Modified gravity

Kazuya Koyama

Institute of Cosmology and Gravitation

University of Portsmouth

VALENCIA, SPAIN JULY 7-12, 2019 (abstract submission 1 March)

Parallel session A3: Modified theory of gravity (theoretical aspects)

Kazuya Koyama & Thomas Sotiriou


30th Texas Symposium on Relativistic Astrophysics
15th - 20th December 2019
Portsmouth, UK

Reading list

- “Dark Energy” Amendola & Tsujikawa, Cambridge University Press
- “Dynamics of Dark Energy” Copeland, Sami & Tsujikawa hep-th/0603057
- “Modified Gravity and Cosmology” Clifton, Ferreira, Padilla and Skordis arXiv:1106.2476
- Beyond the Cosmological standard model” Joyce, Jain, Khoury and Trodden arXiv:1407.0059
- “Cosmological Tests of Modified Gravity” KK arXiv:1504.04623

Observational evidence and cosmological constant

Kazuya Koyama

University of Portsmouth

Basic assumptions (1)

Isotropy and homogeneity

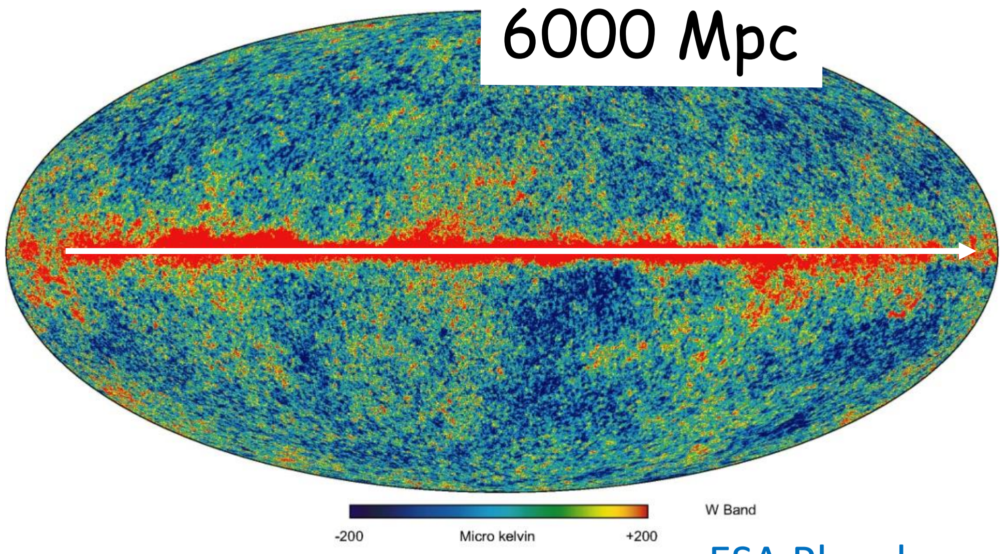
- Isotropy

  CMB fluctuations

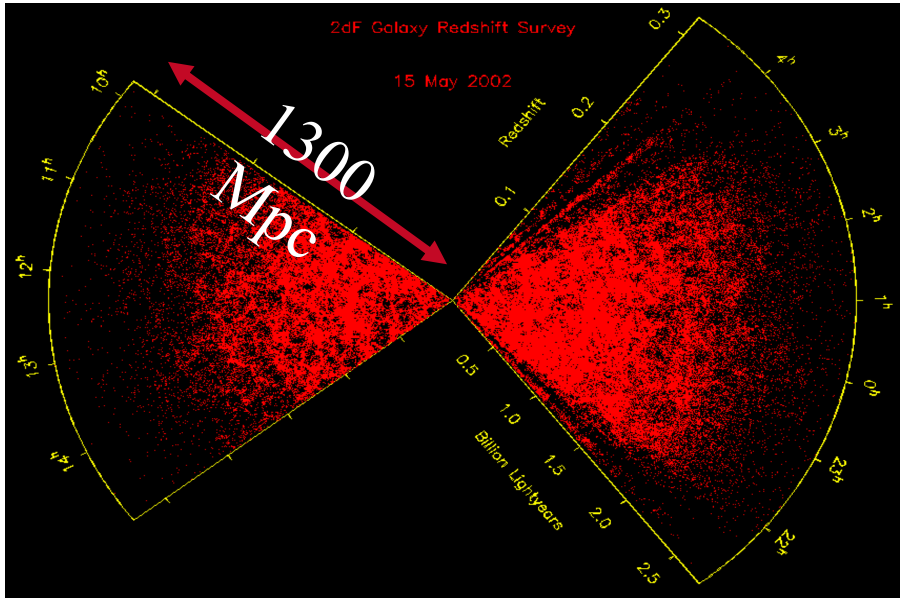
$$\frac{\Delta T}{T} \approx 10^{-5}$$

- Homogeneity

  galaxy distribution



ESA Planck



2dF galaxy redshift survey

- Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = -c^2 dt^2 + a(t)^2 \gamma_{ij} dx^i dx^j$$

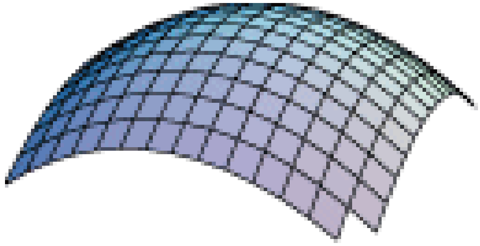
$$ds_3{}^2 = \gamma_{ij} dx^i dx^j = d\chi^2 + \begin{Bmatrix} \sin^2 \chi \\ \chi^2 \\ \sinh^2 \chi \end{Bmatrix} \left(d\theta^2 + \sin^2 \theta d\phi^2 \right)$$

$${}^{(3)}R_{ijkl} = K(\gamma_{ik}\gamma_{jl} - \gamma_{il}\gamma_{jk}) : \text{ 3D curvature}$$

$S^3 : K = 1$

$E^3 : K = 0$

$H^3 : K = -1$



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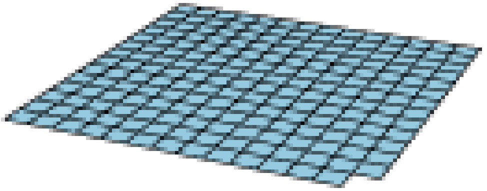

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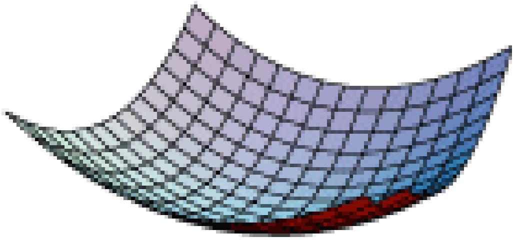

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Basic assumption (2)

- General Relativity (GR)

$$\boxed{G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R} = \boxed{8\pi G T_{\mu\nu}}$$

geometry matter

- Matter

$$T^{\mu}{}_{\nu} = (\rho + P) u^{\mu} u_{\nu} + P \delta^{\mu}_{\nu} \qquad u^{\mu} = (-1, 0, 0, 0)$$

- Bianchi identity

$$\nabla^{\mu} G_{\mu\nu} = 0 \Rightarrow \nabla^{\mu} T_{\mu\nu} = 0$$

Friedman equation

- Einstein equations

$$H(t)^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$$

- Energy-momentum conservation

$$\dot{\rho} + 3H(\rho + P) = 0, \quad \rho = \sum_i \rho_i$$

- Two of these equations are independent

Three unknown quantities a, ρ, P

⇨ we need to specify the equation of state $w = P / \rho$

Basic assumption (3)

- We introduce dark energy in addition to “known” matter such as baryons, cold dark matter and radiation and assume that they satisfy the conservation equation independently

$$\dot{\rho}_i + 3H(1+w_i)\rho_i = 0, \quad w_i = \frac{P_i}{\rho_i} \qquad \rho_i \propto a^{-3(1+w_i)}$$

- equation of state

$$w_r = \frac{1}{3}, \quad w_m = 0, \quad w_{DE} = ?$$

- Density parameter

$$\Omega_i = \frac{8\pi G\rho_i}{3H^2}, \quad \Omega_K = -\frac{K}{(aH)^2} \qquad \sum_i \Omega_i = 1$$

What we measure

- Distance **assumption (1)**

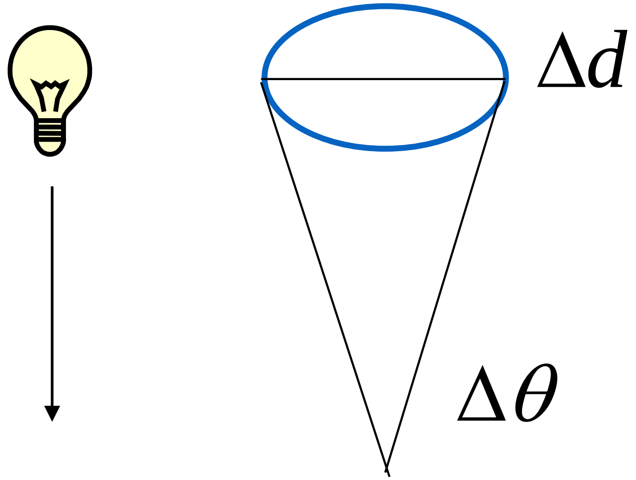
$$ds_3{}^2 = d\chi^2 + f_K(\chi)^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \quad f_K = \frac{1}{\sqrt{-K}}\sinh\left(\sqrt{-K}\chi\right)$$

$$\chi = -\int_{t_0}^{t}\frac{c}{a(t')}dt' = \frac{c}{a_0 H_0}\int_0^z \frac{dz'}{E(z')}, \quad E(z) = \frac{H(z)}{H_0}$$

- Luminosity distance and angular diameter distance

$$d_L = f_K(\chi)(1+z) \qquad \text{redshift} \quad 1+z = \frac{a_0}{a}$$

$$d_A = \frac{d_L}{(1+z)^2}$$



- Age of the universe

$$t = \int_0^{t_0} dt = H_0^{-1} \int_0^{z} \frac{dz'}{E(z)(1+z)}$$

- The present day Hubble parameter

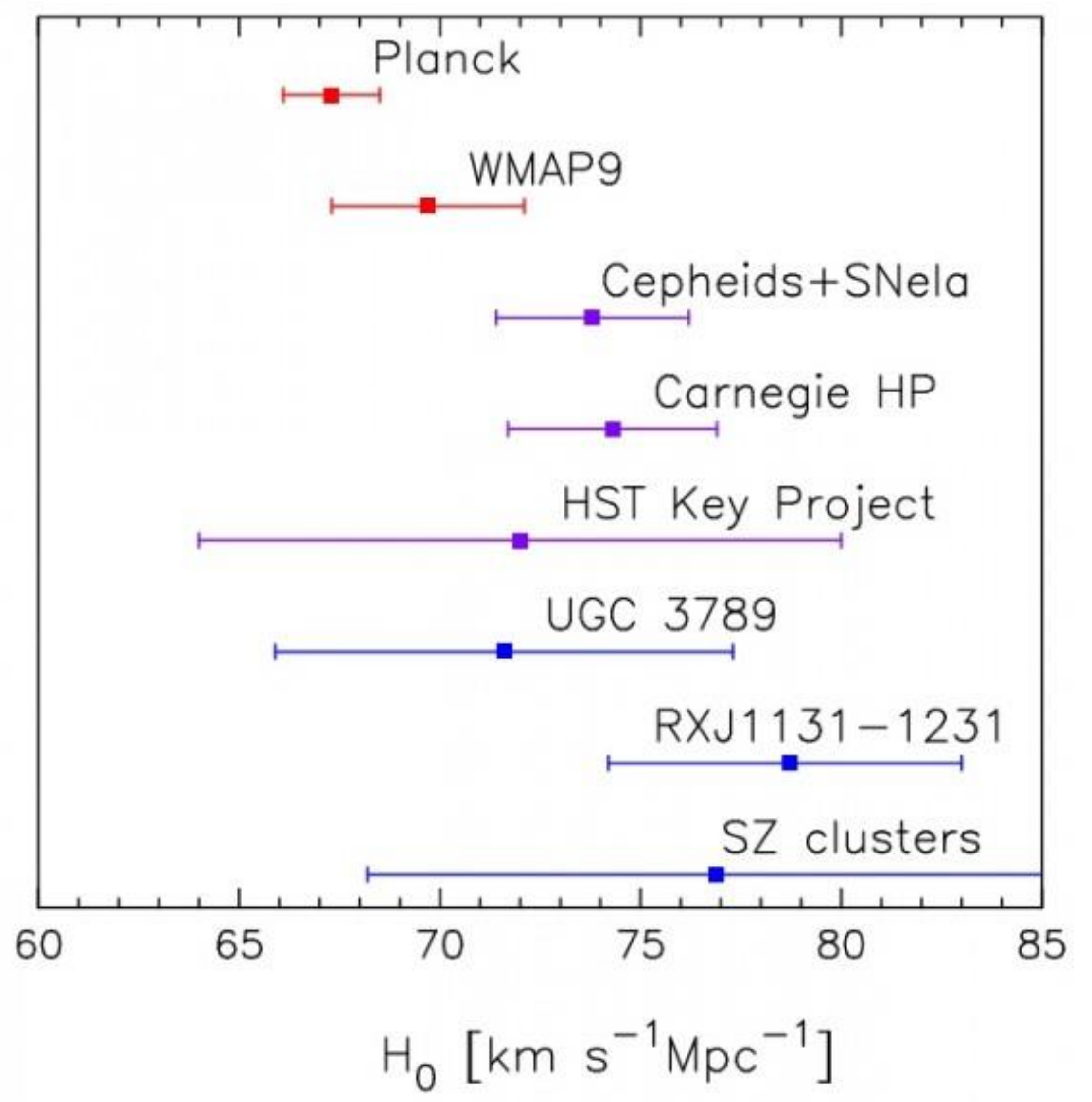
$$H_0^{-1} = 9.78 \times 10^9 h^{-1} \text{ years}$$

$$cH_0^{-1} = 2998\, h^{-1} \text{ Mpc}$$

$$H_0 = 2.13 \times 10^{-42}\ h^{-1} \text{GeV}$$

- Distance at small redshifts

$$d_L \approx d_A \approx c H_0^{-1} z$$

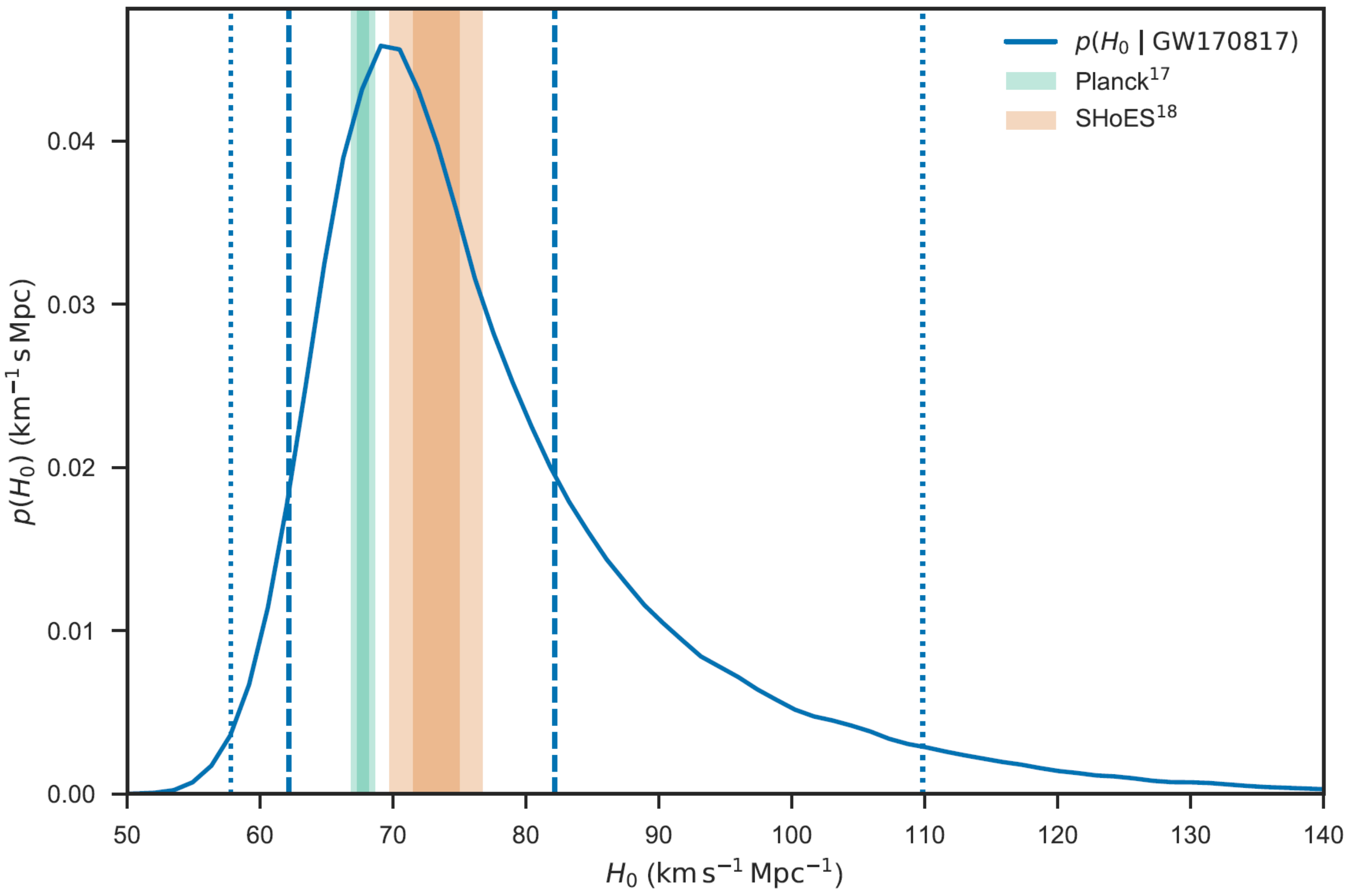


Riess et.al.
arXiv:1604.01424

ESA Planck

A new constraint from gravitational waves

- Gravitational sirens (GW170817)



Distance to a neutron star merger in NGC4993

$$d_L = 43.8^{+2.9}_{-6.9}\,\mathrm{Mpc}$$

Recession velocity

$$v = 3017 \pm 166\ \mathrm{km\ s^{-1}}$$

Theoretical predictions

- Now we use **assumption (2) and (3)**

$$E(z)^2 = \Omega_{m0}(1+z)^3 + \Omega_{r0}(1+z)^4 + \Omega_{K0}(1+z)^2$$
$$+\Omega_{DE}\exp\left[3\int_0^z dz'\frac{1+w_{DE}(z')}{1+z'}\right]$$

- LCDM model

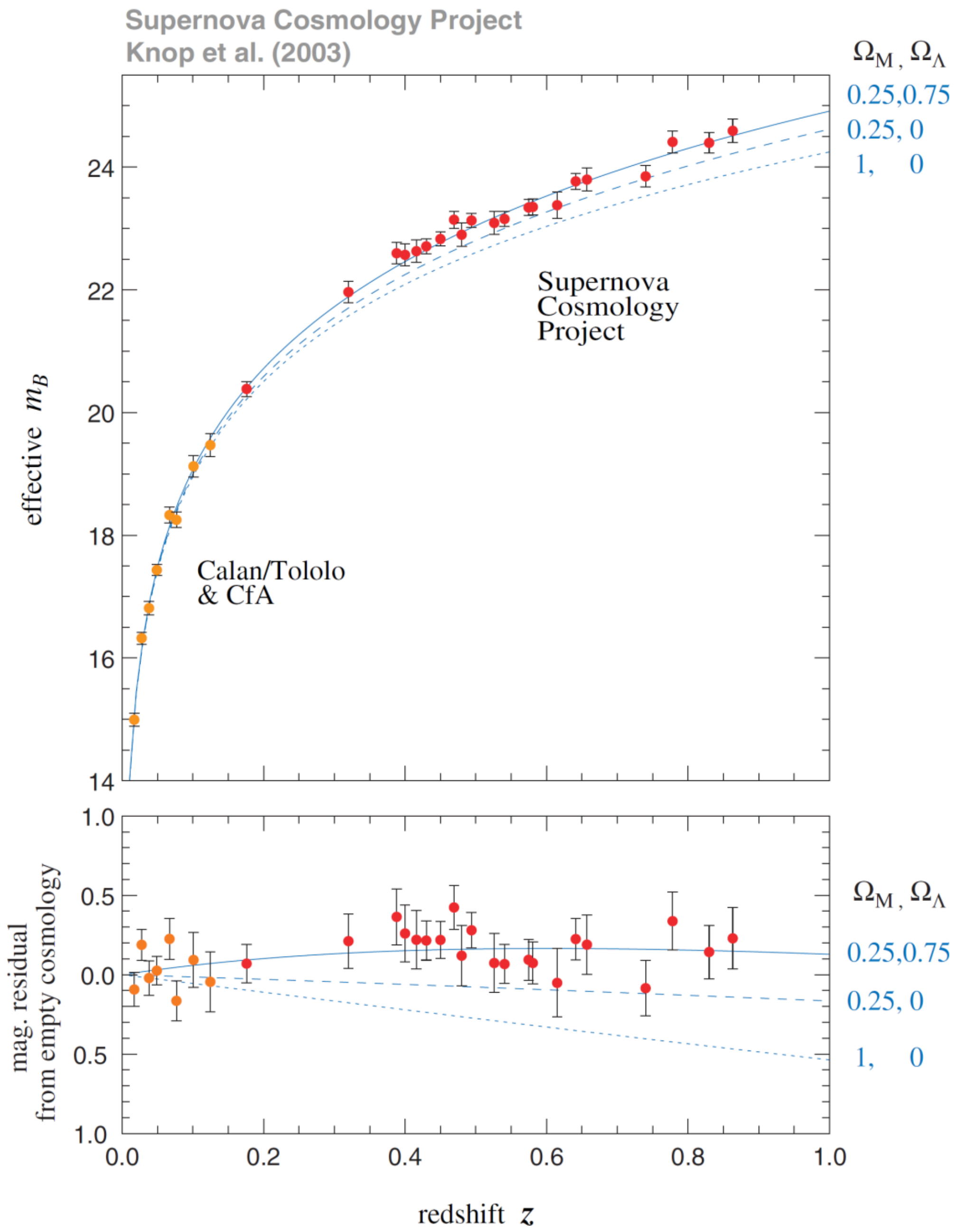
$$w_{DE} = -1, \quad \Omega_{DE0} = \Omega_\Lambda, \quad (\Omega_{r0} = 8\times 10^{-5})$$

