Verão Quântico 2021

Black hole dictionary: Parametrized Black Hole Ringdown

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Outline

- Introduction
  - GW detection
  - Black hole spectroscopy
  - Beyond GR
- Perturbation theory in a nutshell
  - Radial equation
  - Parametrized equations
- Results
- Conclusions

Introduction & motivation

- GWs detection;

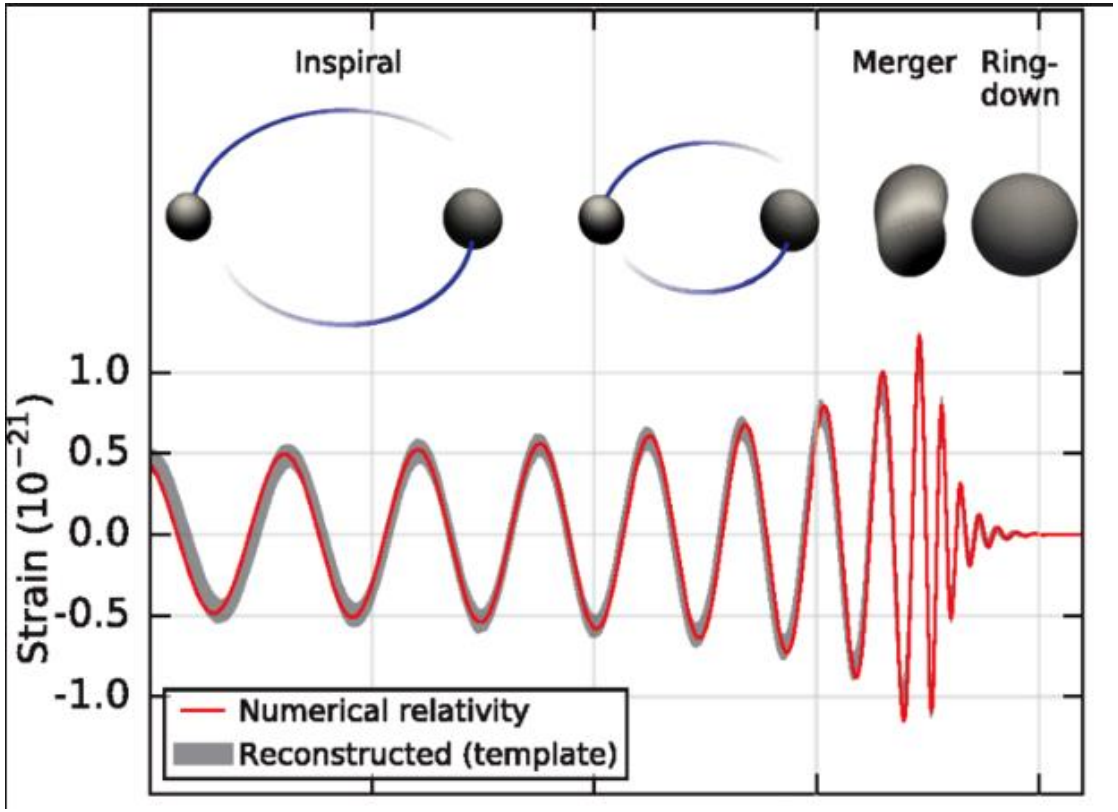
(Abbot *et al*.-VIRGO-LIGO)

- Strong field tests;

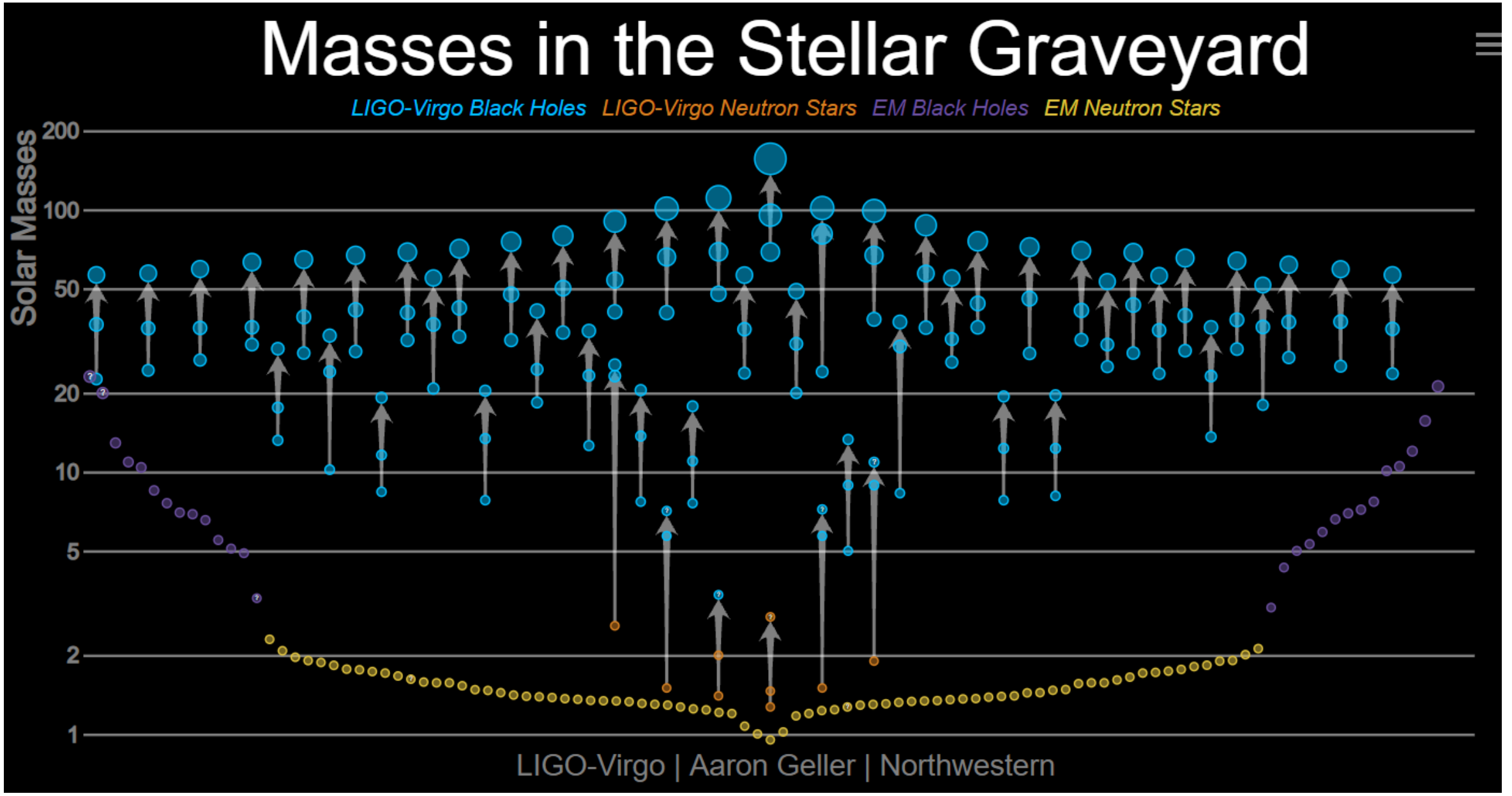
(Berti *et al.,* 2015)

