Rotation curve sample analysis

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FAPES

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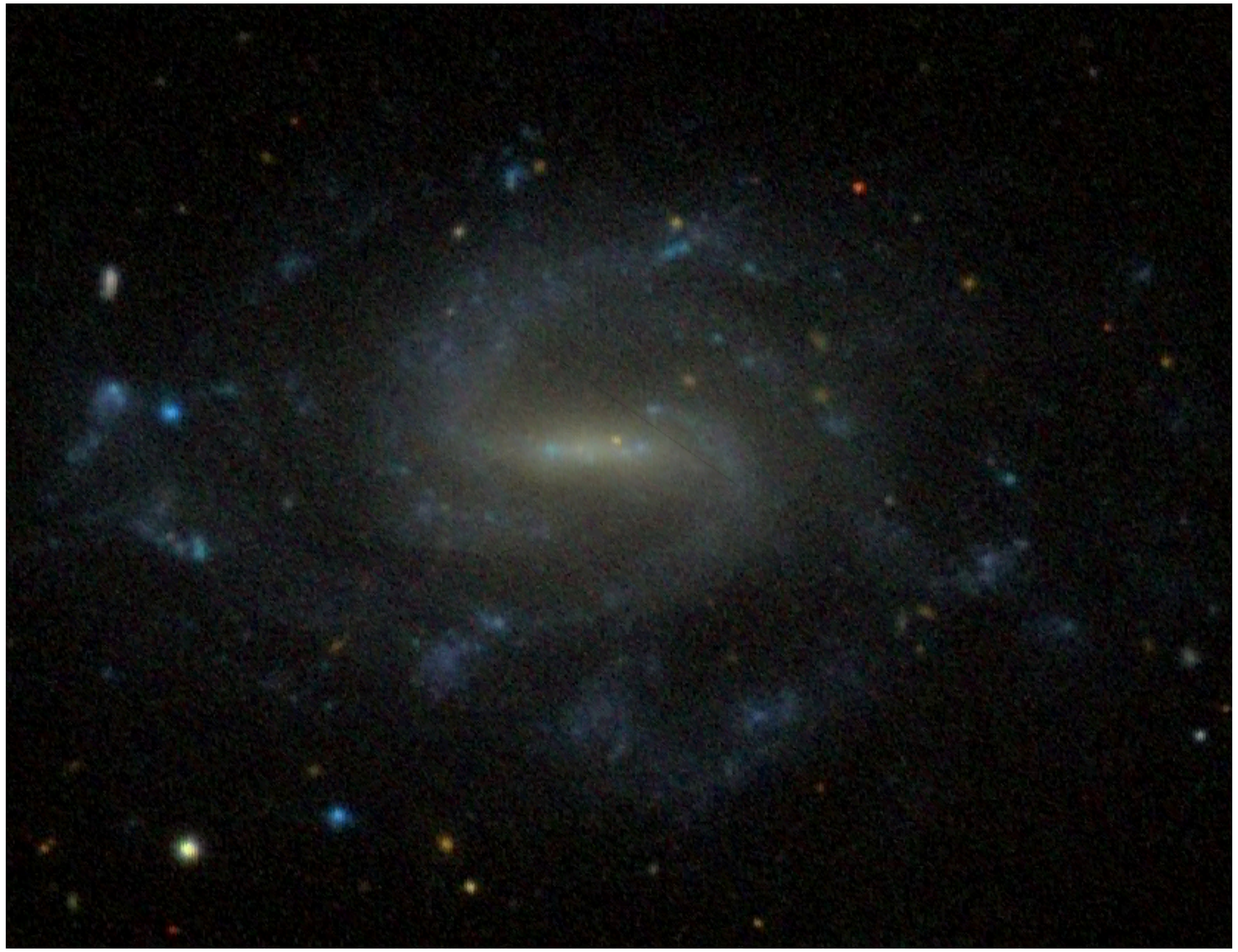
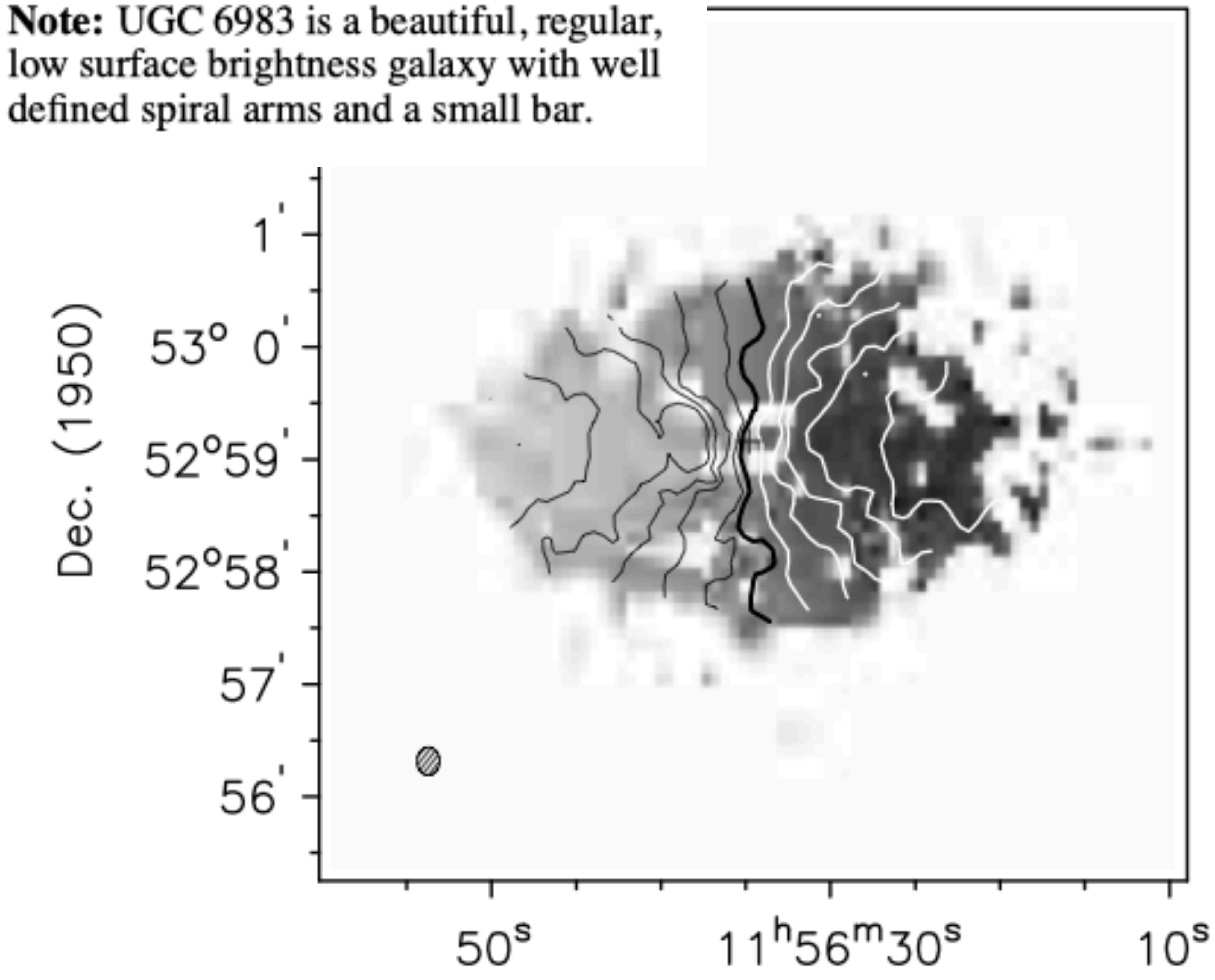
Galaxy rotation curves: basics

Image: SDSS9, Hyperleda



UGC 06983 (LSB, type SBc galaxy)
One of the 173 galaxies in SPARC



[Verheijen & Sancisi, A&A 2001]

The atomic hydrogen (HI) velocity field: no model
The curves are the isovelocity contours
Thick isovelocity: coincides with galaxy global velocity
White isovelocities: receding

Galaxy rotation curves: basics

Gas contribution:

Derived from the 21 cm radiation surface brightness.
Hydrogen density found from hyperfine transition.
Gas density $\rho_{\rm gas} = 1.33\,\rho_{\rm H}$, from BBN.

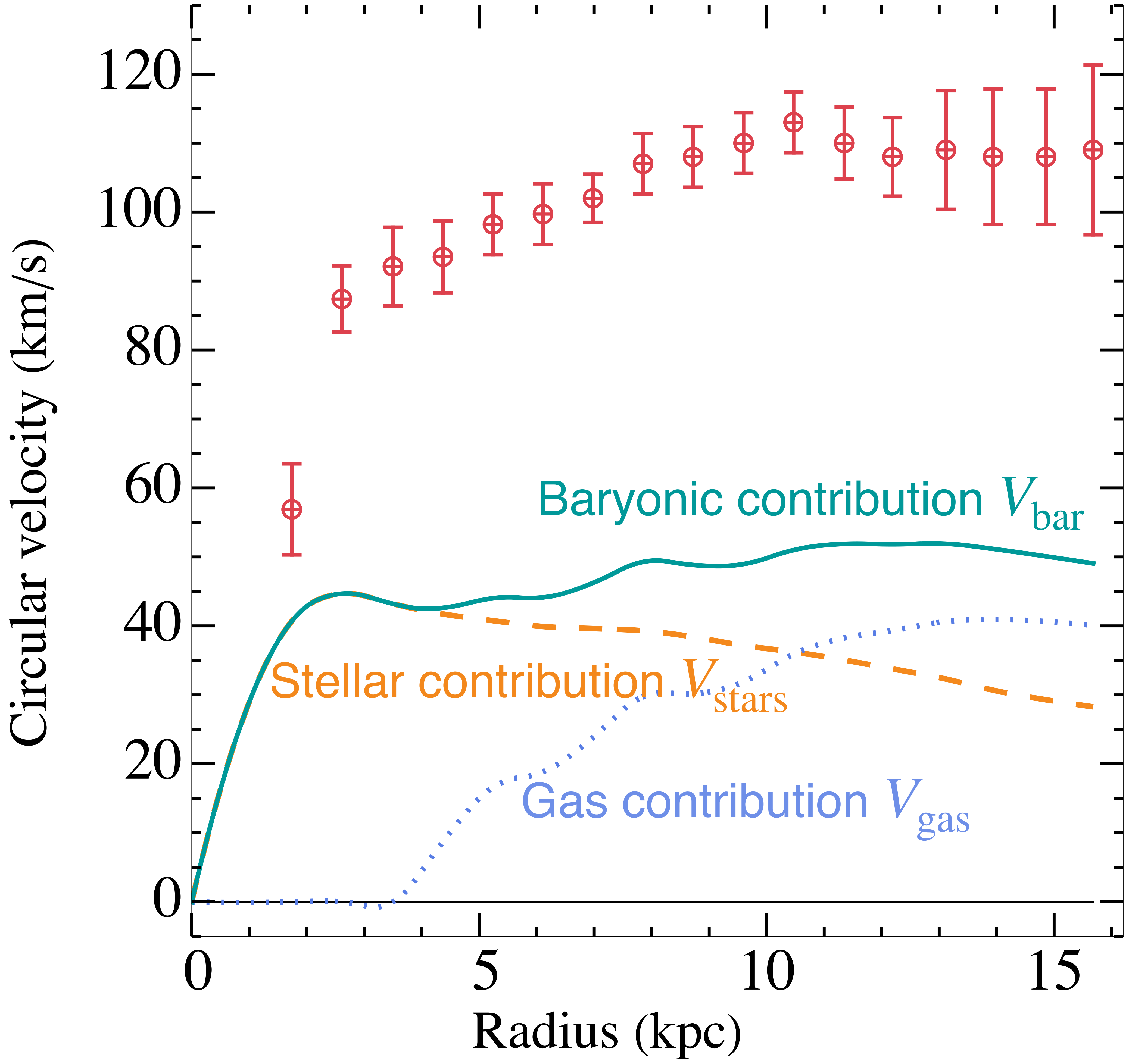
$V^2_{\rm gas} = r\partial_r \Phi_{\rm gas}$, with $\nabla^2 \Phi_{\rm gas} = 4\pi G \rho_{\rm gas}$.