- Distance measurements

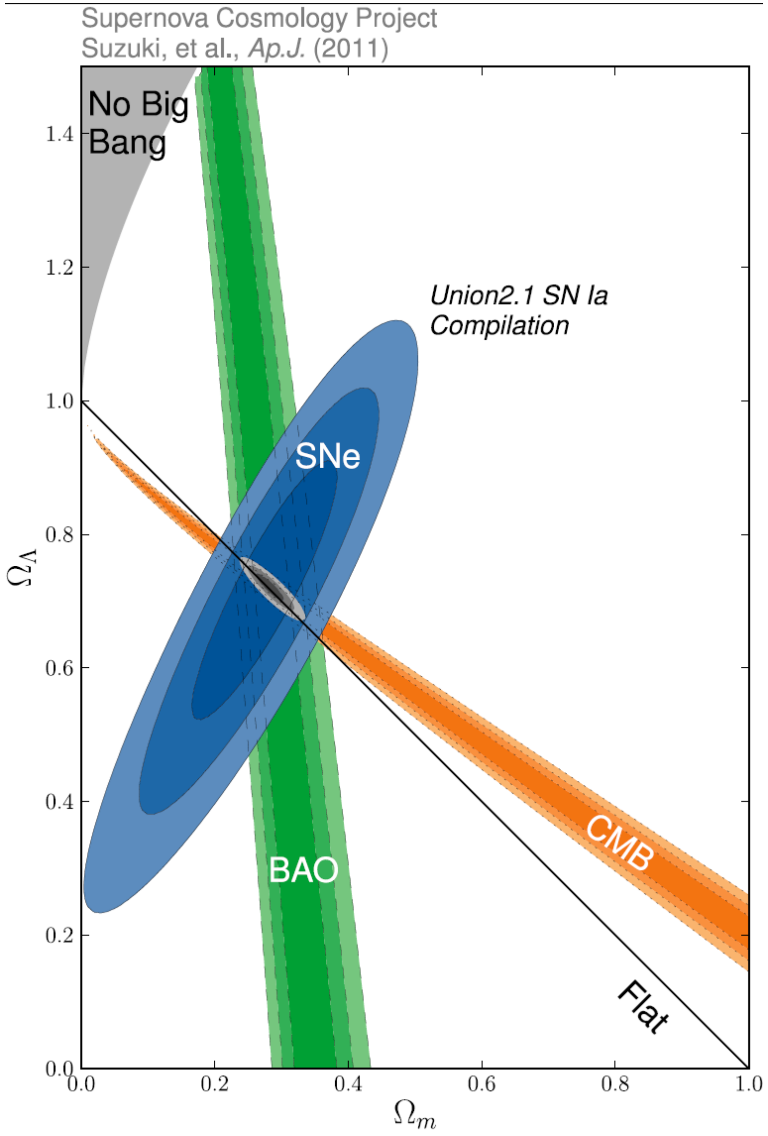
  Supernovae d_L

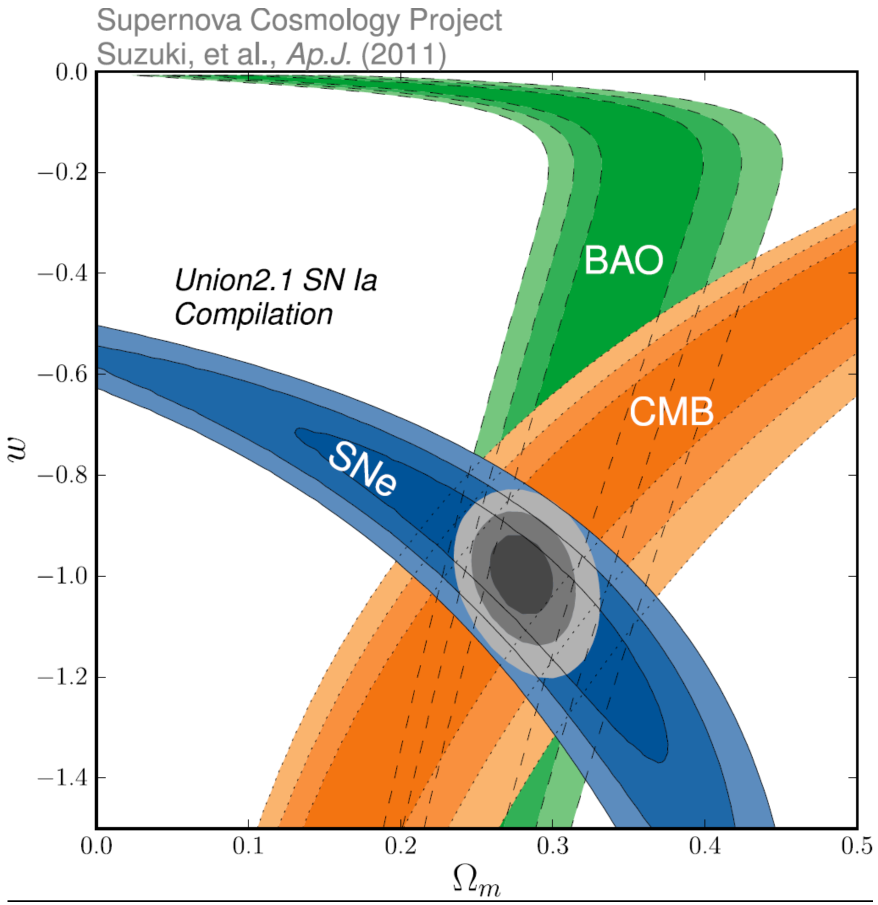
  Cosmic Microwave, d_A

  Baryon Acoustic Oscillations d_A



This is what we found





Cosmological constant

- Action $$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(R - 2\Lambda\right)$$

Einstein equations $$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} + \frac{\Lambda}{3} \qquad \rho_\Lambda = \frac{\Lambda}{8\pi G} = -P_\Lambda$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{\Lambda}{3}$$

cosmological constant does not diminish by the expansion of the universe and the expansion of the Universe accelerates $\ddot{a} > 0$

Why should we bother?

- What's the problem?

  LCDM works well to explain observations

  The cosmological constant can be included in Einstein's GR

- Energy scales (natural unit $\hbar = c = k_B = 1$)

$$m_{pl} = G^{-1/2} = 1.22\times 10^{19}\,\mathrm{GeV} \qquad H_0 = 2.13\times 10^{-42}\ h^{-1}\mathrm{GeV}$$

$$\rho_\Lambda = \frac{\Lambda}{8\pi G} = \frac{m_{pl}^2\Lambda}{8\pi} \approx \frac{m_{pl}^2 H_0^2}{8\pi} \approx 10^{-48}\,\mathrm{GeV}^4 \approx (10^{-3}\,\mathrm{eV})^4$$

Vacuum energy

- Quantum fields have zero-point energy

massive fields (boson and fermion)

$$E = g_i \frac{\hbar\omega}{2} = \frac{1}{2}\sqrt{p^2 + m^2}, \quad g_{boson} = 1, \quad g_{\text{fermion}} = -1$$

vacuum energy

$$\rho_{vac} = \frac{1}{2}\sum_i g_i \int_0^\infty \frac{d^3 p}{(2\pi)^3}\sqrt{p^2 + m_i^2} \qquad\qquad p^2 >> m^2$$

$$\approx \sum_i \frac{g_i}{16\pi^2}\left[p_{\max}^4 + m_i^2 p_{\max}^2 + \frac{1}{2} m_i^4 \ln\left(\frac{m_i}{p_{\max}}\right)\right]$$

This depends on Ultra-Violet (UV) physics but it is robust that there is a contribution of order $O(m^4)$

Vacuum energy is huge

- The observed cosmological constant

$$\rho_{vac} \approx (10^{-3}\mathrm{eV})^4 \qquad m < 10^{-3}\ \mathrm{eV}$$

electron $\rho_{vac} \approx m_e^4 = (0.5\ \mathrm{MeV})^4$

if $m = m_{pl}, \quad \rho_{vac} = m_{pl}^4 = 10^{120}\rho_\Lambda^{obs}$

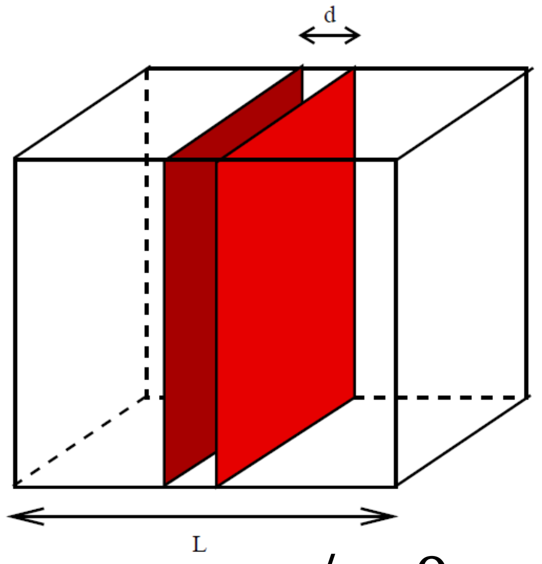
- Phase transition

vacuum energy change by phase transitions

electroweak $\Delta\rho_{vac} \approx (200\ \mathrm{GeV})^4$

QCD $\Delta\rho_{vac} \approx (0.3\ \mathrm{GeV})^4$

Is vacuum energy real?



- Casimir energy

$$\phi(\vec{x}) = \phi(\vec{x} + L\vec{n}), \quad \vec{p} = \left(\frac{n\pi}{d}, p_y, p_z\right), \quad n = 1, 2, 3...$$

$$\phi = 0$$

zero-point energy per unit area

$$\frac{E(d)}{A} = \sum_{n=1}^{\infty} \int \frac{dp_y dp_z}{(2\pi)^2} \left[\frac{1}{2}\sqrt{\left(\frac{n\pi}{d}\right)^2 + p_y^2 + p_z^2}\right] F_{reg}(a), \quad F_{reg}(a) = e^{-a\sqrt{\left(\frac{n\pi}{d}\right)^2 + p_y^2 + p_z^2}}$$

total energy $E_{tot}(d) = E(L-d) + E(d)$ depend on d and diverges as $a \to 0$ but the force between the two plates is finite

$$F = -\frac{1}{A}\frac{\partial E_{tot}(d)}{\partial d} = -\frac{\hbar c \pi^2}{480 d^4}$$

Old cosmological constant problem

- Zero-point energy is not important in quantum field theory in flat spacetime (cf. Casimir force is determined by $\partial E(d)/\partial d$ not $E(d)$)

- In GR, matter curves spacetime including vacuum energy.

$$\rho_{vac} \approx m_e^4 = (0.5\ \mathrm{MeV})^4 \qquad H \approx \frac{m_e^2}{m} = (10^6\,\mathrm{km})^{-1}$$

- Fine tuning

$$\Lambda_{obs} = \Lambda_{vacuum} + \Lambda \qquad S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(R - 2\Lambda\right)$$

vacuum energy is very sensitive to UV physics thus tuning is not stable under radiative corrections

Many attempts

- Symmetry

supersymmetry

$$\rho_{vac} \approx \sum_i \frac{g_i}{32\pi^2} m_i^4 \ln\left(\frac{m_i}{p_{\max}}\right) \qquad \mathrm{g}_{boson} = -g_{fermion} \qquad \rho_{vac} = 0$$

but we know supersymmetry is broken at high energies $M_{SUSY} > \mathrm{TeV}, \quad \rho_{vac} = O(\mathrm{TeV}^4)$

Naturalness

If the theory has an enhanced symmetry with $\Lambda = 0$ that is valid at quantum level, the small $\Lambda_{obs} = \Lambda_{vacuum} + \Lambda$ is technically natural as quantum corrections arise only from non-zero Λ

New cosmological constant problem

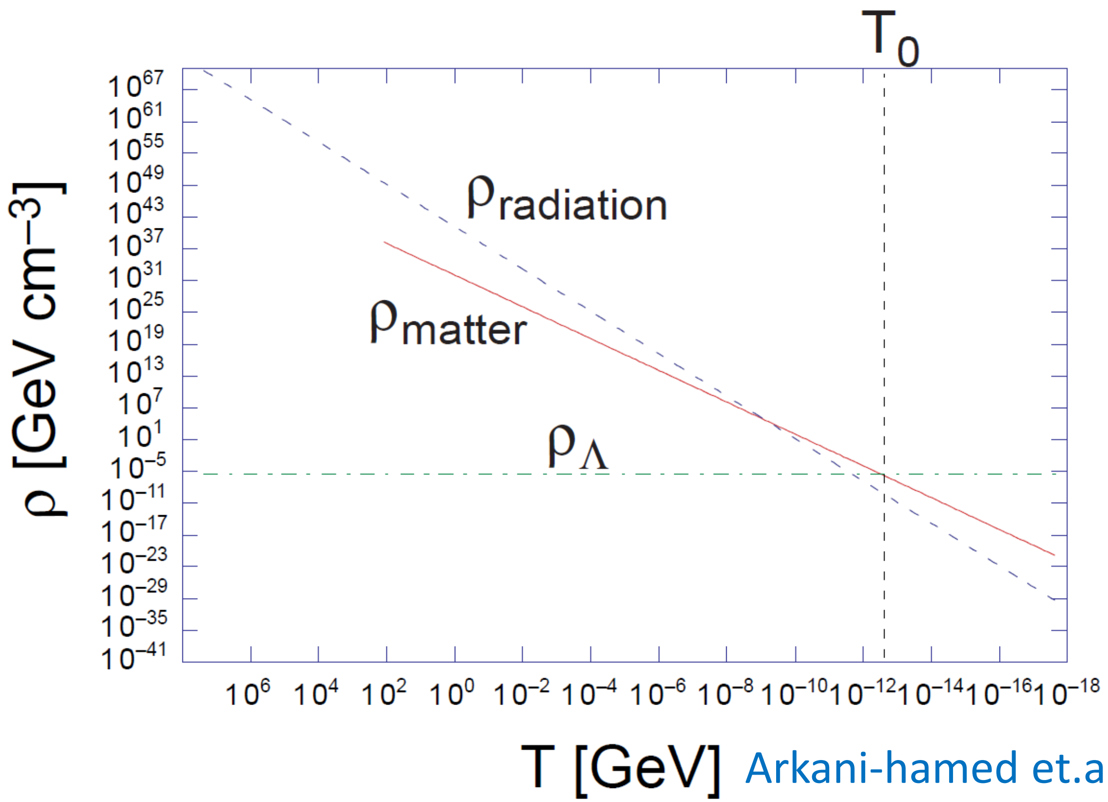
- Assume that the old cosmological constant is solved, we then need to explain why the expansion of the Universe appears to be accelerating now

- Coincidence problem

  why $\Omega_\Lambda \approx \Omega_m$

  anthropic principle

  otherwise we don't exist



Arkani-hamed et.al. astro-ph/0005111

So, we should bother!

We know vacuum energy exists, but it does not gravitate in the way it should in GR. It is important to know whether the acceleration of the Universe is caused by the (fine-tuned) cosmological constant or not.

It is important to reconsider all the assumptions:

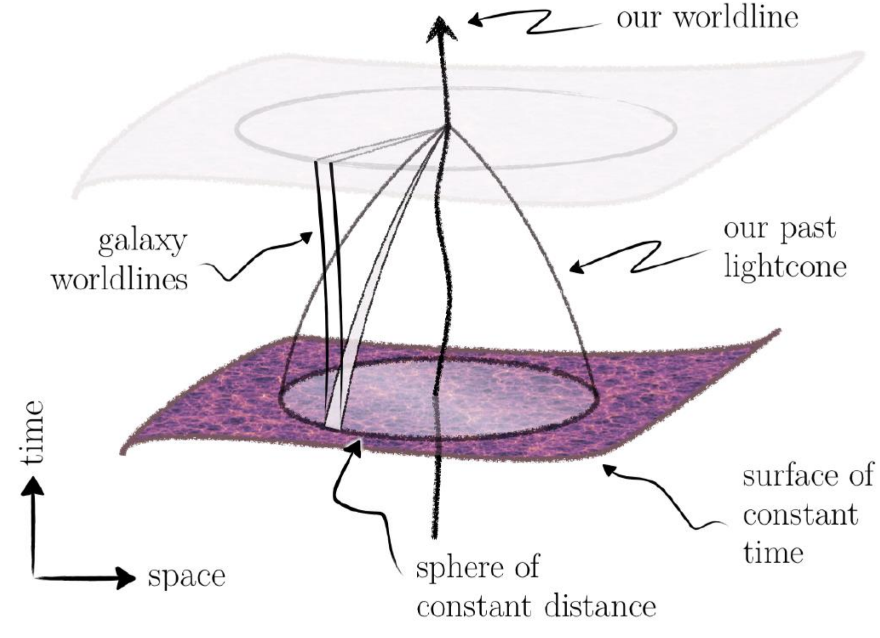
1. Homogeneity and Isotropy
2. General Relativity
3. Matter content of the Universe

Assumption (1)

1. The Copernican principle: we are not at a special location in the universe
2. The cosmological principle: on large scales, the universe is homogeneous and isotropic

- FRW metric

If all observers measure isotropic distance-redshift relation, then the spacetime is FRW

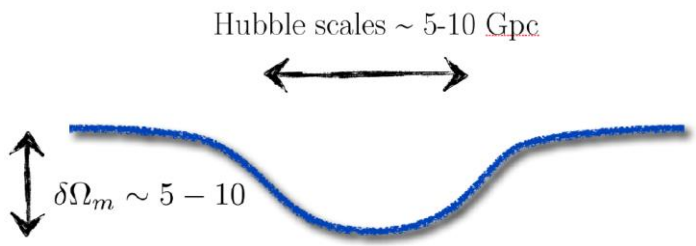


We need the Copernican principle to show the cosmological principle but this is hard to test

Relativistic Cosmology Ellis, Maartens, MacCallum

Clarkson 1204.5505

Assumption (1)

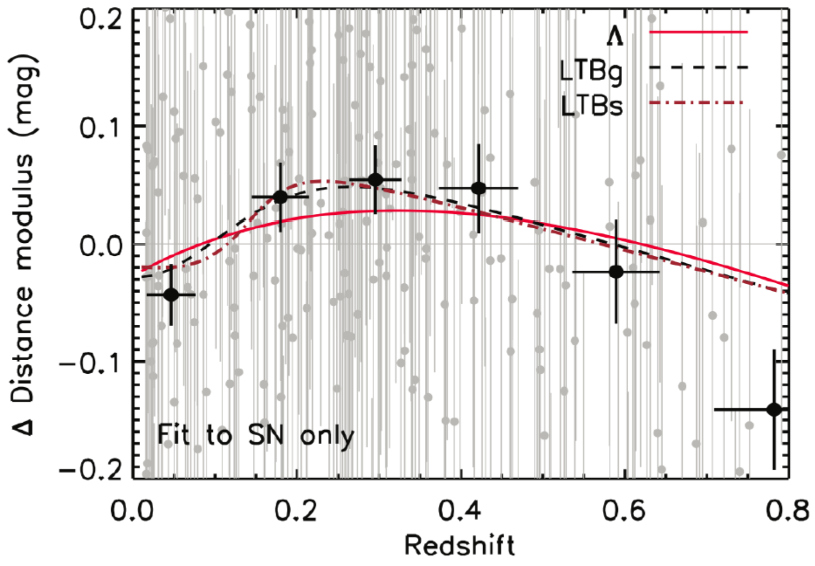


- Void models

If we happen to live inside a void with low densities, the expansion of the universe appears to be accelerating

ex.) Lemaitre-Tolman-Bondi model

$$ds^2 = -dt^2 + \frac{a_{//}{}^2(t,r)}{1-K(r)r^2}dr^2 + a_{\perp}{}^2(t,r)r^2 d\Omega^2$$



a simple mode is ruled out

low H_0, radial velocities of clusters

(kinetic Sunyaev-Zeldovich effect)

Clarkson 1204.5505

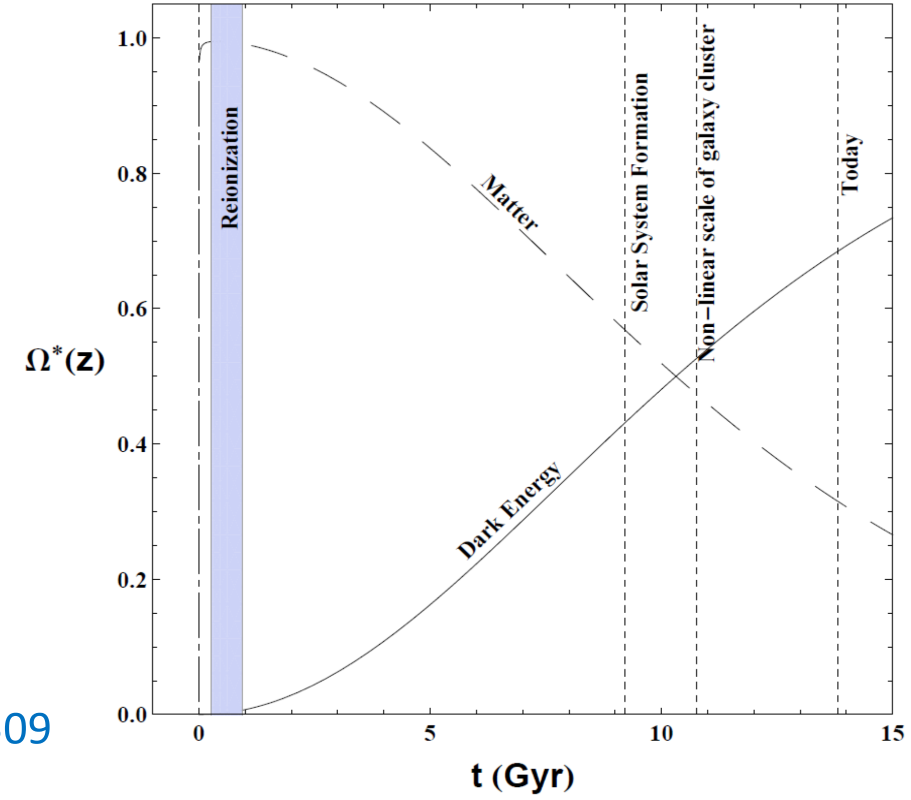
Back-reaction

- The Universe becomes inhomogeneous at late time. If the back-reaction of these imhomogeneities cause the acceleration, we can solve the coincident problem.

long-standing debates on the magnitude of the effect on the expansion of the Universe from small scale inhomogeneity.

