



DESIGNING concordant cosmological distances in the age of precision cosmology

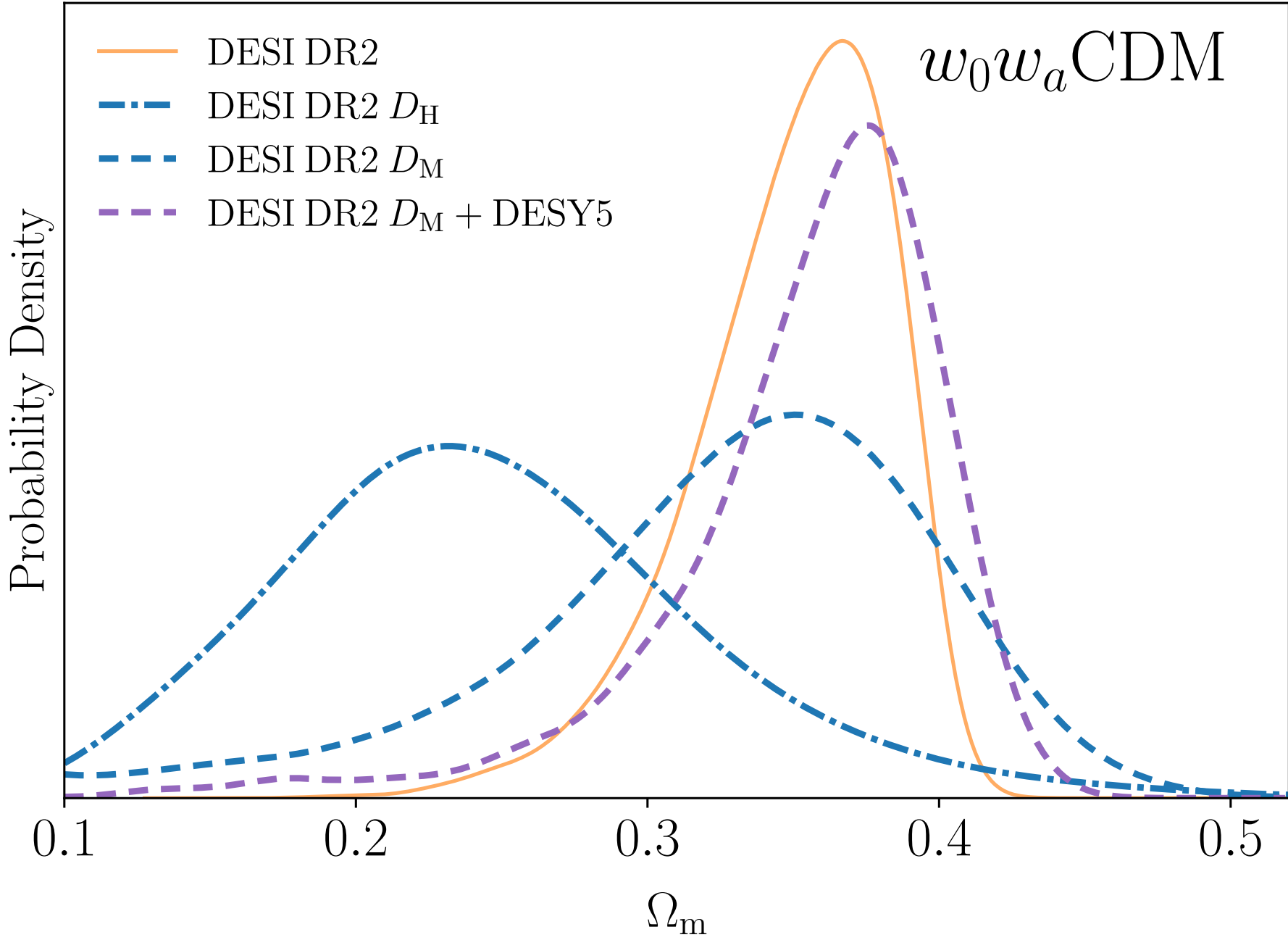
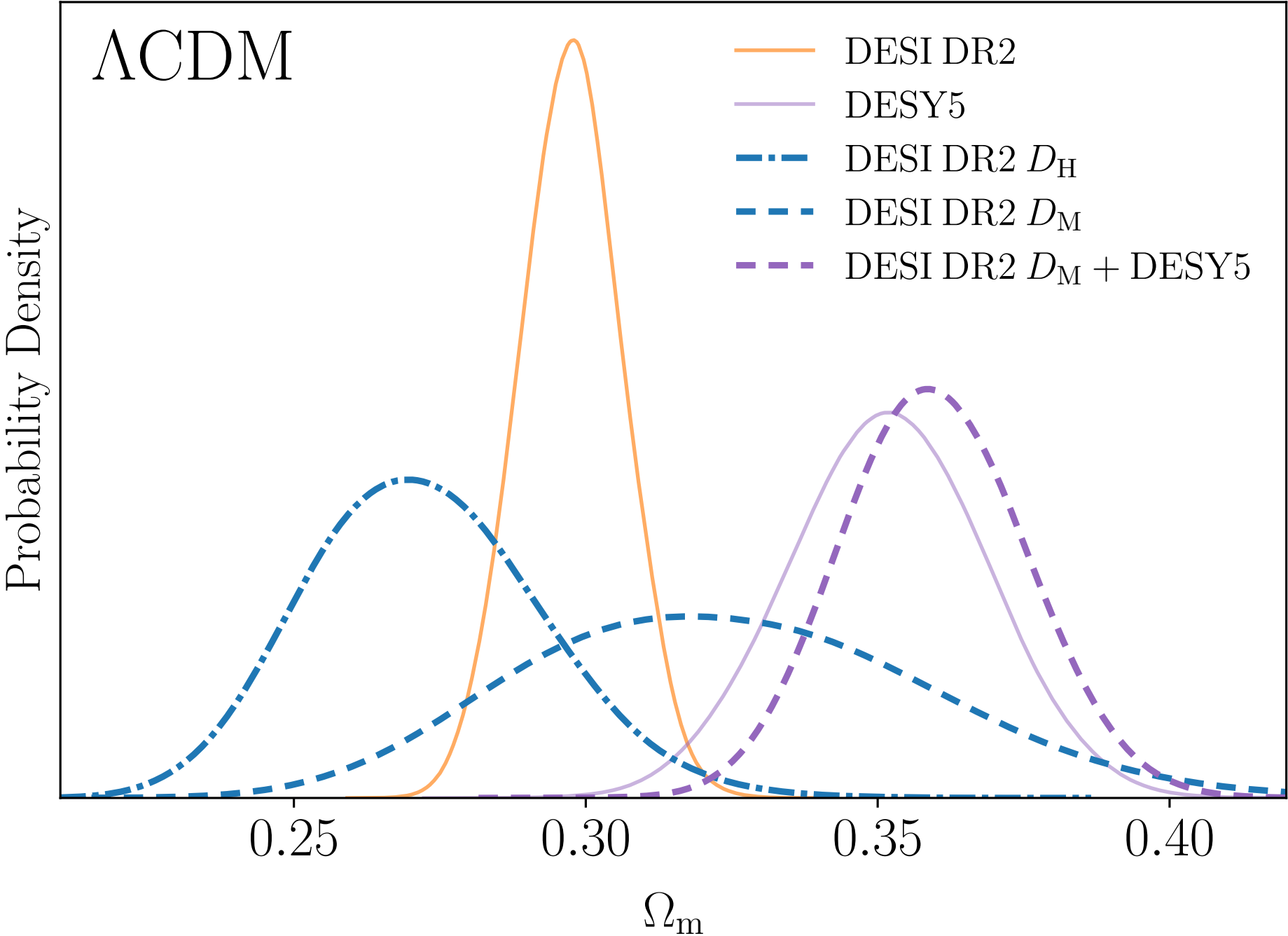
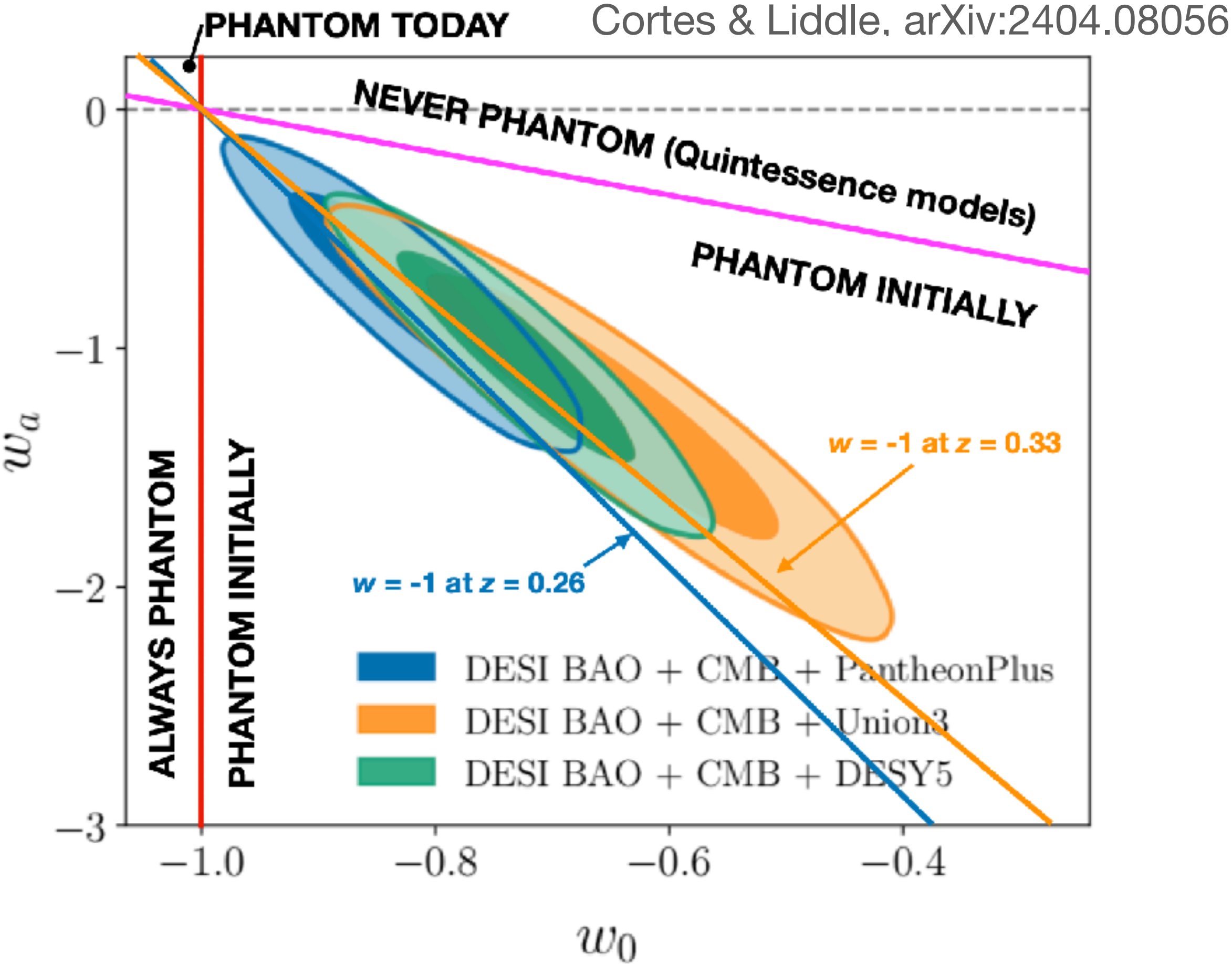
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August 15, 2025

An early-hint for anisotropic expansion



Summary

1. *Inhomogeneous cosmology: the Λ LTB model*
2. *Dynamic dark energy vs inhomogeneities*
3. *Profile likelihood analysis*
4. *Impact on the Hubble tension*
5. *Are inhomogeneous cosmologies a viable option?*
6. *Conclusions*

Inhomogeneous solutions: the LTB metric

We are merely typical observers:

FLRW metric:

$$ds^2 = - dt^2 + \frac{a^2(t)}{1 - kr^2} dr^2 + a^2(t) r^2 d\Omega$$

Inhomogeneous solutions: the LTB metric

‘Inhomo. space-time solutions are not alternatives but extensions to the FLRW paradigm’

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Extra d.o.f

+ radial inhomogeneities



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LTB metric:

$$ds^2 = -dt^2 + \frac{a_{||}^2(r, t)}{1 - K(r)r^2} dr^2 + a_{\perp}^2(r, t)r^2 d\Omega$$