Stellar contribution:

Derived from the stellar surface brightness (near infrared).
Stellar density depends on stellar population models.
For the Spitzer 3.6 μm band (e.g., Meidt et al ApJ 2014):

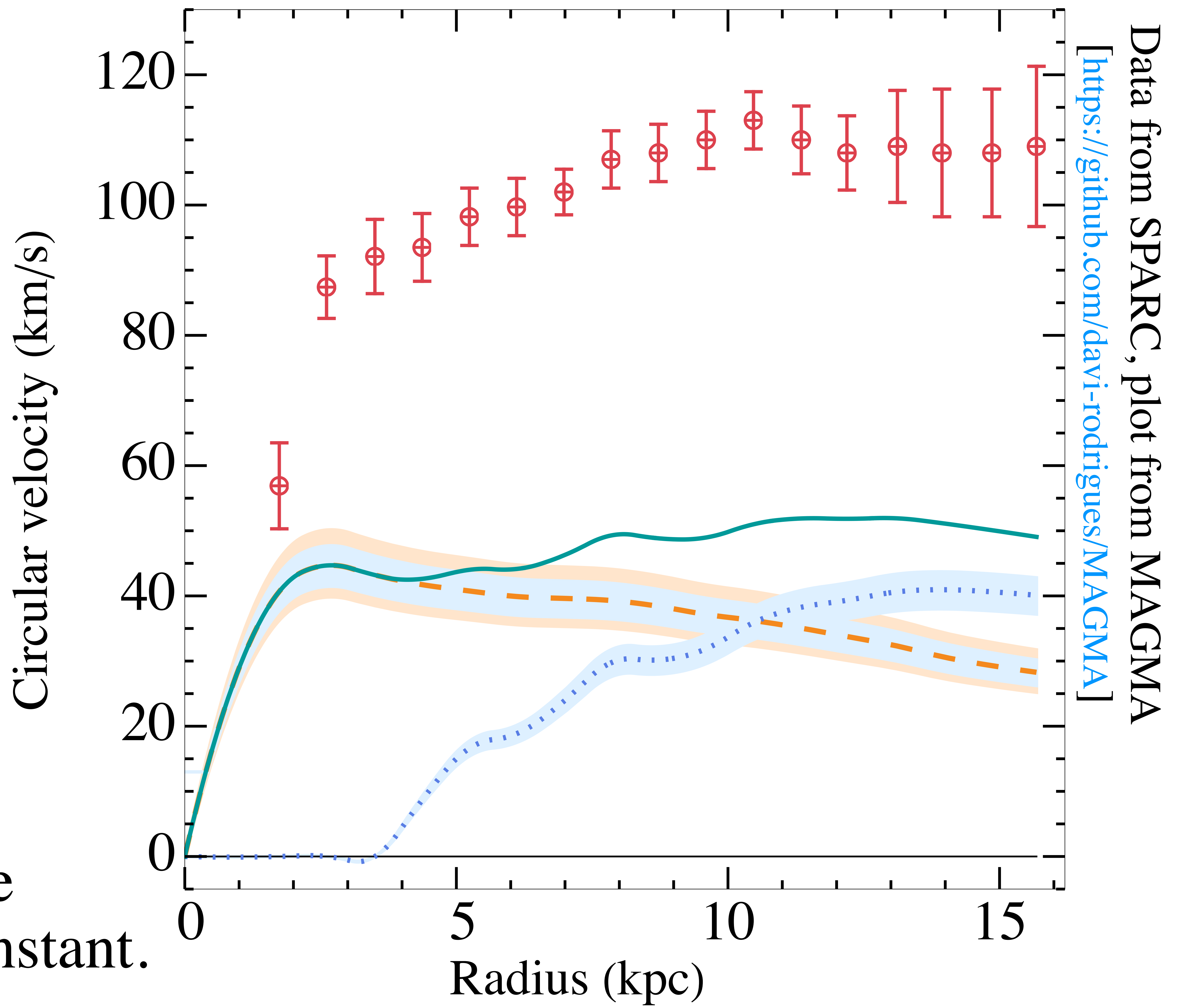
$\Upsilon_* = 0.50^{+0.13}_{-0.10}$ (dimensionless mass-to-light ratio, stellar disk)

$V^2_* = r\partial_r \Phi_*$, with $\nabla^2 \Phi_* = \Upsilon_*\, 4\pi G \rho_{*|\Upsilon_*=1}$.



$$\boxed{V^2_{\rm bar} = V^2_{\rm stars} + V^2_{\rm gas}}$$

The basics of galaxy rotation curves



Main baryonic contribution errors:
Orange shaded area: 1σ Y_* error.
Blue shaded area: 1σ distance error.

Global inclination error only influences the rotation curve itself by a multiplicative constant.

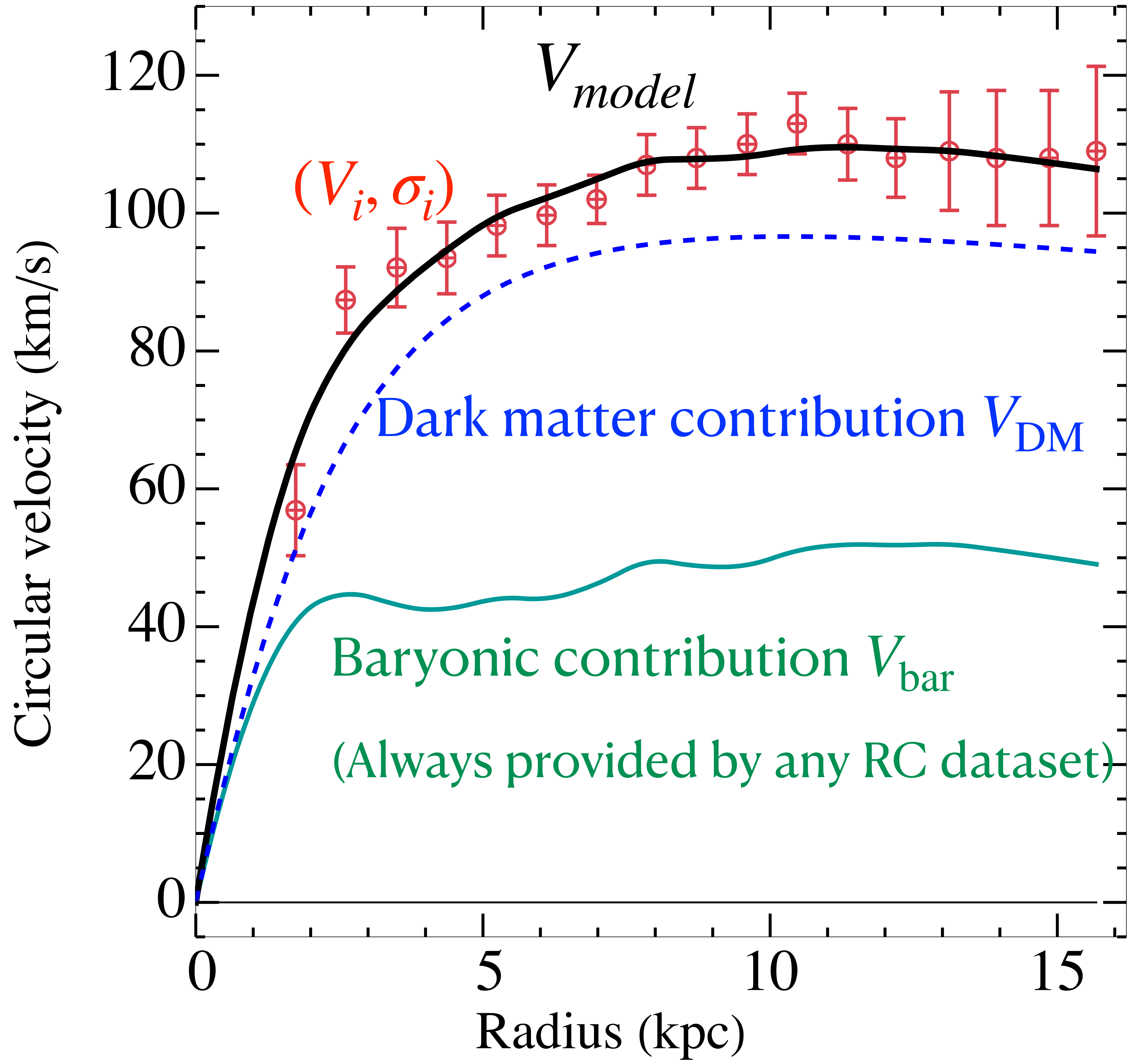
Testing a DM profile for a given galaxy

Typical procedure:

- One assumes a DM profile with free parameters (e.g., NFW, Einasto, Burkert, DC14…)
- For each galaxy RC data, the halo is fitted together with relevant baryonic uncertainties, if any (e.g., Υ_*, δD, δi).

The maximization of the likelihood yields the best fit model parameters (p_a)

$$-\ln \mathscr{L}(p_a) \propto \chi^2(p_a) = \sum_i \left(\frac{V_{model}(p_a, R_i) - V_i}{\sigma_i} \right)^2$$



$$V_{model}^2 = V_{\rm bar}^2 + V_{\rm DM}^2$$

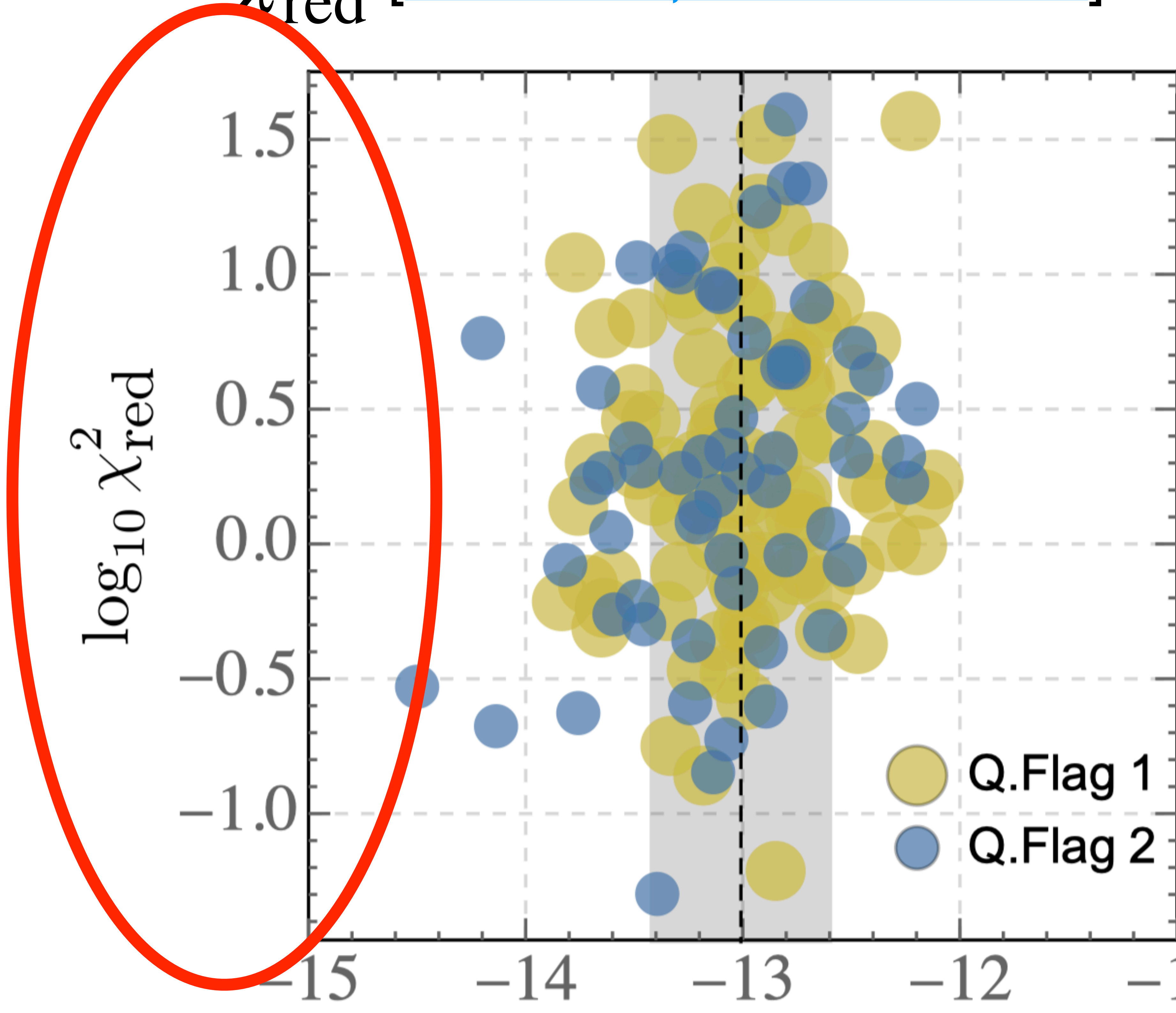
Which model is better?