It is difficult to explain the acceleration although it is also difficult to prove this rigolously.

Velten. et.al. 1410.2509



Assumption (2)

- Why we believe in general relativity?

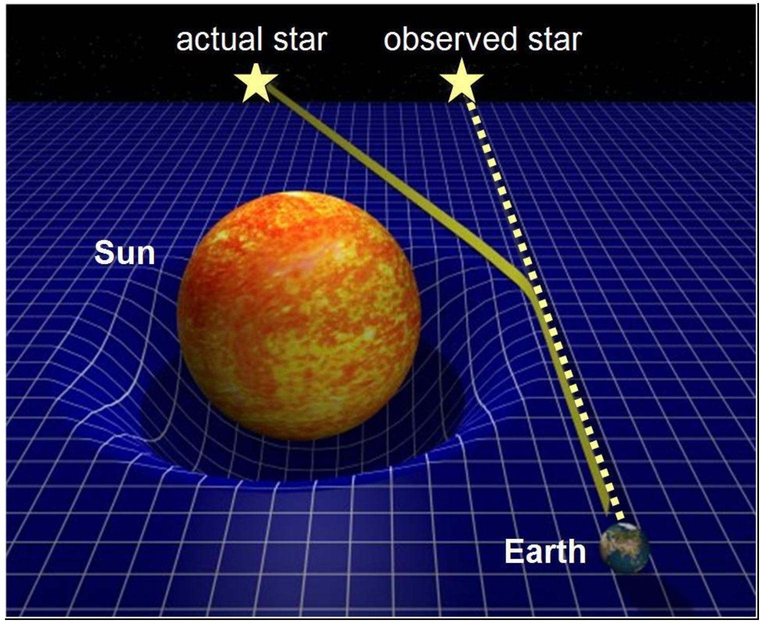
  - Observational point of view

    GR is tested to very high accuracies by solar system experiments and pulsar timing measurements   Will gr-qc/0510072

  - Theoretical point of view

    GR is the unique metric theory in 4D that gives second order differential equations

Solar system tests



- Post-Newtonian parameter

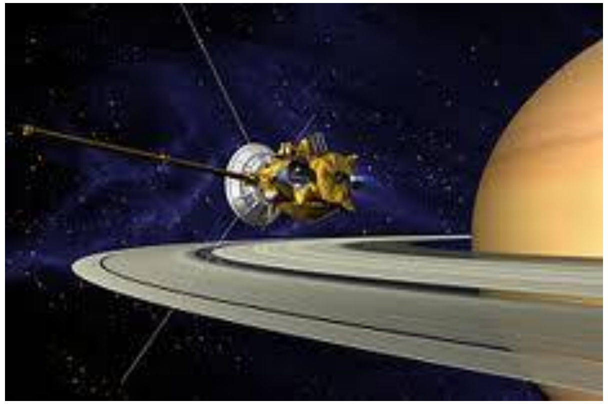
$$g_{00} = -1 + 2GU \qquad U = \frac{M}{r} \qquad \gamma = 0: \text{ "Newtonian"}$$

$$g_{ij} = \delta_{ij}(1 + 2\gamma GU) \qquad \gamma = 1 \; : \; \text{GR}$$

- Bending of lights

$$\theta = 2(1+\gamma)\frac{M_\odot}{r} = \frac{1+\gamma}{2}\theta_{GR}$$

$$\theta = (0.99992 \pm 0.00023) \times 1.75'' \qquad \gamma - 1 = (-1.7 \pm 4.5) \times 10^{-4}$$



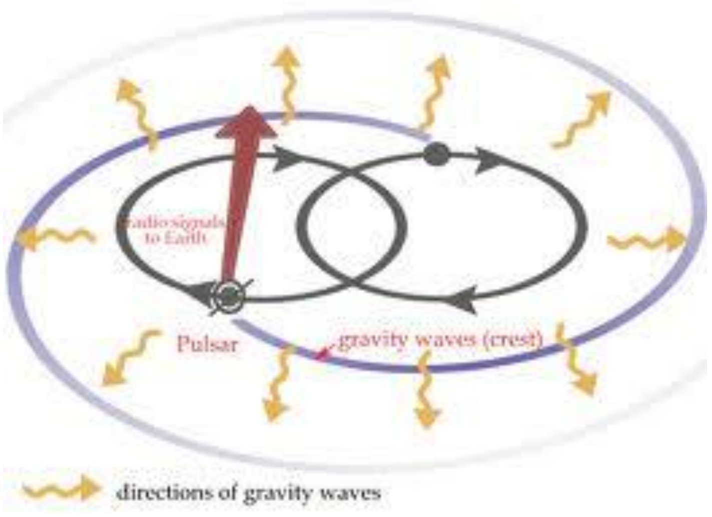
- Shapiro time delay

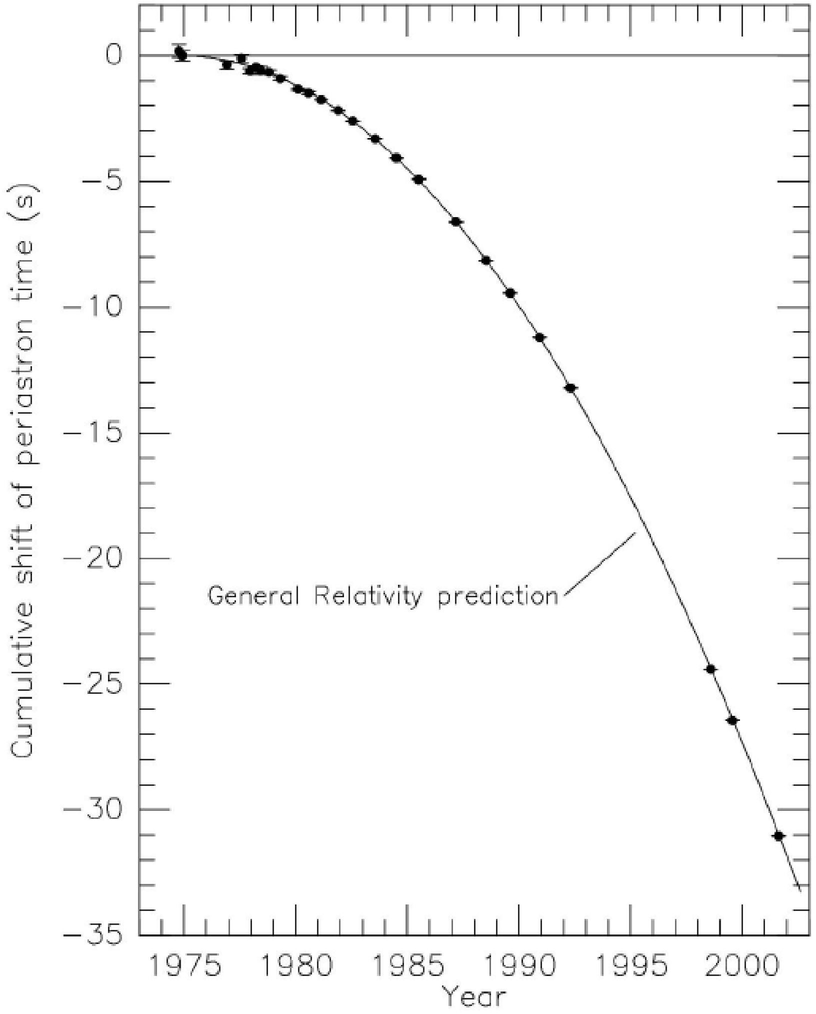
$$\Delta t = (1.00001 \pm 0.0001) \times \Delta t_{GR} \qquad \gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$$

Pulsar timing

- Hulse & Taylor binary pulsar

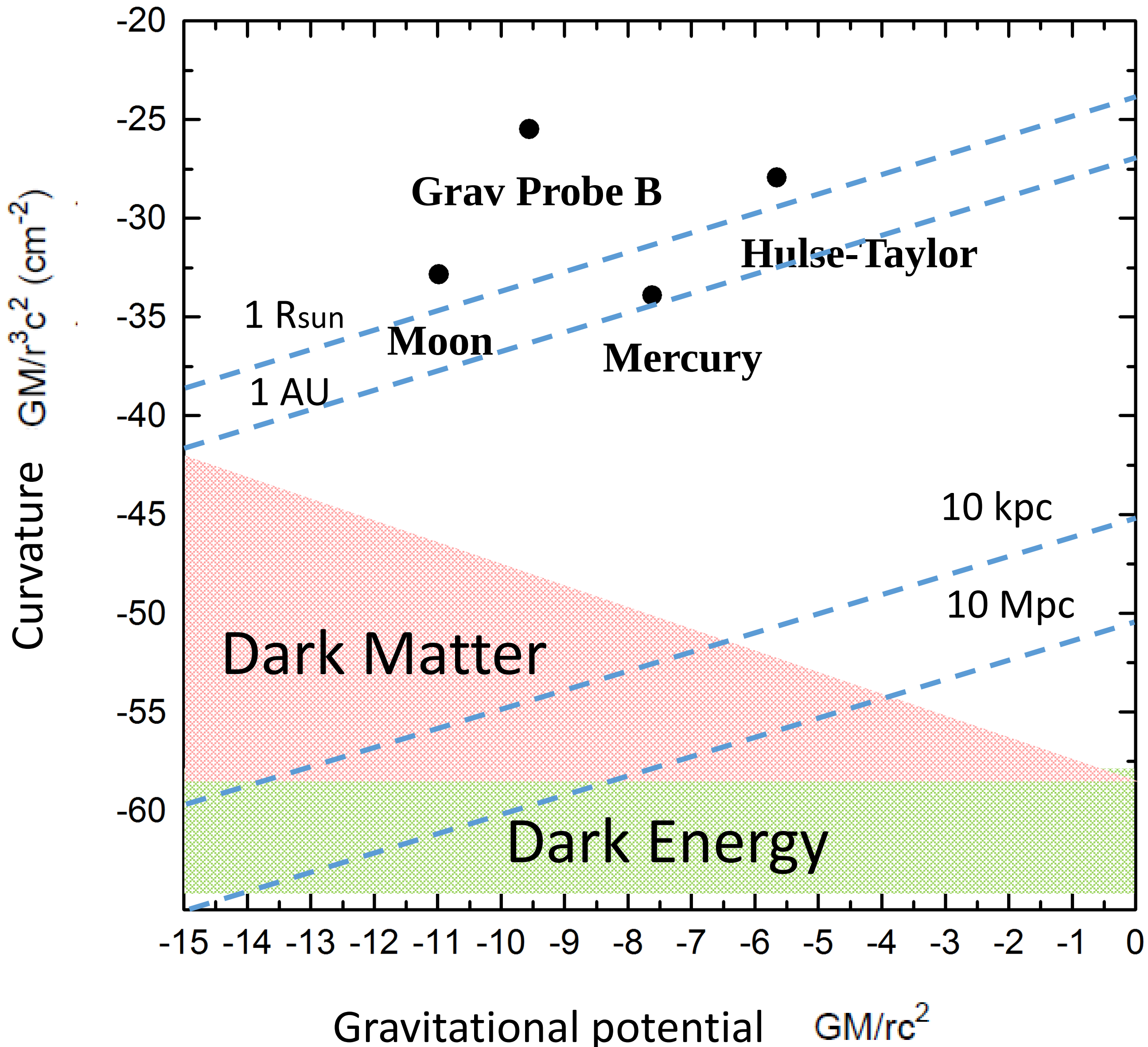
  Orbital decay due to gravitational waves perfectly agrees with GR prediction





Tests of GR

Psaltis Living Rev. Relativity 11 (2008), 9
Baker et.al. ApJ 802 63 (2015)

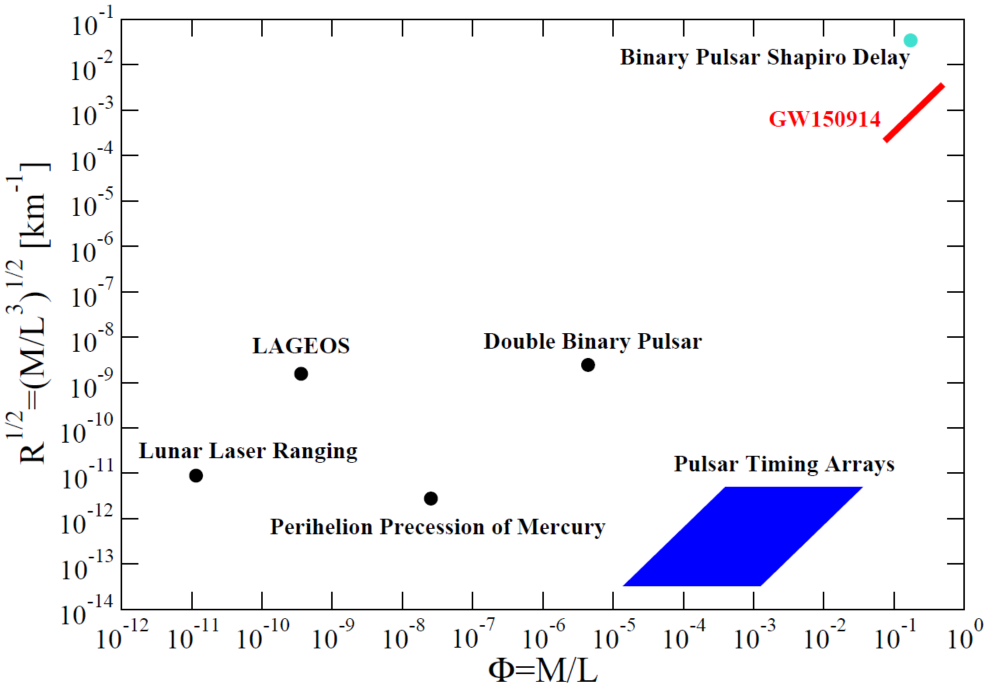


curvature

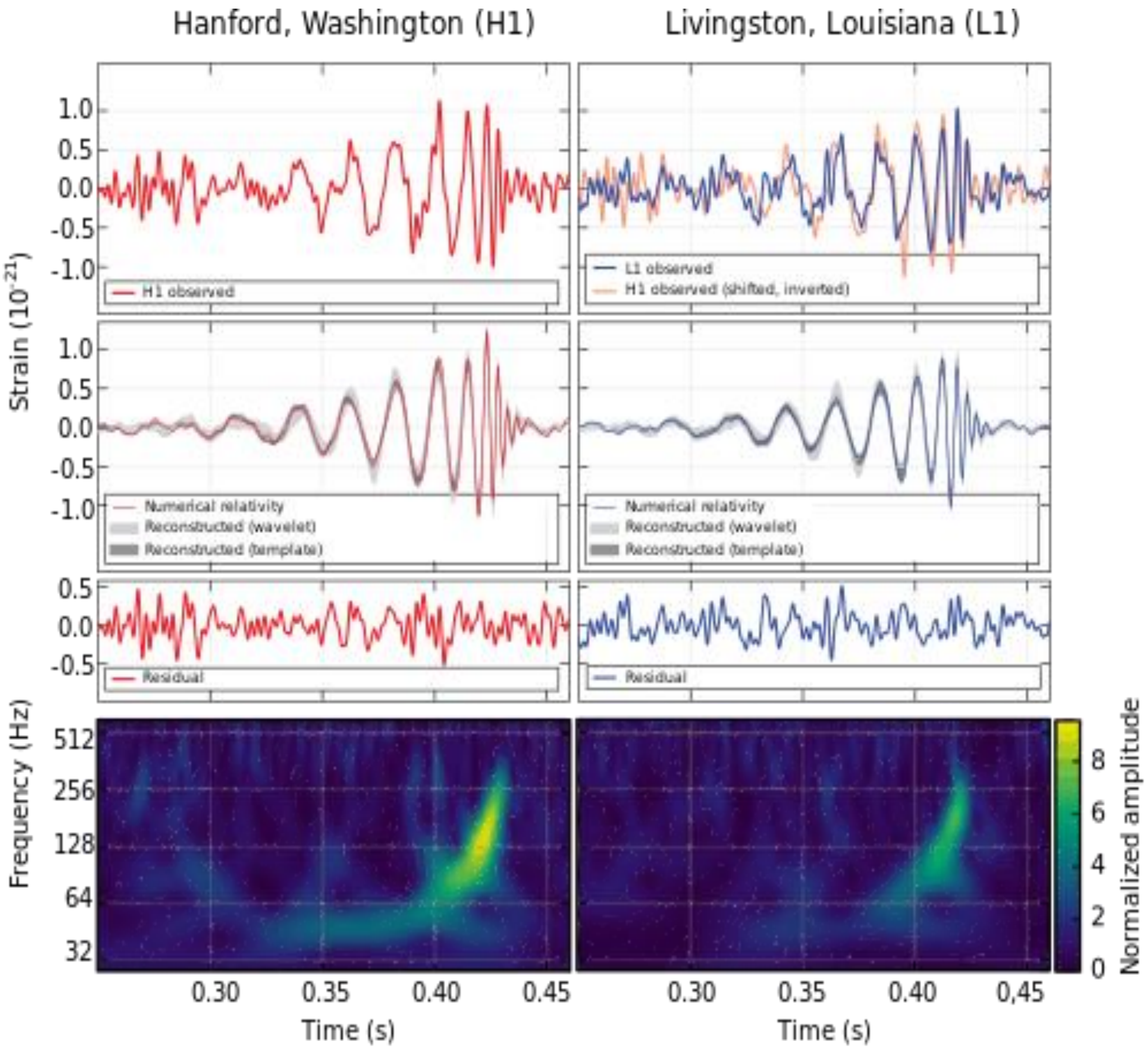
$$R = \frac{GM}{r^3 c^2}$$

potential

$$\Psi = \frac{GM}{rc^2}$$



Yunes et.al. 1603.08955



LIGO collaboration

Assumption (3)

- What is dark energy

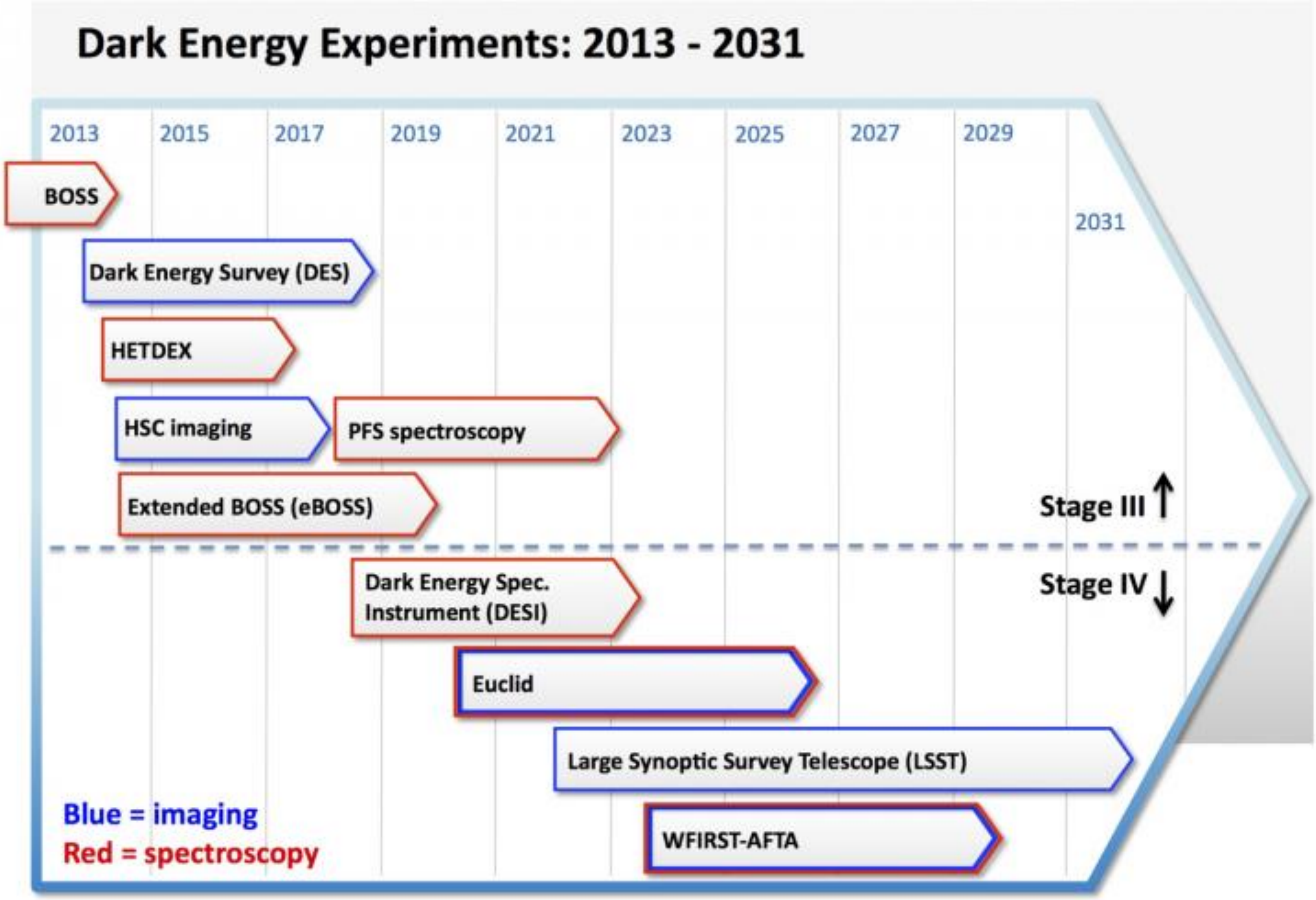
In the background, all information is encoded in the equation of state

$$w_{DE} = \frac{P_{DE}}{\rho_{DE}}$$

what are the candidates for dynamical DE $w_{DE}(z)$

- How to distinguish between DE and modifications of gravity

$$G_{\mu\nu} + G_{\mu\nu}^{MG} = 8\pi G\left(T_{\mu\nu} + T_{\mu\nu}^{DE}\right)$$

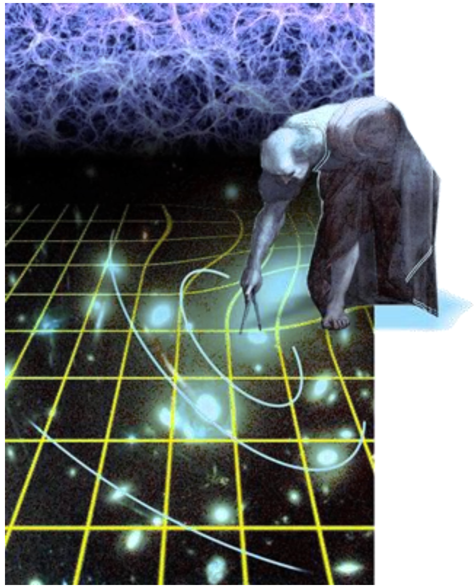


Abazajian et.al. Dark Energy and CMB

Euclid (2020-)

http://sci.esa.int/euclid/

R-L0	The Euclid Mission will by itself allow us to • understand the nature of the apparent acceleration of the Universe and • test gravity on cosmological scales from the measurement of the cosmic expansion history and the growth rate of structures.
R-L0.1	To determine the nature of the apparent acceleration, Euclid will distinguish effects produced by a cosmological constant from those produced by a dynamical dark energy. This must be done by achieving a minimum *FoM*>400 from Euclid data alone.
R-L0.2	To experience effects of gravity on cosmological scales, Euclid will probe the growth of structure and will separately constrain the two relativistic potentials, Ψ and Φ. This can be done by achieving an absolute 1σ precision of 0.02 on the growth index, γ, from Euclid data alone.