FLRW limit: $K(r) \rightarrow \text{const.}$ and $R(r, t) \rightarrow a(t)r$

Three arbitrary functions: $m(r)$, $t_{BB}(r)$, and $K(r)$

Inhomogeneous solutions: a tale about void models

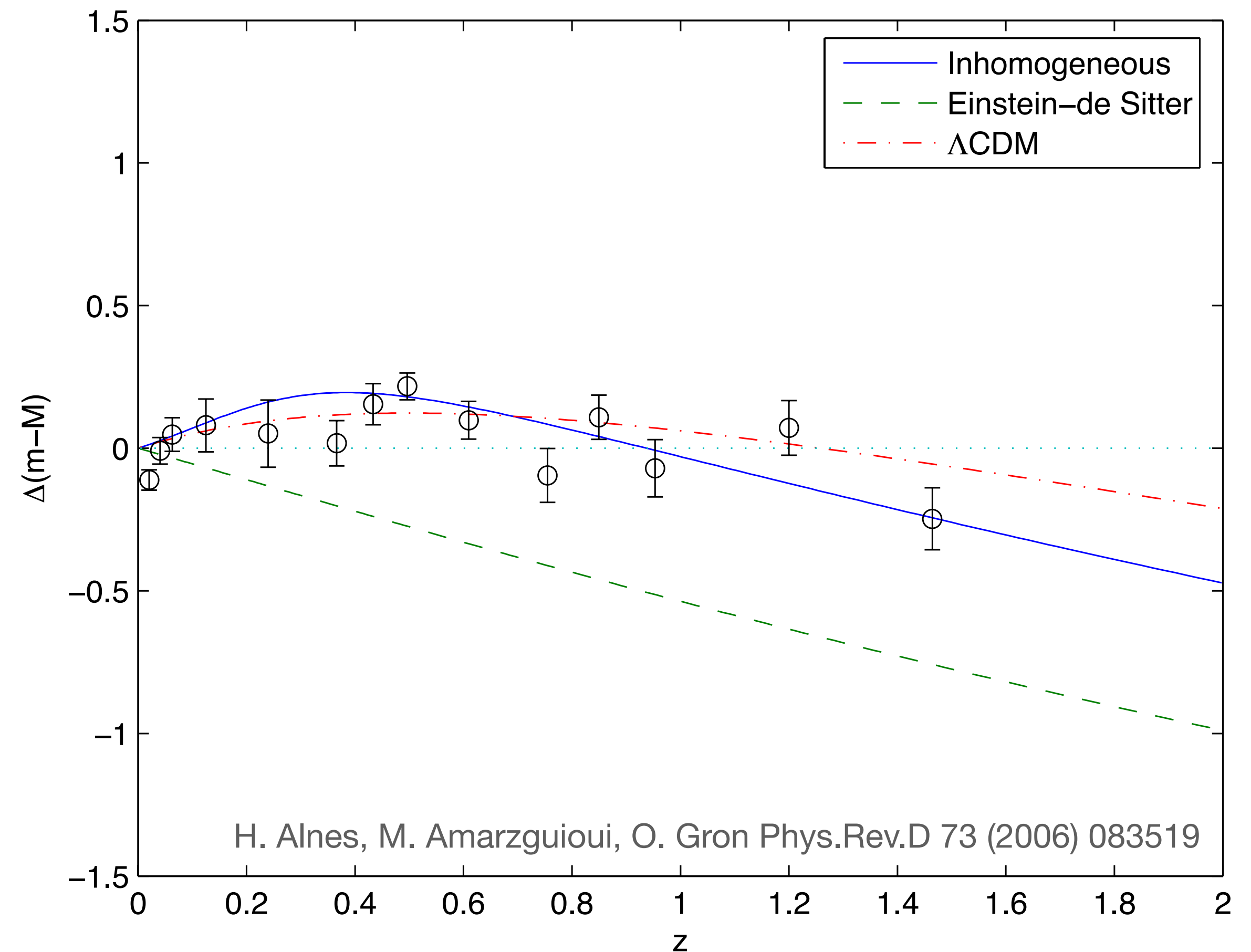
The directional derivative along the past lightcone:

$$\frac{d}{dt} \approx \frac{\partial}{\partial t} - \frac{\partial}{\partial r}$$

The acceleration of the Universe explained by:

$$\frac{\partial H}{\partial r} < 0$$

Apparent acceleration!



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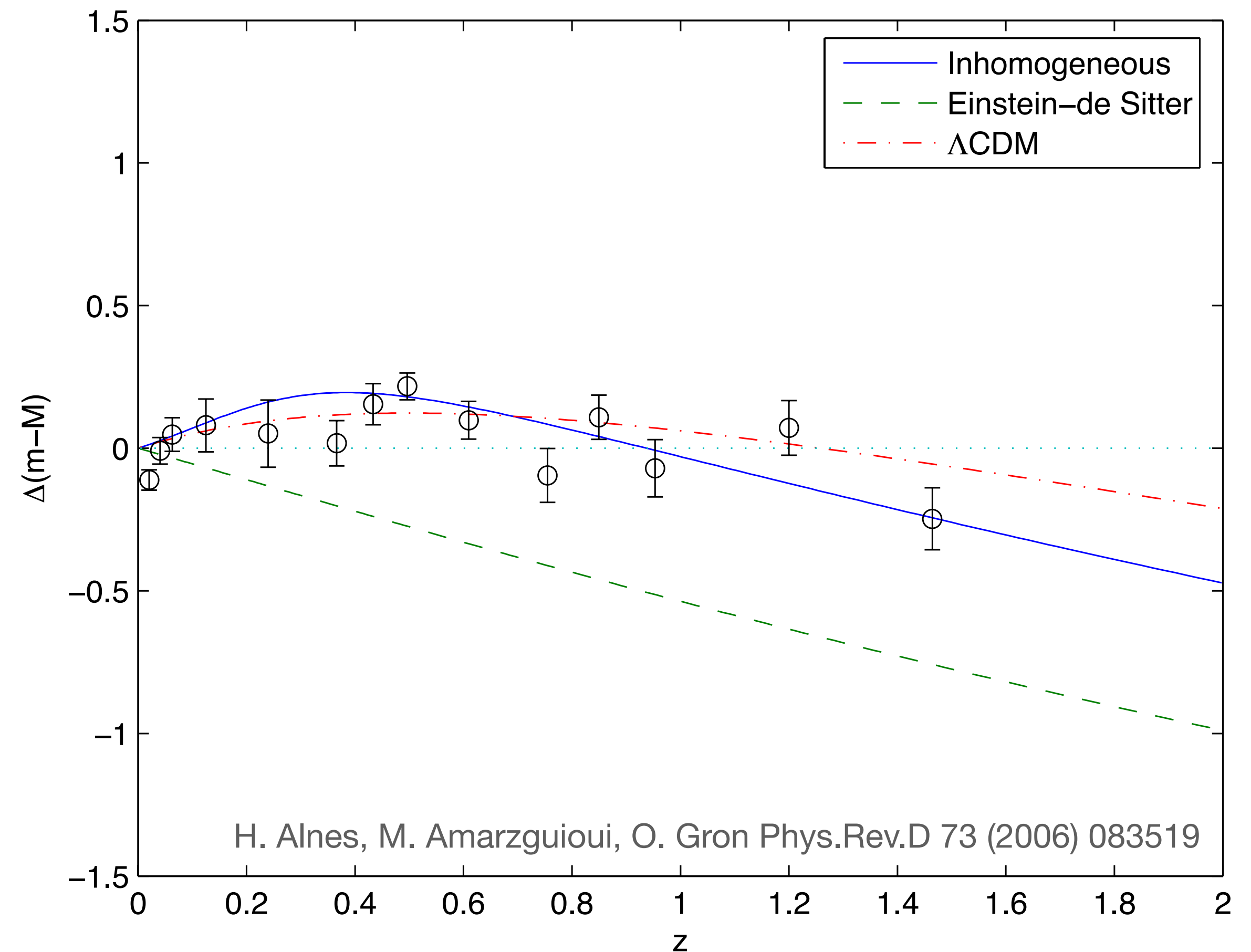
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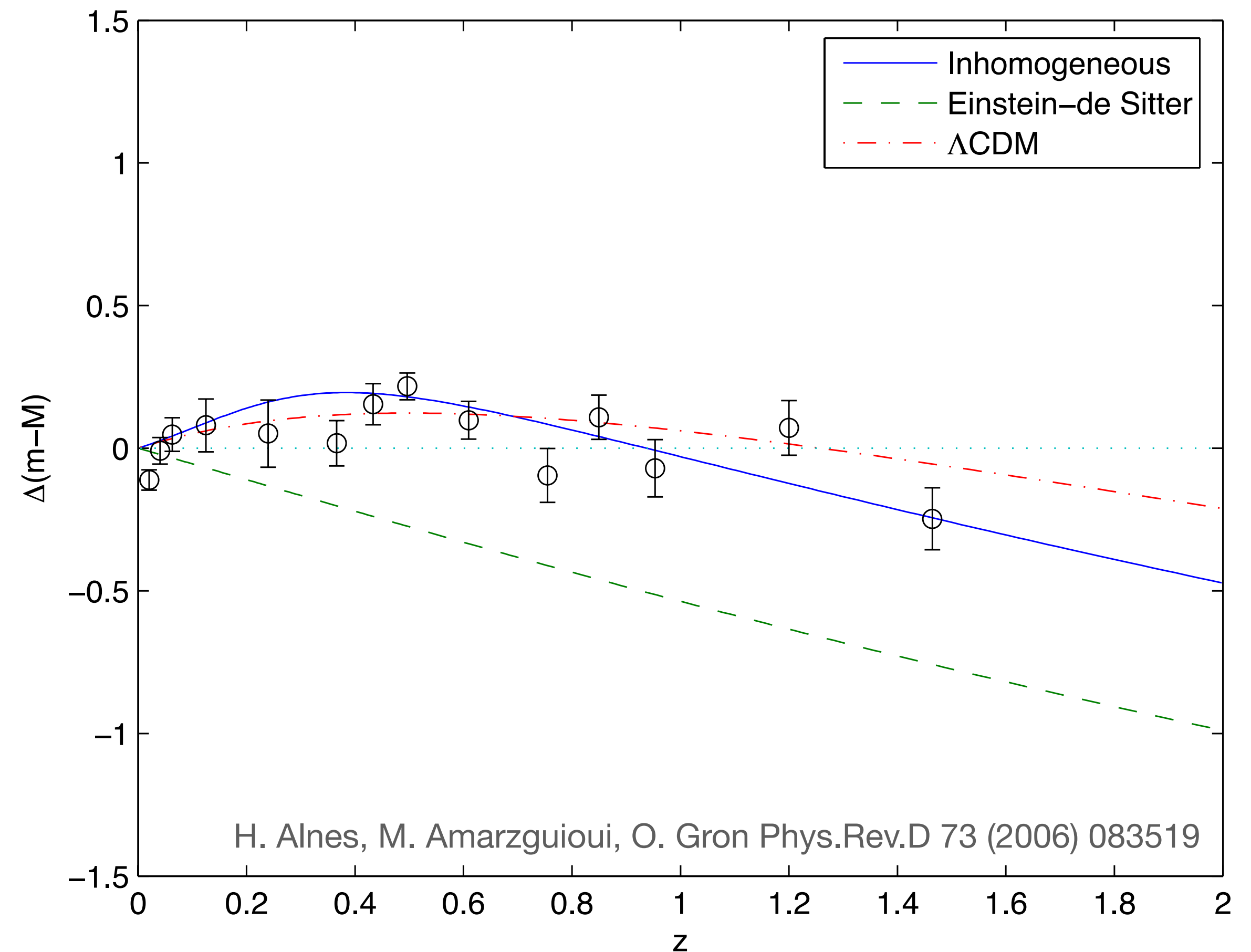
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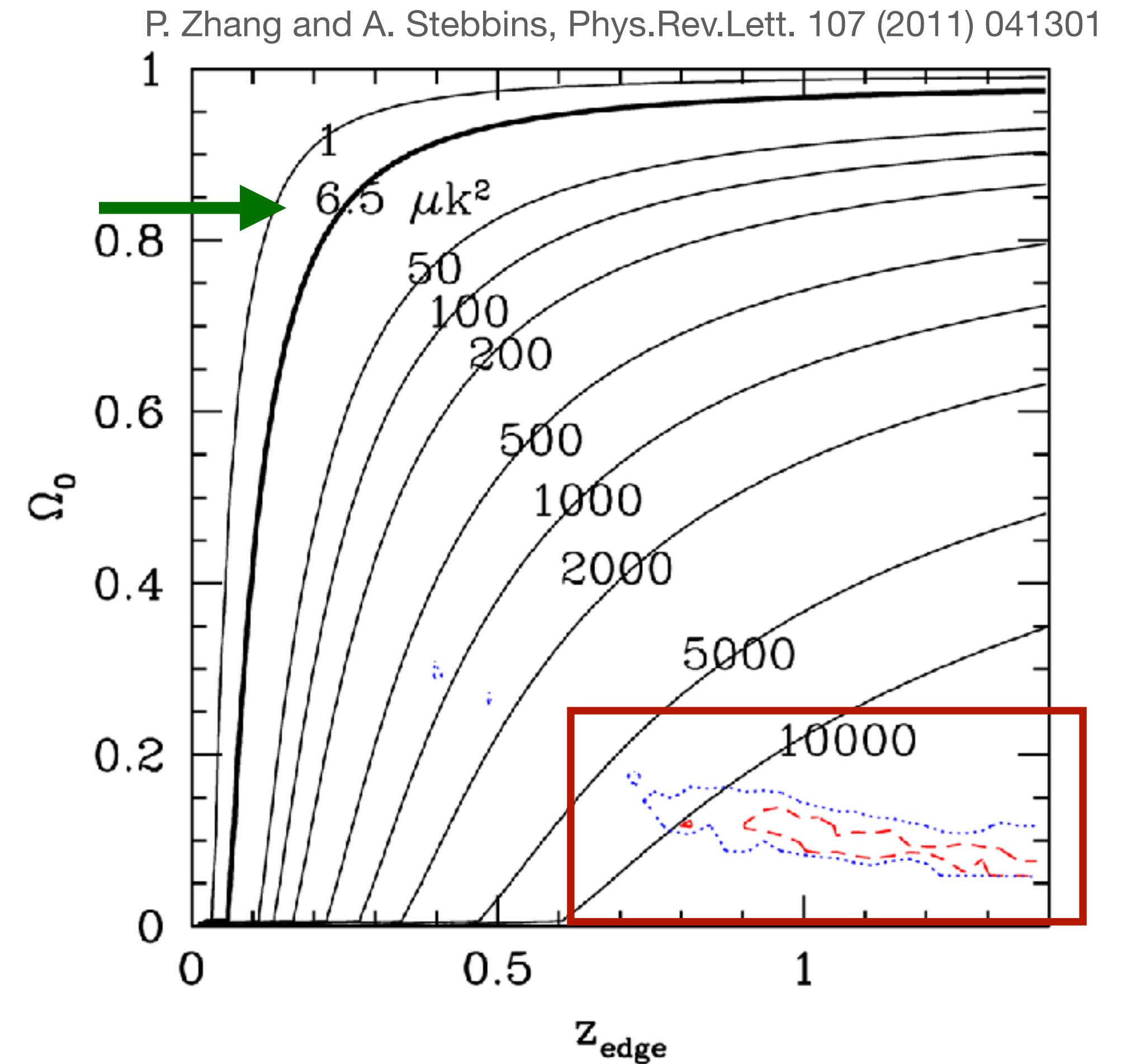
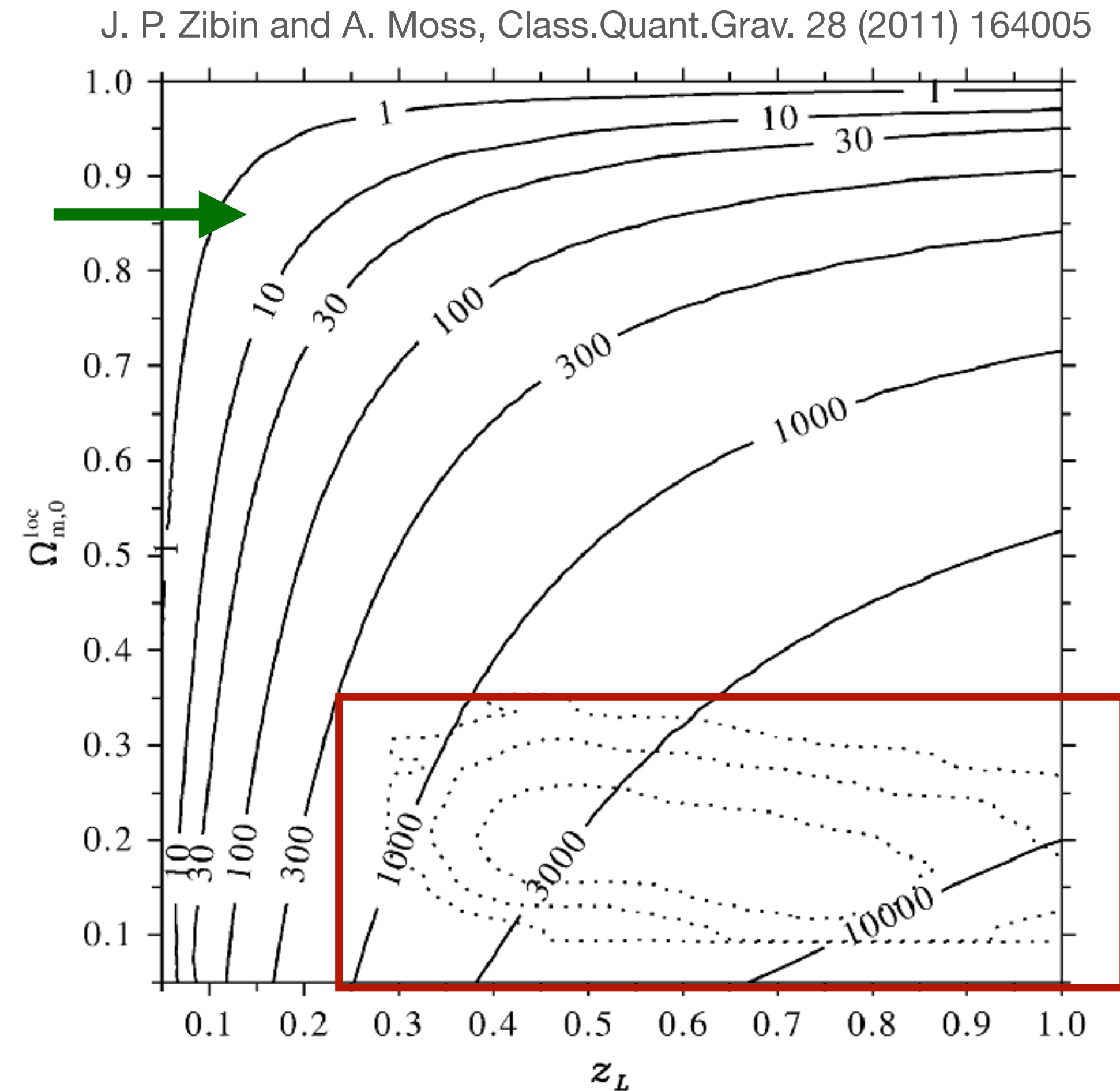