A simple common approach: compute $\chi^2_{\rm red}$ for **each** galaxy.

- Models that yield the minimum average $\langle\, |\chi^2_{\rm red} - 1|\,\rangle$ for a sample are the best.
- Variation: instead of the average, use the CDF distribution of $\chi^2_{\rm red}$ [Katz et al, MNRAS 2016]

Problem: DM halo profiles are nonlinear models, the rule $\chi^2_{\rm red} \sim 1$ is not correct. [Andrae et al 1012.3754].

Issue: Computational time. Each galaxy needs to be carefully *individually* studied, even though the purpose is the overall sample result.



[Rodrigues et al Nat.Ast. 2018]

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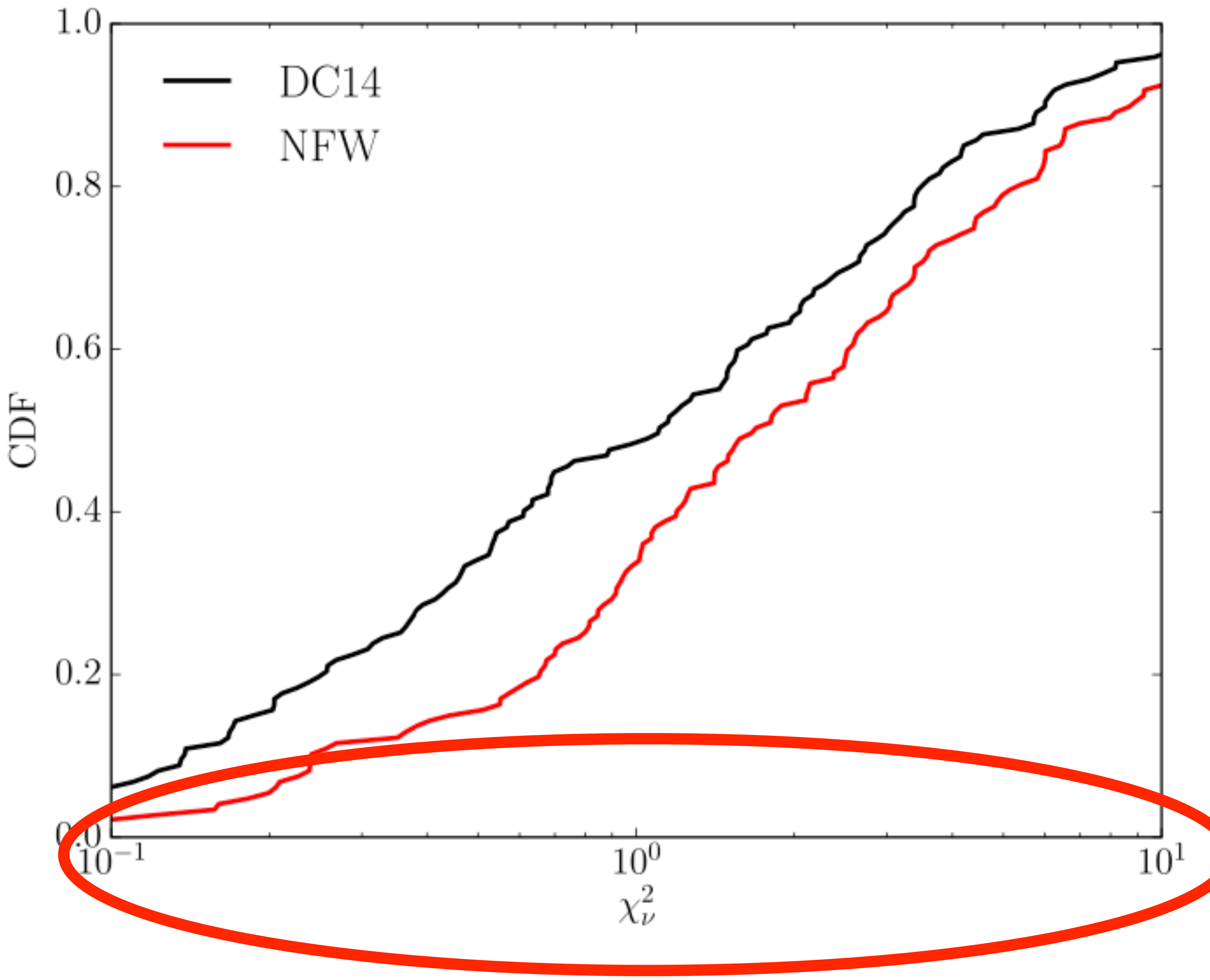


- [Katz et al, MNRAS 2016]

SPARC and modified gravity

- The SPARC galaxy sample [Lelli et al AJ 2016] is a well organized and well known galaxy RC sample. It includes 175 late-type galaxies with high quality data, with emphasis to the stellar component.
- From these, 153 are considered to be specially suitable for galaxy modeling (highly symmetric and with $i \geq 30°$).
- However, there is no public data on the complete $3D$ baryonic distribution.
- [Green, Moffat PDU 2019] develop their own $3D$ baryonic models in part based on the available SPARC data plus analytical approximations that can be found in the literature. This lead to systematical changes in the RC's.
- [Naik et al MNRAS 2019] use exponential profiles to mimic the stellar and the gaseous parts of each galaxy. The latter approximation is less suitable.

Method principles

- The sample distribution is directly studied.
- Good models should be capable of reproducing the observed distribution.
- We focus on one relevant rotation curve feature: its shape

Further details on

Alejandro Hernandez-Arboleda, Davi C. Rodrigues, Aneta Wojnar [2204.03762]
Code available at: https://github.com/davi-rodrigues/NAVanalysis

Normalized Additional Velocity

- One can define the observational (model independent) additional velocity as

$$\Delta V^2_{\rm obs}(r) \equiv V^2_{\rm obs}(r) - V^2_{\rm bar}(r)\ .$$

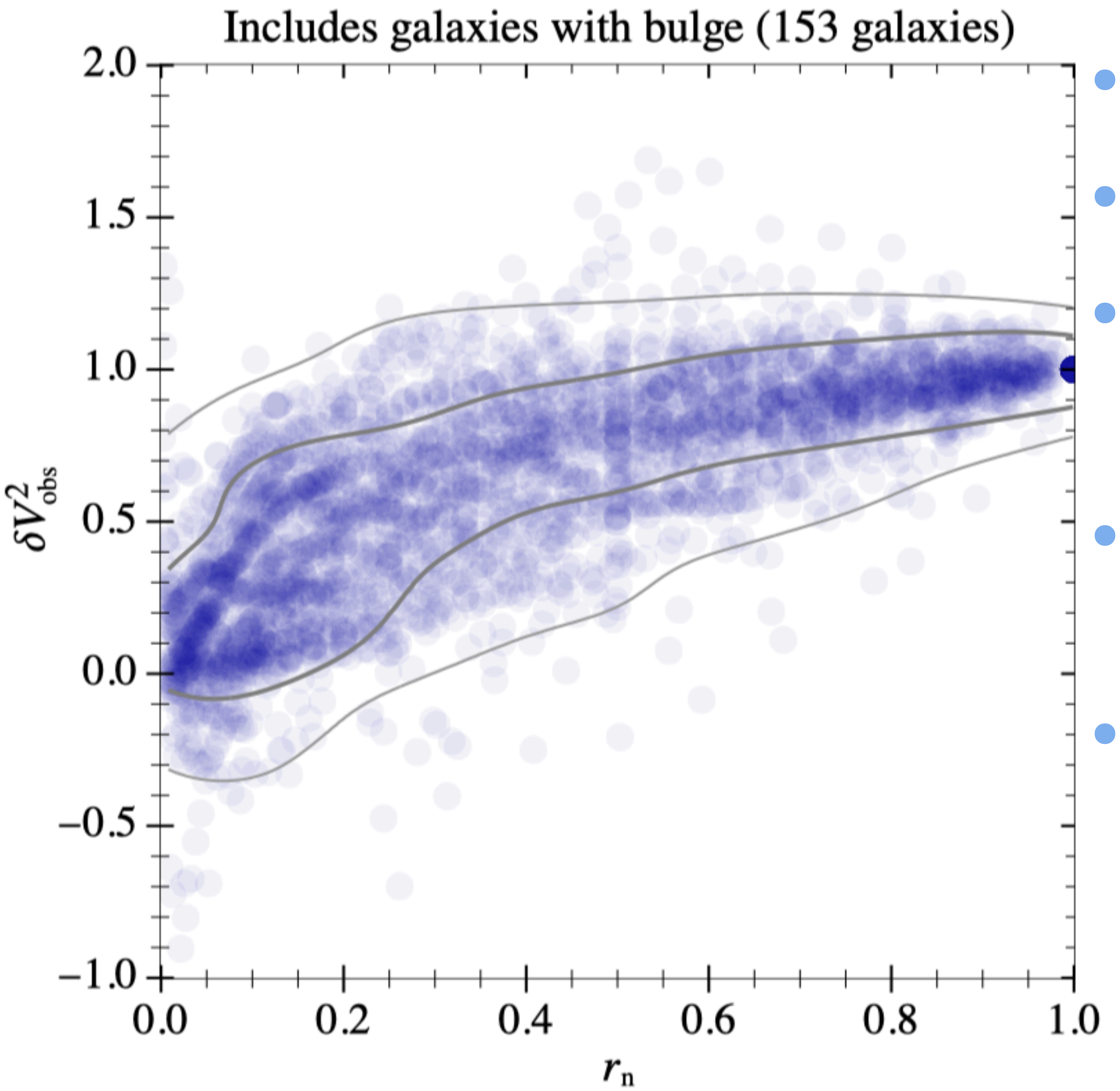
(Newtonian gravity without DM $\Longrightarrow \Delta V^2_{\rm obs} \sim 0$, i.e., small oscillations about zero)

- And the *normalized additional velocity,* as function of $r_n \equiv r/r_{\rm max}$, reads

$$\delta V^2_{\rm obs}(r_n) \equiv \frac{\Delta V^2_{\rm obs}(r_n\, r_{\rm max})}{\Delta V^2_{\rm obs}(r_{\rm max})}\ .$$