Models of dark energy and modified gravity

Kazuya Koyama

University of Portsmouth

Quintessence

- Scalar field

We now know a scale field exists (i.e. Higgs)

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R + S_\phi$$

$$S_\phi = \int d^4x\left[-\frac{1}{2}g^{\mu\nu}\left(\partial_\mu\phi\right)\left(\partial_\nu\phi\right) - V(\phi)\right]$$

energy momentum tensor

$$T^\phi_{\mu\nu} = \left(\partial_\mu\phi\right)\left(\partial_\nu\phi\right) - g_{\mu\nu}\left[\frac{1}{2}g^{\alpha\beta}\left(\partial_\alpha\phi\right)\left(\partial_\beta\phi\right) + V(\phi)\right] \qquad T^\phi_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_\phi)}{\delta g^{\mu\nu}}$$

scalar field equation

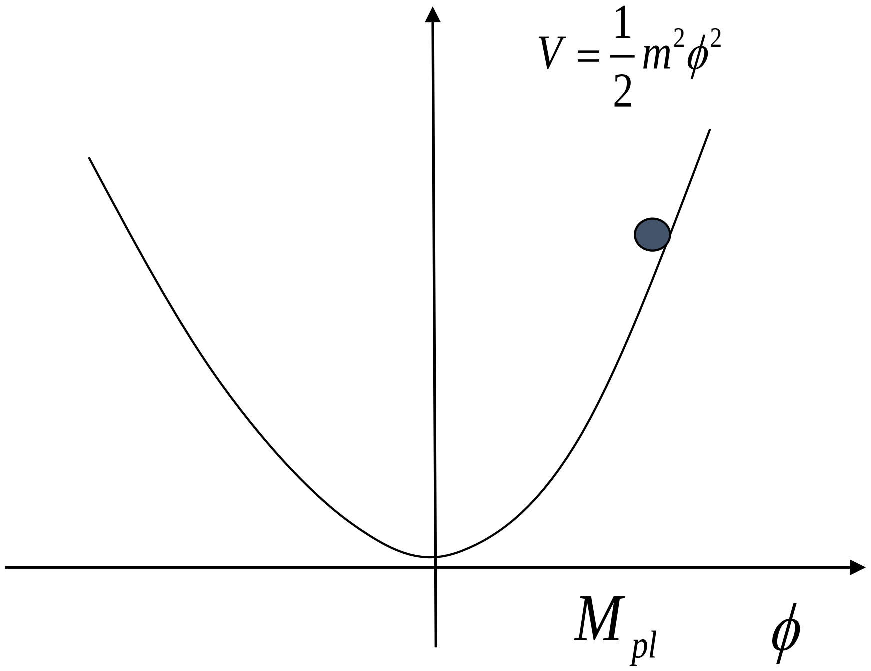
$$\frac{\delta S_\phi}{\delta\phi} = 0 \qquad \nabla^\mu\nabla_\mu\phi - V'(\phi) = 0 \quad \Longleftrightarrow \quad \nabla^\mu T^\phi_{\mu\nu} = 0$$

Background

- Energy density and pressure

$$\rho_\phi = -T_0^0 = \frac{1}{2}\dot{\phi}^2 + V(\phi) \qquad \phi = \phi(t)$$

$$P_\phi = \frac{1}{3}T_i^i = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$



- Scalar field equation

$$\ddot{\phi} + \underbrace{\boxed{3H\dot{\phi}}}_{\text{friction}} + V'(\phi) = 0 \quad \Longleftrightarrow \quad \dot{\rho}_\phi + 3H(\rho_\phi + P_\phi) = 0$$

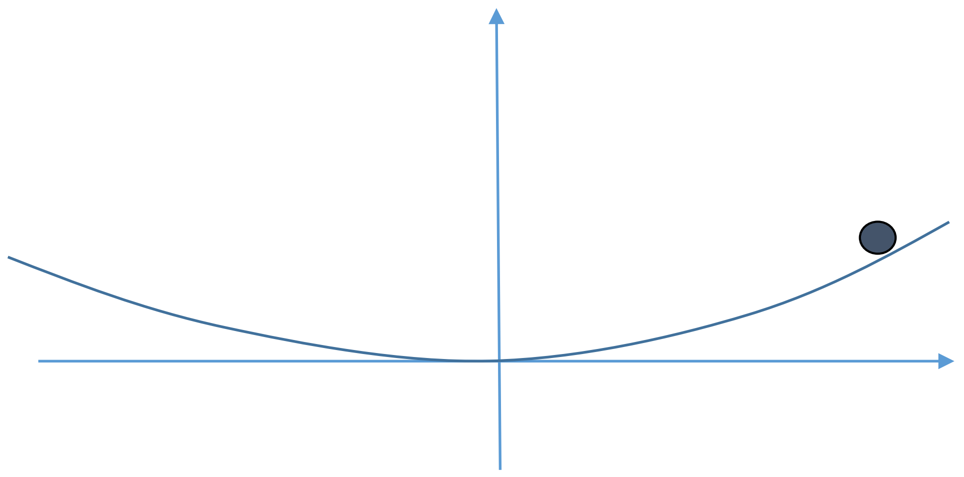
to realise the acceleration, the scalar filed needs to slow-roll the potential

$$\dot{\phi}^2 << V(\phi) \qquad w_\phi = \frac{P_\phi}{\rho_\phi} \approx -1$$

Equation of state

- Equation of state

$$1+w_\phi = \frac{\dot{\phi}^2}{\rho_\phi} = \frac{V'^2}{9H^2(1+\xi_\phi)^2\rho_\phi} > 0, \quad \xi_\phi = \frac{\ddot{\phi}}{3H\dot{\phi}}$$



For $w_\phi \approx -1, \quad \xi_\phi << 1, \quad \rho_\phi \approx V$

$$H^2 = \frac{\kappa^2}{3}(\rho_\phi + \rho_m)$$

$$1+w_\phi \approx \frac{2}{3}\varepsilon_\phi \Omega_\phi(a) \qquad \varepsilon_\phi = \frac{1}{2\kappa^2}\left(\frac{V'}{V}\right)^2, \quad \Omega_\phi = \frac{\rho_\phi}{\rho_\phi + \rho_m} \qquad \kappa^2 = 8\pi G = \frac{1}{M_{pl}^2}$$

Analogous to slow-roll approximations for inflation but the scalar field does not completely dominate the energy density

Quintessence potentials

- Freezing models

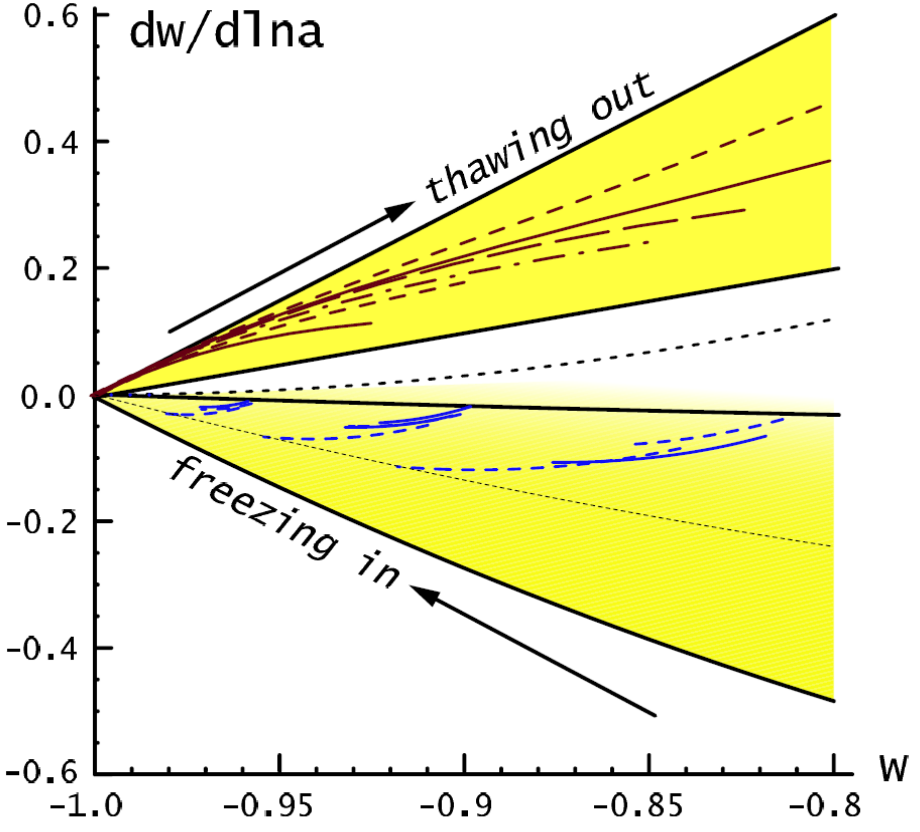
$$V(\phi) = M^{4+n}\phi^{-n}$$

The scalar field dynamics freezes at late time and approaches $w_\phi \approx -1$

- Thawing models

$$V(\phi) = M^4 \cos^2\left(\frac{\phi}{f}\right)$$

The scalar field is initially in the regions of $w_\phi \approx -1$ and later starts to roll down the potential



Linder & Caldwell astro-ph/0505494

Scaling solution

- Tracker solution

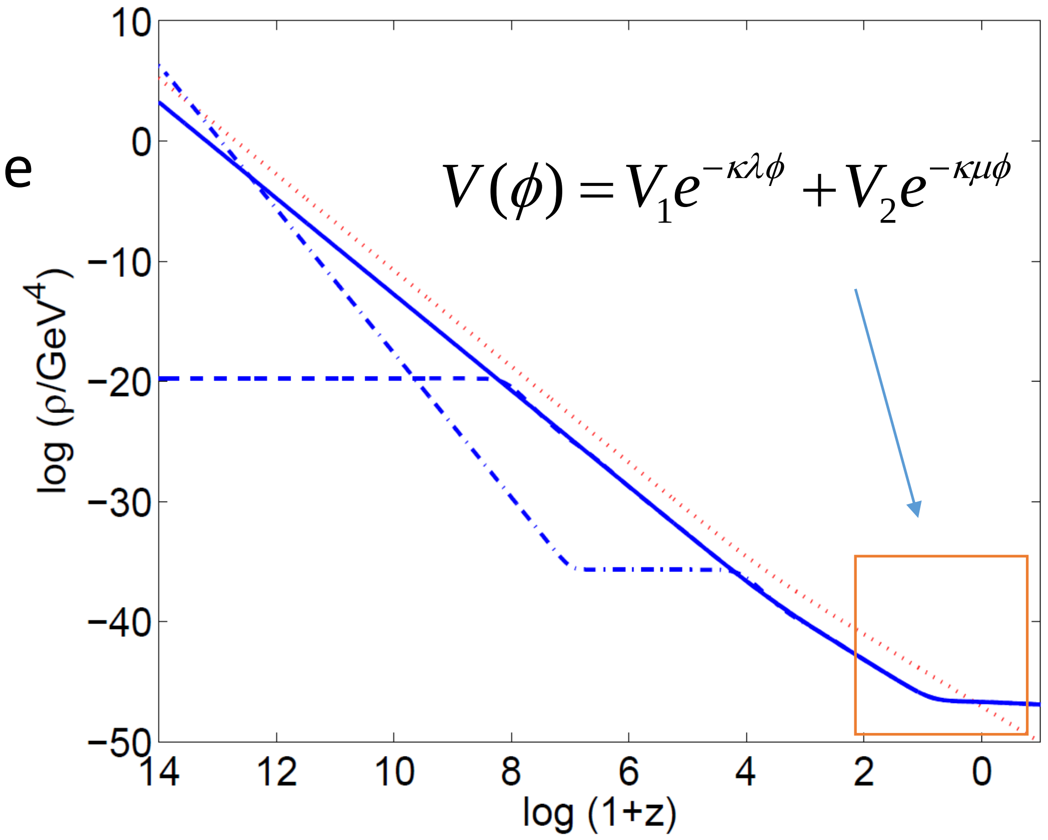
For some potentials, the energy density of the scalar field tracks that of matter/radiation

$$V(\phi) = V_0\, e^{-\kappa\lambda\phi}, \quad \kappa^2 = 8\pi G = \frac{1}{M_{pl}^2}$$

If $\lambda^2 > 3(1+w_M)$

there is an attractor where

$$w_\phi = w_M, \quad \Omega_\phi = \frac{3(1+w_M)}{\lambda^2}$$



Barreiro et.al. astro-ph/9910214

The late time solution is insensitive to the initial density of the scalar field (however we need $\lambda^2 < 2$ to have an accelerating solution)

Fundamental issues

- Consider a massive scalar field

$$V(\phi)=\frac{1}{2}m^2\phi^2 \qquad \varepsilon_\phi=\frac{1}{2\kappa^2}\left(\frac{V'}{V}\right)^2<<1 \quad \rightarrow \quad \phi>M_{pl}=10^{18}\,\mathrm{GeV}$$

$$V(\phi_0)=\rho_{DE}=10^{-48}\,\mathrm{GeV}^4 \quad \rightarrow \quad m=\left(\frac{\rho_{DE}}{\phi^2}\right)^{\frac{1}{2}}=10^{-42}\,\mathrm{GeV}\approx H_0$$

Correction to the potential

$$V(\phi)=\frac{1}{4}\lambda\phi^4<\rho_{DE} \quad \rightarrow \quad \lambda<10^{-120}$$

these small numbers are very difficult to protect against quantum corrections

Modified Gravity

- Lovelock theorem

4D theory of metric and the equations of motion is given by

$$S_{\mu\nu}(g) = \kappa^2 T_{\mu\nu}$$

we require $S_{\mu\nu}(g)$ is

1. a tensor made from the metric and its first and second derivatives
2. symmetric
3. divergence free

$$\Rightarrow \quad S_{\mu\nu}(g) = a\, G_{\mu\nu} + b\, g_{\mu\nu}$$

Beyond Einstein

- Lovelock theorem indicates that if you modify Einstein theory then you must do one or more of the following
  - Consider other fields (scalars, vectors, etc)
  - Accept higher derivatives
  - Work in a space with more than four dimensions
  - Non-local gravity, metric-affine gravity, …

Higher derivative theories

- Ostrogradski ghost

Theories with higher derivatives $L(g,\dot{g},\ddot{g})$

$$\frac{d^2}{d^2t}\left(\frac{\partial L}{\partial \ddot{g}}\right)-\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{g}}\right)+\left(\frac{\partial L}{\partial g}\right)=0$$

We treat $\dot{g}$ as a new variable and $\ddot{g}$ as a new velocity (we need four initial conditions)

If the velocity $\ddot{g}$ can be expressed in terms of momenta $\partial L/\partial \ddot{g}$ (non-degeneracy condition), it can be shown that the Hamiltonian is not bounded from below Woodard astro-ph/0601672

f(R) gravity

- Generalise the Einstein-Hilbert action

$$S=\int d^4x\sqrt{-g}F(R)+S_m\left[g\right],\quad F(R)=R+f(R)$$

- The equations of motion is fourth order

$$F'(R)R_{\mu\nu}-\frac{1}{2}F(R)g_{\mu\nu}-\left(\nabla_\mu\nabla_\nu+g_{\mu\nu}\Box\right)F'(R)=\kappa^2T_{\mu\nu}$$

but this theory violates the assumption of Ostrogradski theorem (i.e. non-degeneracy)

In fact the theory can be rewritten as scalar-tensor gravity

$$S=\int d^4x\sqrt{-g}\left(F(\phi)+(R-\phi)F'(\phi)\right)\qquad \frac{\delta S}{\delta\phi}=(R-\phi)F''(\phi)=0 \;\Rightarrow\; \phi=R\ \ (F''(R)\neq 0)$$

$$S=\int d^4x\left(\psi R-V(\psi)\right)\qquad \psi=F'(\phi),\quad V=RF'-F$$

Brans-Dicke gravity

- Brans-Dicke (BD) gravity (Jordon frame)

$$S = \int d^4x\sqrt{-g}\left(\psi R - \frac{\omega_{BD}}{\psi}\left(\nabla\psi\right)^2 + V(\psi)\right) + S_M[g_{\mu\nu}]$$

f(R) gravity $\omega_{BD} = 0$

- Einstein frame

Conformal transformation $\tilde{g}_{\mu\nu} = \frac{2\psi}{M_{pl}^2} g_{\mu\nu}, \quad \frac{\phi}{M_{pl}} = \frac{1}{2\alpha}\ln\left(\frac{2\psi}{M_{pl}^2}\right), \quad 2w_{BD} + 3 = 1/2\alpha^2$

$$S = \int d^4x\sqrt{-\tilde{g}}\left(\frac{M_{pl}^2}{2}\tilde{R} - \frac{1}{2}\left(\tilde{\nabla}\phi\right)^2 - \tilde{V}(\phi)\right) + S_M[A^2(\phi)\tilde{g}_{\mu\nu}] \qquad A(\phi) = \exp\left(\frac{\alpha\,\phi}{M_{pl}}\right)$$

the scalar field is coupled to matter

Horndeski theory

- The most general scalar tensor theory with 2nd order equations of motion

$$ {}^{H}S = \int d^4x \sqrt{-g} \left({}^{H}\mathcal{L}_2 + {}^{H}\mathcal{L}_3 + {}^{H}\mathcal{L}_4 + {}^{H}\mathcal{L}_5 \right) $$

$$ \begin{aligned} {}^{H}\mathcal{L}_2 &= K(\phi, X)\,, \\ {}^{H}\mathcal{L}_3 &= G_3(\phi, X)\Box\phi\,, \\ {}^{H}\mathcal{L}_4 &= G_4(\phi, X)R - 2G_{4X}(\phi, X)\left[(\Box\phi)^2 - \phi_{\mu\nu}\phi^{\mu\nu}\right]\,, \\ {}^{H}\mathcal{L}_5 &= G_5(\phi, X)G^{\mu\nu}\phi_{\mu\nu} + \frac{1}{3}G_{5X}(\phi, X)\left[(\Box\phi)^3 - 3\Box\phi\,\phi_{\mu\nu}\phi^{\mu\nu} + 2\,\phi_{\mu\nu}\phi^{\nu\rho}\phi^{\mu}_{\rho}\right] \end{aligned} $$

$$ X \equiv \partial_\mu\phi\,\partial^\mu\phi \qquad \phi_{\mu\nu} \equiv \nabla_\mu\nabla_\nu\phi $$

