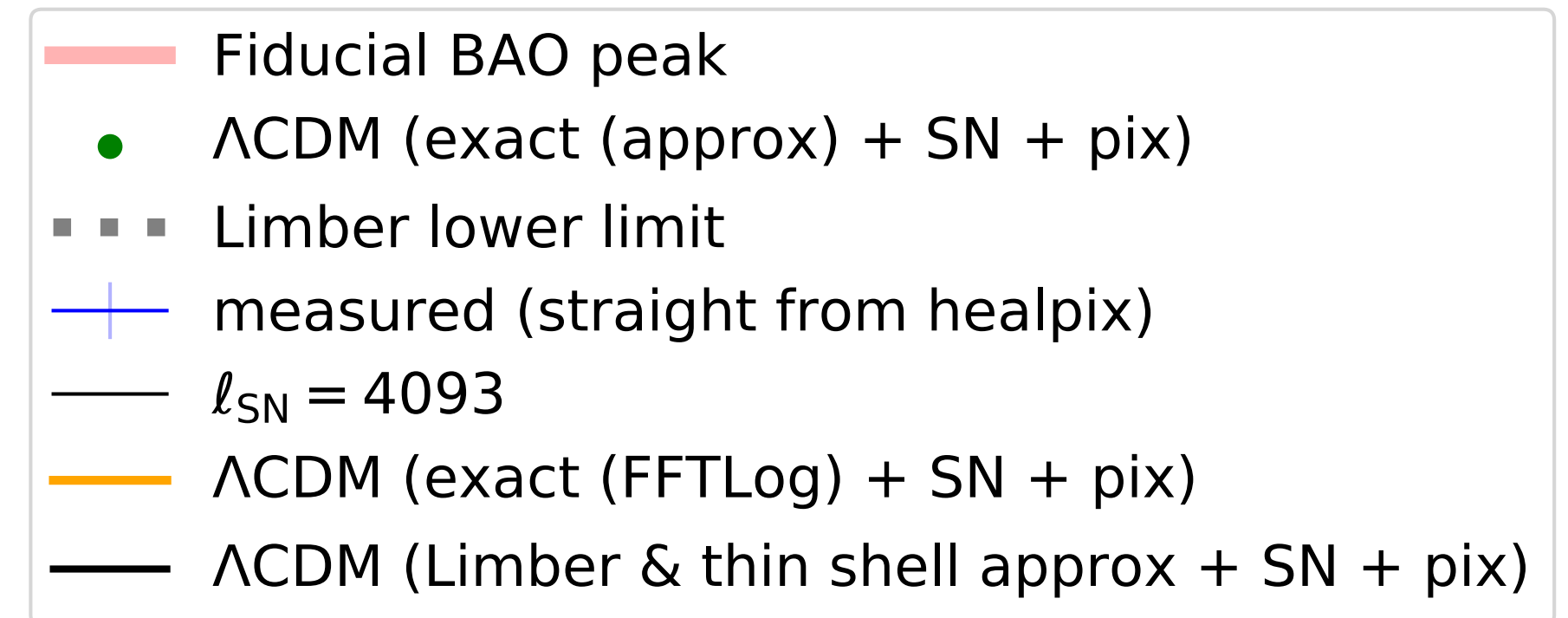


Clustering: simulation

$z=0.0$ - r_{bin_4}



PRE-PRELIMINARY!