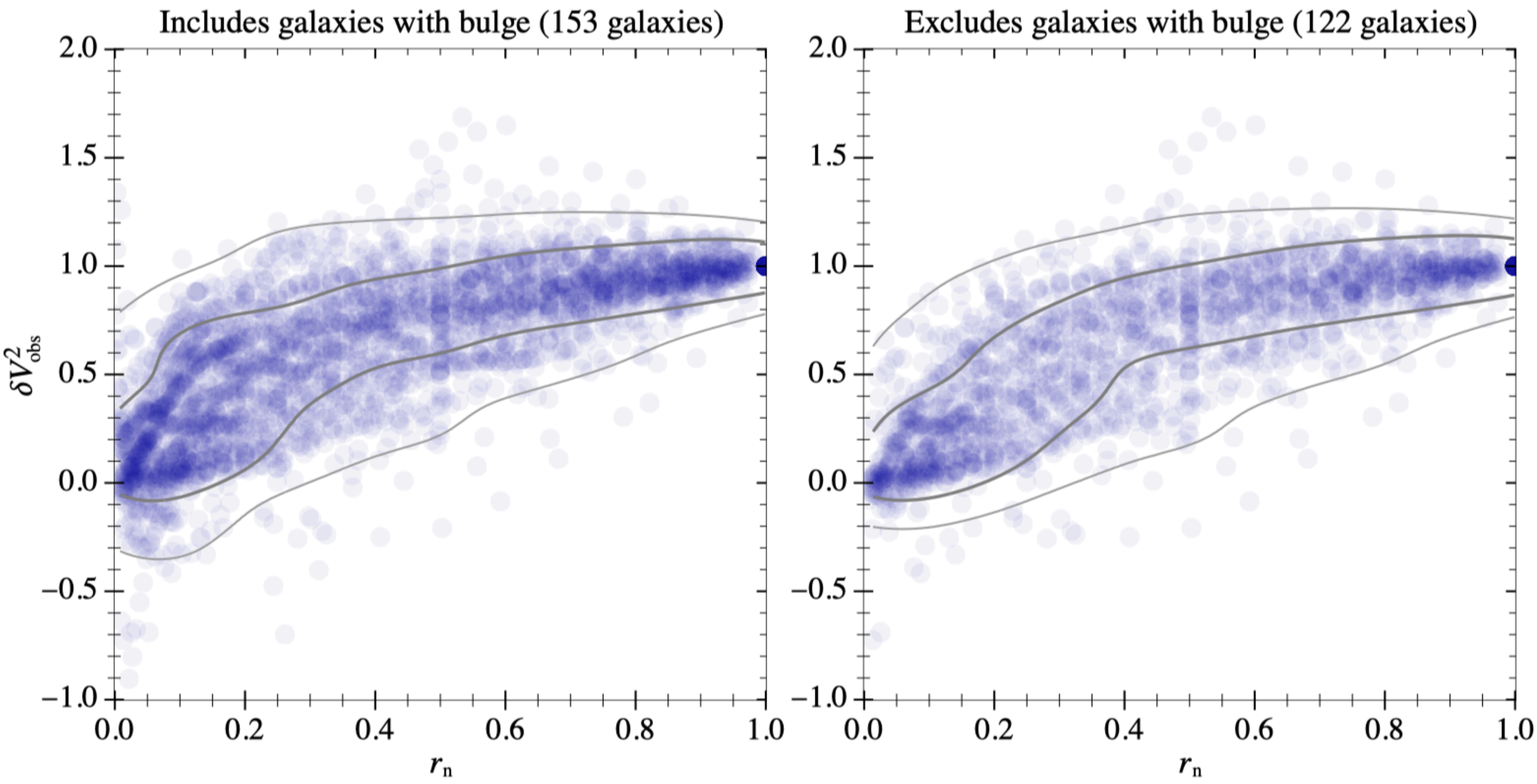
(Newtonian gravity without DM $\Longrightarrow \delta V^2_{\rm obs}(r_n)$ oscillate with arbitrary magnitude)

The NAV distribution form the SPARC data



- This plot is model independent.
- Each blue circle is a data point from a galaxy.
- All the 153 galaxies have a point at $(r_n, \delta V^2_{\mathrm{obs}}) = (1,1)$.
- The gray curves delimit the 1σ and 2σ *highest density regions*.
- If there was no dark matter, this plane should be randomly populated, without preference for $\delta V^2_{\mathrm{obs}} > 0$.

The NAV distribution form the SPARC data



NFW profile's NAV - analytical part

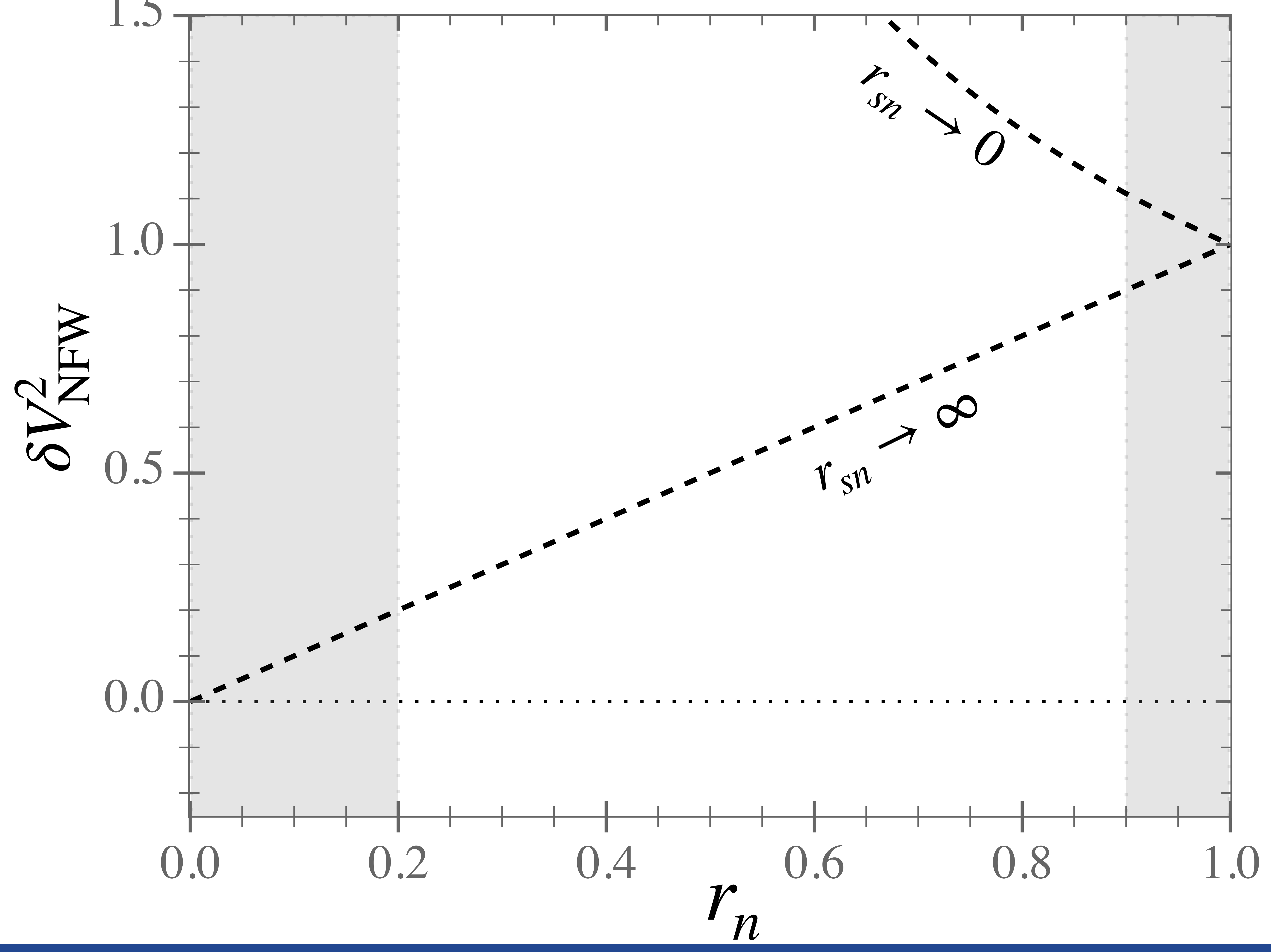
$$\rho_{\mathrm{NFW}}(r) = \frac{\rho_s}{\frac{r}{r_s}\left(1+\frac{r}{r_s}\right)^2} \longrightarrow M_{\mathrm{NFW}}(r) = 4\pi r^3 \rho_s \left[\ln\left(\frac{r+r_s}{r_s}\right) - \frac{r}{r+r_s}\right]$$

$$\Delta V^2_{\mathrm{NFW}}(r) = G\frac{M_{\mathrm{NFW}}(r)}{r}$$

$$\delta V^2_{\mathrm{NFW}}(r_n) = \frac{\Delta V^2_{\mathrm{NFW}}(r_n\, r_{\max})}{\Delta V^2_{\mathrm{NFW}}(r_{\max})} = \frac{1}{r_n}\,\frac{\frac{r_n}{r_n+r_{sn}} + \ln\frac{r_{sn}}{r_n+r_{sn}}}{\frac{1}{1+r_{sn}} + \ln\frac{r_{sn}}{1+r_{sn}}} \qquad r_{sn} \equiv \frac{r_s}{r_{\max}}$$

NFW profile's NAV - analytical part

$$\lim_{r_{sn}\to\infty} \delta V^2_{\rm NFW}(r_n) = r_n \qquad \lim_{r_{sn}\to 0} \delta V^2_{\rm NFW}(r_n) = \frac{1}{r_n}$$



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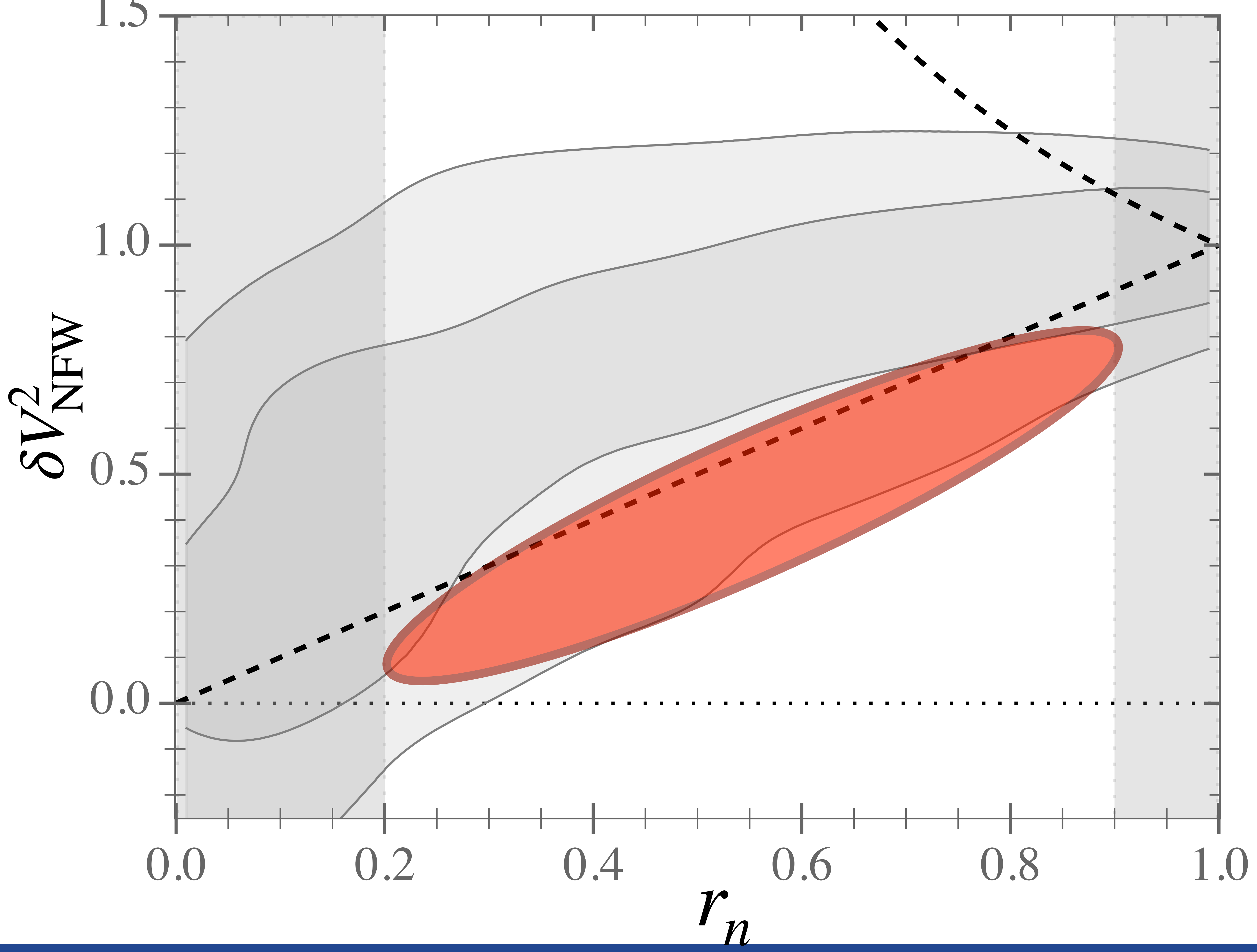


- Most of the data from galaxies can be promptly covered.

