Post-Newtonian γ-like parameters and the slip: differences and consequences for future observations

Universidade Federal do Espírito Santo

Cosmo-ufes
Center for Astrophysics and Cosmology

www.cosmo-ufes.org

Davi C. Rodrigues
Universidade Federal do Espírito Santo
Vitória, ES - Brazil

FAPES
FUNDAÇÃO DE AMPARO À PESQUISA DO ESPÍRITO SANTO

CNPq
Conselho Nacional de Desenvolvimento Científico e Tecnológico

Mainly based on:
Júnior Toniato, Davi C. Rodrigues, *PRD* (2021) [2106.12542]
Júnior Toniato, Davi C. Rodrigues, Aneta Wojnar, *PRD* (2020) [1912.12234]

What is "γ_{PPN}"?

- The Parametrized Post-Newtonian (PPN) formalism, in its "modern" form, due to Will and Nordtvedt, depends on 10 parameters.
- The PPN perturbation order is counted using powers on the matter velocity field v.
- The matter is described as relativistic perfect fluids (up to the first PN order).
- For Newtonian physics, only the g_{00} component needs to be known up to $O(v^2)$.
- For light propagation, all the metric components need to be known up to $O(v^2)$, in this case,

PPN metric up to $O(v^2)$

$$g_{00}^{\text{PPN}} = -1 + 2U + O(v^4),$$

$$g_{0i}^{\text{PPN}} = 0 + O(v^3),$$

$$g_{ij}^{\text{PPN}} = \delta_{ij} + 2\gamma U \delta_{ij} + O(v^4).$$

$$U(x,t) \equiv \int \frac{\rho(\mathbf{x}', t)}{|\mathbf{x} - \mathbf{x}'|} d^3x'$$

What is "$\gamma_{\rm PPN}$"?

PPN metric
up to $O(v^2)$

$$g_{00}^{\rm PPN} = -1 + 2U + O(v^4),$$

$$g_{0i}^{\rm PPN} = 0 + O(v^3),$$

$$g_{ij}^{\rm PPN} = \delta_{ij} + 2\gamma U \delta_{ij} + O(v^4).$$

$$U(x,t) \equiv \int \frac{\rho(\mathbf{x}', t)}{|\mathbf{x} - \mathbf{x}'|} d^3x'$$

Within the PPN formalism, γ is constrained by two phenomena:

- Light deflection
- Shapiro time delay

For GR: $\gamma = 1$ always, it cannot change.
Physical bounds in the solar system: $|\gamma - 1| \lesssim 10^{-5}$

What is "γ_{PPN}"?

PPN metric up to $O(v^2)$

$$g_{00}^{\mathrm{PPN}} = -1 + 2U + O(v^4),$$
$$g_{0i}^{\mathrm{PPN}} = 0 + O(v^3),$$
$$g_{ij}^{\mathrm{PPN}} = \delta_{ij} + 2\gamma U \delta_{ij} + O(v^4).$$

$$U(x,t) \equiv \int \frac{\rho(\mathbf{x}', t)}{|\mathbf{x} - \mathbf{x}'|} d^3x'$$

Within PPN, γ is a constant and it is defined from this metric.

Within the PPN formalism, γ is constrained by two phenomena:

- Light deflection
- Shapiro time delay

For GR: $\gamma = 1$ always, it cannot change.
Physical bounds in the solar system: $|\gamma - 1| \lesssim 10^{-5}$

What is "γ_{PPN}"?

The Newtonian limit is assumed.

PPN metric up to $O(v^2)$

$$g_{00}^{\text{PPN}} = -1 + 2U + O(v^4),$$

$$g_{0i}^{\text{PPN}} = 0 + O(v^3),$$

$$g_{ij}^{\text{PPN}} = \delta_{ij} + 2\gamma U \delta_{ij} + O(v^4).$$

$$U(x,t) \equiv \int \frac{\rho(\mathbf{x}', t)}{|\mathbf{x} - \mathbf{x}'|} d^3x'$$

Within PPN, γ is a constant and it is defined from this metric.