Dvali-Gadadze-Porrati (DGP) braneworld model

- 5D braneworld model

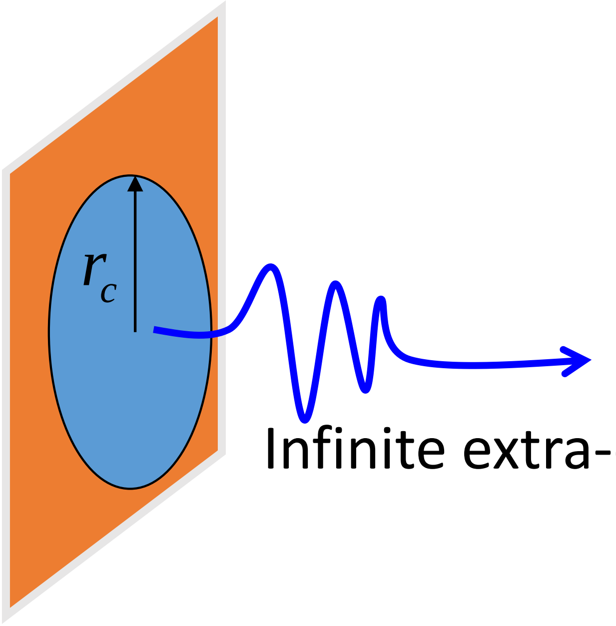
  Standard model particles are confined to a 4D brane in a 5D bulk spacetime

$$S = \frac{1}{32\pi G r_c}\int d^5x\sqrt{-g_{(5)}}R_{(5)} + \frac{1}{16\pi G}\int d^4x\sqrt{-g}R + \int d^4x\sqrt{-g}L_m$$

cross over scale r_c

$$\pm\frac{H}{r_c} = H^2 - \frac{8\pi G}{3}\rho$$

in the + branch, the expansion of the universe accelerates without the cosmological constant (self-accelerating solution)



Infinite extra-dimension

Dynamics of dark energy
hep-th/0603057

Modified Gravity and Cosmology
arXiv:1106.2476

Summary

- DE/MG models

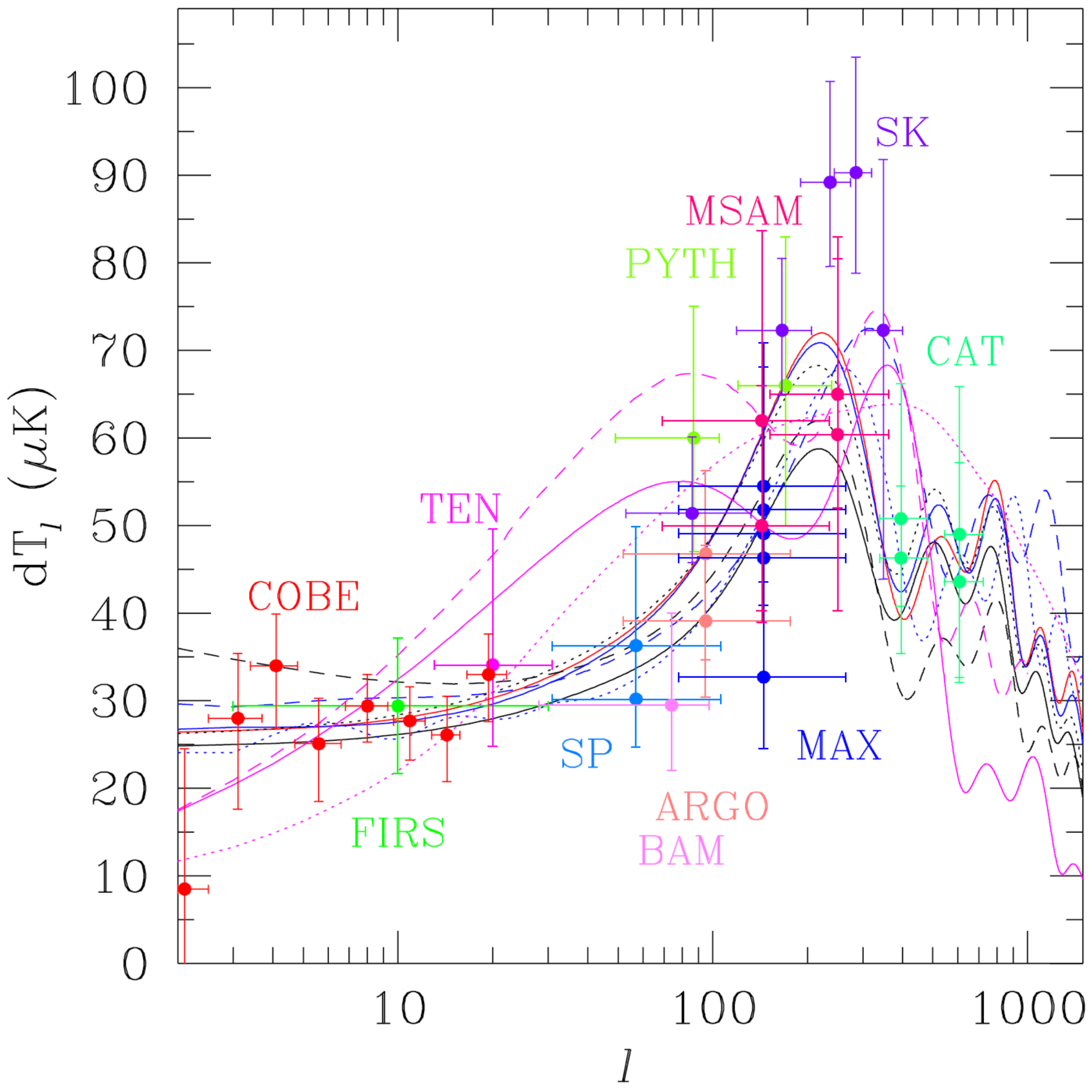
There are many models but we still do not have a compelling alternative to LCDM

Observation may give us a clue:

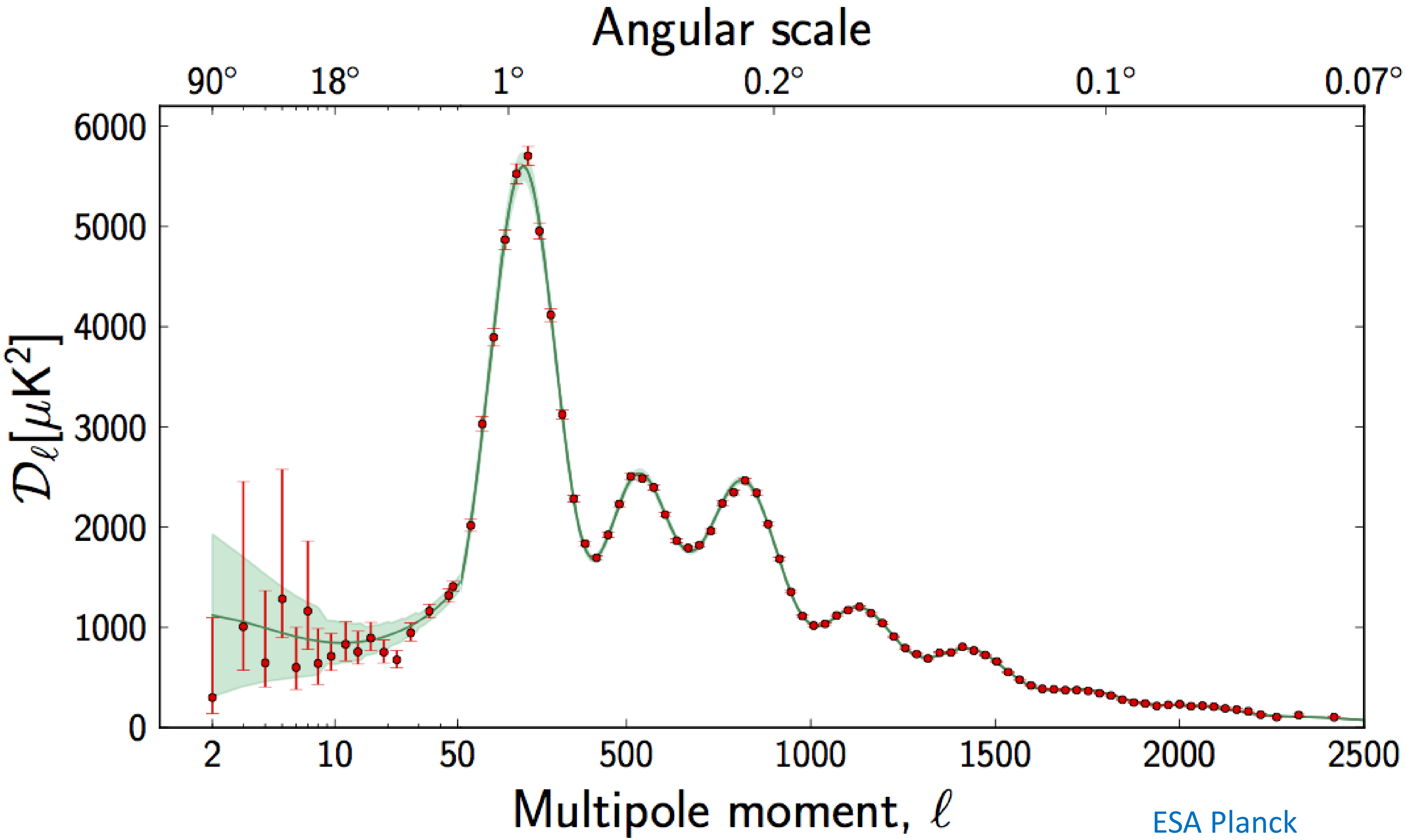
Is it the cosmological constant or a light degree of freedom?

Does it cluster or couple to matter?

cf. CMB (before WMAP)



Gawiser & Silk astro-ph/9806197


Angular scale
90°
18°
1°
0.2°
0.1°
0.07°
6000
5000
4000
3000
2000
1000
0
$\mathcal{D}_\ell[\mu K^2]$
2
10
50
500
1000
1500
2000
2500
Multipole moment, ℓ

ESA Planck

GW170817 with GRB170817A

- Almost simultaneous detections of GWs and gamma ray bursts from a neutron star merger put a strong constraint

$$\frac{c_{GW}}{c} - 1 < 10^{-15}$$

- implications for Horndeski theories

without fine-tuning

$$G_{4X} = G_{5X} = G_{5\phi} = 0$$

$$^{H}\mathcal{L}_2 = K(\phi, X)\,,$$

$$^{H}\mathcal{L}_3 = G_3(\phi, X)\Box\phi\,,$$

$$^{H}\mathcal{L}_4 = G_4(\phi, \cancel{X})R - \cancel{2G_{4X}(\phi, X)\left[(\Box\phi)^2 - \phi_{\mu\nu}\phi^{\mu\nu}\right]}\,,$$

$$\cancel{^{H}\mathcal{L}_5 = G_5(\phi, X)G^{\mu\nu}\phi_{\mu\nu} + \frac{1}{3}G_{5X}(\phi, X)\left[(\Box\phi)^3 - 3\Box\phi\,\phi_{\mu\nu}\phi^{\mu\nu} + 2\,\phi_{\mu\nu}\phi^{\nu\rho}\phi_{\rho}^{\mu}}$$

Fundamental issues

- Theoretical consistency

  modified gravity models often suffer from instabilities

  (cf. the self-accelerating solution in DGP model suffers from ghost instabilities)

- Small numbers

  cf. viable f(R) models

$$F(R) = R - 2\Lambda_{eff} - \frac{R_0^{n+1}}{R^n}, \quad \Lambda_{eff} \approx R_0 \approx H_0^2$$

DGP models $r_c \approx H_0^{-1}$

Ultra-light scalar field

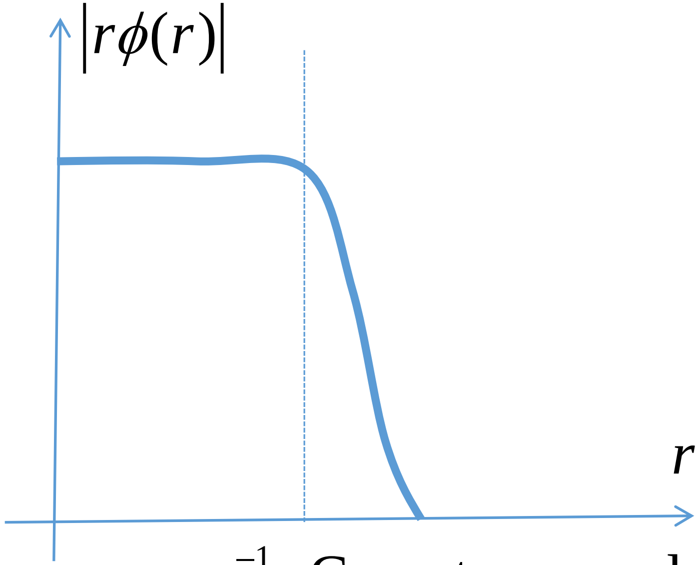
- In most of DE/MG models, there exists a scalar field with tiny mass

$$S = \int d^4x \left[-\frac{1}{2}\left(\partial_\mu \phi\right)\left(\partial^\mu \phi\right) - \frac{1}{2} m^2 \phi^2 \right] - \frac{\alpha}{M_{pl}} \int d^4x \, \phi \, \rho$$

static source

$$(\nabla^2 - m^2)\phi = \frac{\alpha}{M_{pl}} \rho, \qquad \frac{\phi}{M_{pl}} = -\left(\frac{\alpha}{4\pi M_{pl}^2}\right)\frac{M}{r} \exp(-m\,r)$$

$$\Rightarrow \begin{cases} \phi = -\left(\dfrac{\alpha}{4\pi M_{pl}^2}\right)\dfrac{M}{r}, & r < m^{-1} \\ \phi \to 0, & r > m^{-1} \end{cases}$$



m^{-1} : Compton wavelength

if $\quad m^{-1} \approx H_0^{-1} \quad$ the scalar field mediates a long range force

Fifth force

- Due to the coupling between the scalar and matter, the geodesic equation is modified as

$$\ddot{\vec{a}} = -\nabla\Psi - \frac{\alpha}{M_{pl}}\nabla\phi = F_G + \boxed{F_5}$$

Fifth force

$$F_5 = \frac{2\alpha^2}{8\pi M_{pl}^2}\frac{M}{r^2} \quad \text{for} \quad r < m^{-1}$$

$$F_G = \frac{1}{8\pi M_{pl}^2}\frac{M}{r^2}$$

Fifth force is strongly constrained in the solar system

$$\alpha^2 < 10^{-5}, \quad w_{BD} > 40,000 \quad (3 + 2w_{BD}) = 1/2\alpha^2$$

Way out

- Quintessence

  the coupling to matter is assumed to be zero $\alpha^2 = 0$

- Violation of weak equivalence principle

  the solar system constraints come from coupling to baryons $\alpha^2_{baryon} = 10^{-5}$

  the coupling to cold dark matter can be large $\alpha^2_{cdm} = O(1)$

  interacting dark energy

$$S = \int d^4 x \sqrt{-\tilde{g}} \left(\frac{M_{pl}^2}{2} \tilde{R} - \frac{1}{2} \left(\tilde{\nabla} \phi \right)^2 - \tilde{V}(\phi) \right) + S_{cdm}[A^2(\phi) \tilde{g}_{\mu\nu}] + S_{baryon}[\tilde{g}_{\mu\nu}]$$

Screening mechanism

- The coupling constant or mass need to be scale dependent

$$m\big|_{\cos mo} = O(H_0) \qquad m\big|_{local} >> H_0$$

$$\alpha^2\big|_{\cos mo} = O(1) \qquad \alpha^2\big|_{local} \to 0$$

- Two representative mechanisms

1. Chameleon mechanism
2. Vainshtein mechanism

Screening mechanism

Kazuya Koyama

University of Portsmouth

Screening mechanism

- The coupling constant or mass need to be scale dependent

$$m\big|_{\cos mo} = O(H_0) \qquad m\big|_{local} >> H_0$$

$$\alpha^2\big|_{\cos mo} = O(1) \qquad \alpha^2\big|_{local} \to 0$$

- Two representative mechanisms

1. Chameleon mechanism
2. Vainshtein mechanism

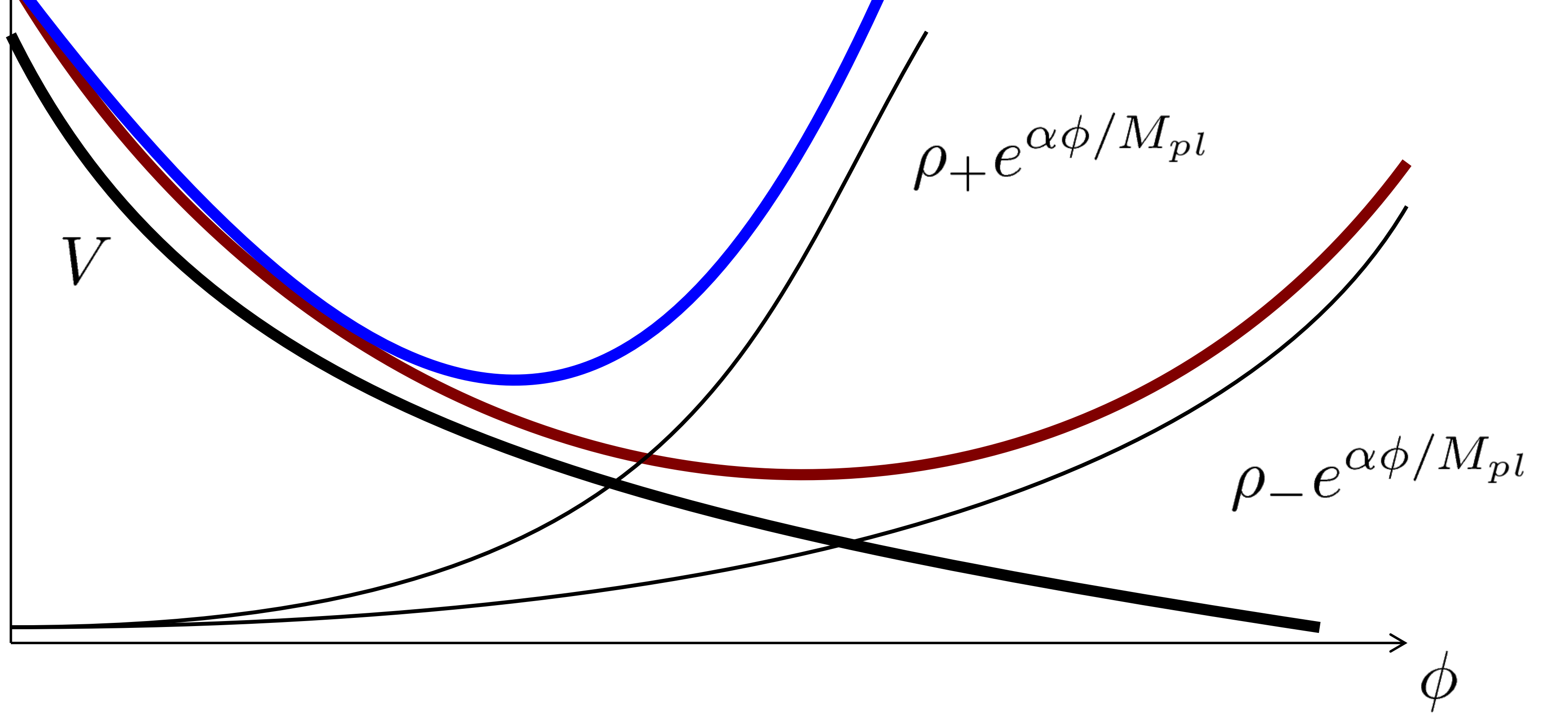
Chameleon mechanism

Khoury & Weltman astro-ph/0309300

$$S = \int d^4x \sqrt{-\tilde{g}} \left(\frac{M_{pl}^2}{2} R - \frac{1}{2} (\nabla \phi)^2 - V(\phi) \right) + S_m[A^2(\phi) g_{\mu\nu}] \qquad A(\phi) = \exp\left(\alpha \frac{\phi}{M_{pl}} \right)$$

$$\nabla^2 \phi = \partial_\phi V + \frac{\alpha}{M_{pl}} \rho e^{\alpha\phi/M_{pl}}$$

$$V_{eff}(\phi) = V + \rho e^{\alpha\phi/M_{pl}}$$

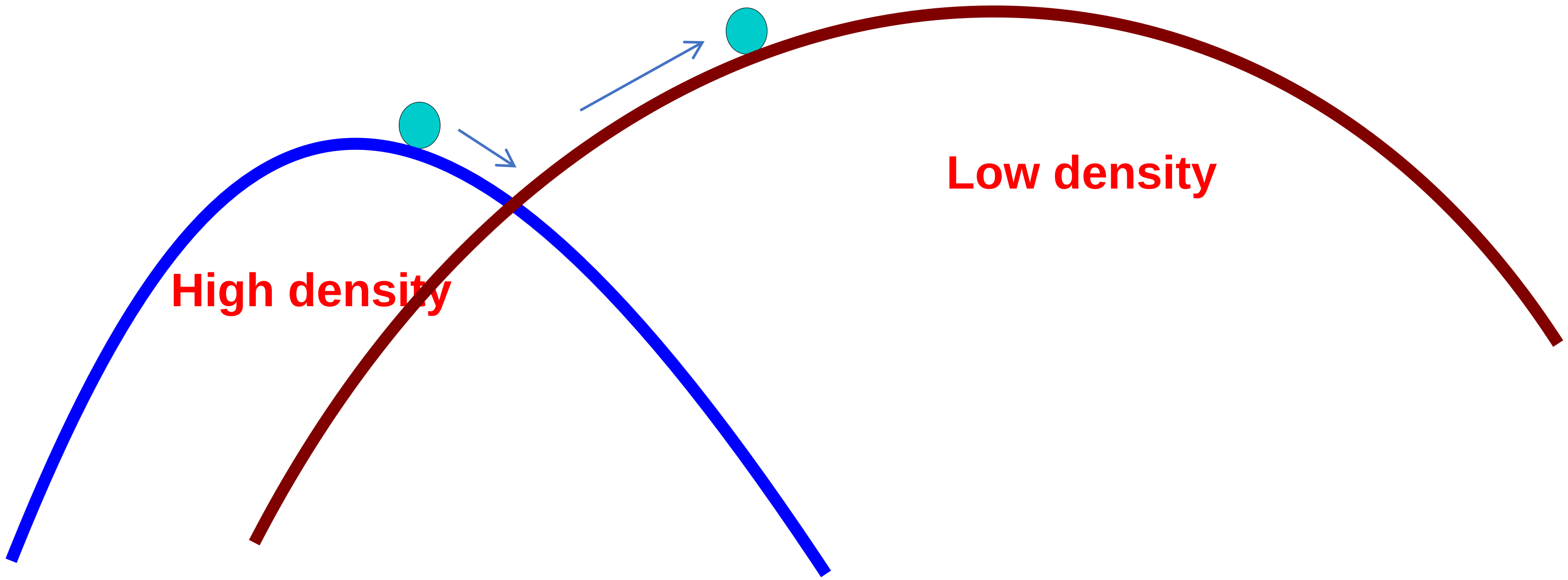


Analytic solution

- Analytic solution - spherically symmetric case

$$\phi'' + \frac{2}{r}\phi' = V'_{eff}(\phi) \qquad \ddot{\phi} + \frac{2}{t}\dot{\phi} = -V'_{eff}(\phi)$$