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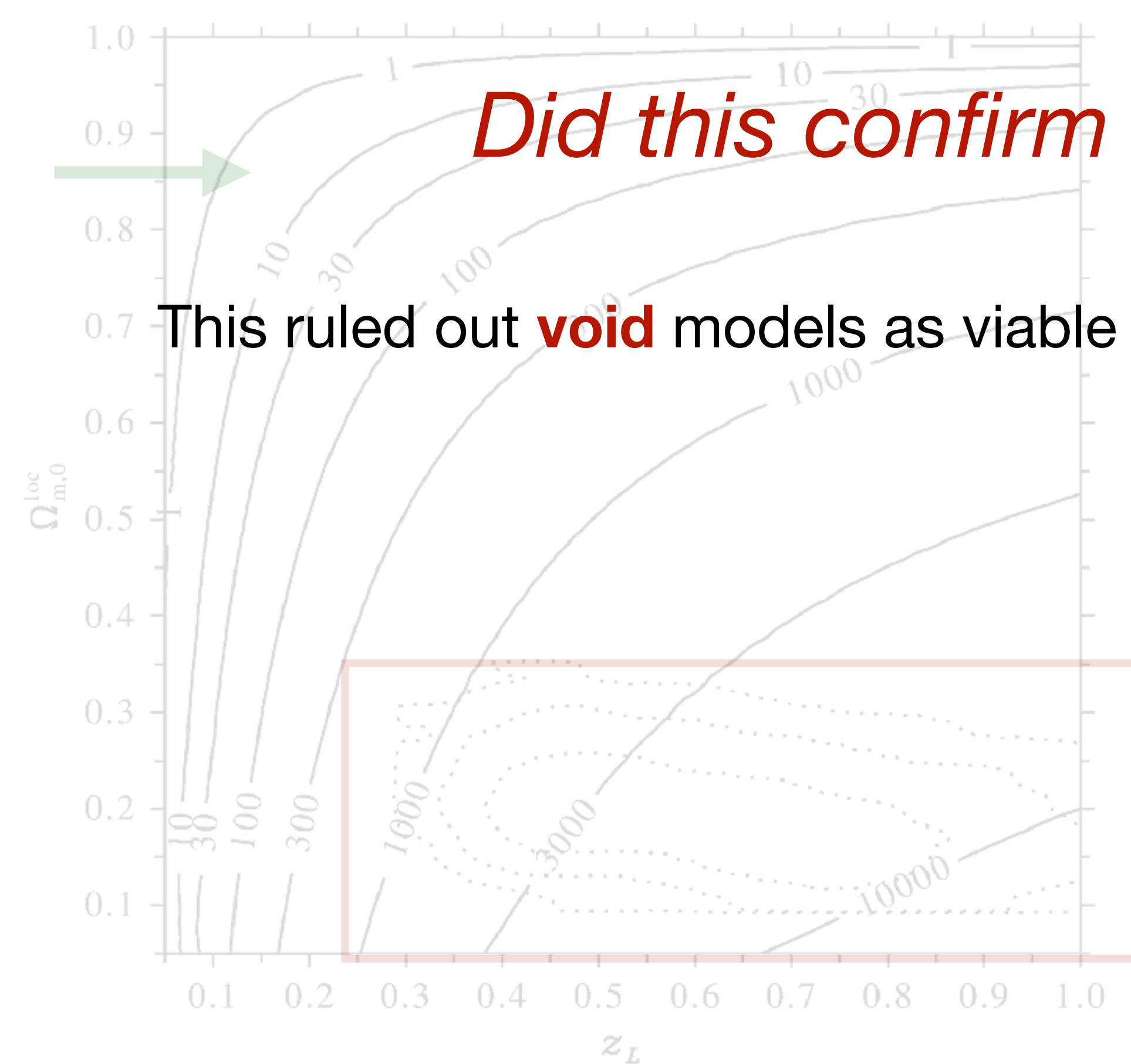
$$\Lambda \xrightarrow{\text{replace by}} \frac{\partial H}{\partial r} < 0$$

Inhomogeneous solutions: a tale about void models

Despite its ability to explain the dimming of the luminosity of Supernovae
LTB **void** models were shown to be inconsistent with the several observations

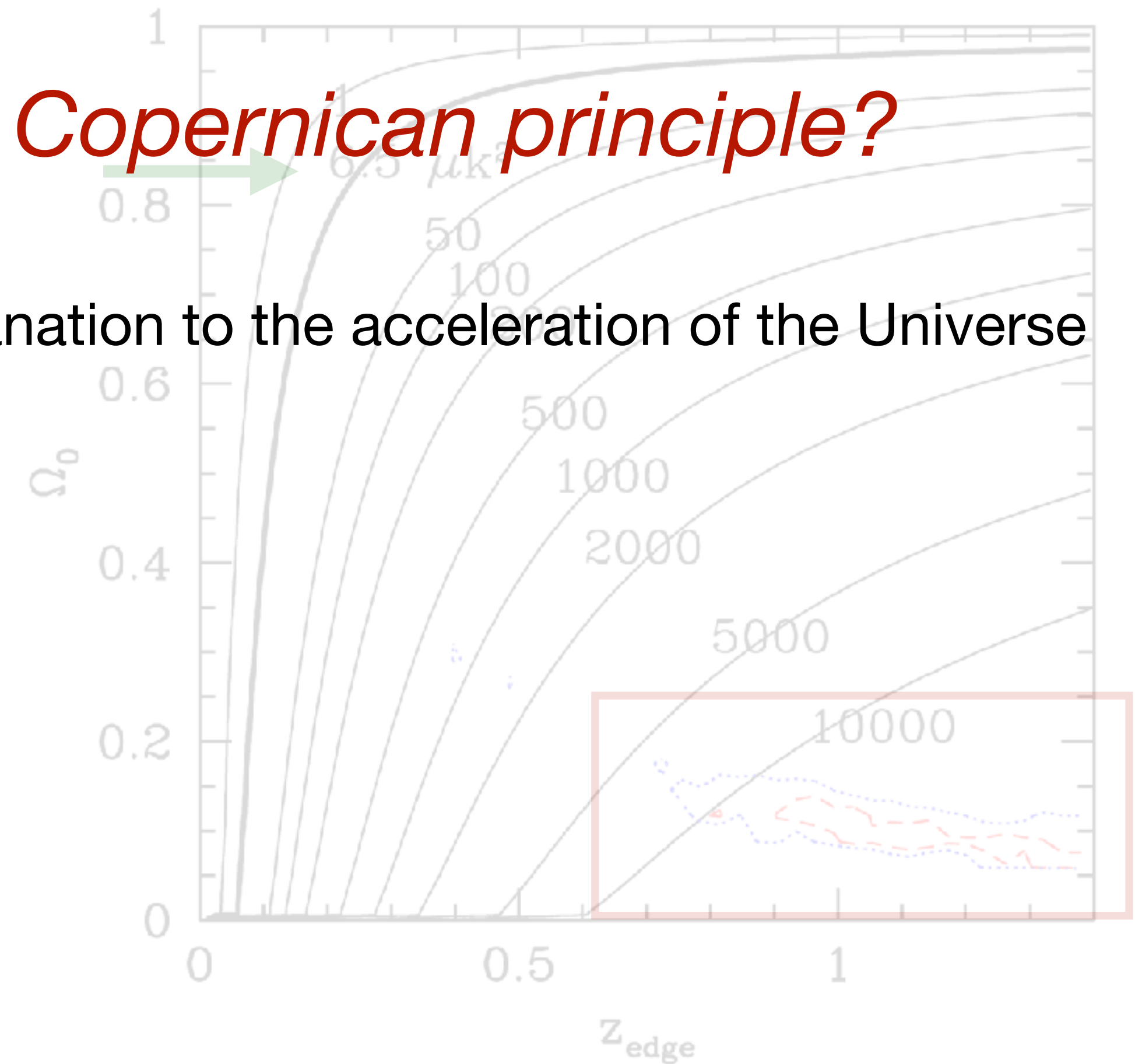


Inhomogeneous solutions: a tale about void models



Did this confirm the Copernican principle?