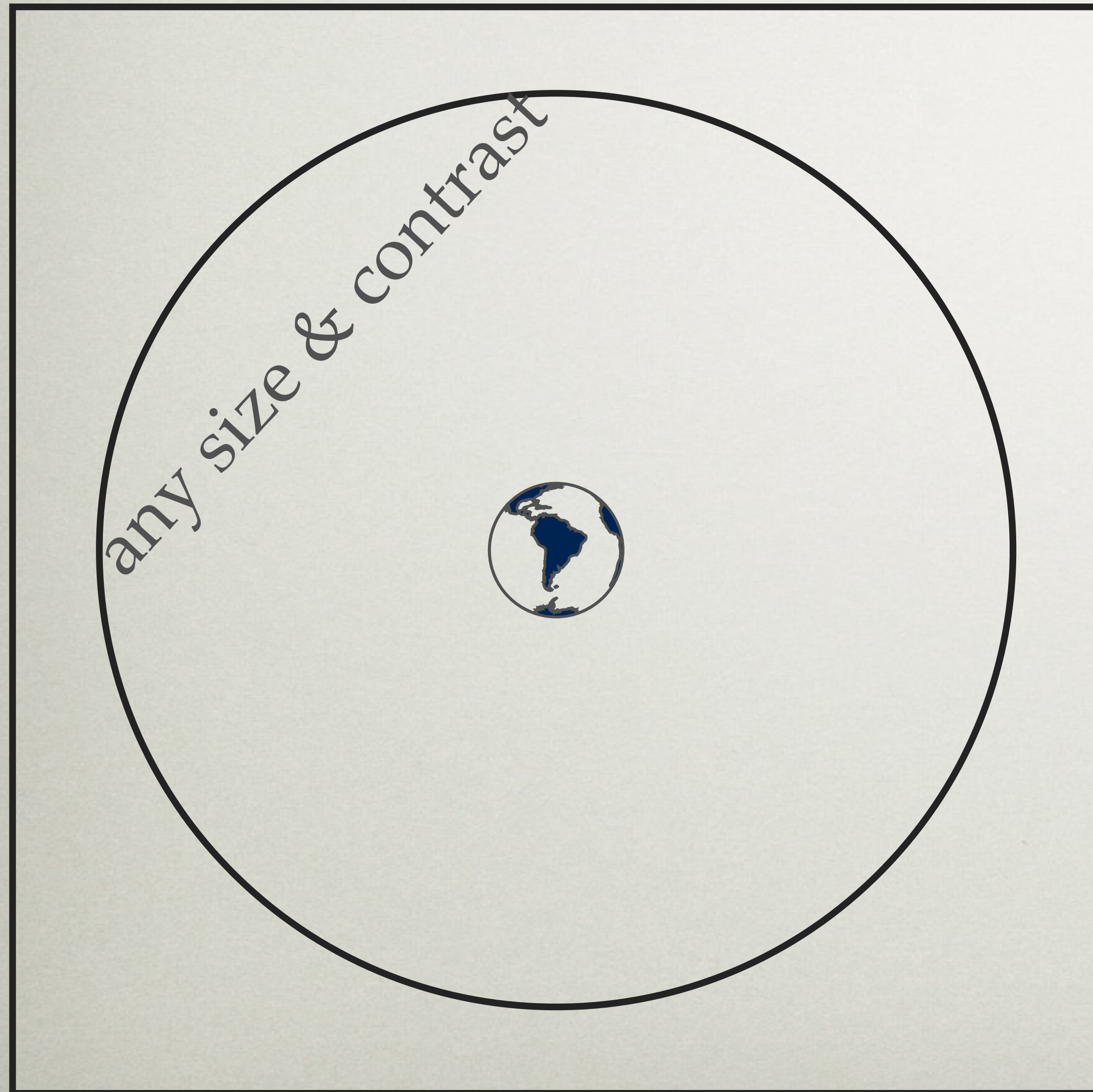


BEHOMO papers

- **paper 1:** *The BEHOMO project: Λ LTB N-body simulations*; Marra et al 2203.04009
- **paper 2:** *The BEHOMO project: linear-scale evolution on a Λ LTB background*
- **paper 3:** 3D PLC analysis
- **paper 4:** nonlinear perturbations, lensing
- **paper 5:** halo abundances, bias (linear and secondary) and clustering
- **paper 6:** halo profile, concentrations (properties), merger trees
- **paper 7+:** ..., dipole/bulk motion (off-center observer), fractal dimensions, genus, Minkowski functionals, ..., **you name it!**

A future perspective on inhomogeneous cosmology

Early-FLRW cosmologies (cont'd)



spacetime inhom: LTB

justification: 

[Λ LTB model: Λ CDM endowed with a spherical over/underdensity]

observer: at the center

justification:

- if nonlinear, observer is constrained near center by CMB
- if linear, observer's observations averaged over angles

Early-FLRW cosmologies (cont'd)



spacetime inhom: LTB

justification:



observer: off-center

missing: off-center ray tracing module

important constraints from CMB



Early-FLRW cosmologies (cont'd)