Analogy - a ball rolling on upside down potential

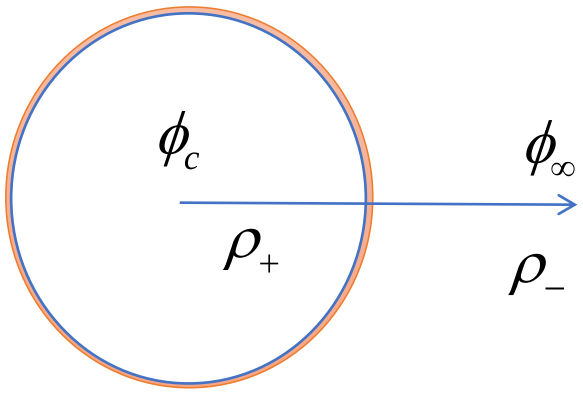


Thin shell condition

- If the thin shell condition is satisfied, only the shell of the size ΔR_c contributes to the fifth force

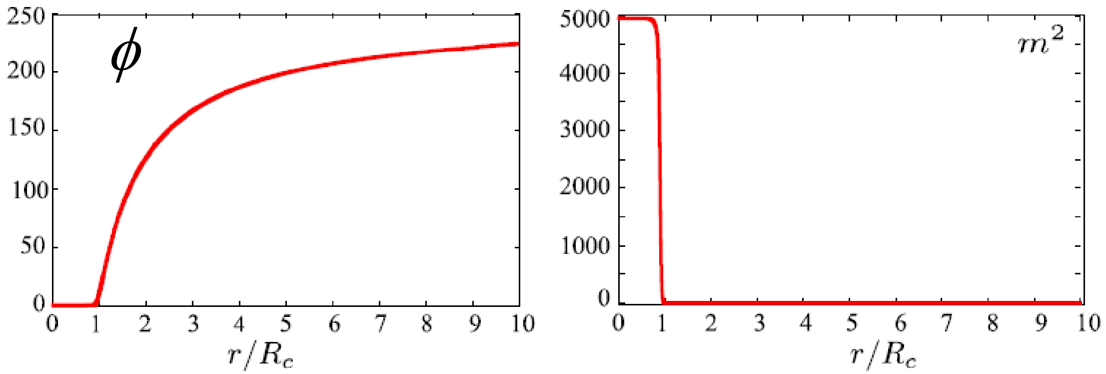
Khoury & Weltman astro-ph/0309411

$$V_{eff}(\phi) = V + \rho e^{\alpha\phi/M_{pl}}$$



$$\phi(r) = -\left(\frac{\alpha}{4\pi M_{pl}}\right)\left(\frac{3\Delta R_c}{R_c}\right)\frac{M\exp\left(-m_\infty(r-R_c)\right)}{r} + \phi_\infty$$

$$\frac{\Delta R_c}{R_c} = \frac{(\phi_\infty - \phi_c)/M_{pl}}{6\alpha\Psi_c} << 1 \qquad \Psi_c = \frac{GM}{R_c}$$



Silvestri 1103.4013

Screening is determined by the gravitational potential of the object

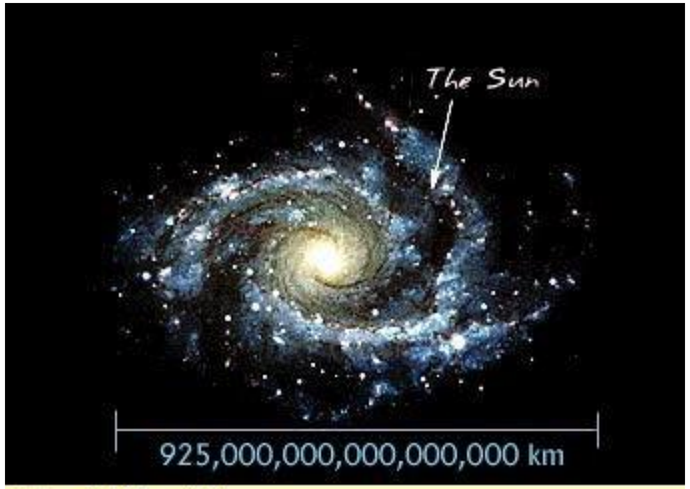
Solar system constraints

- Solar system constraints

$$\rho_{\odot} \sim 10 \text{ g cm}^{-3}$$

$$\rho_{gal} \sim 10^{-24} \text{g cm}^{-3}$$

$$\frac{\Delta R_c}{R_c} = \frac{\phi_{gal} - \phi_{\odot}}{6\alpha M_{pl} \Psi_{\odot}} \sim \frac{\phi_{gal}}{6\alpha M_{pl} \Psi_{\odot}}$$



The Milky Way

The sun has a potential $\Psi_{\odot} \sim 10^{-6}$

The thin shell suppression eases the constraints $\alpha = O(1)$

$$\frac{\Delta R_c}{R_c} \leq 10^{-5} \Rightarrow \frac{\phi_{gal}}{M_{pl}} < 5 \times 10^{-11}$$

This is a model (potential) independent constraint

From galaxy to cosmology

- Example

$$V = V_0 - M^4 \left(\phi / M_{pl}\right)^{1/2} \qquad \frac{\phi_{gal}}{M_{pl}} = \left(\frac{M^4}{\alpha \rho_{gal}}\right)^2$$

$$\rho_{gal} \sim 10^{-24}\,\mathrm{g\ cm^{-3}}$$

$$\rho_{crit} \sim 10^{-29}\,\mathrm{g\ cm^{-3}}$$

Solar system constraints

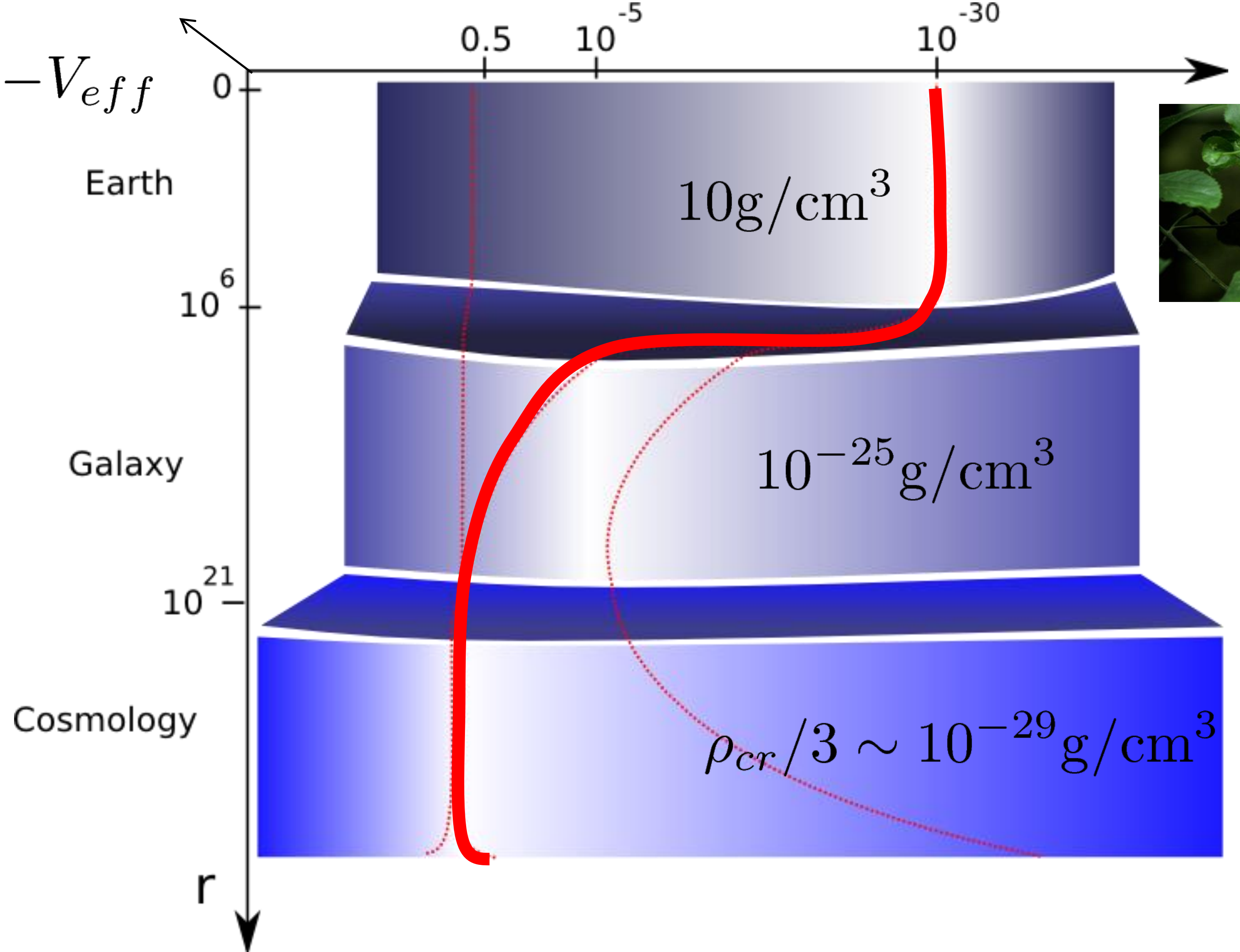
$$\frac{\phi_{gal}}{M_{pl}} < 10^{-11} \qquad \frac{\phi_{\cos mo}}{M_{pl}} = \left(\frac{M^4}{\alpha \rho_{crit}}\right)^2 \simeq 10^{10} \frac{\phi_{gal}}{M_{pl}} \leq 10^{-1} \qquad M \simeq 10^{-3}\mathrm{eV}$$

Galaxy $\dfrac{\Delta R_c}{R_c} = \dfrac{\phi_{\cos mo} - \phi_{gal}}{6\alpha M_{pl} \Psi_{gal}} \sim \dfrac{\phi_{\cos mo}}{6\alpha M_{pl} \Psi_{gal}}$

The Milky way galaxy $\Psi_{Milk} \sim 10^{-6}$

in order to screen the Milky way, we need $\dfrac{\phi_{\cos mo}}{M_{pl}} < 10^{-6}$

From solar system to cosmology



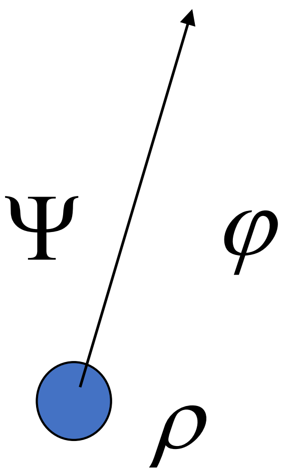
Vainshtein mechanism

- Vainshtein mechanism

originally discussed in massive gravity

rediscovered in DGP brane world model

linear theory $\omega_{BD} = 0$



$$3\nabla^2\varphi = -8\pi G\rho$$

$$\nabla^2\Psi = 4\pi G\rho - \frac{1}{2}\nabla^2\varphi \qquad ds^2 = -(1+2\Psi)dt^2 + a(t)^2(1-2\Phi)d\vec{x}^2 \quad \psi = \psi_0 + \varphi$$

even if gravity is weak, the scalar can be non-linear $r_c \sim m^{-1} \sim H_0^{-1}$

$$3\nabla^2\varphi + r_c^2\left\{\left(\nabla^2\varphi\right)^2 - \partial_i\partial_j\varphi\,\partial^i\partial^j\varphi\right\} = 8\pi G a^2\rho$$

Vainshtein radius

- Spherically symmetric solution for the scalar

$$\frac{d\varphi}{dr} = \frac{r_g}{r^2}\Delta(r), \quad \Delta(r) = \frac{2}{3}\left(\frac{r}{r_V}\right)^3 \left(\sqrt{1+\left(\frac{r_V}{r}\right)^3} - 1\right) \qquad r_V = \left(\frac{8r_c^2 r_g}{9}\right)^{\frac{1}{3}}, \quad r_g = 2GM$$

r_g r_V r_c

4D Einstein | 4D BD

Screening is determined by the curvature of the object

$$\Phi = \frac{r_g}{2r} + \sqrt{\frac{r_g r}{2r_c^2}}, \qquad \Psi = -\frac{r_g}{2r} + \sqrt{\frac{r_g r}{2r_c^2}}$$

$$\Phi = \frac{r_g}{2r}\left(\frac{2}{3}\right), \qquad \Psi = -\frac{r_g}{2r}\left(\frac{4}{3}\right)$$

$$\left(\frac{r_V}{r}\right)^{-3/2} \propto \left(\frac{GM}{r^3} r_c^2\right)^{-3/2}$$

2.95km   0.1 kpc   3000Mpc   for the Sun

Solar system constraints

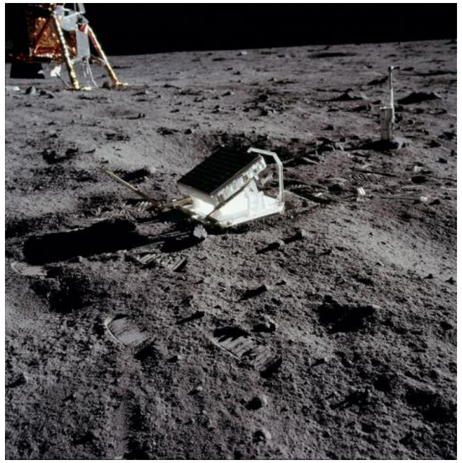
- The fractional change in the gravitational potential $\varepsilon = \dfrac{\delta\Psi}{\Psi}$

The anomalous perihelion precession

$$\delta\phi = \pi r \frac{\partial}{\partial r}\left[r^2 \frac{\partial}{\partial r}\left(\frac{\varepsilon}{r}\right)\right]$$

The vainshtein radius is shorter for a smaller object

Lunar laser ranging: the Erath-moon distance $r_{E-M} = 4.1\times 10^5\,\mathrm{km}$



$$\delta\phi = \frac{3\pi}{4}\left(\frac{{r_{E-M}}^3}{2GM_{\oplus} r_c^2}\right)^{1/2} < 2.4\times 10^{-11} \quad \Rightarrow \quad r_c > H_0^{-1}$$

Screening mechanisms

- They look contrived but some theories naturally have these mechanisms

f(R) gravity

$$F(R) = R - 2\Lambda_{eff} - \frac{R_0^{n+1}}{R^n}$$

we expect to recover GR in the limit $R / R_0 \gg 1$

In fact, in Einstein frame this is nothing but chameleon with $V = V_0 - M^4 \left(\phi / M_{pl}\right)^{1/2}$

DGP gravity

$$S = \frac{1}{32\pi G r_c} \int d^5x \sqrt{-g_{(5)}} R_{(5)} + \frac{1}{16\pi G} \int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g} L_m$$

we expect to recover GR in the limit $r / r_c \gg 1$ and this is realised by Vainshtein

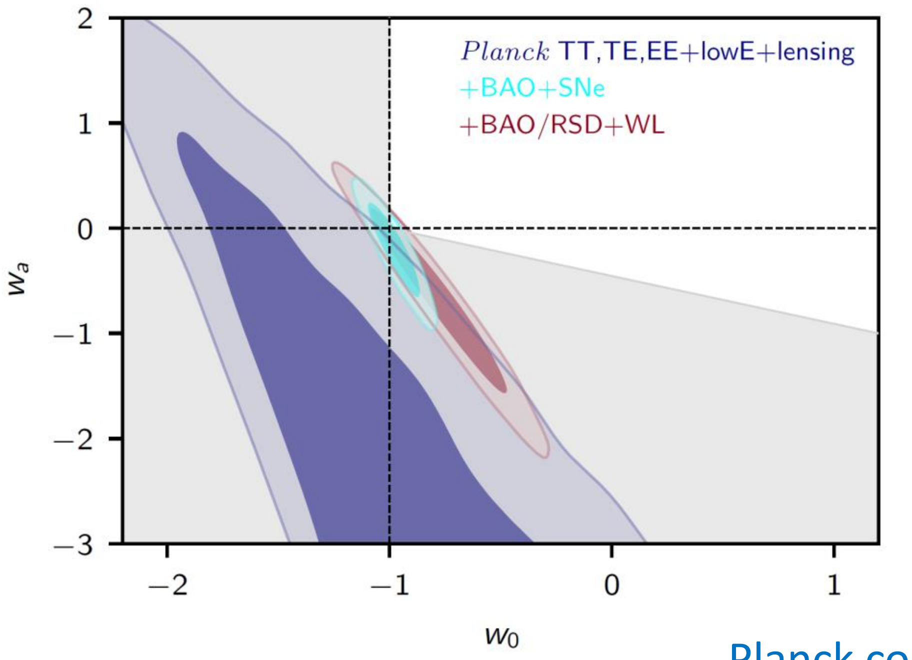
Observational tests

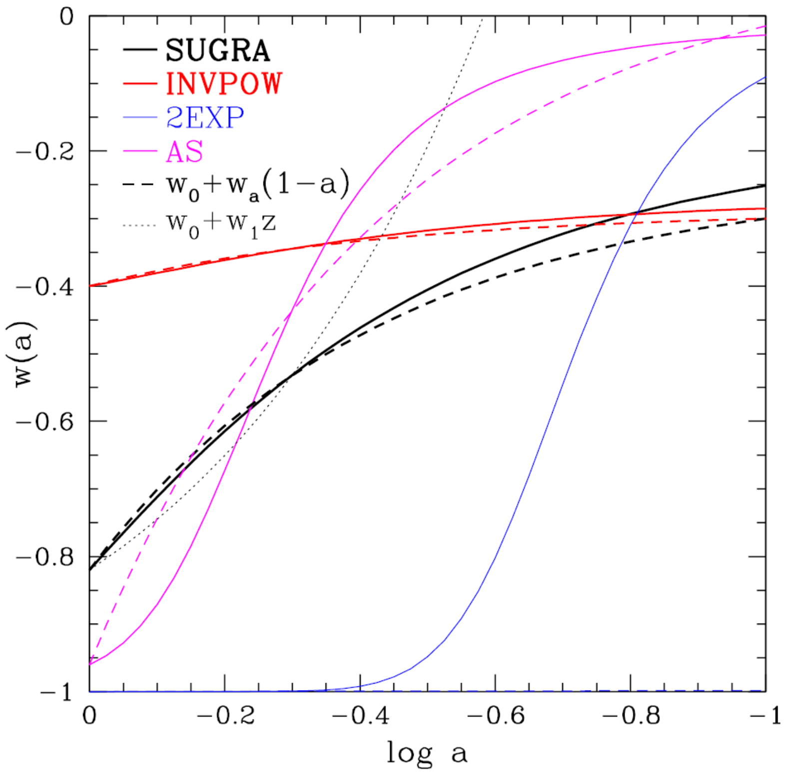
Background expansion history

- Background expansion is determined by the equation of state

Parametrisation

$$w_{DE} = w_0 + w_a(1-a) = w_0 + w_a \frac{z}{1+z}$$





Planck collaboration 1807.06209

Linder astro-ph/0311403

Model independent approach

- Principal component analysis

Approximate $w_{DE}(z)$ with many stepwise constant values

$$1 + w_{DE}(z) = \sum_{i=1}^{N} w_i \theta_i(z)$$

Errors on w_i are highly correlated. We diagonalise the covariant matrix

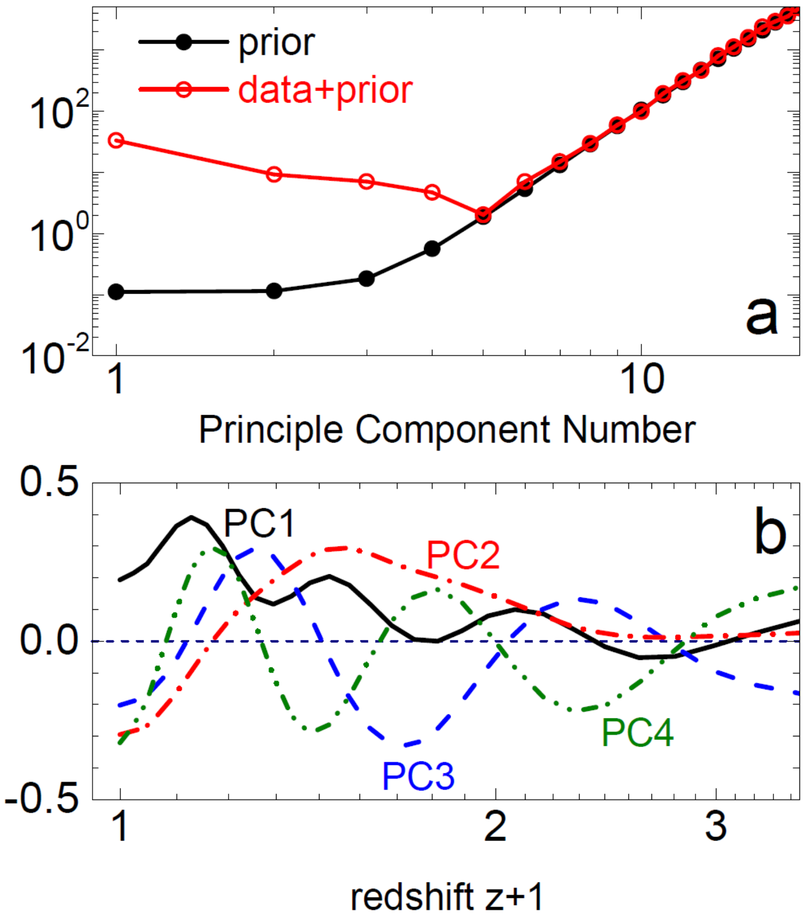
$$C_{ij} = \left\langle (w_i - \bar{w}_i)(w_j - \bar{w}_j) \right\rangle \quad C = W\,\Lambda^{-1}\,W^T, \quad W = (\vec{e}_1, \vec{e}_2, ...,), \quad \Lambda_{ij} = \lambda_i \delta_{ij}$$