This ruled out **void** models as viable explanation to the acceleration of the Universe



The Λ LTB model

We model a violation to the Copernican principle by using the FLRW metric + *radial inhomogeneities*

Three arbitrary functions: $m(r)$, $t_{BB}(r)$, and $K(r)$

$$\frac{\dot{R}^2}{R} = \frac{8\pi G}{3}\rho_{\Lambda} + \frac{2m(r)}{a_{\perp}^3 r^3} - \frac{K(r)}{a_{\perp}^2}$$

$$\rho_m(r, t) = \frac{m'(r)}{4\pi G a_{\parallel} a_{\perp}^2}$$

$$t_0 - t_{BB}(r) = \frac{1}{H_{\perp}(r, t_0)} \int \frac{x}{\sqrt{\Omega_{m0}(r)x^{-1} + \Omega_{k0}(r) + \Omega_{\Lambda0}(r)x^2}}$$

Cosmological distances in the Λ LTB model

$$ds^2 = - dt^2 + \frac{a_{\parallel}^2(r, t)}{1 - K(r)r^2} dr^2 + a_{\perp}^2(r, t) r^2 d\Omega$$

Inhomogeneity yields to anisotropic expansion rate!

$$D_H(z) = \frac{1}{H_{\parallel}(r(z), t(z))}$$

$$D_M(z) = (1 + z) r^2(z) a_{\perp}(r(z), t(z))$$

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$$D_M(z) = (1 + z)r^2(z)a_{\perp}(r(z), t(z))$$

$$\frac{dt}{dz} = - \frac{1}{(1 + z)H_{\parallel}(r, t)}$$

$$\frac{dr}{dz} = \frac{1 - K(r)r^2}{(1 + z)\dot{a}_{\parallel}(r, t)}$$

BAO distances in the Λ LTB model

BAO measurements are typically presented as ratios between coming distances:

$$\frac{D_H}{r_d} \rightarrow \frac{D_H}{(1+z)l_{\parallel}(z)}$$
$$\frac{D_M}{r_d} \rightarrow \frac{D_M}{(1+z)l_{\perp}(z)}$$

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The anisotropic expansion deforms in a direction dependent way these ratios!

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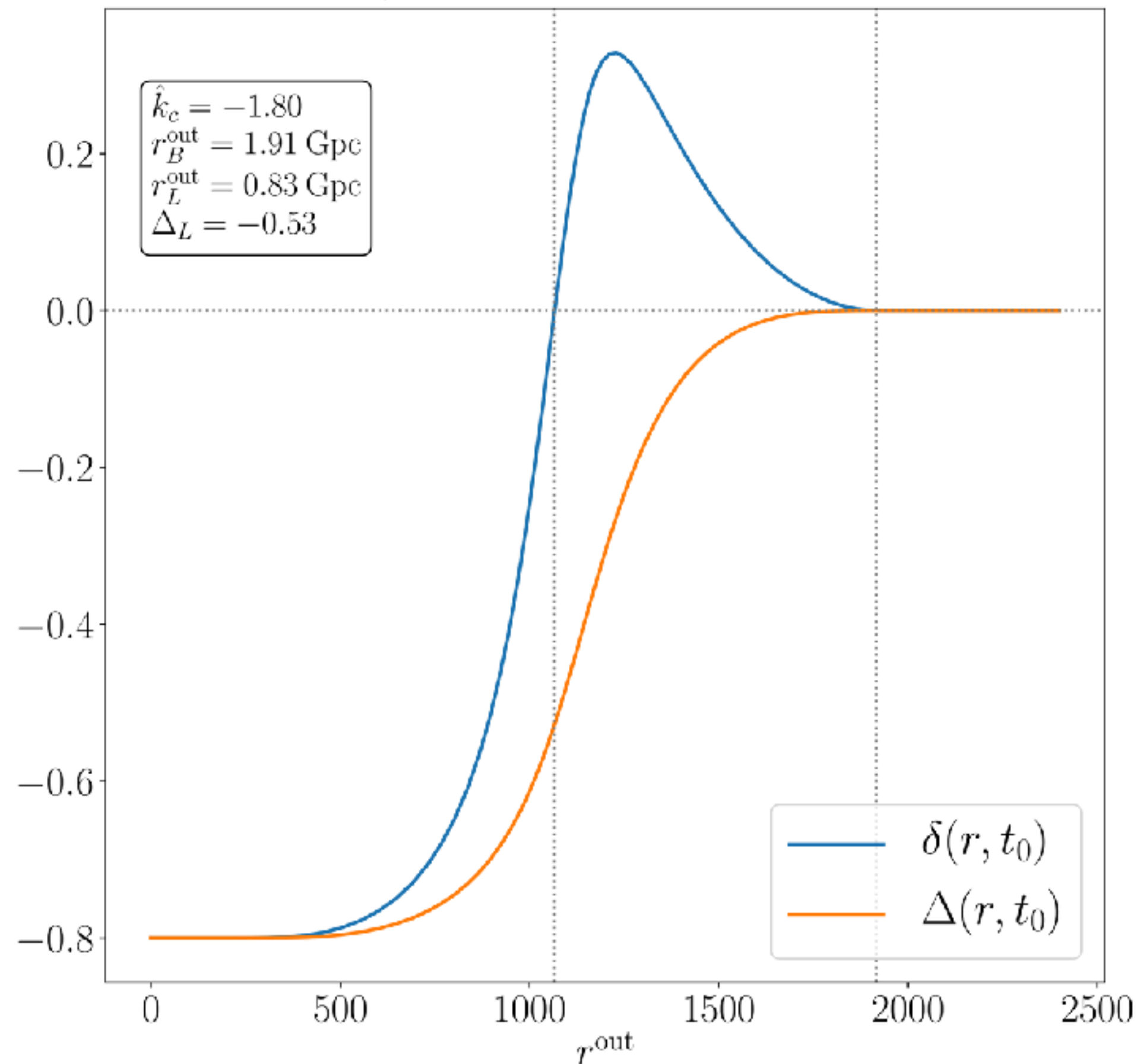
The anisotropic expansion deform in a direction dependent way these ratios!

$$l_{\perp,\parallel}(z) = \frac{a_{\perp,\parallel}(r(z), t(z))}{a_{\perp,\parallel}(r(z), t_d)} \frac{r_d}{(1+z_d)}$$

The Λ LTB model

δ_0, r_B + 6 Λ CDM param.

$\delta_0 = -0.80$ and $z_B = 0.50$



Λ CDM background endowed
with a spherical over/underdensity

Early-FLRW cosmologies:

- ♦ Mass function: $m(r) = m_0 r^3$
- ♦ Homogeneous Big Bang time: $t_{\text{BB}}(r) = 0$
- ♦ Compensated curvature profile:

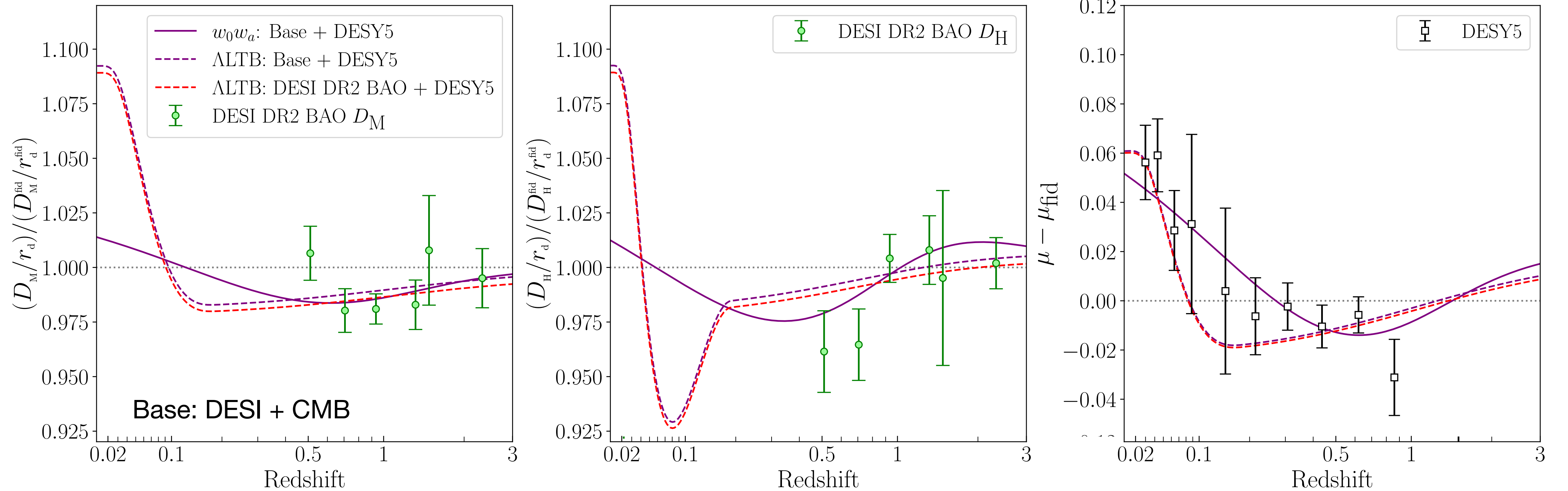
$$k(r) = k_B + (k_C - k_B)P_3(r/r_B)$$

$$P_3(x) = \begin{cases} 1 - \exp[-(1-x)^3/x] & \text{if } 0 \geq x < 1 \\ 0 & \text{if } 1 \geq x \end{cases}$$

$$\delta(r, t) \equiv \frac{\rho_m(r, t)}{\rho_m(r_B, t)} - 1 \quad \Delta(r, t) \propto \int_0^r d\bar{r} \delta(\bar{r}, t) a_{\perp}^2 a_{\parallel} r^2$$

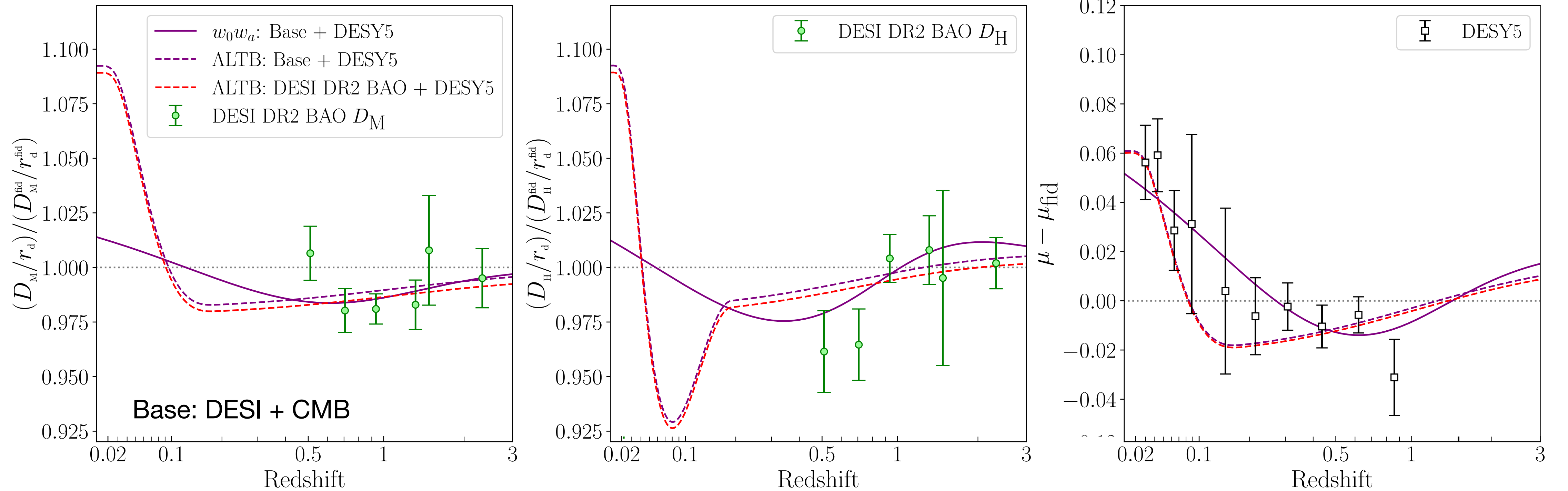
Matter and mass dens. Contrast

Dynamic dark energy vs inhomogeneities



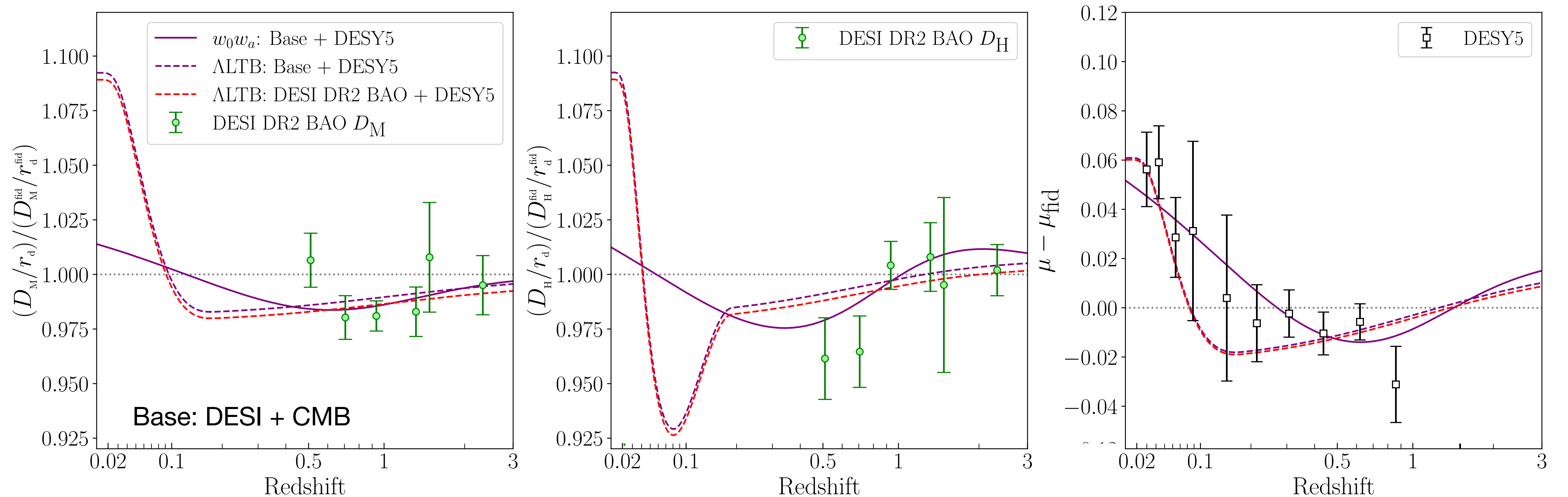
Dynamic dark energy vs inhomogeneities

Local inhomogeneities bring back the agreement between different probes!