- But another part requires adjusting galaxy parameters and $r_s \to \infty$.

- Several individual galaxy fits should lead to $r_s \to \infty$.

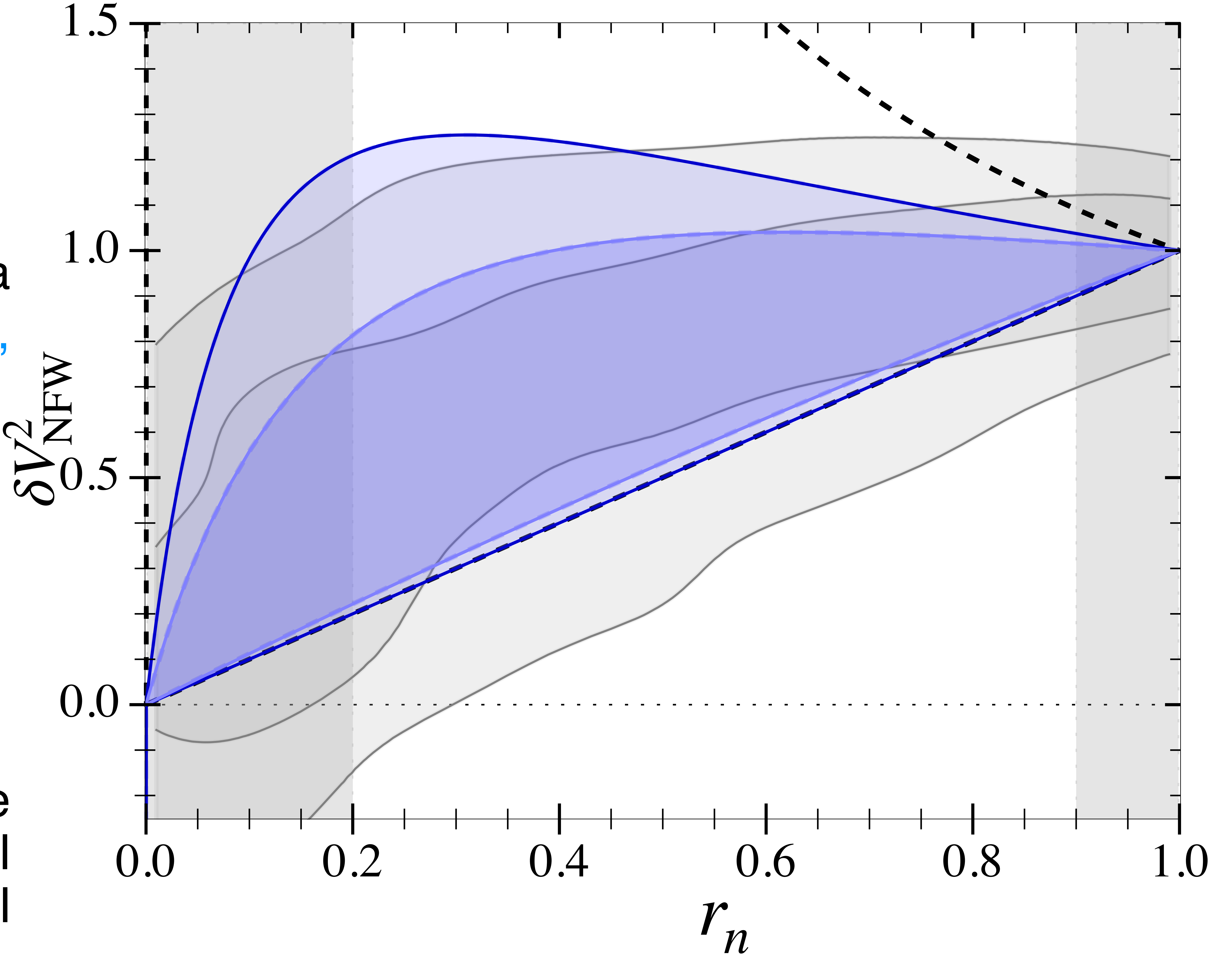
NFW profile's NAV - numerics

$$\rho_{\rm NFW} = \frac{\rho_s}{\frac{r}{r_s}\left(1+\frac{r}{r_s}\right)^2}$$

- Are there $r_{\rm sn}$ values that can cover a large part of the $\delta V^2_{\rm obs}$ region? ANS: Yes, but problems in the lower part of the plot.

- The most probable values of r_{sn}
  $0.29 < r_{\rm sn} < 9.9$

- Efficiency coefficient: 0.67
  (measures the covering of the observational region, *minus* model regions that are outside the observational region, for $0.2 < r_n < 0.9$).



Results valid for any ρ_s

Burkert profile

Phenomenological halo profile that in general fits better galaxies than NFW [e.g., DCR et al MNRAS 2017]

$$\rho_{\rm Bur}(r) = \frac{\rho_{\rm c}}{(1+r/r_{\rm c})(1+r^2/r_{\rm c}^2)} \, .$$

Two free parameters

In the above, r is the spherical radial coordinate, while $r_{\rm c}$ and $\rho_{\rm c}$ are constants that can change from galaxy to galaxy.

The internal mass of the Burkert profile reads,

$$M_{\rm Bur}(r) = 4\pi \int_0^r \rho_{\rm Bur}(r) r^2 \, dr$$
$$= 2\pi \, \rho_{\rm c} r_{\rm c}^3 \, \xi\left(\frac{r}{r_{\rm c}}\right) , \qquad (10)$$

where

$$\xi(x) \equiv \ln\left((1+x)\sqrt{1+x^2}\right) - \tan^{-1}(x) \, . \qquad (11)$$

It is convenient to introduce the normalized core ra

$$r_{\rm cn} \equiv \frac{r_{\rm c}}{r_{\rm max}} \, .$$

Using that, for a spherical mass distribution, $V^2(r$

we can now compute $\Delta V^2_{\rm mod}$ and $\delta V^2_{\rm mod}$ as

$$\Delta V^2_{\rm Bur}(r_{\rm n}) = 2\pi \frac{G\rho_{\rm c} r_{\rm c}^3}{r_{\rm max}} \frac{1}{r_{\rm n}} \xi\left(\frac{r_{\rm n}}{r_{\rm cn}}\right) ,$$

$$\boxed{\delta V^2_{\rm Bur}(r_{\rm n}) = \frac{1}{r_{\rm n}} \frac{\xi(r_{\rm n}/r_{\rm cn})}{\xi(1/r_{\rm cn})} \, .}$$

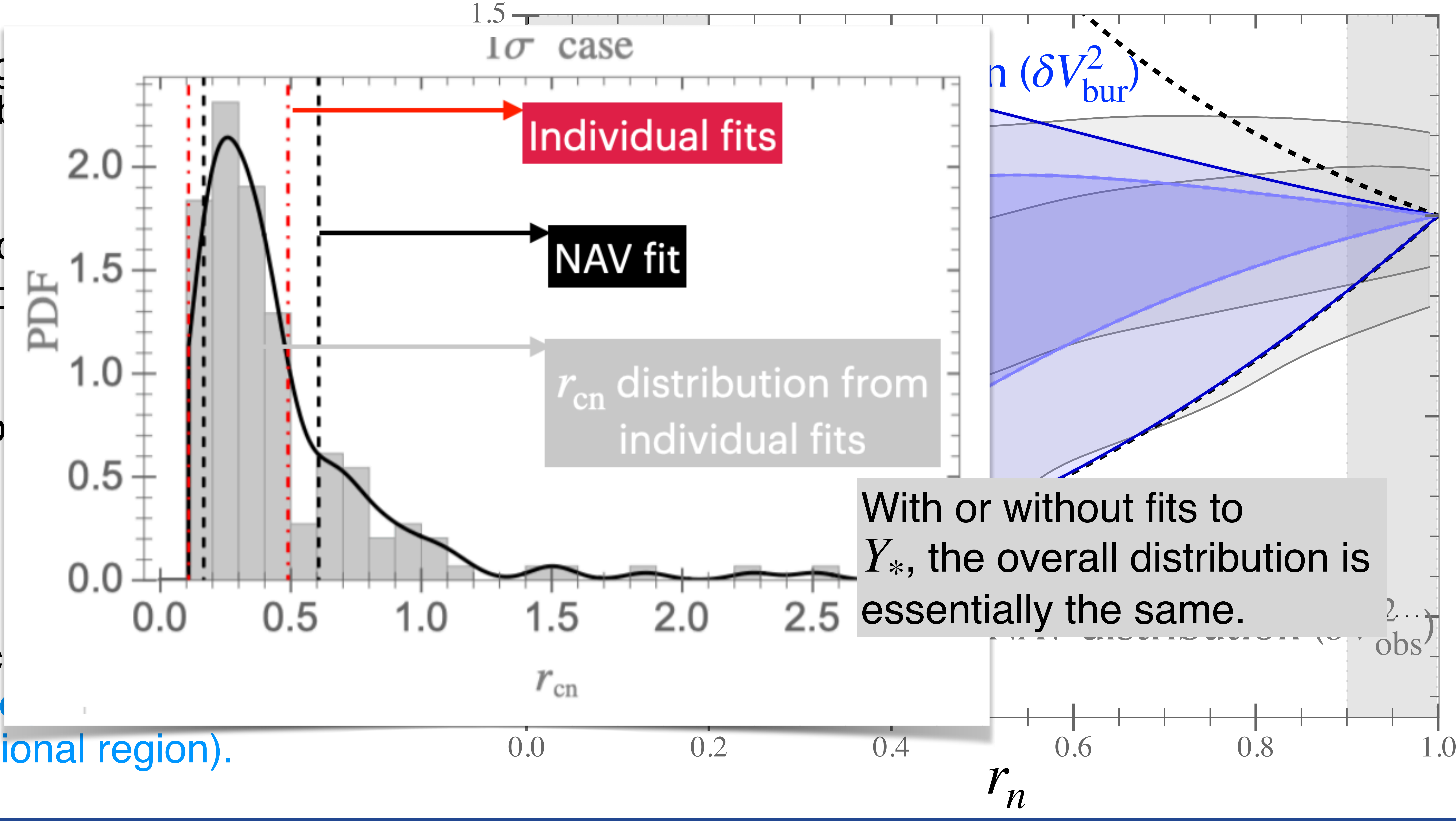
One free parameter

$$\boxed{\lim_{r_{cn}\to\infty} \delta V^2_{\rm Bur} = r_n^2}$$

Burkert profile's NAV

Results valid for any ρ_c

- Instead of fitting one fits the distrib
- $r_{\rm cn}$ is fitted to distribution as clo
- The most prob $0.2 < r_{\rm cn} < 0.6$ $0.1 < r_{\rm cn} < 0.5$
- Efficiency coeffic $\propto$ (Intersection re beyond observational region).