- Different ways to probe gravity;
- Ringdown stage: The final scream;

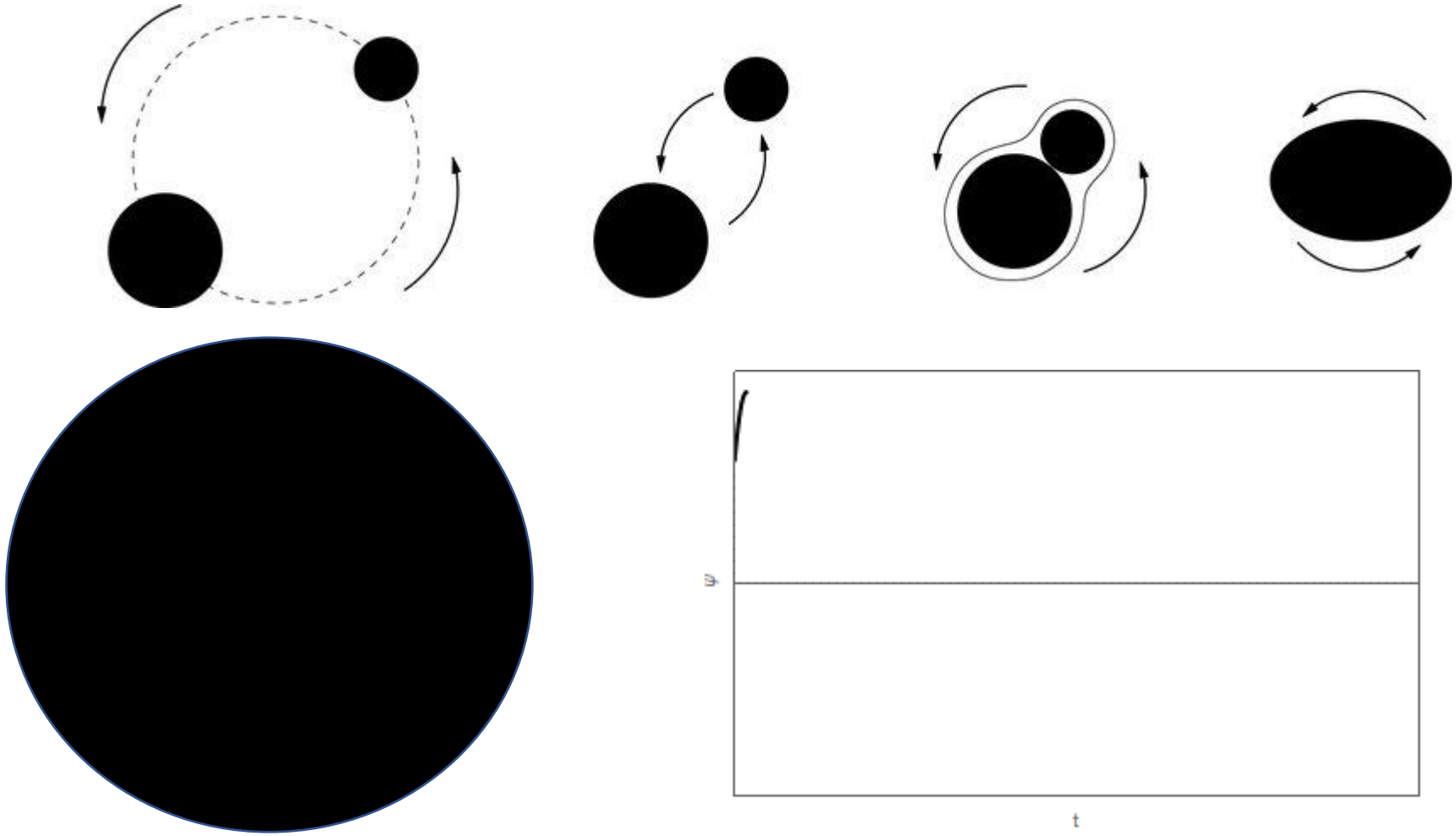
(Berti, Cardoso & Starinets; Kokkotas;…)



Introduction & motivation



Ringdown stage and BH spectroscopy



Theory	Structure			Collapse	Sensitivities	Stability	Geodesics
	NR	SR	FR				
Extra scalar field							
Scalar-Tensor	[26, 114–118]	[116, 119, 120]	[121–123]	[124–131]	[132]	[133–143]	[122, 144]
Multiscalar	?	?	?	?	?	?	?
Metric $f(R)$	[145–157]	[158]	[159]	[160, 161]	?	[162, 163]	?
Quadratic gravity							
Gauss-Bonnet	[164]	[164]	[82]	?	?	?	?
Chern-Simons	$\equiv$ GR	[27, 45, 165–167]	?	?	[166]	?	?
Horndeski	?	?	?	?	?	?	?
Lorentz-violating							
Æ-gravity	[168, 169]	?	?	[170]	[48, 49]	[162]	?
Khronometric/							
Hořava-Lifshitz	[171]	?	?	?	[48, 49]	?	?
n-DBI	?	?	?	?	?	?	?
Massive gravity							
dRGT/Bimetric	[172, 173]	?	?	?	?	?	?
Galileon	[174]	[174]	?	[175, 176]	?	?	?
Nondynamical fields							
Palatini $f(R)$	[177–181]	?	?	?	–	?	?
Eddington-Born-Infeld	[182–188]	[182, 183]	?	[183]	–	[189, 190]	?