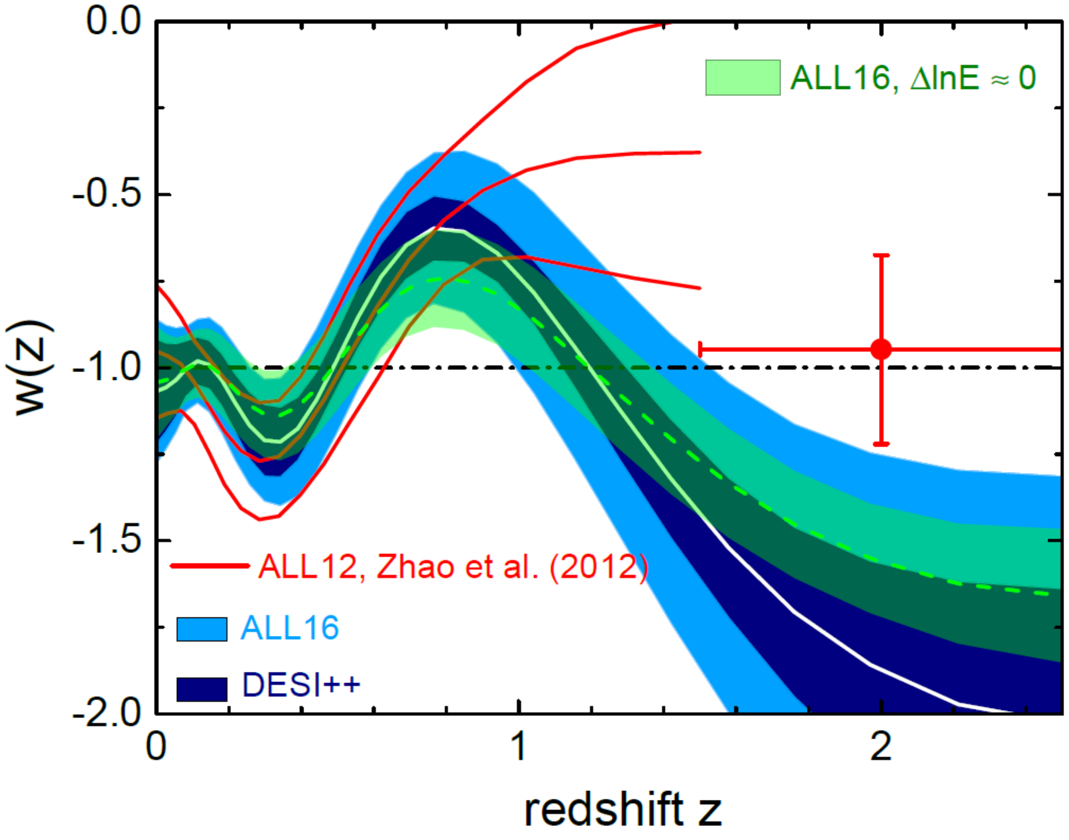
$$1 + w_{DE}(z) = \sum_{i=1}^{N} \alpha_i e_i(z)$$

Errors on new parameters $\alpha_i = \sum_j W_{ij}(w_j - \bar{w}_j)$ are uncorrelated and given by $\sqrt{\lambda_i}$

Reconstructed equation of state

- Latest results (2016) Zhao et.al. arXiv:1701.08165





Model based parametrisation

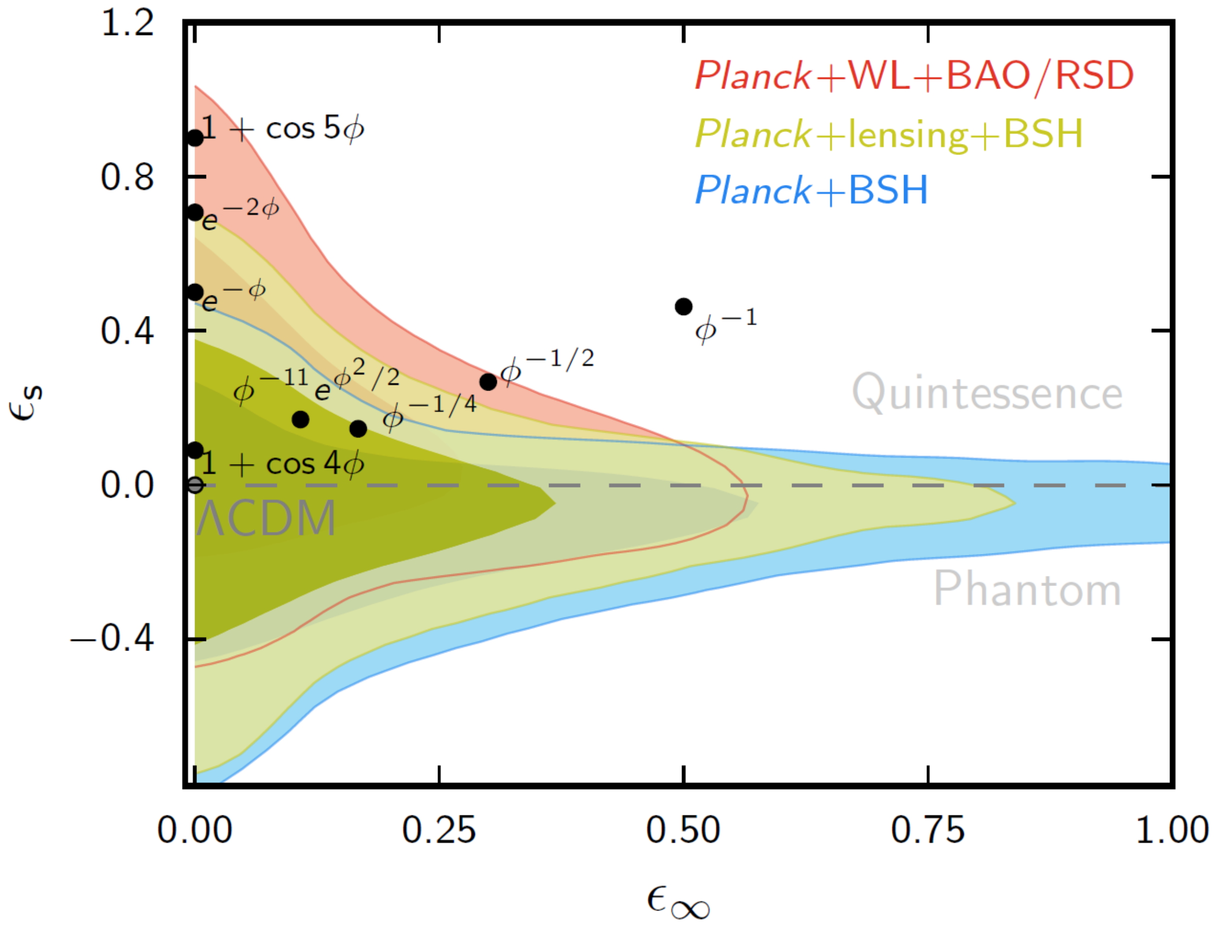
- Parametrisation

$$1+w_\phi \approx \frac{2}{3}\varepsilon_\phi \Omega_\phi(a)$$

$$\varepsilon_\phi = \frac{1}{2\kappa^2}\left(\frac{V'}{V}\right)^2, \quad \Omega_\phi = \frac{\rho_\phi}{\rho_\phi + \rho_m}$$

$$\varepsilon_s = \varepsilon_\phi(\rho_m = \rho_{DE})$$

$$\varepsilon_\infty = \varepsilon_\phi \Omega_\phi(a \to 0)$$

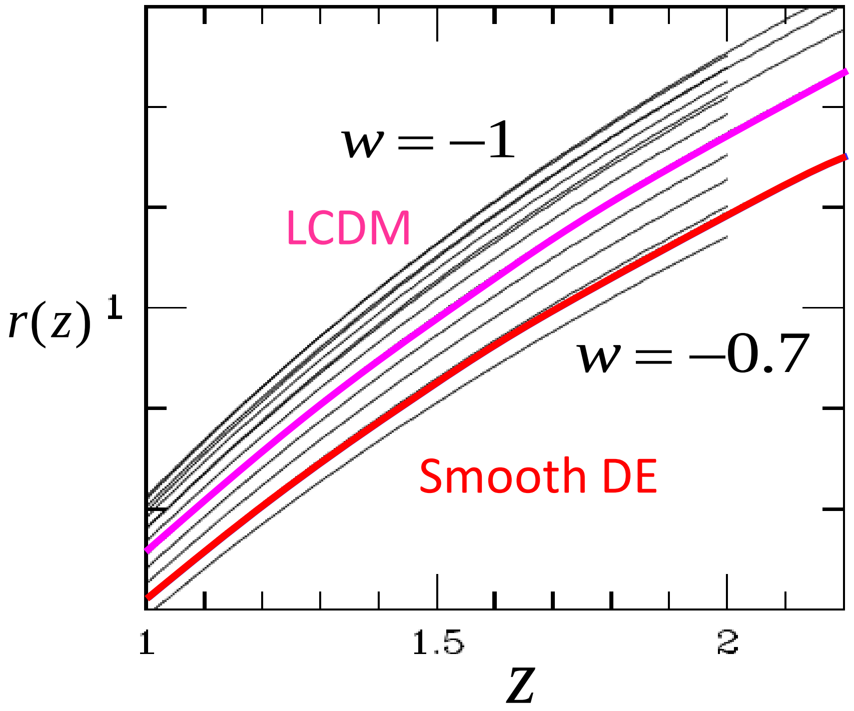


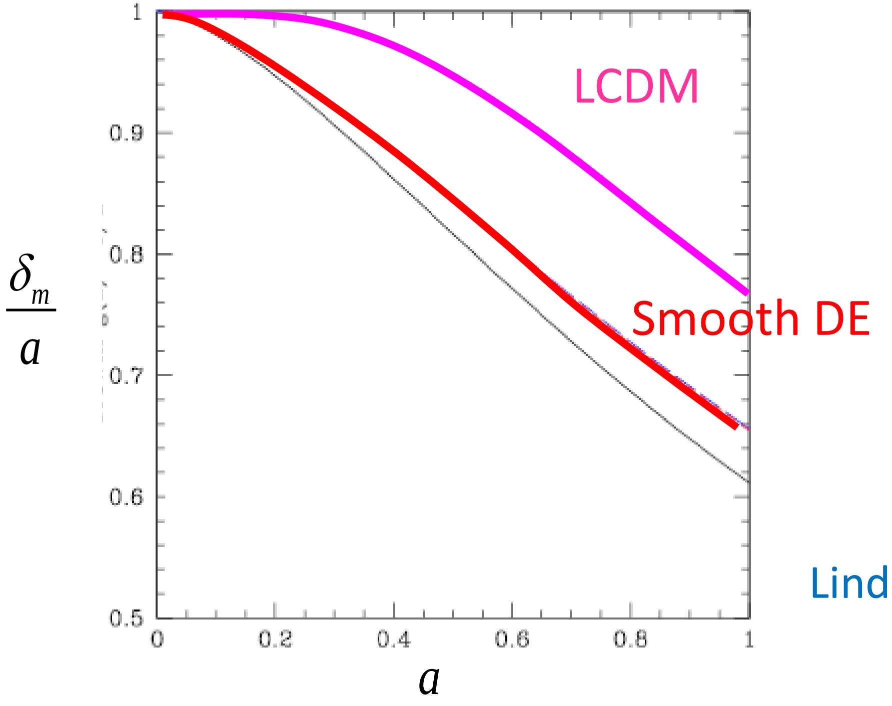
Planck 1502.01590

Expansion history v structure growth

- LCDM/Smooth DE

There is a one-to-one correspondence between background expansion history and growth of structure





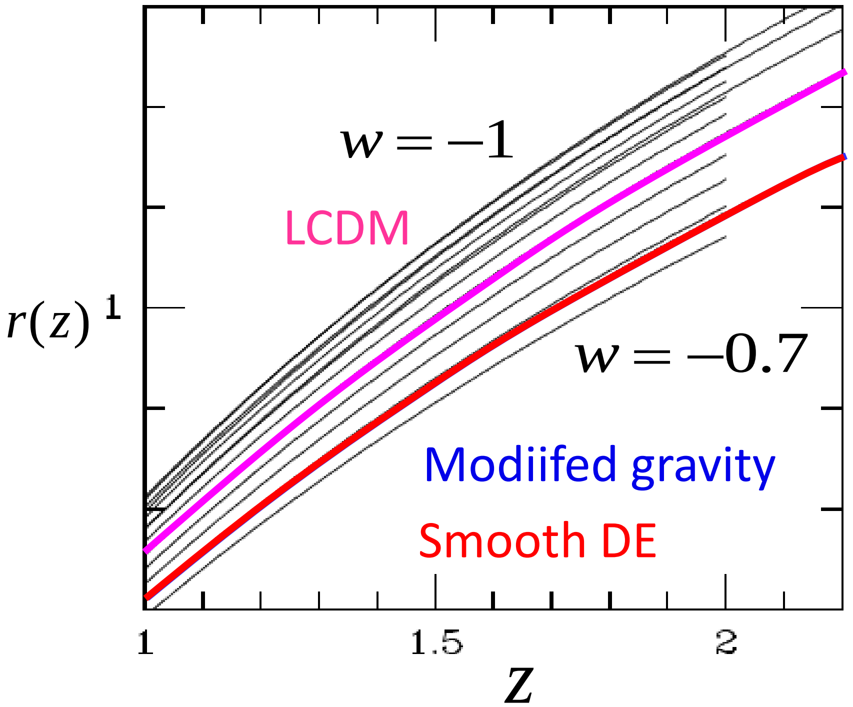
Linder astro-ph/0507263

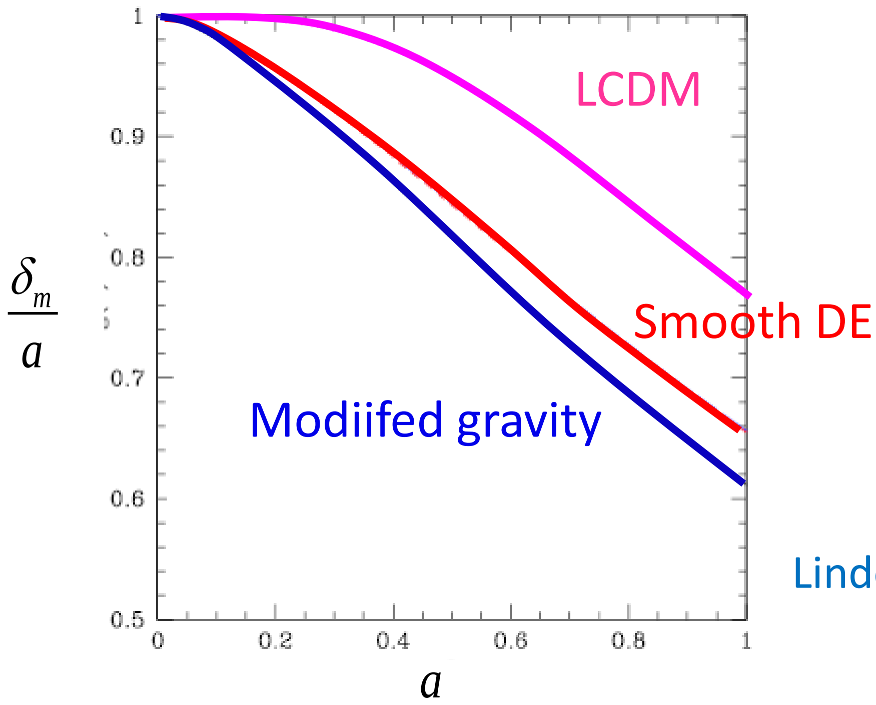
Expansion history v structure growth

- Modified gravity

Structure growth is controlled also by a modified gravitational force

Even if it has the same expansion history as smooth DE, structure growth is different



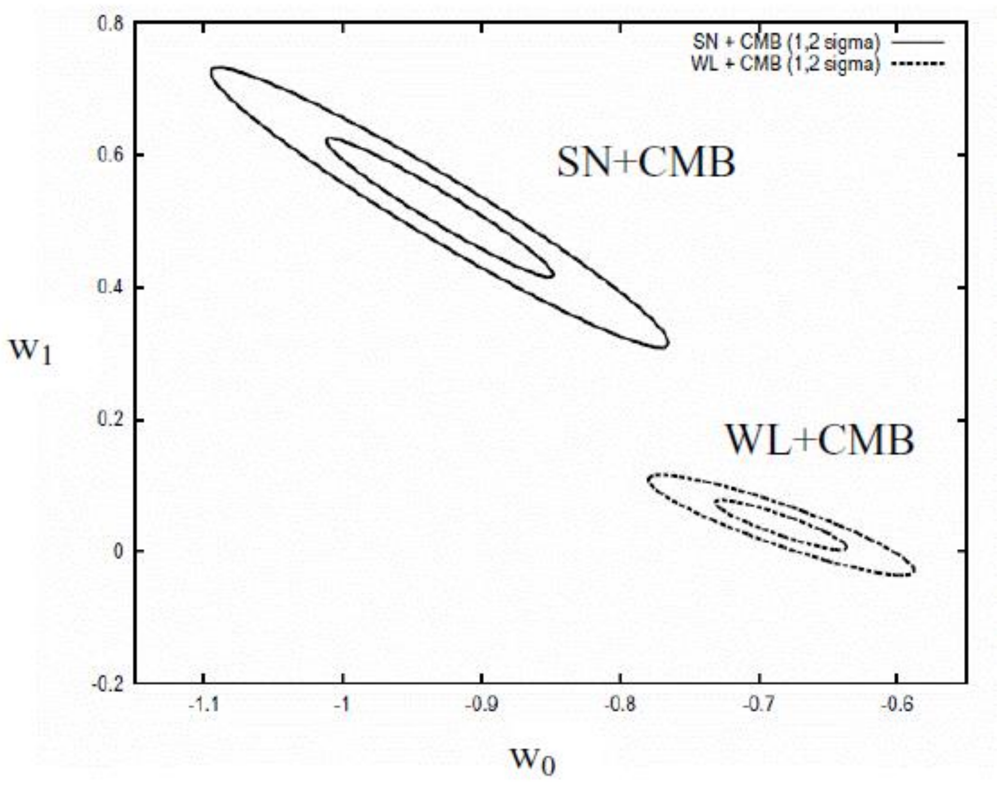


Linder astro-ph/0507263

Consistency test

Assume that the Universe is described by a modified gravity model but we still try to fit the date using smooth DE

$$w(z) = w_0 + w_1 z,$$



SNe+CMB

SNe+weak lensing

Inconsistent!