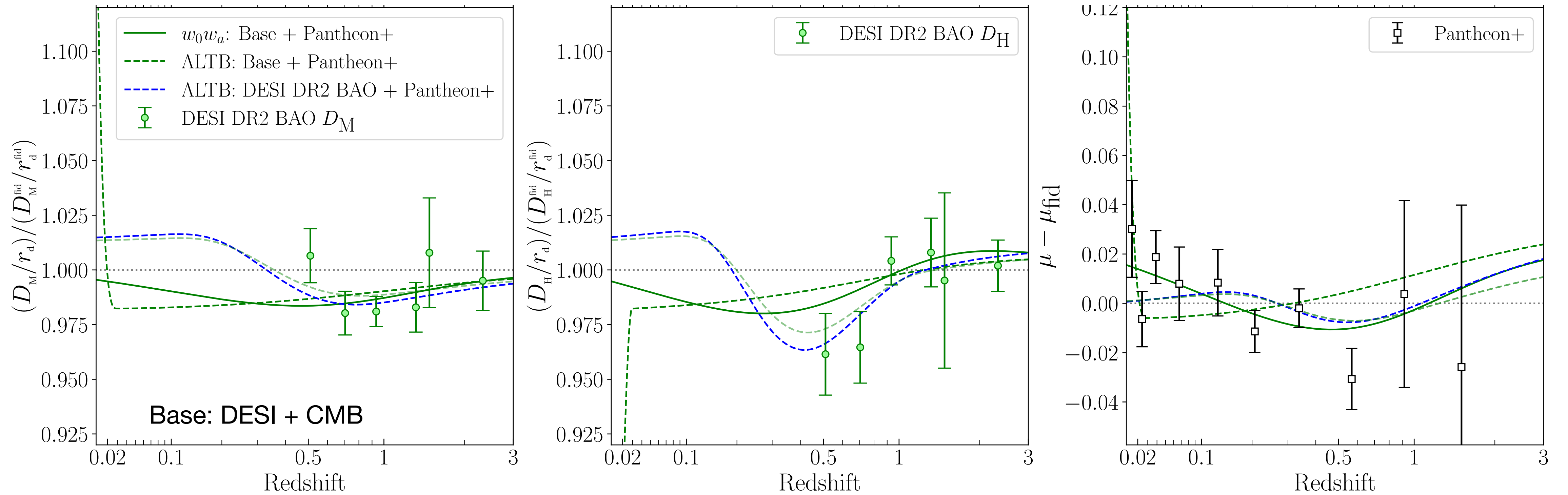
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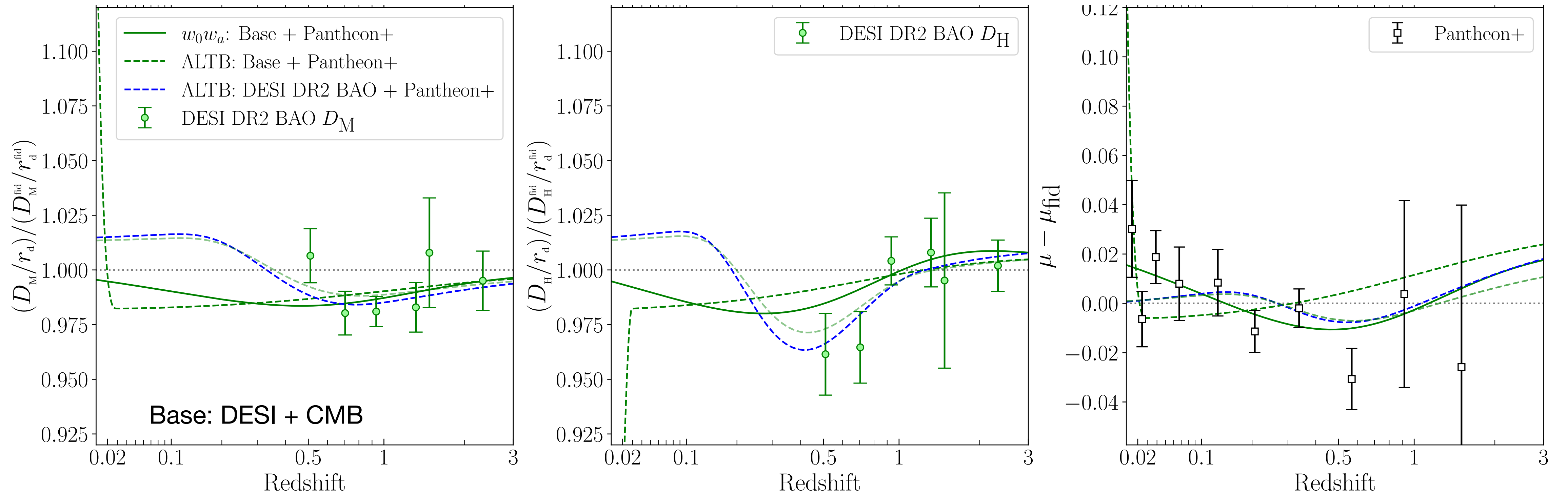
BF parameters: $\delta_0 \approx 0.2$ & $r_B \approx 800$ Mpc ($z_B \approx 0.19$)

Dynamic dark energy vs inhomogeneities



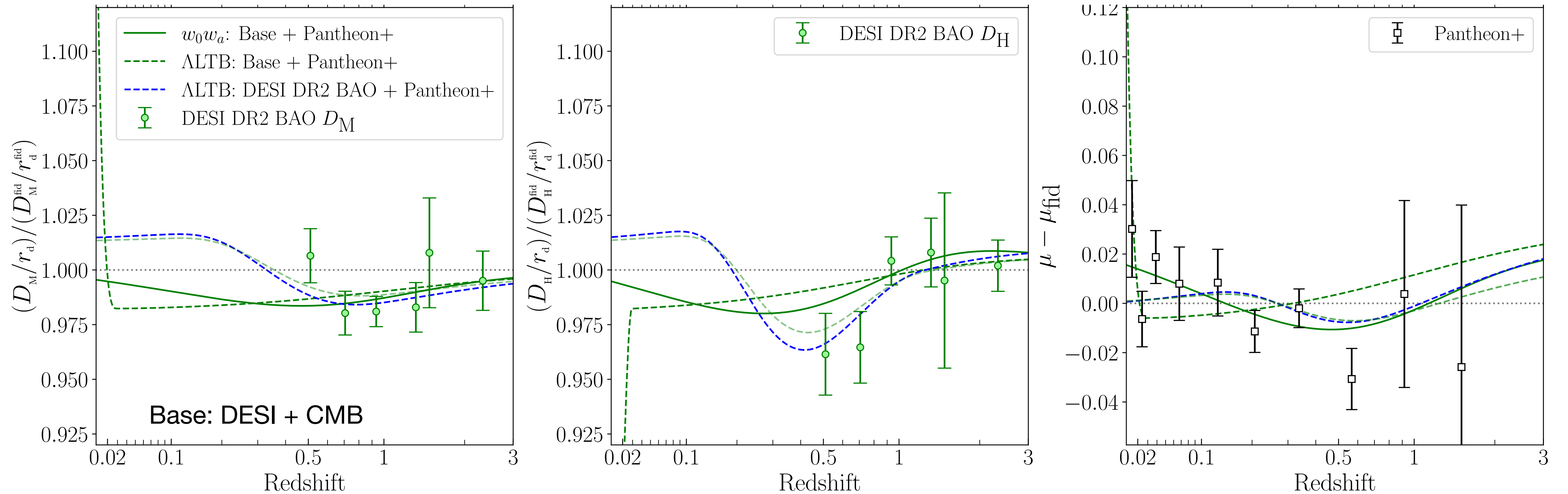
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Dynamic dark energy vs inhomogeneities

Local inhomogeneities bring back the agreement between different probes!



BF parameters: $\delta_0 \approx 0.6$ & $r_B \approx 130$ Mpc ($z_B \approx 0.03$)