With or without fits to Y_*, the overall distribution is essentially the same.

DC14 profile NAV

$$\rho_{\rm DC14} = \frac{\rho_s}{\frac{r^\gamma}{r_s^\gamma}\left(1+\frac{r^\alpha}{r_s^\alpha}\right)^{(\beta-\gamma)/\alpha}}$$

[Di Cintio et al MNRAS 2014]

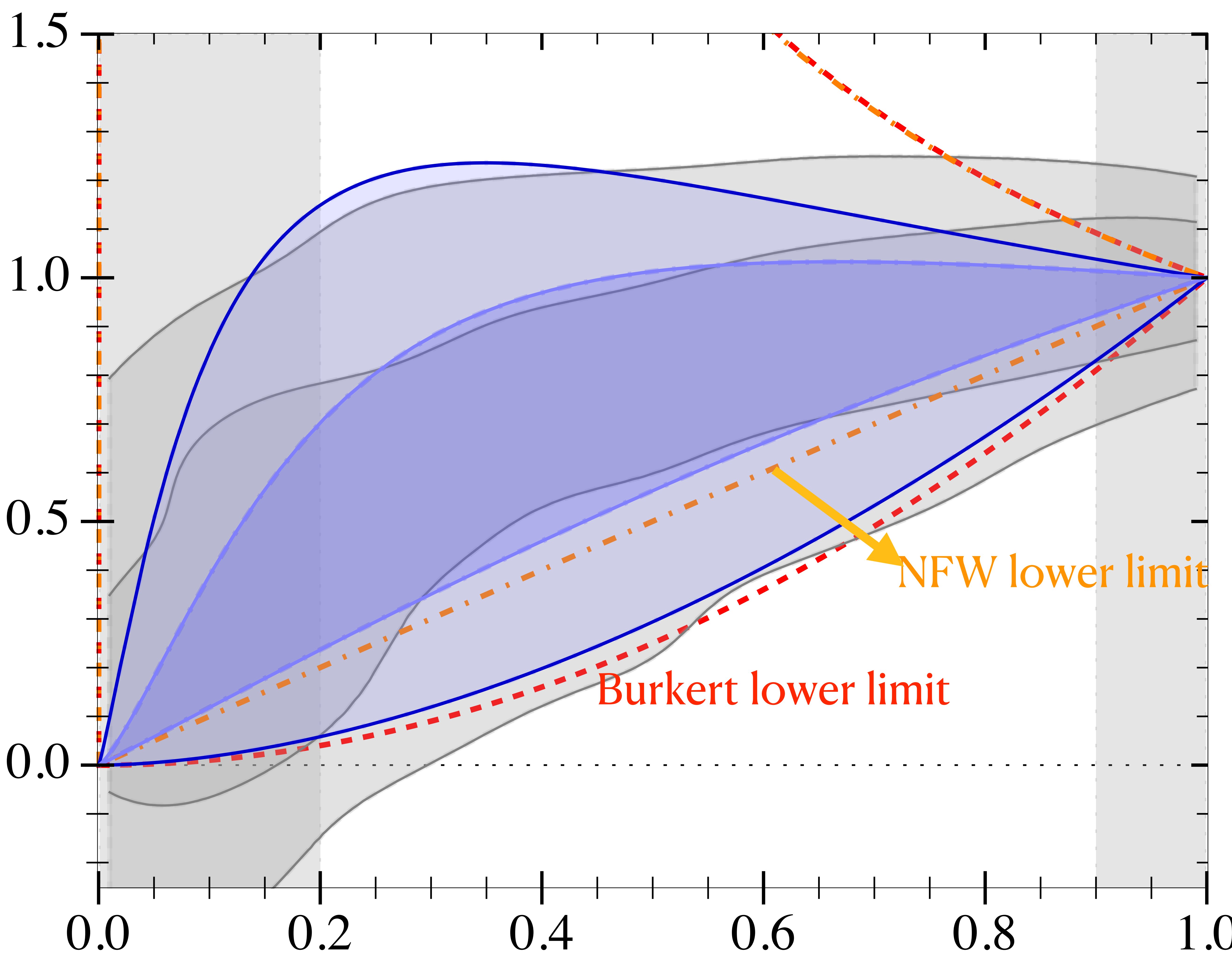
$$\alpha = 2.94 - \log_{10}\left(\left(10^{X+2.33}\right)^{2.29} + \frac{1}{\left(10^{X+2.33}\right)^{1.08}}\right)$$

$$\beta = 0.26X^2 + 1.34X + 4.23$$

$$\gamma = \log_{10}\left(10^{X+2.56} + \frac{1}{\left(10^{X+2.56}\right)^{0.68}}\right) - 0.06$$

$$-4.1 < X < -1.3$$

- Is there $r_{\rm sn}$ values that can cover a large part of the $\delta V^2_{\rm obs}$ region? ANS: Yes.
- Efficiency: 0.80



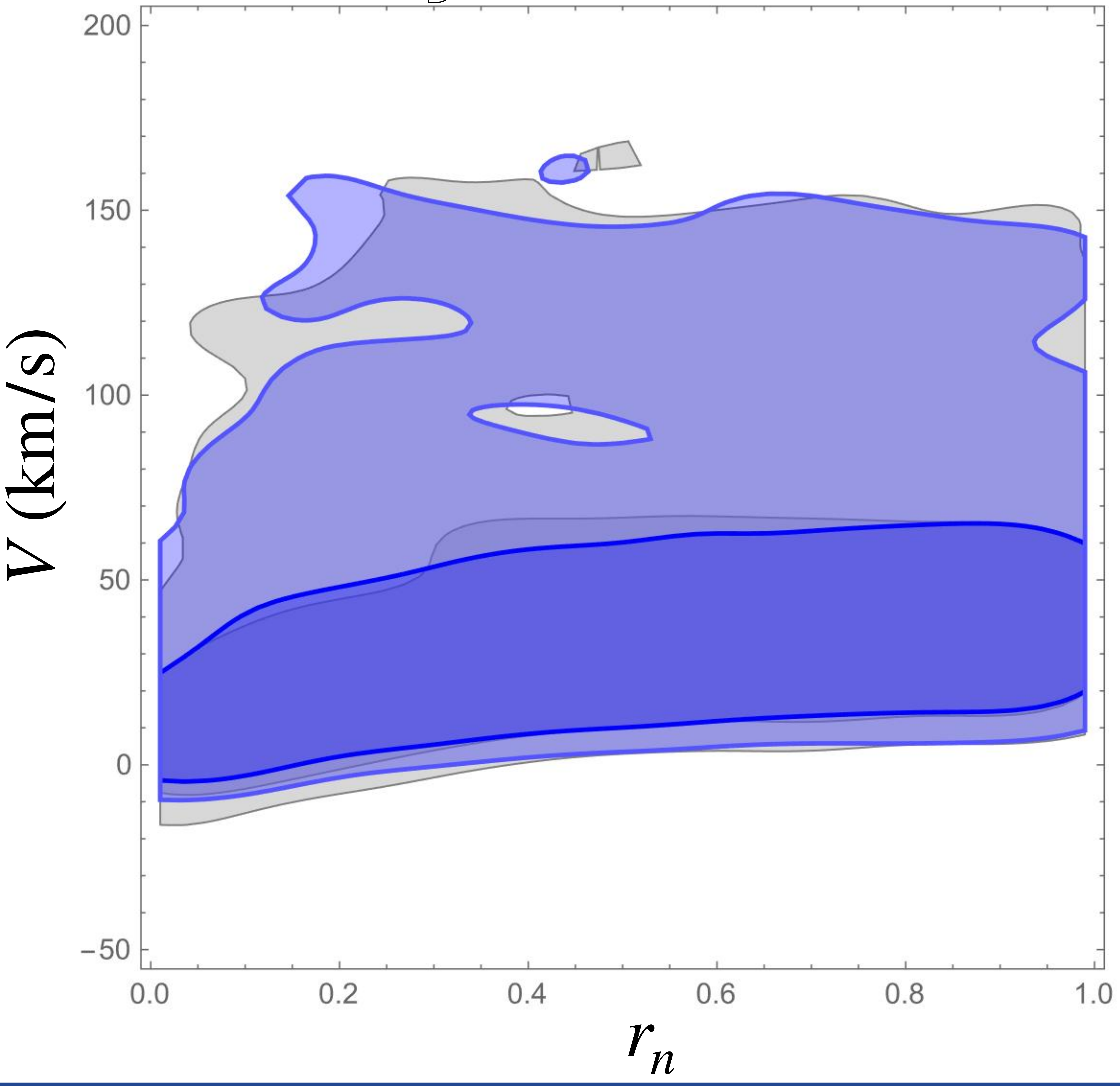
Using NAV to test modified gravity with approximations

- Provides much quicker tests than individual fits
- The 3D density distributions need not to be known.

Approximations for modified gravity

- We remove all the galaxies with a bulge, this reduces that sample size from 153 to 122 galaxies
- For the stellar component, we use an exponential profile, and this is very reasonable [Freeman 1970; van der Kruit & Freeman 2011].
- For the gas component, we also use an exponential profile. For individual galaxies, this is not a good approximation, but for a sample it is much better.

Baryonic rotation curve, comparison



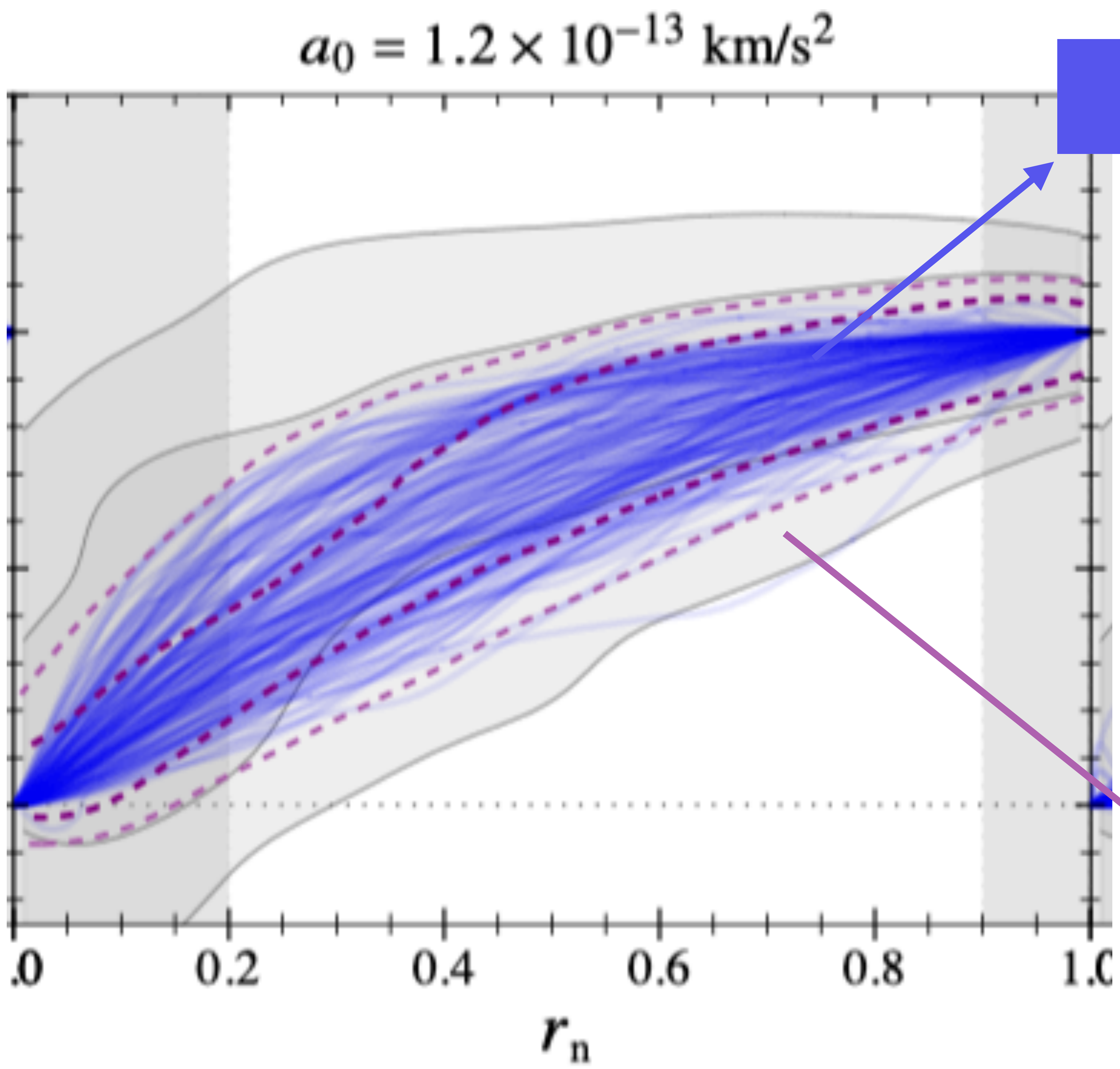
Sample: 122 high quality galaxies without bulge from SPARC.