Within the PPN formalism, γ is constrained by two phenomena:

- Light deflection
- Shapiro time delay

For GR: $\gamma = 1$ always, it cannot change.
Physical bounds in the solar system: $|\gamma - 1| \lesssim 10^{-5}$

The gravitational slip η

- The gravitational slip (η) is commonly introduced in a cosmological context, sometimes contextualized as a parameterized post-Friedmann (PPF) approach.
- Let

$$ds^2 = a^2(\tau)\left[-(1-2\psi)d\tau^2 + (1+2\phi)d\mathbf{x}^2\right].$$

- A common definition for the slip in physical space is

$$\eta \equiv \frac{\phi}{\psi}\,.$$

- If $T_{ij} = 0, \forall i \neq j$, then GR field equations imply $\eta = 1$.
- By particularizing the cosmological metric above to be the PPN metric up to $O(v^2)$,

$$\eta\,|_{\mathrm{PPN}} = \gamma.$$

The gravitational slip η

- The gravitational slip (η) is commonly introduced in a cosmological context, sometimes contextualized as a parameterized post-Friedmann (PPF) approach.
- Let

$$ds^2 = a^2(\tau)\left[-(1-2\psi)d\tau^2 + (1+2\phi)d\mathbf{x}^2\right].$$

- A common definition for the slip in physical space is

$$\eta \equiv \frac{\phi}{\psi}.$$

- If $T_{ij} = 0, \forall i \neq j$, then GR field equations imply $\eta = 1$.
- By particularizing the cosmological metric above to be the PPF metric up to $O(v^2)$,