Dynamic dark energy vs inhomogeneities

Dynamic dark energy vs inhomogeneities

$$\Delta\chi^2 \equiv \chi^2_{\text{model}} - \chi^2_{\Lambda\text{CDM}}$$

	Base	Base + DESY5	Base + Pantheon+
<i>w₀w_a</i> CDM			
$\Delta\chi^2_{\text{BAO}}$	-4.18	-5.57	-2.72
$\Delta\chi^2_{\text{CMB}}$	-3.36	-1.66	-1.96
$\Delta\chi^2_{\text{SNe}}$	-	-21.73	-3.47
$\Delta\chi^2_{\text{min}}$	-7.54	-28.96	-8.15
ΛLTB			
$\Delta\chi^2_{\text{BAO}}$	-3.50	-2.18	-0.04 (-4.07)
$\Delta\chi^2_{\text{CMB}}$	0.41	1.45	0.25 (0.94)
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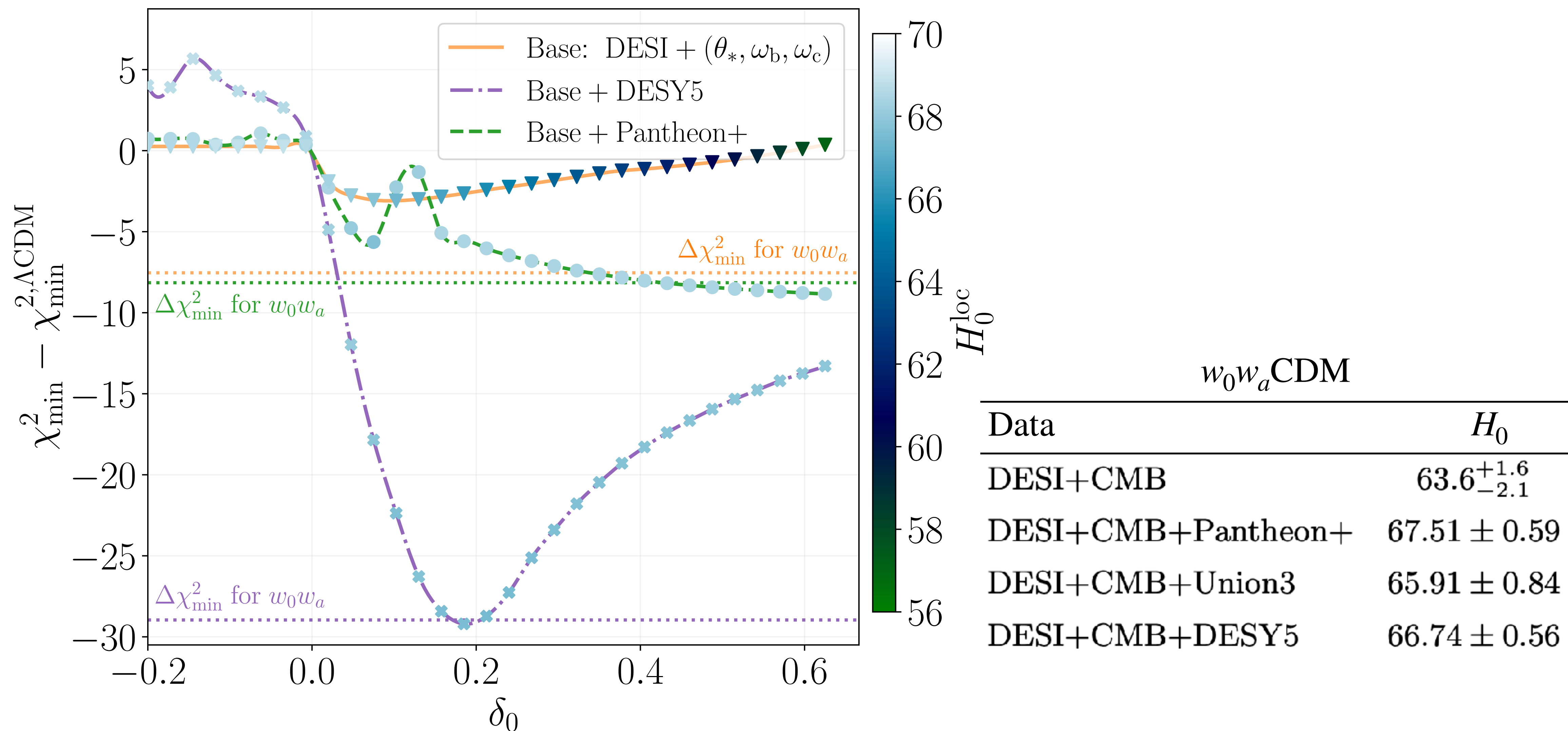
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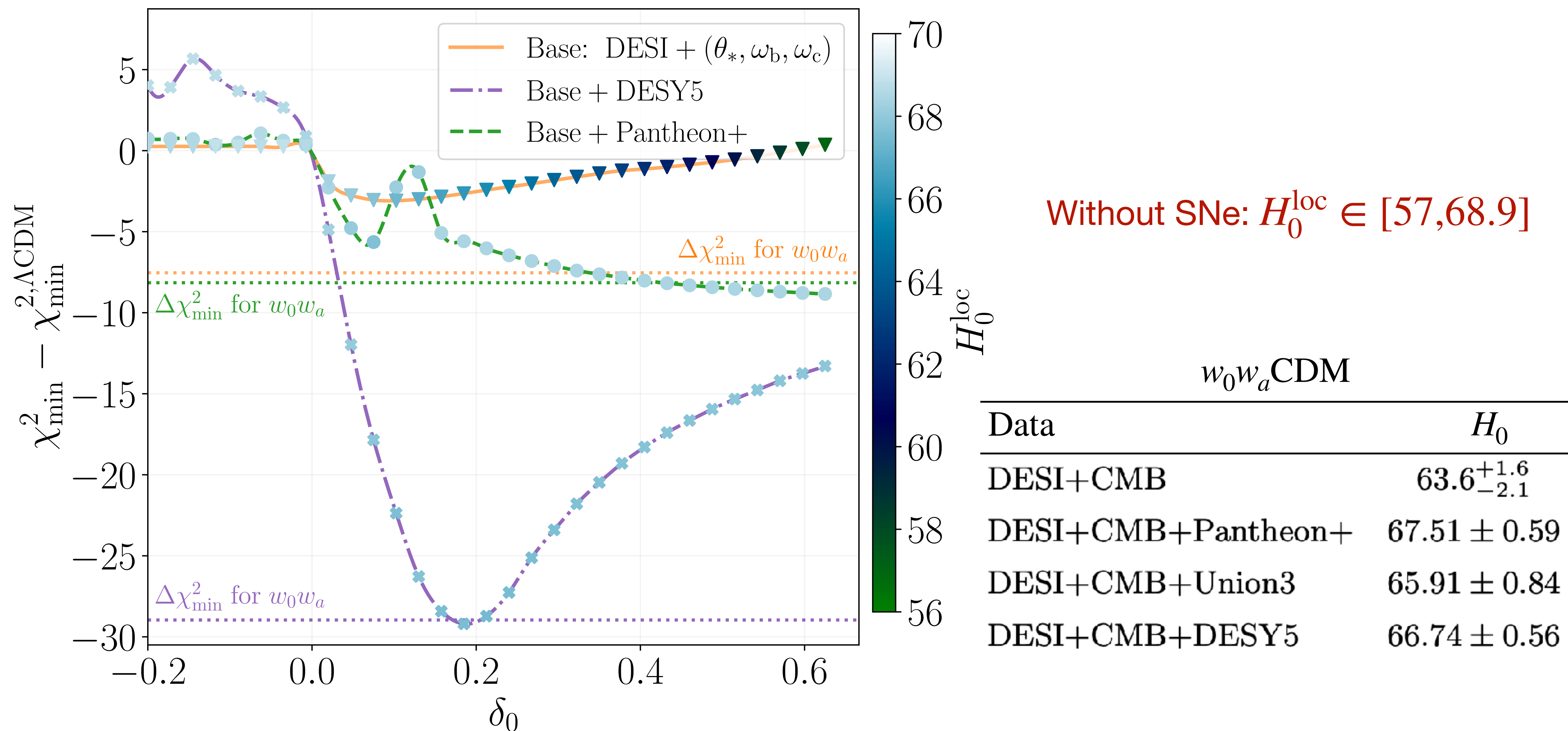
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When considering SNe, fits obtained with inhomogeneous models are statistically indistinguishable from the dynamical dark energy scenario!

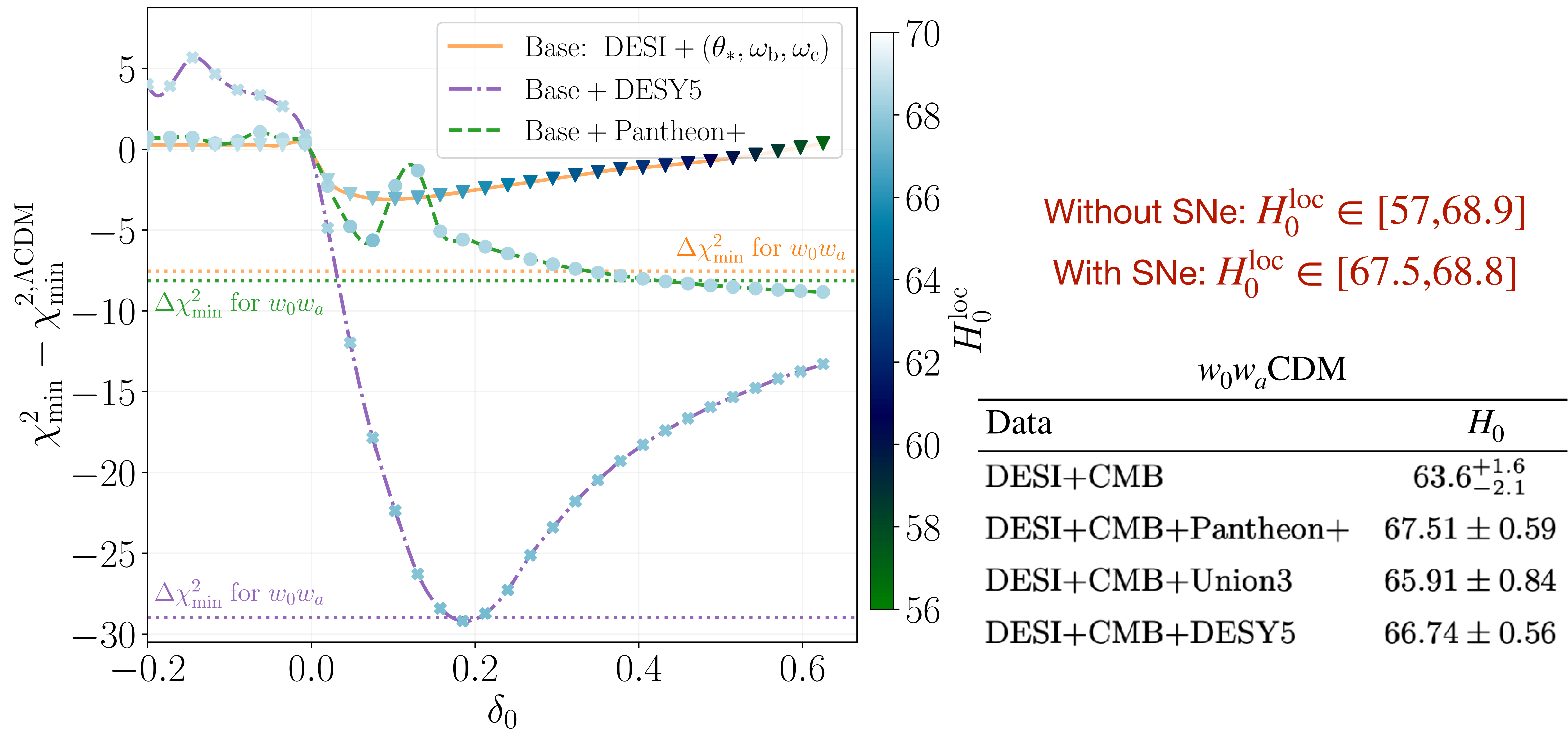
Profile likelihood analysis and impact on H_0



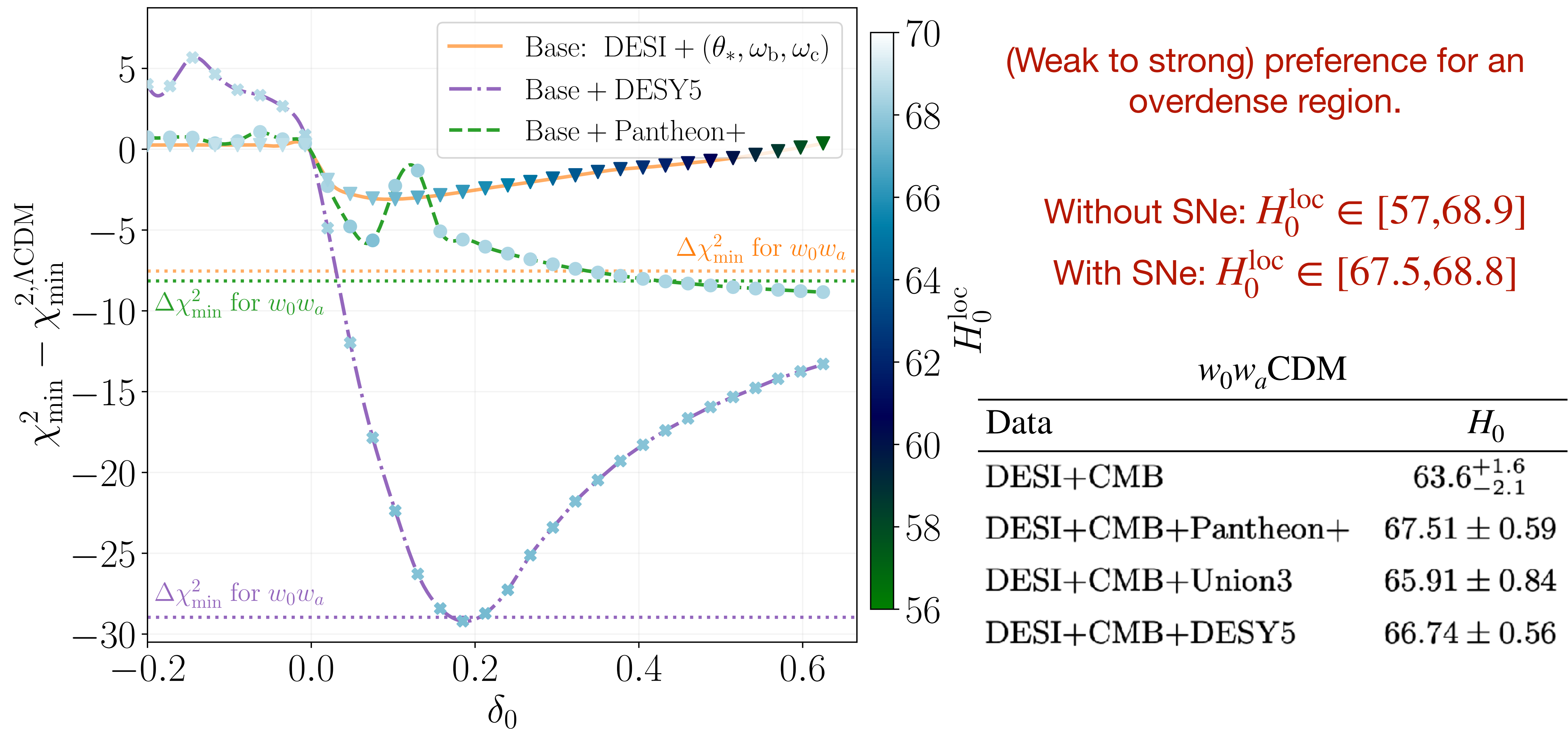
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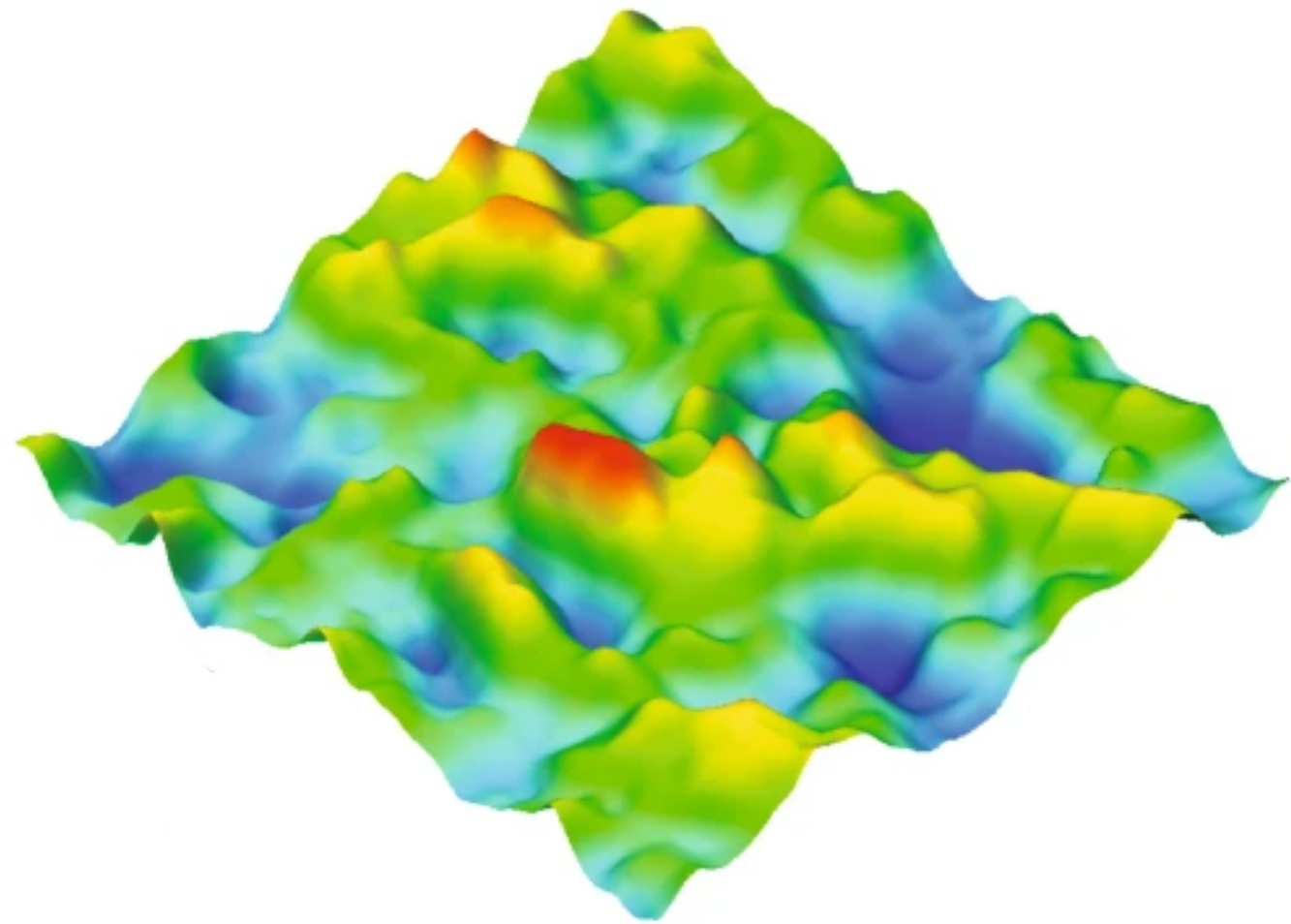