How to include beyond GR effects in a generic effective way?

Perturbation theory in a nutshell

(Regge, Wheeler, Zerilli, Moncrief, Teulkosky, Kodama, Ishibashi)

- One starts with small deviations from the background metric

$$g_{\mu\nu} = g^0_{\mu\nu} + \epsilon\, h_{\mu\nu}$$

- Further simplification can be made by separating the angular part (!)
- The main goal: A radial Schrodinger-like equation

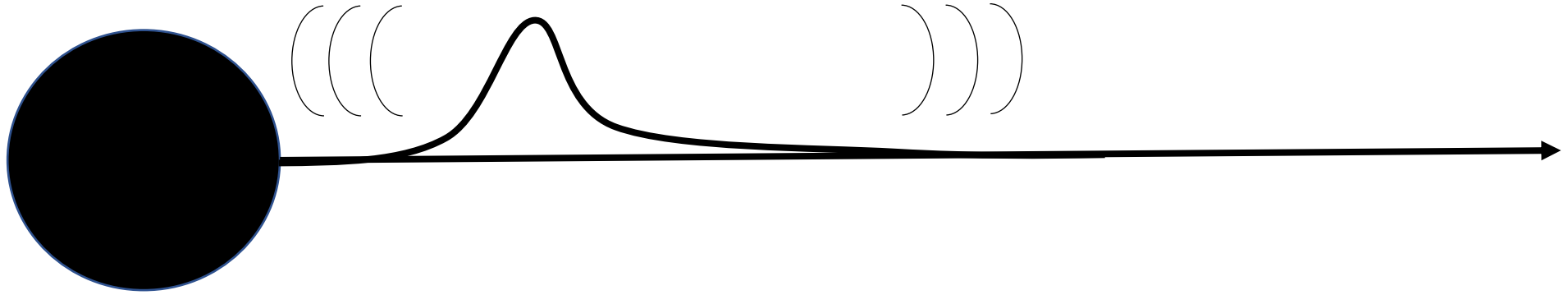
$$f\frac{d}{dr}\left(f\frac{d\Phi}{dr}\right) + [\omega^2 - fV_{\pm}]\Phi = 0$$

$$f = 1 - r_H/r$$

Polar and axial

Computation of QNMs

- QNMs are physically motivated solutions, obtained by solving a BVP.



- Many methods were explored to compute QNMs; (Pani, arXiv:1305.6759)
- Other types of perturbations (scalar and vector) can also be described in the same way.

Parametrized equations: Deviations from GR

- We may consider that the spacetime deviates slightly from Schwarzschild and write the potential as

$$V = V_{\pm} + \delta V_{\pm},$$

$$\delta V_{\pm} = \frac{1}{r_H^2} \sum_{j=0}^{\infty} \alpha_j^{\pm} \left(\frac{r_H}{r} \right)^j$$

$$V = V_s + \delta V_s,$$

$$\delta V_s = \frac{1}{r_H^2} \sum_{j=0}^{\infty} \beta_j^s \left(\frac{r_H}{r} \right)^j$$

Measure deviations from GR.
Depend on the theory.

Finding the radial equation beyond GR is already a laborious task...

Parametrized equations: Deviations from GR

- By assuming that we find the deviations from GR, we can compute the QNM corrections and write the modes as

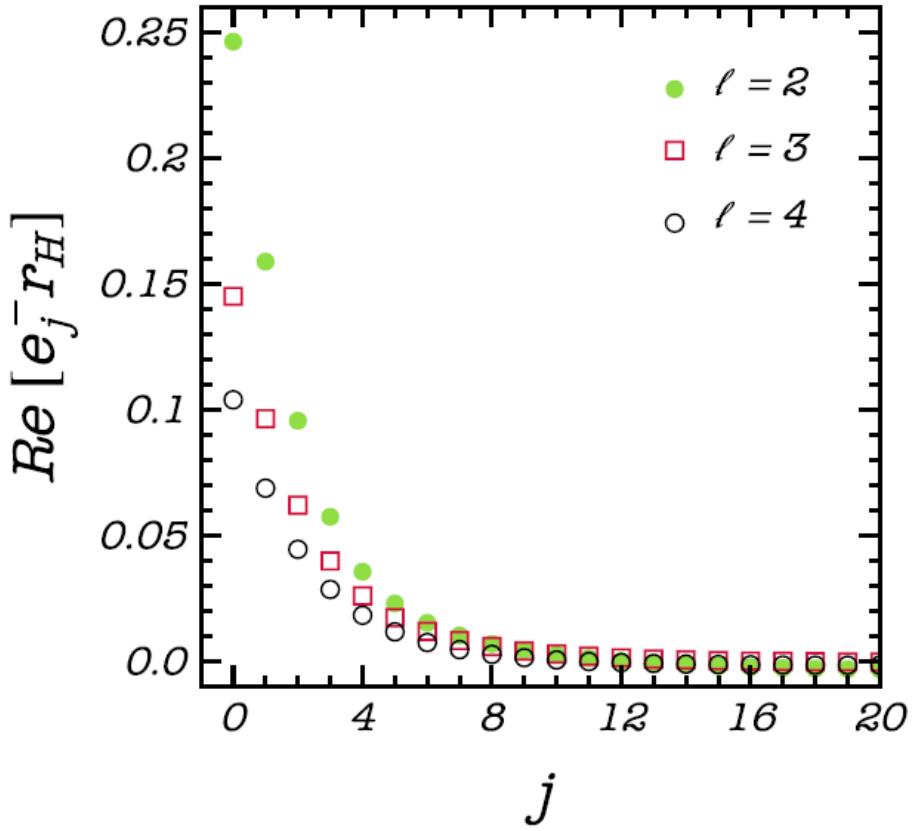
$$\omega^{\pm}_{\rm QNM} = \omega^{\pm}_0 + \sum_{j=0}^{\infty} \alpha^{\pm}_j e^{\pm}_j ,$$