Gray: expected (SPARC) baryonic curve distribution.

Blue: approximated baryonic curve distribution

MOND's NAV

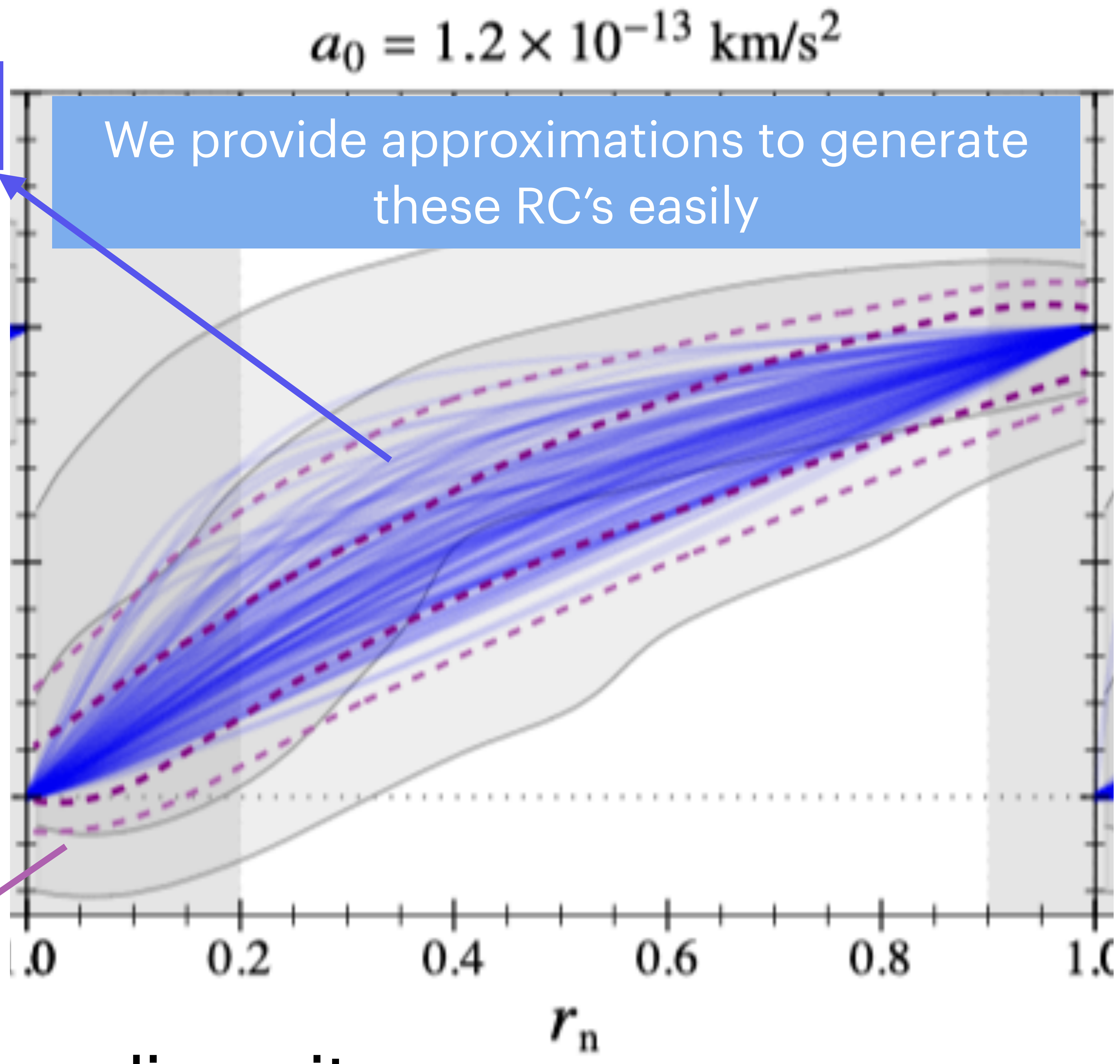
Without modified gravity approximations



MOND RC's

MOND 1σ and 2σ regions

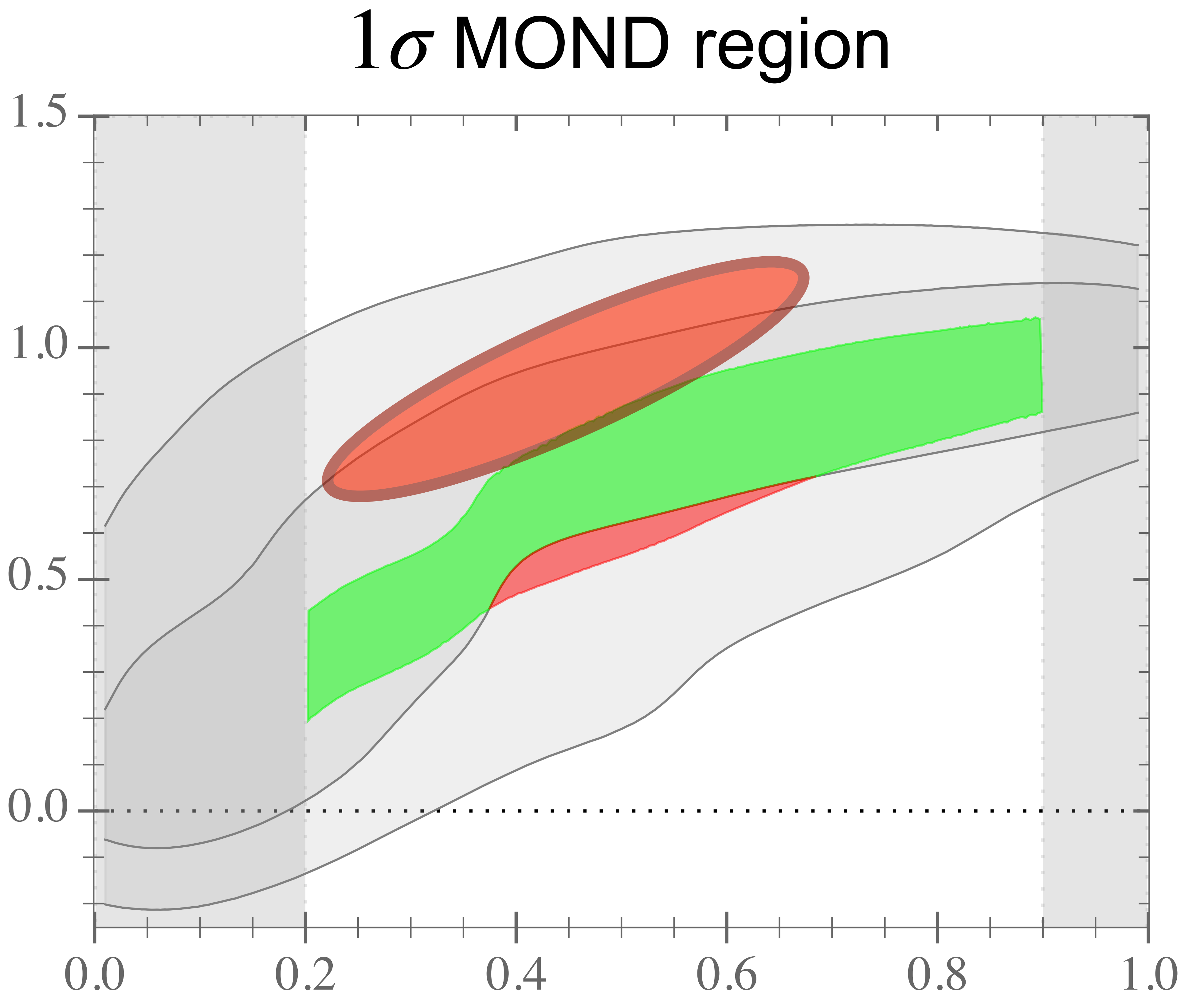
With modified gravity approximations

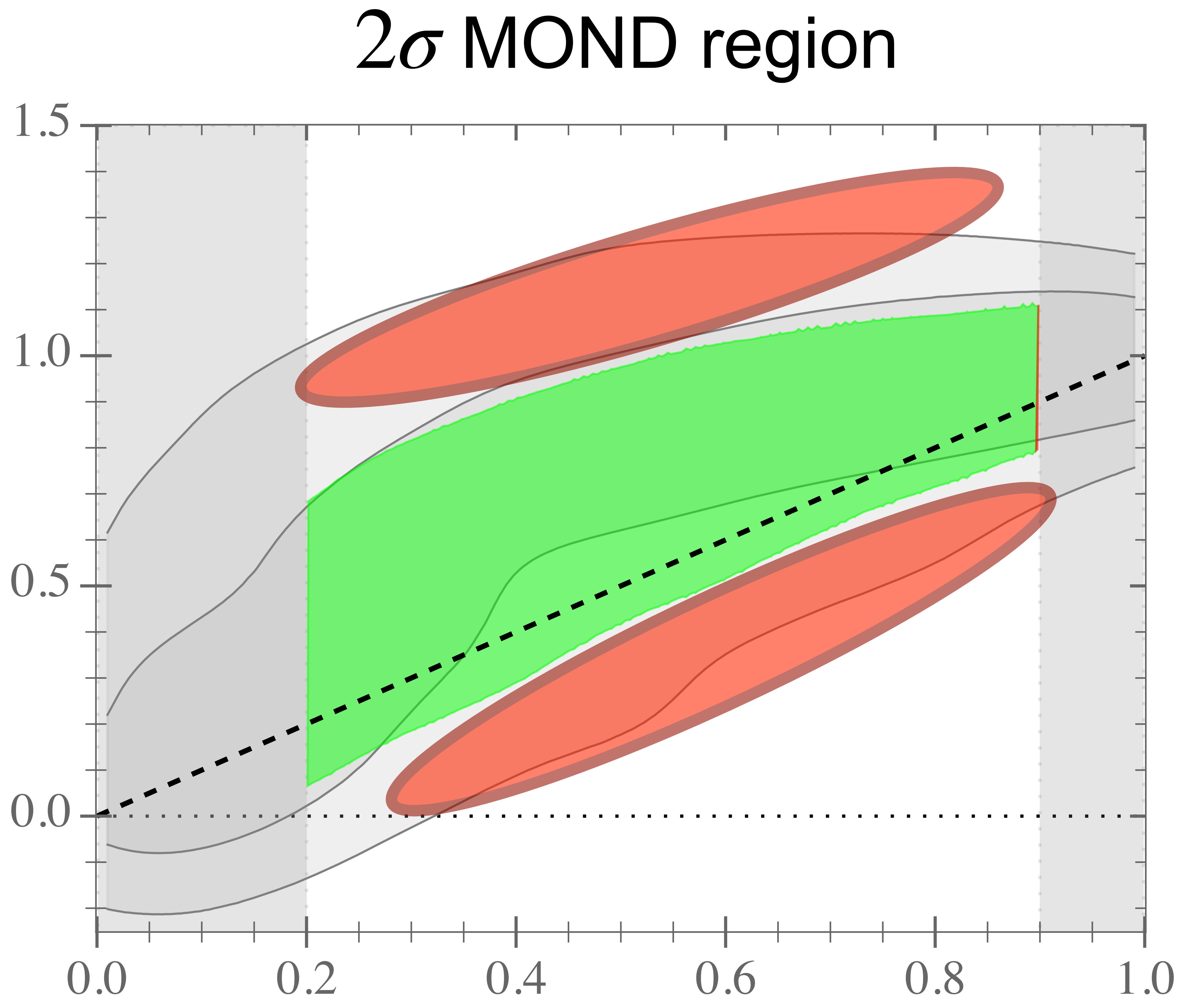


We provide approximations to generate these RC's easily

- MOND's result is excessively concentrated: much less diversity. Efficiency: 0.53. (χ^2 values of MOND are much larger than those of DM).
- With or without approximations, the results are the same.

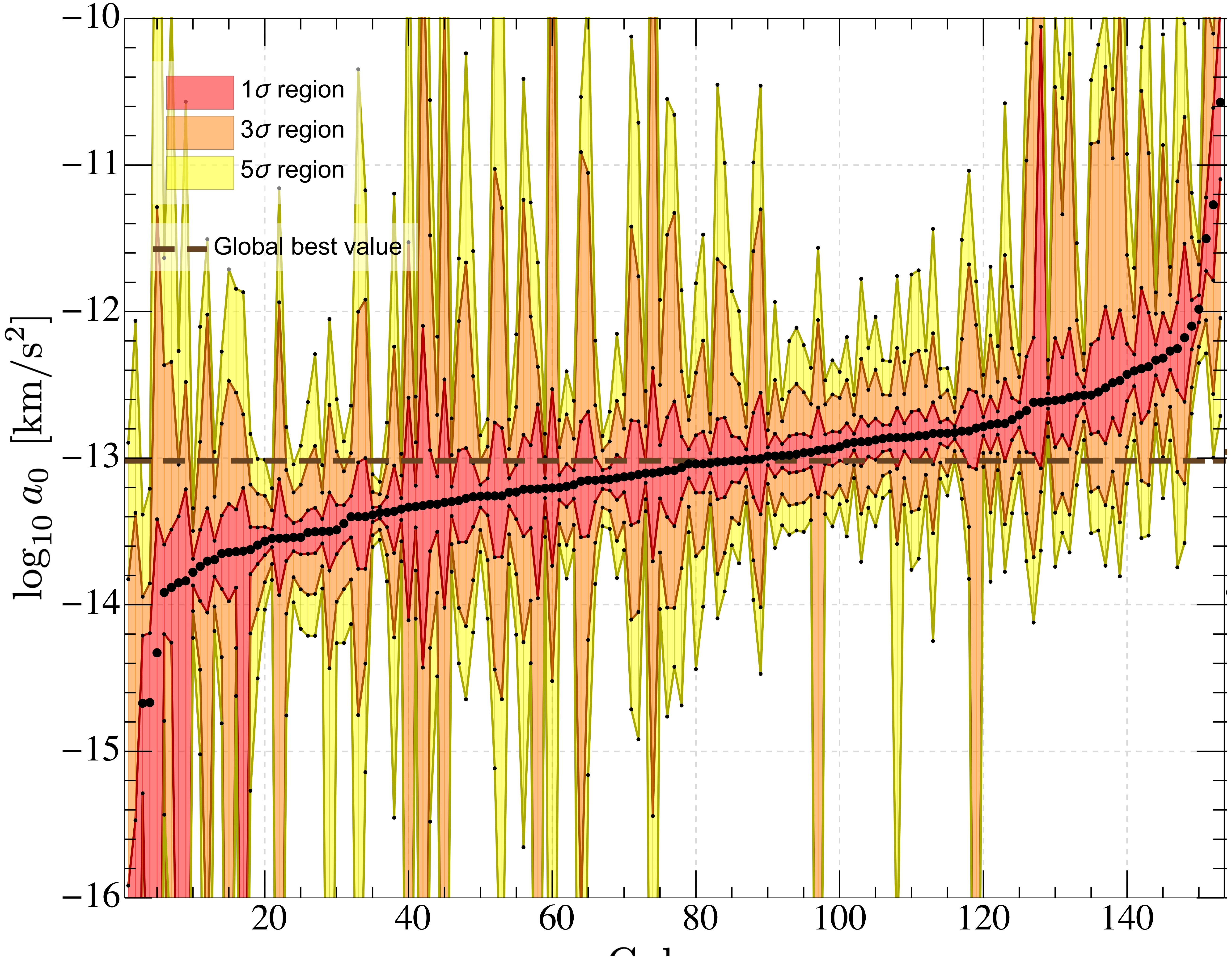
MOND's NAV





- Efficiency: 0.53.

Comment: The deepest problem with MOND



This applies the original form of MOND (inertia) but for different interpolation functions.

LETTERS

https://doi.org/10.1038/s41550-018-0498-9

nature astronomy

Absence of a fundamental acceleration scale in galaxies

Davi C. Rodrigues[1,2*], Valerio Marra[1,2*], Antonino del Popolo[3,4,5] and Zahra Davari[6]

Monthly Notices of the Royal Astronomical Society

MNRAS **494,** 2875–2885 (2020)
Advance Access publication 2020 April 7

A fundamental test for MOND

Valerio Marra,[1★] Davi C. Rodrigues[1★] and Álefe O. F. de Almeida[1,2★]