How viable are these structures?

Given a nearly-invariant adiabatic power spectrum, how likely is to find these Λ LTB structure?

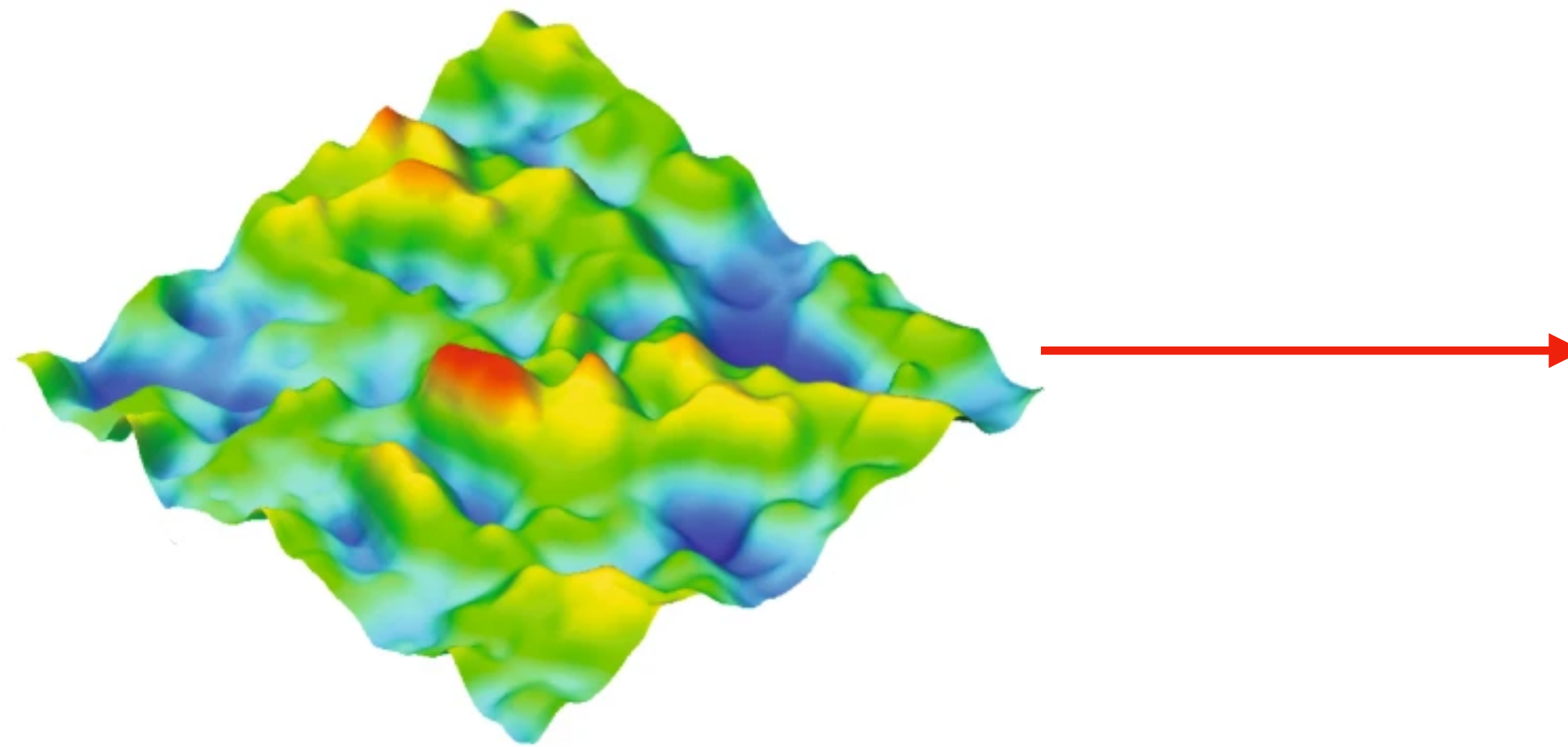
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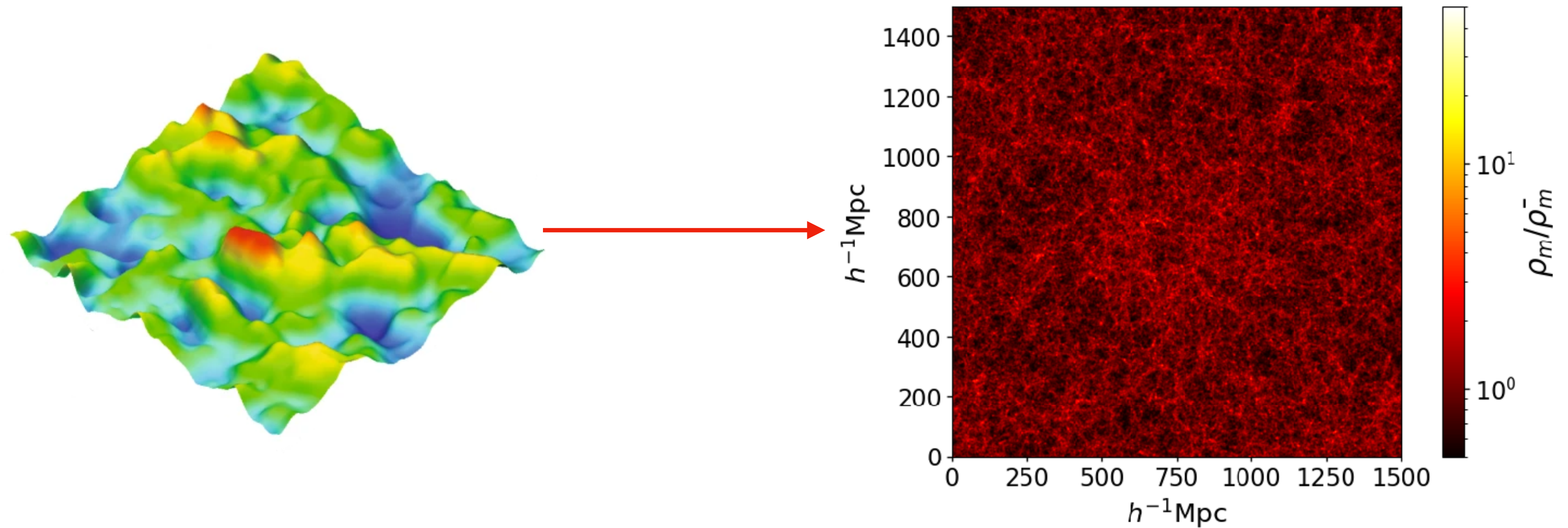
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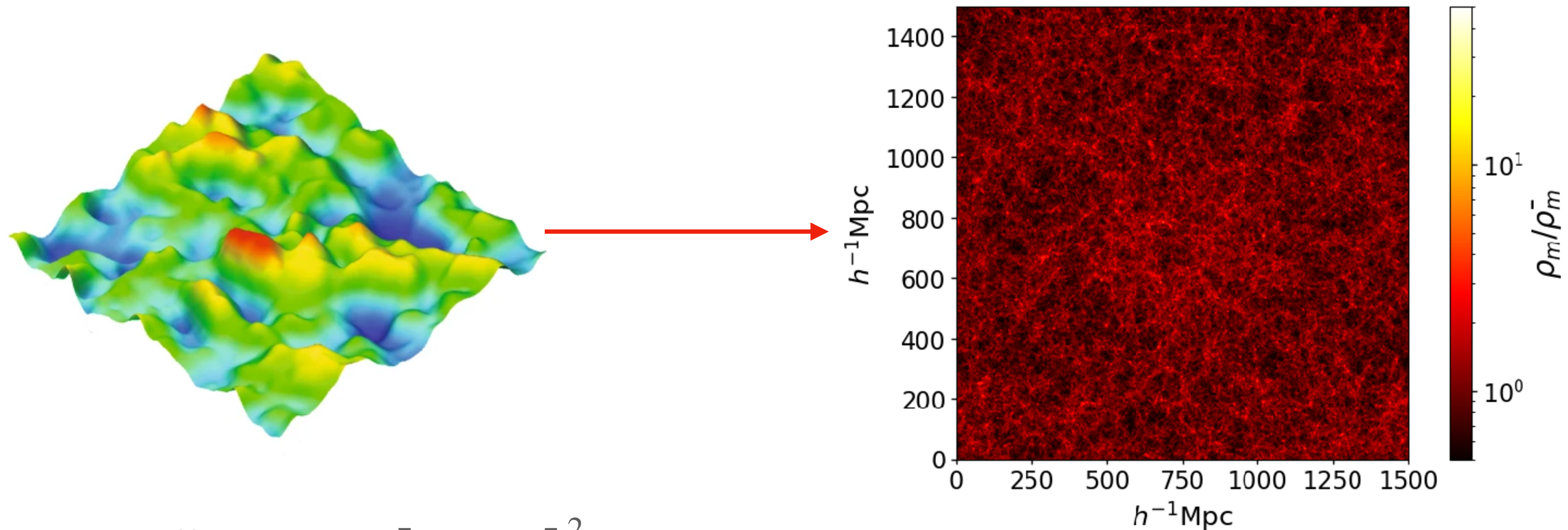
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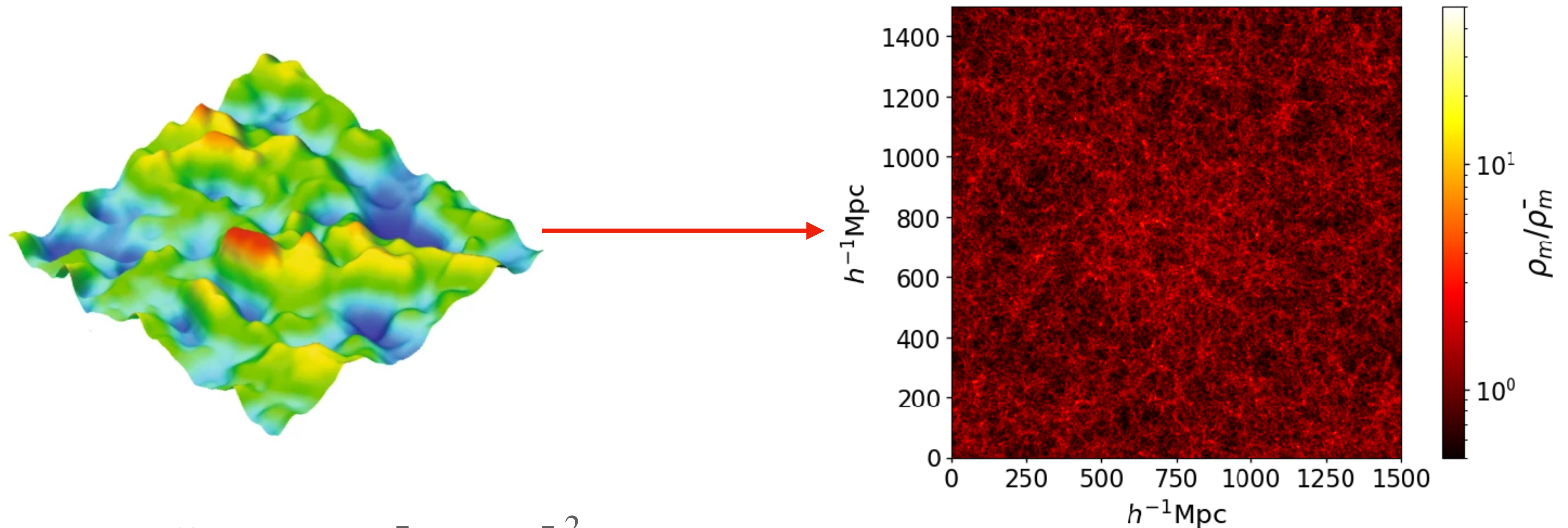


$$\sigma^2(r) = \int_0^\infty \frac{dk}{k} \Delta_{m0}^2(k) \left[\frac{3j_1(rk)}{rk} \right]^2$$

$$\mathcal{P}(\Delta, r) \propto \exp \left[-\frac{1}{2} \frac{\Delta^2(r)}{\sigma^2(r)} \right]$$