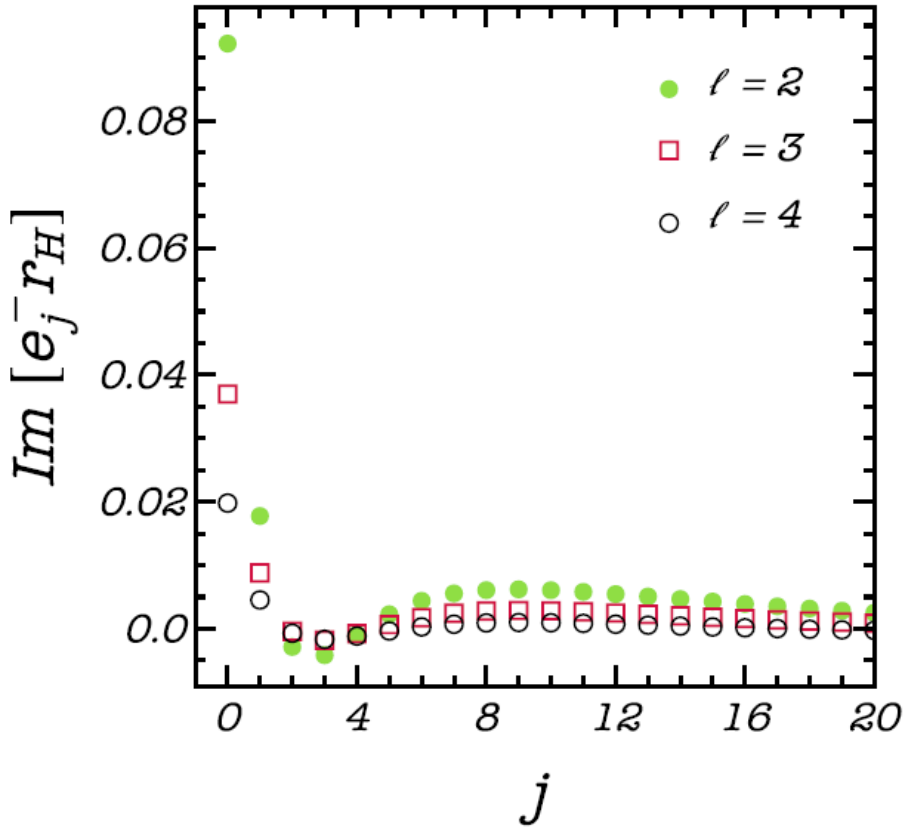
$$\omega^{s}_{\rm QNM} = \omega^{s}_0 + \sum_{j=0}^{\infty} \beta^{s}_j d^{s}_j .$$

Determined numerically. For your convenience, we've computed them and provided data files. See **arXiv:1906.05155** and **arXiv:1901.01265.**

For simplicity, I'll let the discussion of the coupled wave equations to the final slides…

Results. Representative cases





Results. Representative cases

j	$r_H e_j^-$	$r_H d_j^0$
0	$0.24725 + 0.092643\mathrm{i}$	$0.15782 + 0.054078\mathrm{i}$
1	$0.15985 + 0.018208\mathrm{i}$	$0.11307 + 0.015119\mathrm{i}$
2	$0.096632 - 0.0024155\mathrm{i}$	$0.076570 + 0.00016782\mathrm{i}$
3	$0.058491 - 0.0037179\mathrm{i}$	$0.051121 - 0.0032973\mathrm{i}$
4	$0.036679 - 0.00043870\mathrm{i}$	$0.034527 - 0.0024724\mathrm{i}$
10	$0.0036853 + 0.0065244\mathrm{i}$	$0.0050350 + 0.0037363\mathrm{i}$

Results. Isospectrality is a fragile thing

- In GR, axial and polar gravitational perturbations are **isospectral.**

$$fV_{\pm} = W_0^2 \mp f\frac{dW_0}{dr} - \frac{\lambda^2(\lambda+2)^2}{36r_H^2},$$

$$W_0 = \frac{3r_H(r_H-r)}{r^2(3r_H+\lambda r)} - \frac{\lambda(\lambda+2)}{6r_H}.$$