Ishak et.al. astro-ph/0507184

Parametrisation

- Parametrisation of Einstein equations

$$k^2\Psi = -4\pi G a^2 \mu(k,a)\rho_m\Delta_m \qquad ds^2 = a^2(\eta)\left[-(1+2\Psi)d\eta^2 + (1-2\Phi)\delta_{ij}dx^i dx^j\right]$$

$$\Phi = \eta(k,a)\Psi$$

equivalently, we can also parametrise the lensing potential

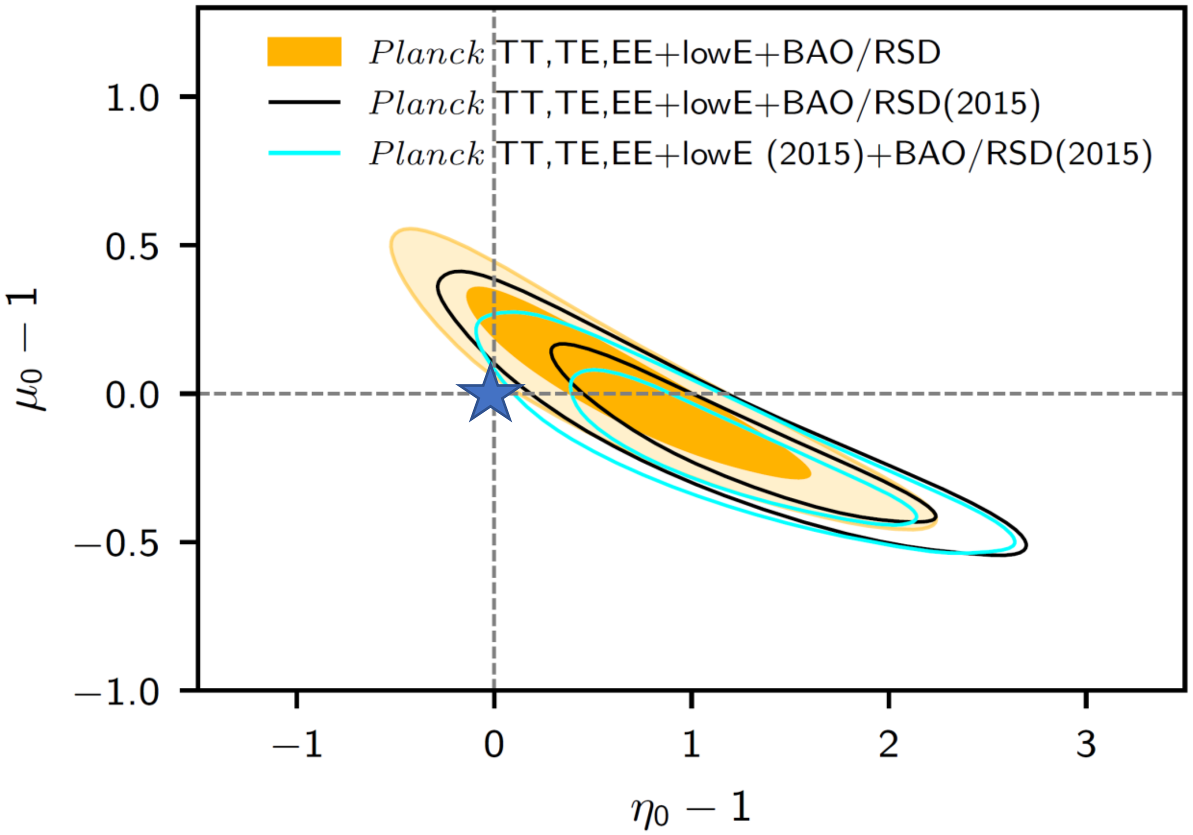
$$k^2\Psi = -4\pi G a^2 \mu(k,a)\rho_m\Delta_m$$

$$k^2\frac{(\Psi+\Phi)}{2} = -4\pi G a^2 \Sigma(k,a)\rho_m\Delta_m, \quad \Sigma = \frac{\mu(1+\eta)}{2}$$

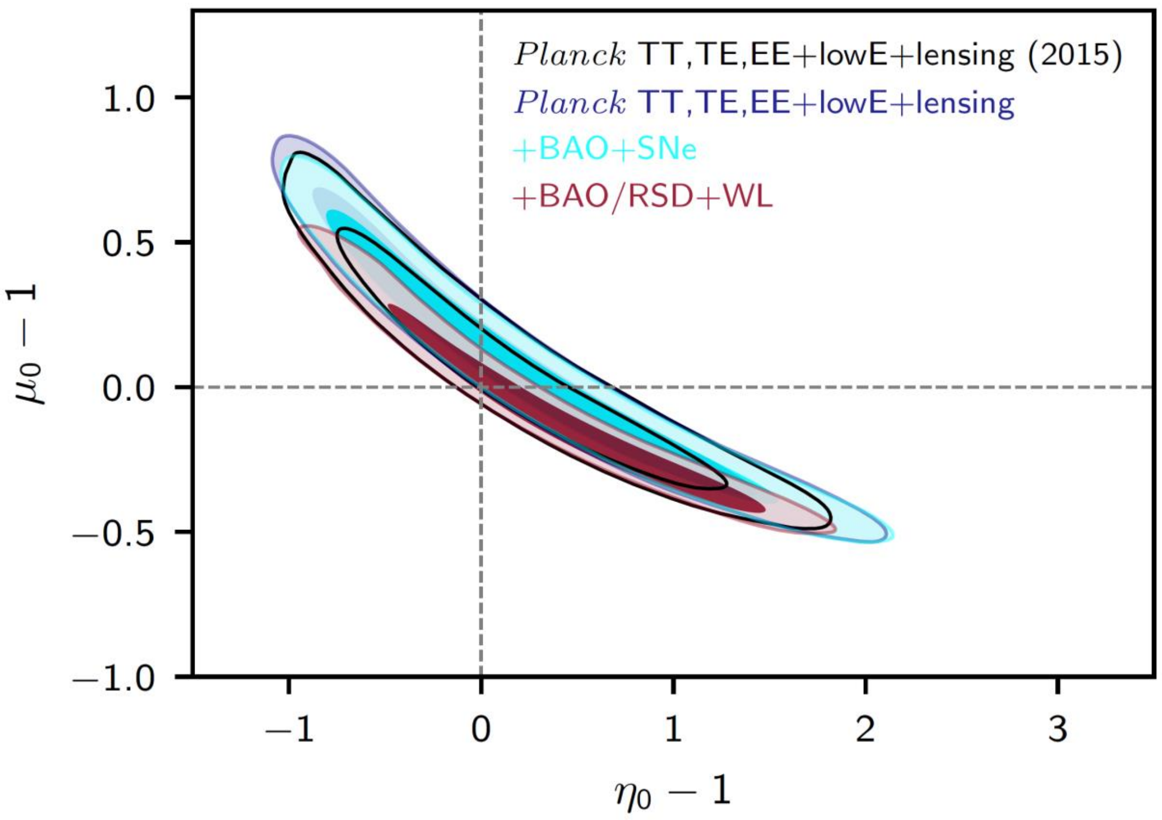
$$\mu = \eta = \Sigma = 1 \text{ for smooth DE}$$

Planck constraints

Planck collaboration 1807.06209



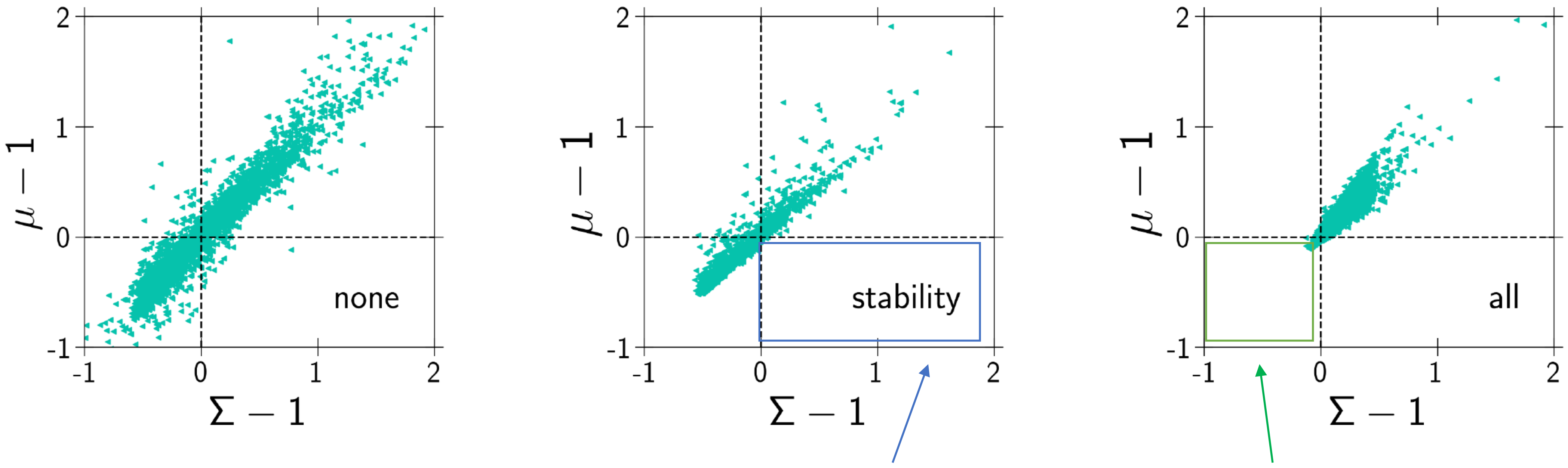
with CMB lensing



Predictions in Horndeski theory

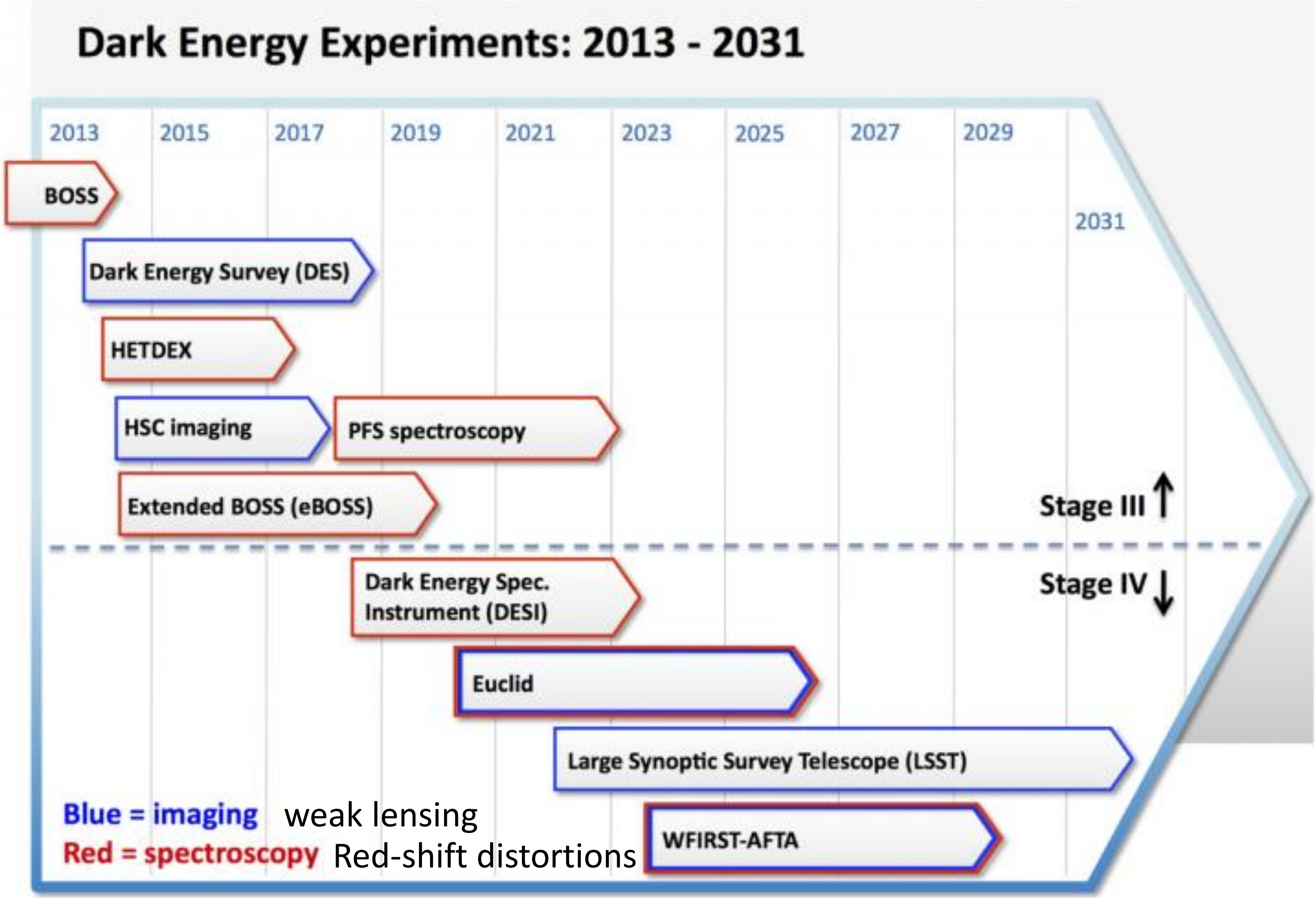
- Models with $c_{GW} = 1$

Peirone et.al. arXiv:1712.00444



Stability condition removes this part

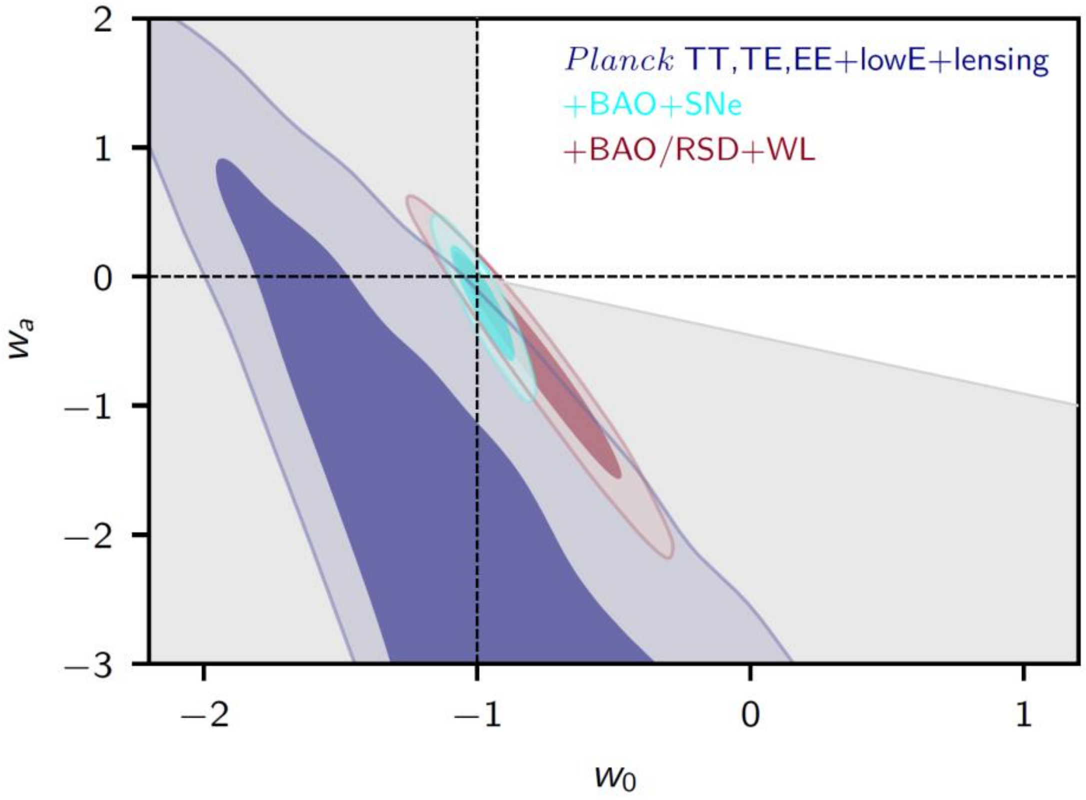
Observational condition removes this part



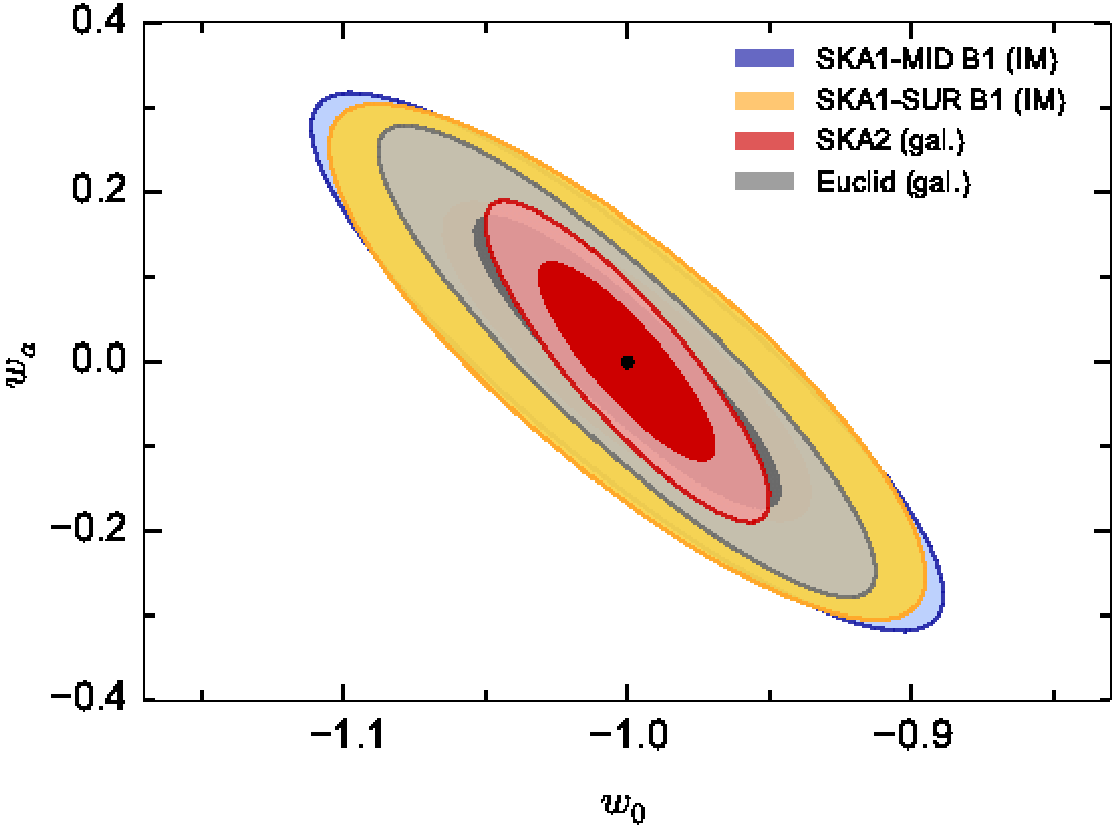
Abazajian et.al. Dark Energy and CMB

Future forecasts

current constraints

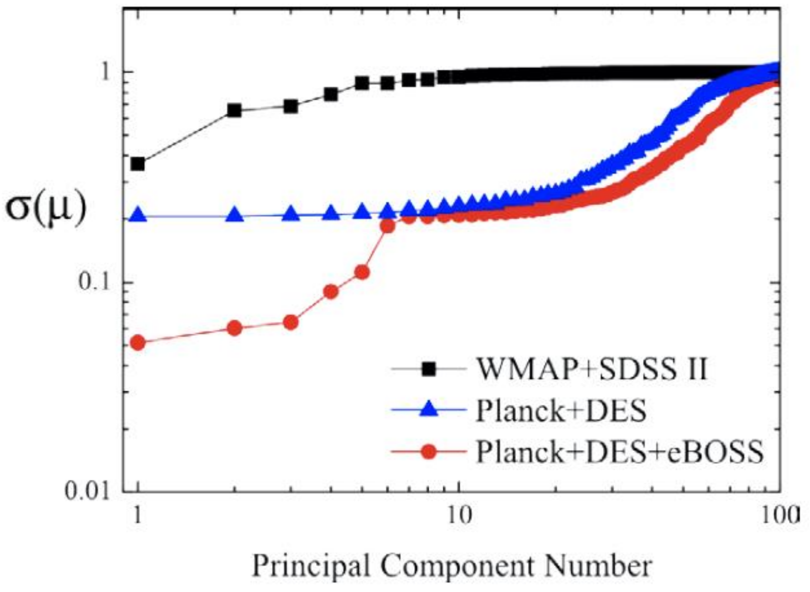


next 10 years



Future forecasts

Next few years



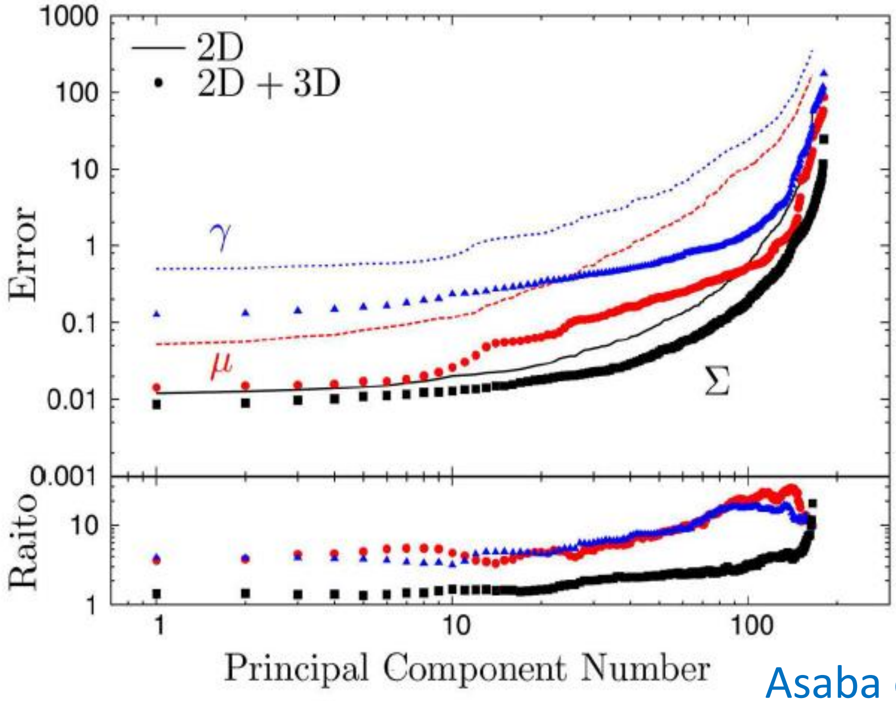
Zhao et.al. 1510.08216

DES (-2019) imaging

eBOSS (-2019) spectroscopic

Several parameters at the 5-10% level

Next 10 years

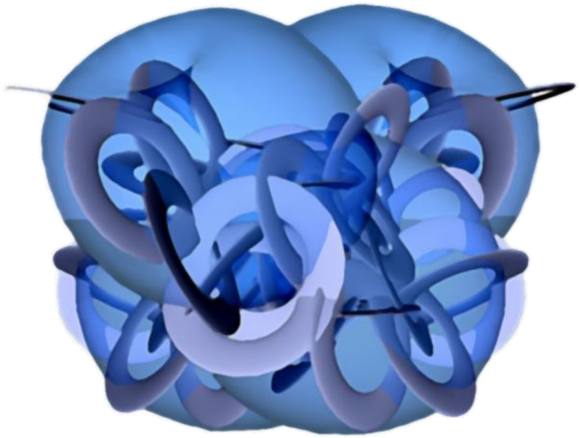


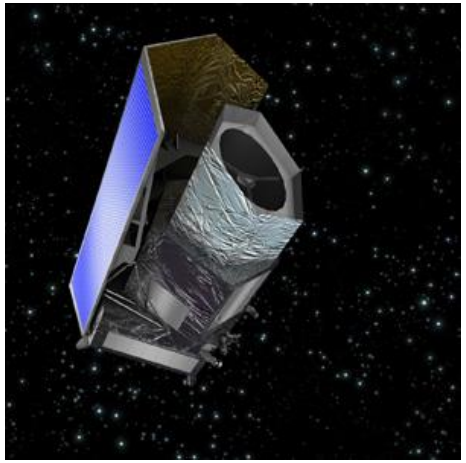
Asaba et.al. 1306.2546

Euclid (2021-)

10 parameters at the 1% level

From theory to observations





fluid approach
Effective Field Theory

many free functions

$$\mu(k_i, z_i), \Sigma(k_i, z_i)$$

$$C_\ell^{IJ}(z)$$

galaxy count
Lensing
ISW

systematics