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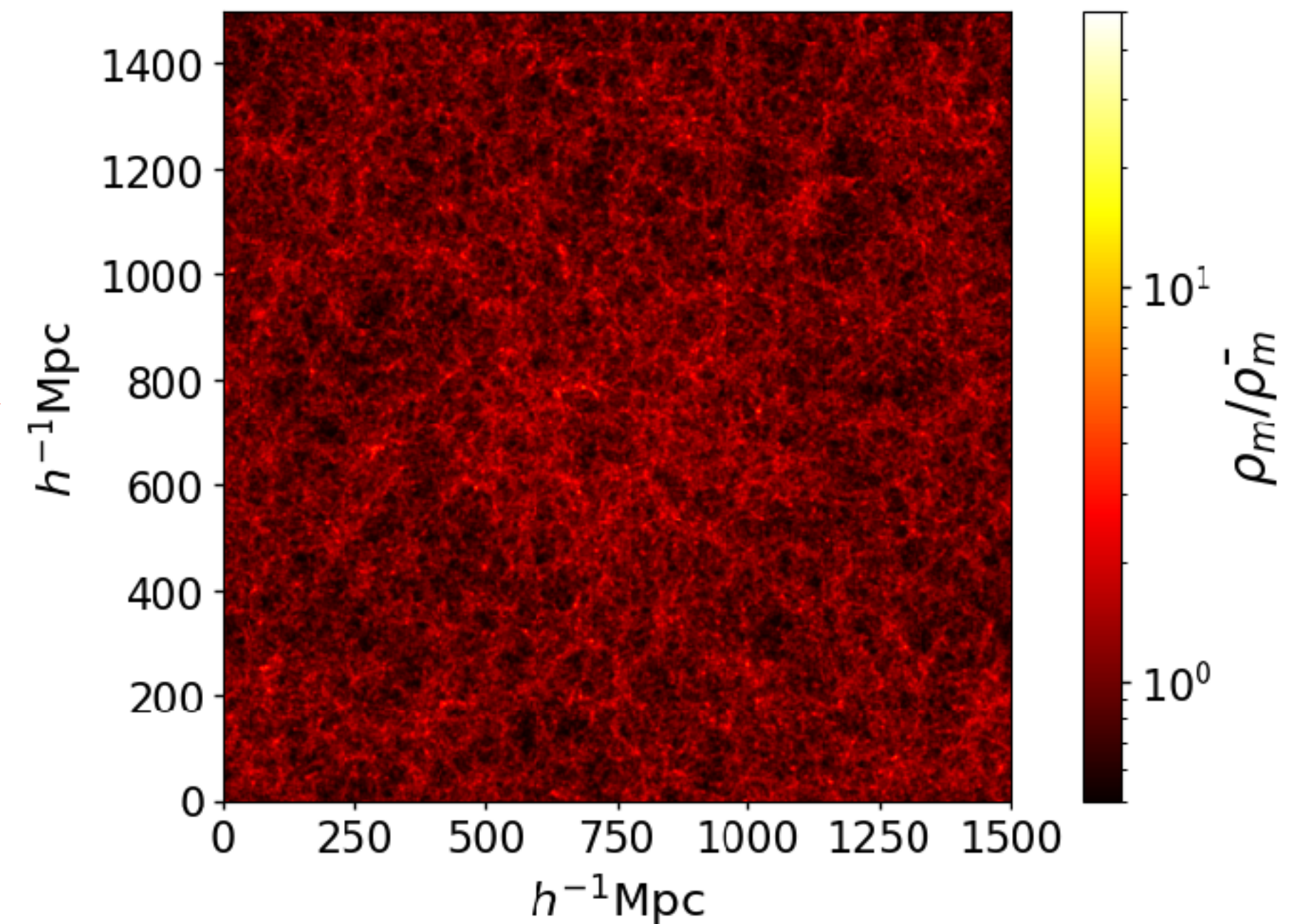
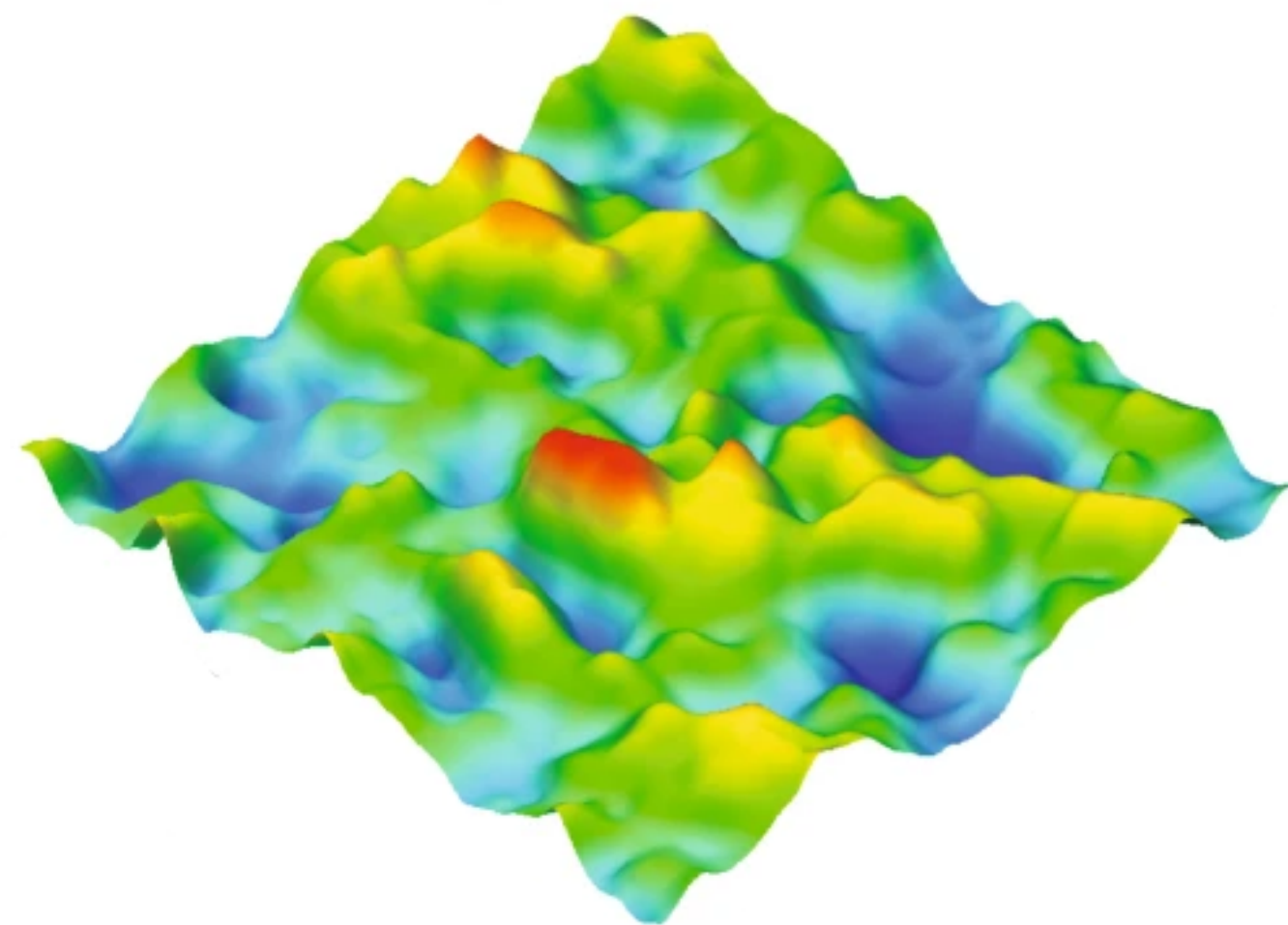


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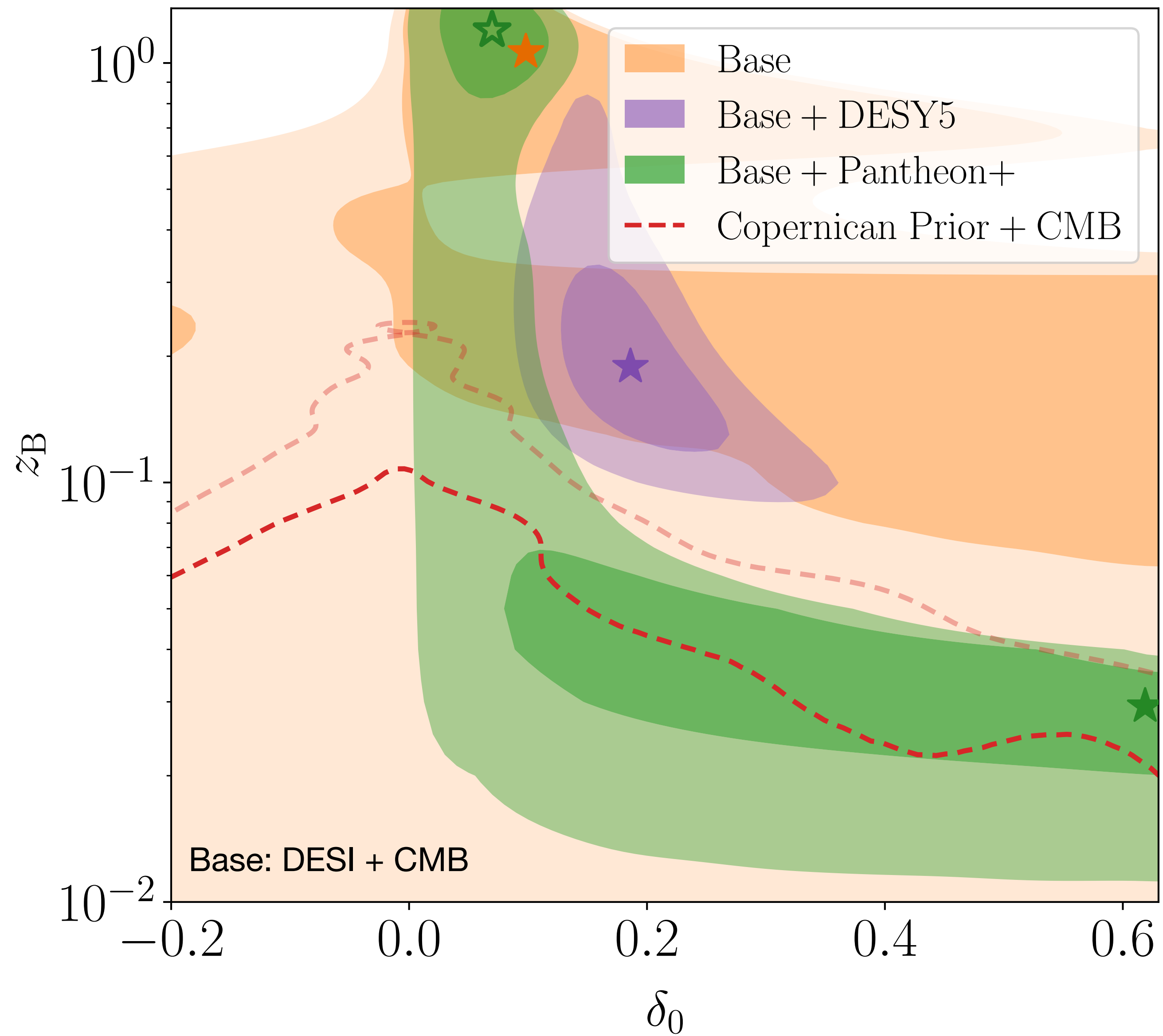


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Constraints on the Λ LTB
parameter space
(Copernican prior)

How viable are these structures?



Conclusions

- *Non-standard features can be explained by deviation of the FLRW paradigm: density fluctuations impact the way we measure distances.*
- *Data allows a wide range of scenarios: both Copernican and non-Copernican fluctuations are allowed.*
- *Current cosmological distances favor overdense regions: with a significance of 2.8σ when considering Pantheon+ and at more than 5.2σ when using DESY5.*
- *Draws the attention to whether the perfectly homogeneous and isotropic FLRW paradigm can still be assumed to accurately predict cosmological distances in the era of precision cosmology.*