- If we write the modifications, we have

$$2\frac{d\delta W}{dr} = \delta V_- - \delta V_+,$$

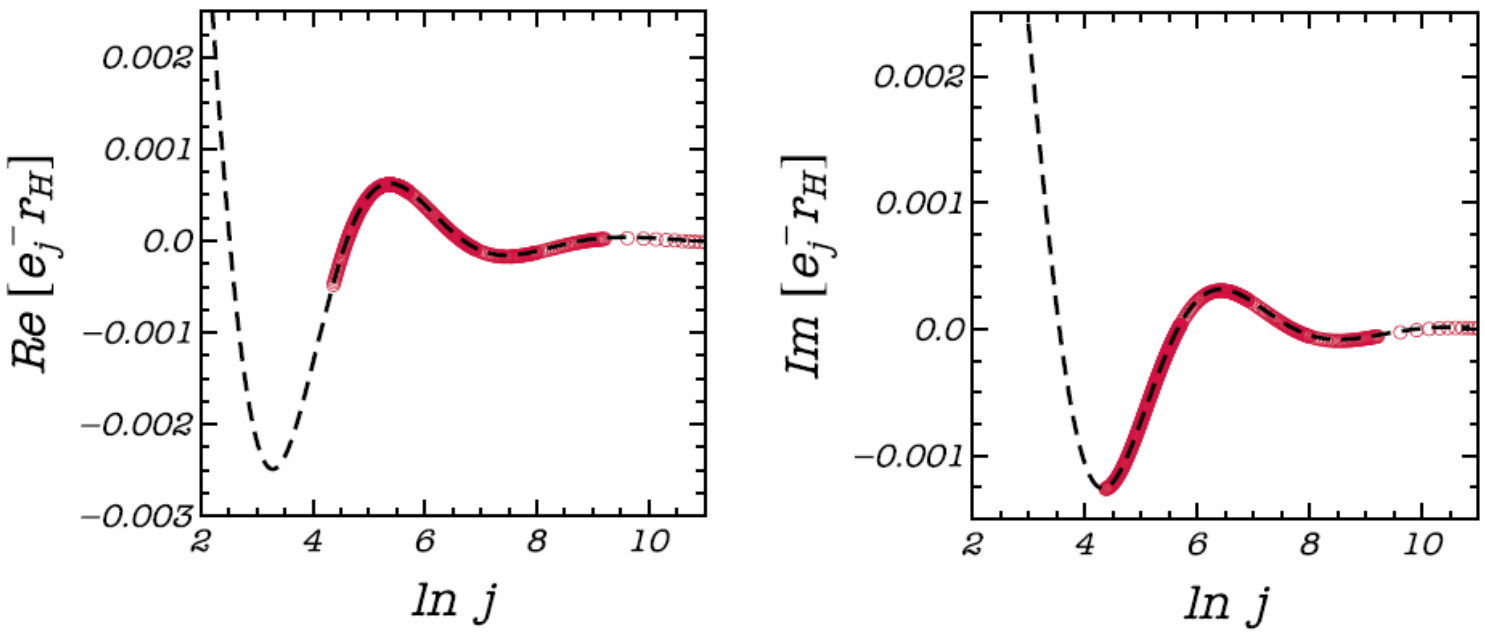
$$4\frac{W_0}{f}\delta W = \delta V_+ + \delta V_-,$$

$$\alpha_0^+ = \alpha_0^-, \qquad \alpha_1^+ = \alpha_1^-,$$

$$\alpha_2^+ = \alpha_2^- + \frac{6(\alpha_0^- - \alpha_1^-)}{\lambda(\lambda+2)}, \ldots.$$

Results. Convergence

$$e_j \sim \frac{\kappa}{j^{\beta}} \sin(\gamma \ln j + \zeta)$$



Results. tests

✓Effective field theory;(Endlich *et al.,* Cardoso *et al.*)

✓Reissner-Nordstron BHs;

Q/M	Δ_R	Δ_I
0.00	0%	0%
0.05	0.11%	0.042%
0.10	0.43%	0.17%
0.20	1.7%	0.66%
0.30	3.8%	1.5%
0.40	6.8%	2.6%
0.50	11%	4.2%

✓Slowly rotating BHs;

Beyond single master equations

Because everything ***gravitates***

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{\kappa} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}\bar{B}^{\mu\nu}B_{\mu\nu} - \frac{1}{2}g^{\mu\nu}\bar{D}_\mu\bar{\Phi}D_\nu\Phi \right.$$
$$\left. -\frac{1}{2}V_\Phi(|\Phi|^2) - \frac{1}{2}V_V(|C|^2)\right].$$

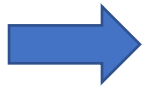
And gravity could couple ***directly*** to something new

$$S = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left[R - \frac{1}{2}\nabla_a\varphi\nabla^a\varphi - V(\varphi) + f(\varphi)\mathcal{G}\right]$$

Beyond single equations: Coupling perturbations

- In general, extensions of GR couple gravity with other fields. This also extends to the perturbations. We have

$$f\frac{d}{dr}\left(f\frac{d\mathbf{\Phi}}{dr}\right)+[\omega^2-f\mathbf{V}]\mathbf{\Phi}=0\,.$$



$$V_{ij}=V_{ij}^{\rm GR}+\delta V_{ij}\,,$$

$$\delta V_{ij}=\frac{1}{r_H^2}\sum_{k=0}^{\infty}\alpha_{ij}^{(k)}\left(\frac{r_H}{r}\right)^k\,.$$

Side note: EdGB

(Blazquez-Salcedo *et al.* 2015)

$$S = \int d^4x \frac{\sqrt{-g}}{16\pi}\left(R - \frac{1}{2}\partial_a\phi\partial^a\phi + \frac{\alpha}{4}e^{\phi}\mathcal{R}^2_{\rm GB}\right)$$