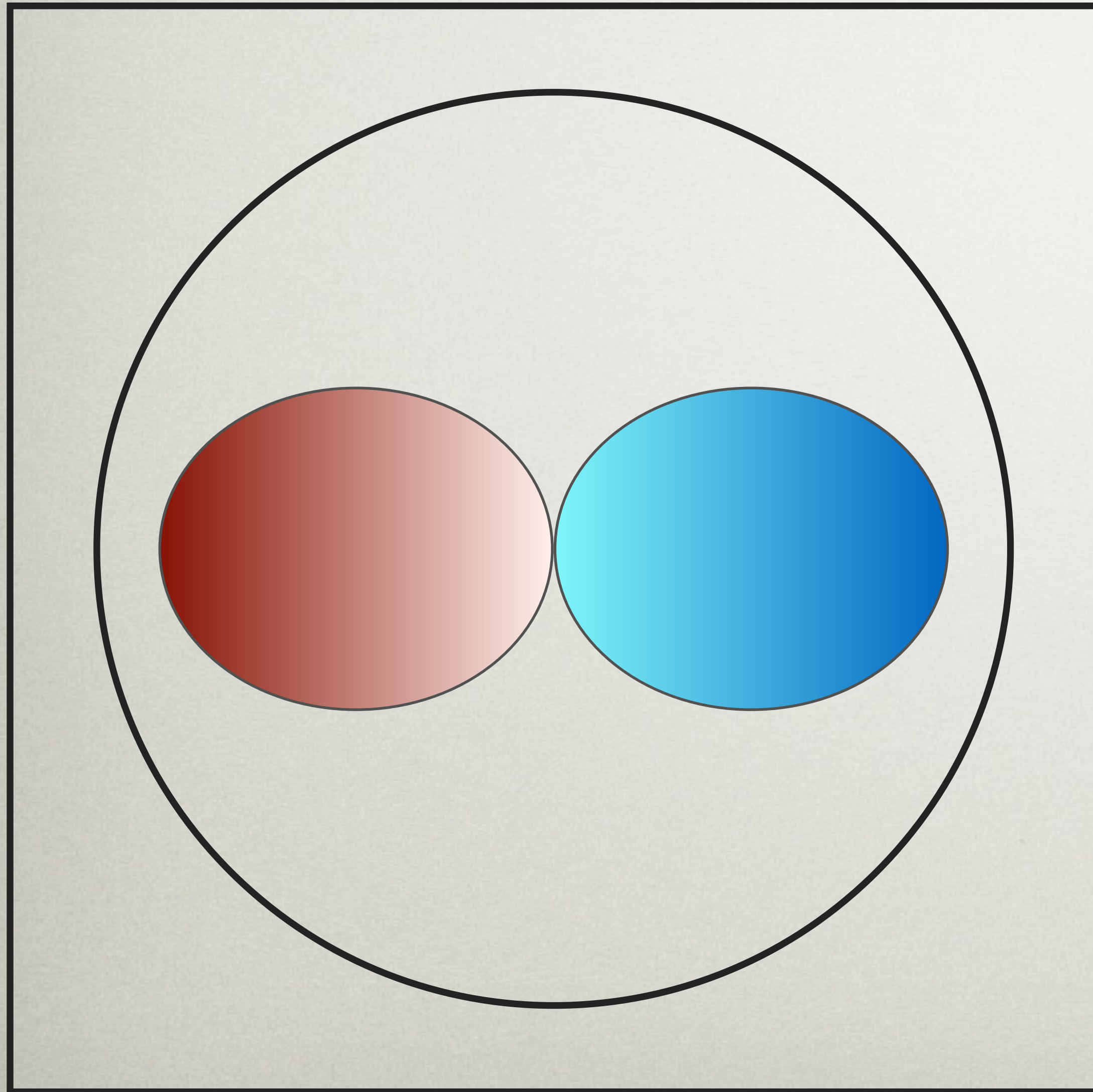
spacetime inhom: LTB

justification: swiss-cheese universe,
bypassing the limitations of the LTB
monotonic spherical profile

construction: trivial thanks to
matching conditions in model and
simulations



Early-FLRW cosmologies (cont'd)



Time to retire the LTB metric?

Szekeres is interesting (dipole inhom), but is it worth the trouble?

Early-FLRW cosmologies (cont'd)

Exact GR solutions are useful for nonlinear departures from Λ CDM. However, nonlinear structures are ruled out at large scales and linear modeling can be tested consistently a posteriori.

The next step could be a perturbative modeling of large-scale deviations from FLRW; Newtonian codes could be tested against GR codes for the background evolution.

thanks!