MOND is incompatible with a single a_0 value at more than 5σ (or more than 10σ under certain gaussianity approximation)

f(R) Palatini NAV (identical results for EiBI)

$$S[g,\Gamma,\Psi] = \frac{1}{2\kappa}\int f(R(\Gamma,g))\sqrt{-g}\, d^4x + S_{\text{matter}}[g,\Psi]\,,$$

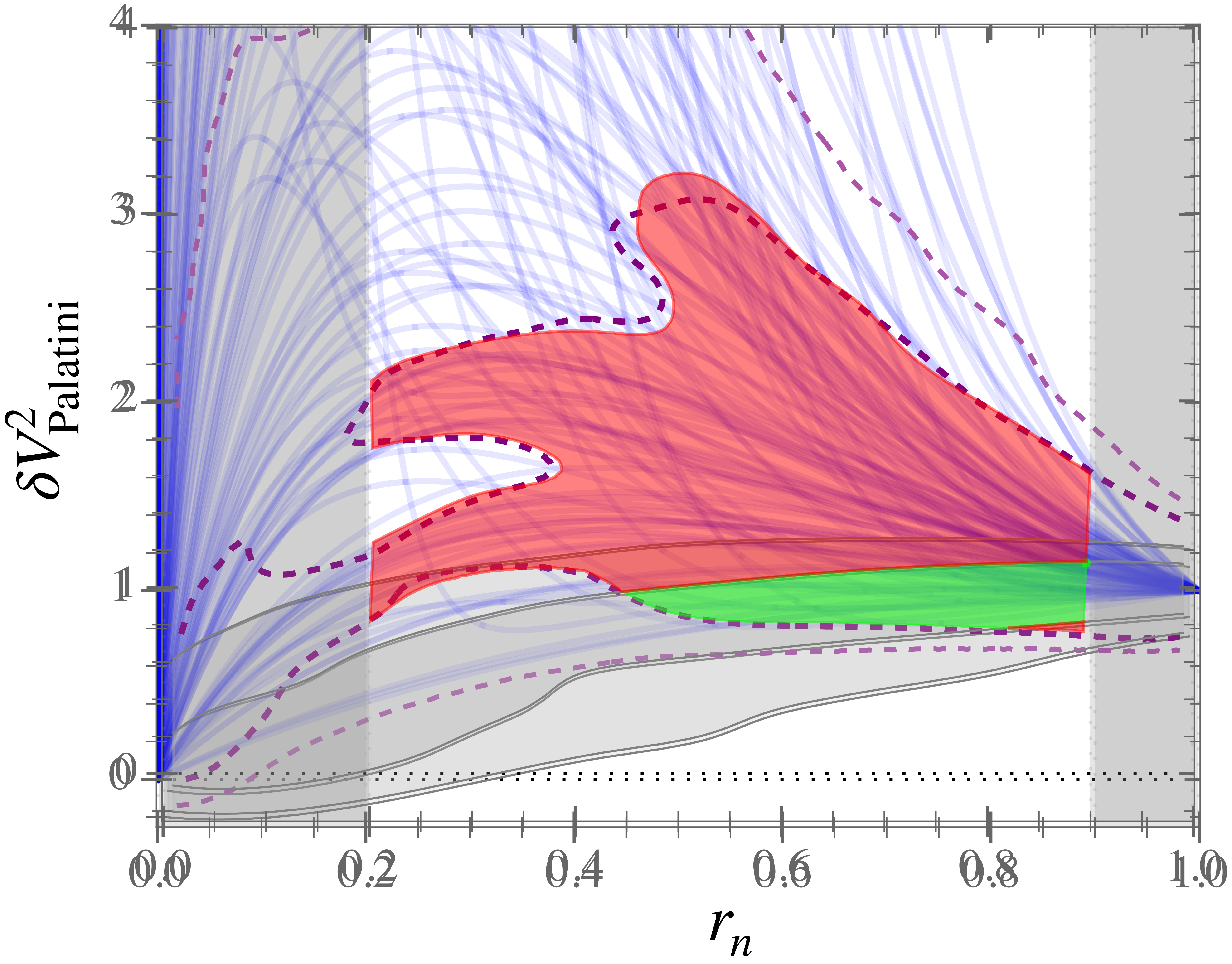
$$\frac{V_c^2}{r} = \partial_r U + \tilde{a}_2 \partial_r \rho$$

$$\Delta V_c^2 = r\,\tilde{a}_2 \partial_r \rho$$

$$\delta V^2_{\text{Palatini}}(r_{\text{n}}) = r_{\text{n}} \frac{\rho'(r_{\text{n}})}{\rho'(1)}$$

Efficiency: < -2.0

Independent from $\tilde{a}_2$



Conclusions

- For large samples, efficient ways to compare DM and MG models are relevant.
- It is particularly difficult to study MG in individual galaxies
- The NAV method provides fast results for an important RC feature: its shape.
- Results from the NAV plane:
  - Most efficient DM models for NAV: DC14 > Burkert > NFW
    Besides cusp/core, it is relevant to consider the intermediate radial dependence.
  - MOND RC's lack diversity. For a 5σ+ problem: [Rodrigues et al Nat.Ast. 2018]
  - Palatini $f(R)$ and EiBI gravity: they cannot be used to replace DM in galaxies.
  - For covering of the NAV region: NFW > MOND >> f(R) Palatini.
- Further details in [Hernandez-Arboleda, Rodrigues, Wojnar 2204.03762].
  The code can be found here: https://github.com/davi-rodrigues/NAVanalysis