$$K' + \frac{\alpha(B-1)e^{\phi_0}}{r^2\left(\alpha B e^{\phi_0}\phi_0' - r\right)}\phi_1' + \left(\frac{1}{r} - \frac{A'}{2A}\right)K - i\frac{(\Lambda+1)}{r^2\omega}H_1 + \frac{\left(\alpha(3B-1)e^{\phi_0}\phi_0' - 2r\right)}{2r\left(r - \alpha B e^{\phi_0}\phi_0'\right)}H_2$$
$$+ \frac{\left(\alpha(B-1)e^{\phi_0}\left(rA' + A\left(2 - 2r\phi_0'\right)\right) + Ar^3\phi_0'\right)}{2Ar^3\left(r - \alpha B e^{\phi_0}\phi_0'\right)}\phi_1 = \frac{4\sqrt{2}\pi r A^{(1)}_{lm}}{r\omega - \alpha B\omega e^{\phi_0}\phi_0'},$$
$$H_1' + \left(\frac{2(B-1)B\left(rA' - 2A\right)}{Ar} - \frac{r\left(B^2 r\phi_0'^2 + 2(B-1)B'\right)}{r - \alpha B e^{\phi_0}\phi_0'} - 4B'\right)\frac{H_1}{4(B-1)B} + \frac{i\omega}{B}H_2$$
$$- ir\omega\left(\frac{4B'}{B} + \frac{r^2\phi_0'^2}{r - \alpha B e^{\phi_0}\phi_0'}\right)\frac{K}{4(B-1)} - \frac{i\alpha\omega e^{\phi_0}B'}{Br^2 - \alpha B^2 r e^{\phi_0}\phi_0'}\phi_1 = 0,$$
$$H_0' + \frac{r\left(\alpha B e^{\phi_0}A'\phi_0' - 2A\right)}{2A\left(r - \alpha B e^{\phi_0}\phi_0'\right)}K' + \left(\frac{2\left(A - rA'\right)}{r - \alpha B e^{\phi_0}\phi_0'} + 3A'\right)\frac{H_2}{2A} + \frac{\alpha B e^{\phi_0}A'\left(r\phi_0' - 2\right) - Ar^2\phi_0'}{Ar^2\left(r - \alpha B e^{\phi_0}\phi_0'\right)}\phi_1$$
$$+ \frac{\alpha B e^{\phi_0}A'}{Ar^2 - \alpha ABre^{\phi_0}\phi_0'}\phi_1' + \left(\frac{A'}{2A} - \frac{1}{r}\right)H_0 + \frac{i\omega}{A}H_1 = 0,$$
$$H_2\left(2A - 3\alpha B^2 e^{\phi_0}A'\phi_0'\right) - BrK'\left(A'\left(r - 3\alpha B e^{\phi_0}\phi_0'\right) + 2A\right) + K\left(\alpha B e^{\phi_0}\phi_0'\left(2r\omega^2 - \Lambda A'\right) + 2A\Lambda - 2r^2\omega^2\right)$$
$$+ \frac{\phi_1\left(\alpha e^{\phi_0}\left(B\left(2r\omega^2 - A'\left((1-3B)r\phi_0' + 3B + 2\Lambda + 1\right)\right) - 2r\omega^2\right) - ABr^2\phi_0'\right)}{r^2}$$
$$+ \phi_1'\left(\frac{\alpha B(3B-1)e^{\phi_0}A'}{r} + ABr\phi_0'\right) - \frac{2AH_0(\Lambda+1)\left(r - \alpha B e^{\phi_0}\phi_0'\right)}{r} + ABH_0'\left(2r - \alpha(3B-1)e^{\phi_0}\phi_0'\right)$$
$$+ 2iBH_1\omega\left(2r - \alpha(3B-1)e^{\phi_0}\phi_0'\right) = -16\pi ABr^2 A_{lm},$$

$$H_2 - \frac{AH_0\left(4B'\left(r - \alpha B e^{\phi_0}\phi_0'\right) + Br^2\phi_0'^2\right)}{2(B-1)\left(\alpha B e^{\phi_0}A'\phi_0' - 2A\right)} + \frac{\alpha e^{\phi_0}\phi_1\left(AA'B' - B\left((A')^2 - 2AA''\right)\right)}{Ar\left(\alpha B e^{\phi_0}A'\phi_0' - 2A\right)} = 0,$$
$$\frac{H_2\left(4\alpha B e^{\phi_0}\phi_0'\left((B-1)(\Lambda+1) - rB'\right) + 4r\left(B\left(rB' + B - \Lambda - 3\right) + \Lambda + 2\right) + B^2r^3\phi_0'^2\right)}{4(B-1)Br^2\left(r - \alpha B e^{\phi_0}\phi_0'\right)}$$
$$+ \frac{K'\left(\frac{r\left(B^2(-r)\phi_0'^2 - 2(B-1)B'\right)}{r - \alpha B e^{\phi_0}\phi_0'} - 4B' + \frac{8(B-1)B}{r}\right)}{4(B-1)B} + \frac{\Lambda K\left(\frac{4B'}{B} + \frac{r^2\phi_0'^2}{r - \alpha B e^{\phi_0}\phi_0'}\right)}{4(B-1)r}$$
$$+ \frac{\phi_1'\left(Br^3\phi_0' - \alpha e^{\phi_0}\left(B\left(3rB' + 4(B-1)r\phi_0' - 4B + 4\right) - rB'\right)\right)}{2Br^3\left(r - \alpha B e^{\phi_0}\phi_0'\right)}$$
$$+ \frac{\phi_1\left(2\alpha e^{\phi_0}\left((2\Lambda+1)rB' + B\left(3rB' + 4(B-1)r\phi_0' - 4B + 4\right)\right) - r^2\left(4rB' + B\left(r\phi_0'\left(r\phi_0' + 2\right) + 4\right) - 4\right)\right)}{4Br^4\left(r - \alpha B e^{\phi_0}\phi_0'\right)}$$
$$+ \frac{H_2'\left(\alpha(3B-1)e^{\phi_0}\phi_0' - 2r\right)}{2r\left(r - \alpha B e^{\phi_0}\phi_0'\right)} + \frac{\alpha(B-1)e^{\phi_0}\phi_1''}{r^2\left(\alpha B e^{\phi_0}\phi_0' - r\right)} + K'' = -\frac{8\pi r A^{(0)}_{lm}}{ABr - \alpha AB^2 e^{\phi_0}\phi_0'},$$

$$\frac{d^2\phi_1}{dr_*^2} + \left(\frac{BrA'\left(r\phi_0' + 1\right) + A\left(rB'\left(r\phi_0' + 1\right) + 2Br\left(r\phi_0'' + 2\phi_0'\right) + 4(\Lambda+1)\right)}{2r^2} - \omega^2\right)\phi_1 + \alpha B^2 e^{\phi_0}A'K''$$
$$- \frac{\alpha A(B-1)Be^{\phi_0}H_0''}{r} - \frac{H_0'\left(\alpha e^{\phi_0}\left(2(B-1)BA' + A(3B-1)B'\right) + ABr^2\phi_0'\right)}{2r} - \frac{BH_2'\left(\alpha(3B-1)e^{\phi_0}A' + Ar^2\phi_0'\right)}{2r}$$
$$+ \alpha H_2 e^{\phi_0}\left(B^2r\left(A'\right)^2 + A\left(-B\left(2BrA'' + A'\left(3rB' + 2\Lambda + 2\right) - 2r\omega^2\right) - 2r\omega^2\right)\right) + \frac{\alpha AH_0(\Lambda+1)e^{\phi_0}B'}{r^2}$$
$$+ \frac{BK'\left(\alpha e^{\phi_0}\left(A\left(2BrA'' + A'\left(3rB' + 4B\right)\right) - Br\left(A'\right)^2\right) + 2A^2r^2\phi_0'\right)}{2Ar} - iH_1\left(\frac{\alpha(3B-1)\omega e^{\phi_0}B'}{r} + Br\omega\phi_0'\right)$$
$$+ \frac{\alpha Ke^{\phi_0}\left(AB'\left(2r\omega^2 - \Lambda A'\right) + B\Lambda\left(\left(A'\right)^2 - 2AA''\right)\right)}{2Ar} - \frac{2i\alpha(B-1)B\omega e^{\phi_0}H_1'}{r} = 0. \qquad (1$$

Beyond single equations: Coupling perturbations

- Likewise, the decoupled equations, we have that the modes can be written as

$$\omega \approx \omega_0 + \alpha_{ij}^{(k)} d_{(k)}^{ij} + \alpha_{ij}^{(k)} \alpha'^{(s)}_{pq} d_{(k)}^{ij} d_{(s)}^{pq} + \frac{1}{2} \alpha_{ij}^{(k)} \alpha_{pq}^{(s)} e_{(ks)}^{ijpq}$$

Example: dCS gravity

(Molina *et al.* (2010))

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\Big\{R + \frac{\alpha}{4}\varphi\,{}^*RR - \frac{1}{2}[g^{ab}\nabla_a\varphi\nabla_b\varphi + V_s(\varphi)]\Big\} + S_{\rm mat},$$

The spacetime is Schwarzschild

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2$$

And the perturbations can be written in terms of a pair of equations

$$V_{11} = V_-\,,$$

$$V_{12} = V_{21} = \frac{1}{r_H^2}\frac{12}{\sqrt{\beta}r_H^2}\sqrt{\pi\frac{(\ell+2)!}{(\ell-2)!}}\left(\frac{r_H}{r}\right)^5\,,$$

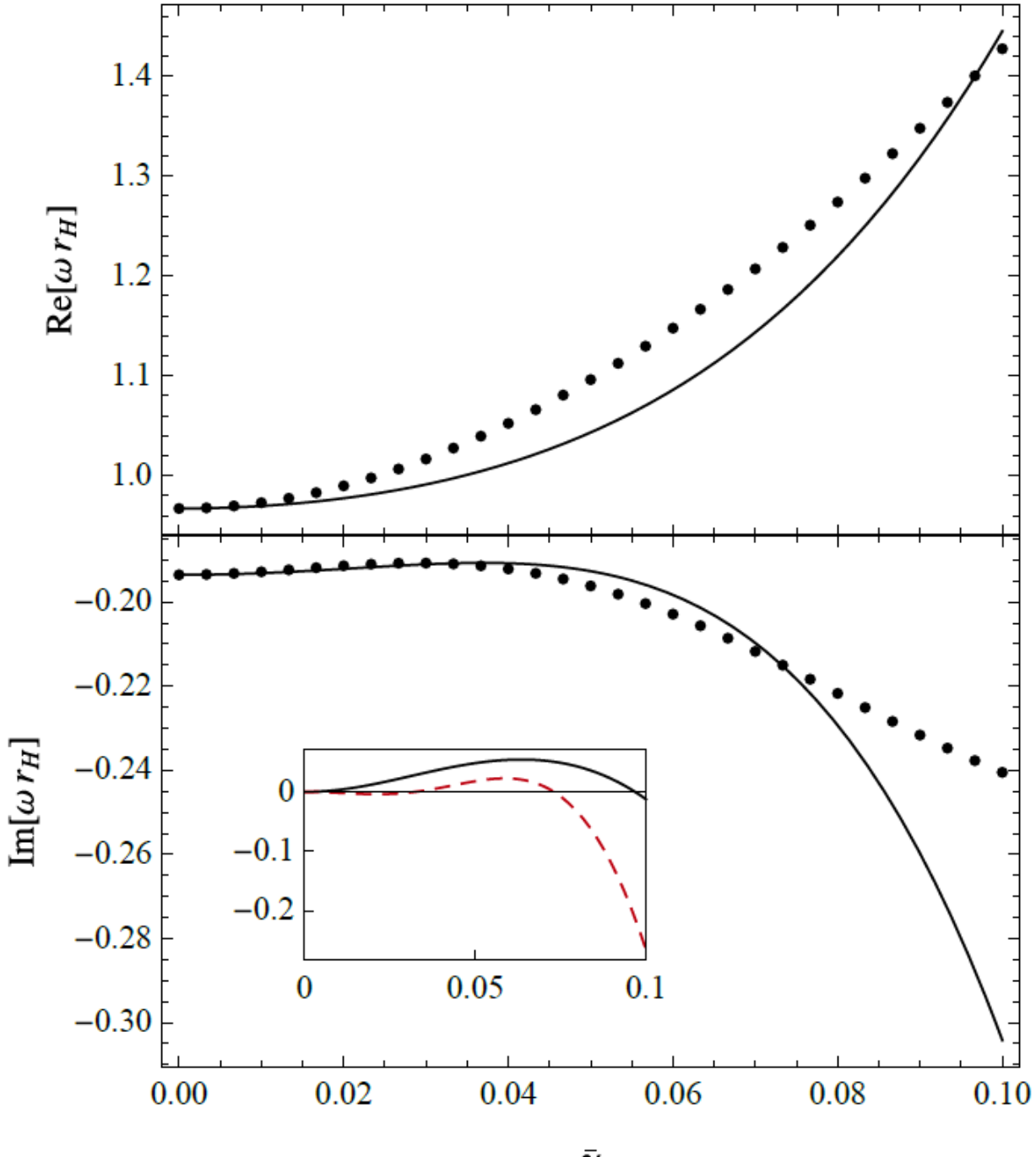
$$V_{22} = V_{s=0} + \frac{1}{r_H^2}\frac{144\pi\ell(\ell+1)}{\beta r_H^4}\left(\frac{r_H}{r}\right)^8\,.$$

Example: dCS gravity

$$V_{11} = V_{-}\,,$$

$$V_{12} = V_{21} = \frac{1}{r_H^2}\frac{12}{\sqrt{\beta} r_H^2}\sqrt{\pi\frac{(\ell+2)!}{(\ell-2)!}}\left(\frac{r_H}{r}\right)^5\,,$$

$$V_{22} = V_{s=0} + \frac{1}{r_H^2}\frac{144\pi\ell(\ell+1)}{\beta r_H^4}\left(\frac{r_H}{r}\right)^8\,.$$



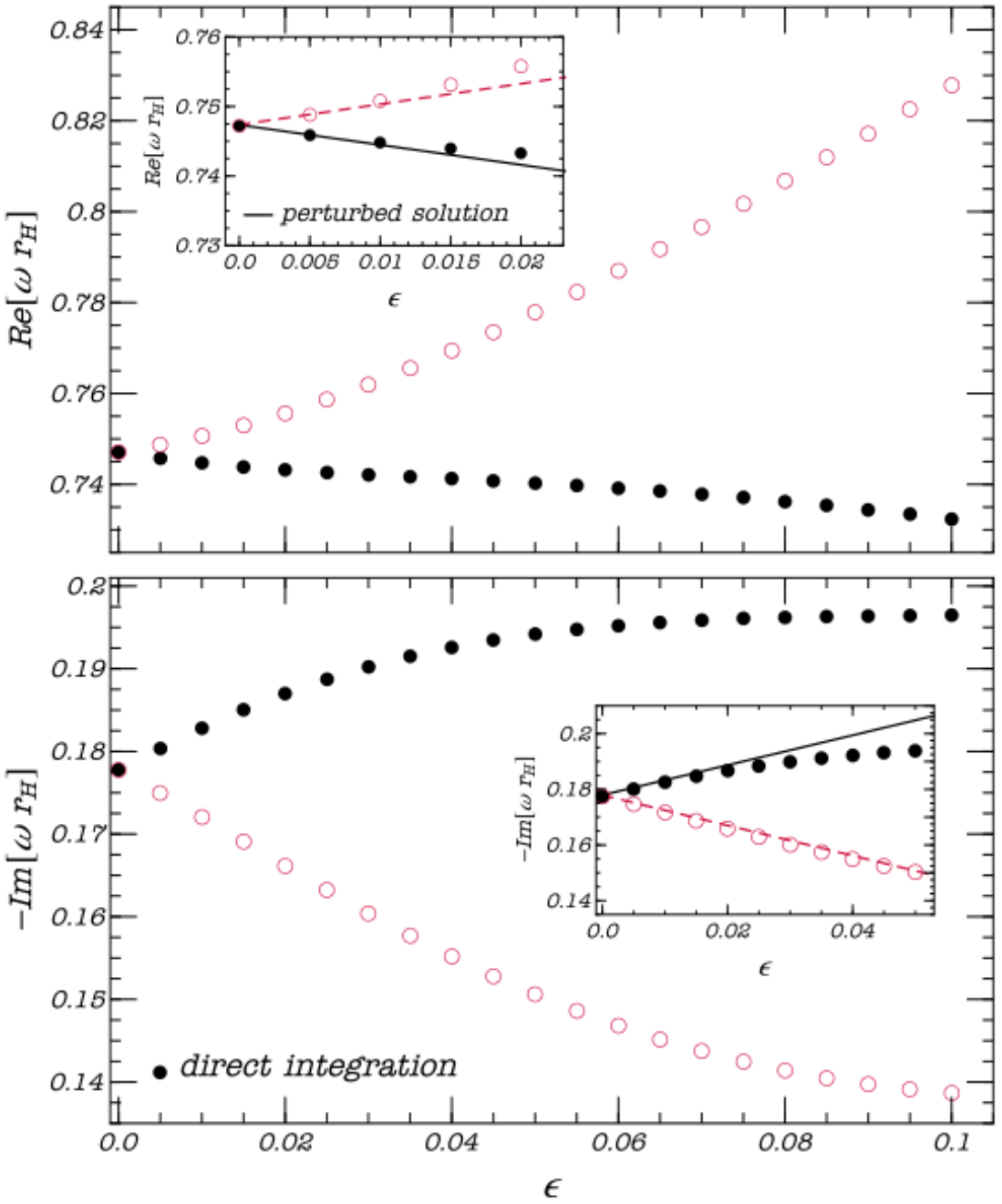
Example: EFT

(Cardoso *et al.* 2018 | Endlich et. *al* (2017))

$$S_{\rm eff} = \int d^4x\sqrt{-g}\,2M_{\rm pl}^2\left(R - \frac{\mathcal{C}^2}{\Lambda^6} - \frac{\tilde{\mathcal{C}}^2}{\tilde{\Lambda}^6} - \frac{\tilde{\mathcal{C}}\mathcal{C}}{\Lambda_-^6}\right)$$

$$\mathcal{C} \equiv R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}\,, \quad \tilde{\mathcal{C}} \equiv R_{\alpha\beta\gamma\delta}\tilde{R}^{\alpha\beta\gamma\delta}\,,$$

$$V_{11} = V_+\,,$$
$$V_{22} = V_-\,,$$
$$V_{12} = V_{21} = \epsilon V(r)\,.$$



Conclusions

- We presented a parametric expansion to compute QNMs.

- The recipe:
  - Find master eqs -> eliminate first order terms -> identify coefficients -> use the formulas (data available).

- Limitations:
  - Additional modes, quasibound states, rotation…
  - It works only for parametric *small deviations of Schwarzschild perturbations*.

Obrigado!