$$\eta|_{\mathrm{PPN}} = \gamma.$$

✓

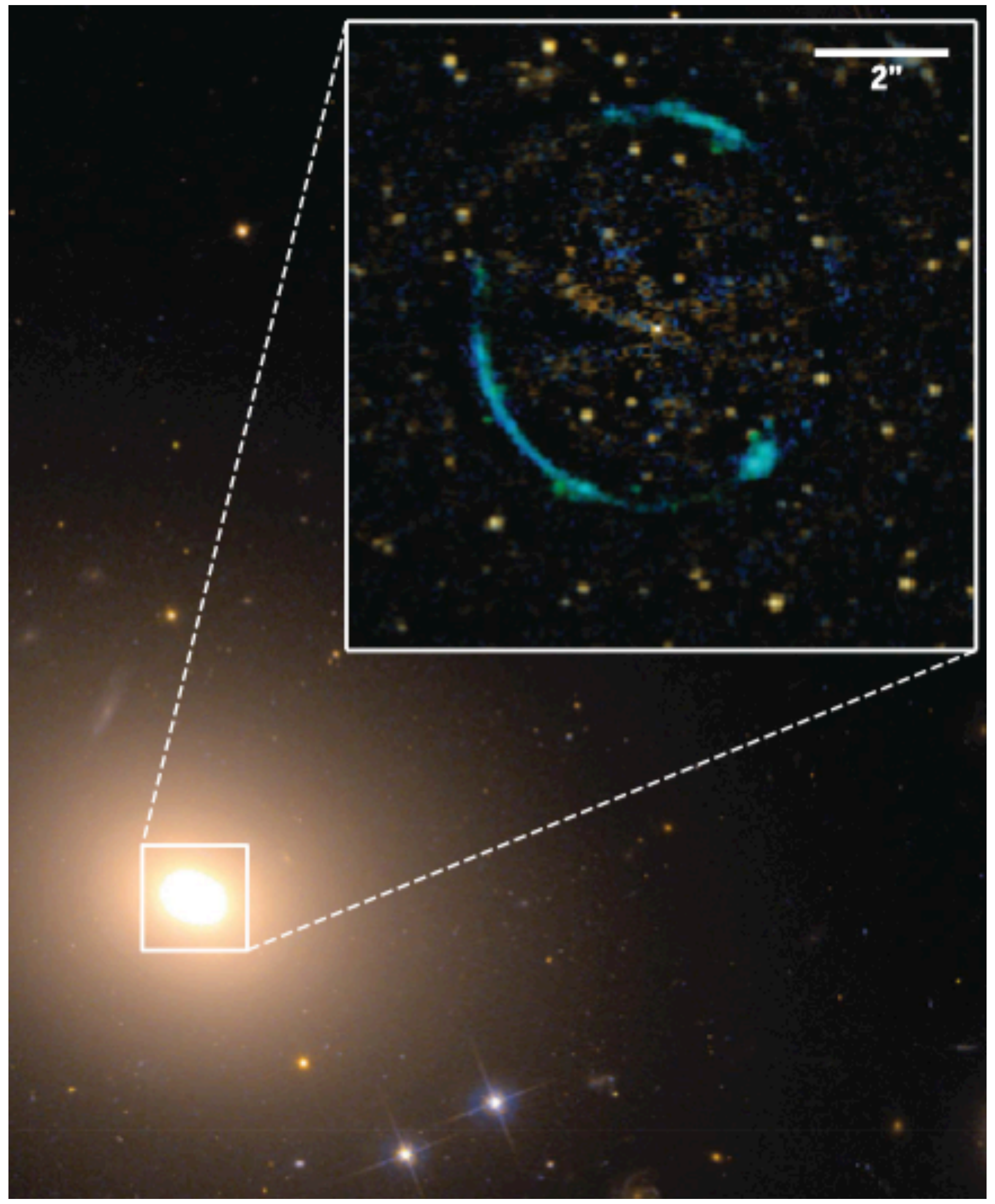
$\not\Rightarrow \eta = \gamma$

The root of (almost) all evil

Example: Collet et al Science 2018 did it right

- This work (among others) looks for constraining γ from galaxy data, by using simultaneously the internal dynamics and lensing data from the same galaxy.

- At first, what Collet el al call γ is actually η, but they demand that:
  - η is a constant and
  - There is a Newtonian limit (they include a dark matter halo).
- Hence, they are indeed finding bounds on γ (within their hypothesis).
- On the other hand, they cite $f(R)$ theories as the main motivation to measure arbitrary values for γ in their context. We will see that this is not the case.



$$M_{\rm dyn} = \frac{1+\gamma}{2} M^{\rm GR}_{\rm lensing}$$

$\gamma = 0.97 \pm 0.09$ at 68%

From [Collet et al Science 2018]

Introducing γ_Σ

- Let

$$ds^2 = -(1 - 2\alpha_e U)dt^2 + (1 + 2\gamma_e U)d\mathbf{x}^2 \,,$$

where α_e and γ_e are *arbitrary functions*. In the above, using U is just a matter of convenience.

- From the geodesics for electromagnetic waves,

$$\frac{dn^i}{dt} = (\delta^{ij} - n^i n^j)\partial_j[(\alpha_e + \gamma_e)U].$$

- n^i determines the light direction of propagation.
- We can introduce, see also [Berry, Gair, PRD (2011)],

$$\gamma_\Sigma \equiv \alpha_e + \gamma_e - 1 \,.$$

This is true even in the absence of a Newtonian limit (i.e. α_e being not const.).

- The GR solution for the light deflection requires $\alpha_e + \gamma_e = 2$, hence $\gamma_\Sigma = 1$.
- Only if γ_Σ is a constant it can fully parameterize the bending of light and the time delay. Otherwise, these phenomena locally depend on the derivative of γ_Σ as well.

Application: Brans-Dicke with a potential

$$S = \frac{1}{2\kappa}\int \left[\Phi R - \frac{\omega(\Phi)}{\Phi}\nabla_\mu\Phi\,\nabla^\mu\Phi - V(\Phi)\right]\sqrt{-g}\,d^4x + S_m\,,$$

- We expand on powers of v, following the PPN framework
  - $\Phi = \varphi_0 + \varphi, \quad$ with $\varphi_0 > 0$ and $|\varphi| \lesssim O(v^2)$;
  - $V(\Phi) \approx V_2\varphi^2$, asymptotic flatness requires $V_0 = 0$, and we use here $V_1 = 0$ (see our paper for $V_1 \neq 0$).
  - $\omega(\Phi) \approx \omega_0 + \omega_1\varphi + \omega_2\varphi^2$.

- Scalar field mass:

$$m_\varphi^2 = \frac{2V_2\varphi_0}{3 + 2\omega_0}\,.$$

- A Newtonian limit can only be found if the scalar field is either sufficiently large or small.

Results: Brans-Dicke with a potential

$$\gamma_\Sigma = \begin{cases} \frac{\kappa}{4\pi\varphi_0} - 1\,, & \text{in general}, \\ \gamma = \begin{cases} \frac{1+\omega_0}{2+\omega_0}, & \text{if } m_\varphi^2 \ell^2 \lesssim O(v^1), \\ 1\,, & \text{if } e^{-m_\varphi \ell} \lesssim O(v^1). \end{cases} \end{cases}$$

Results: Brans-Dicke with a potential

$$\gamma_\Sigma = \begin{cases} \frac{\kappa}{4\pi\varphi_0} - 1\,, & \text{in general,} \\ \gamma = \begin{cases} \frac{1+\omega_0}{2+\omega_0}\,, & \text{if } m_\varphi^2 \ell^2 \lesssim O(v^1)\,, \\ 1\,, & \text{if } e^{-m_\varphi \ell} \lesssim O(v^1)\,. \end{cases} \end{cases}$$

Small mass case. ℓ is the system length scale.

Large mass case.

Results: Brans-Dicke with a potential

$$\gamma_\Sigma = \begin{cases} \frac{\kappa}{4\pi\varphi_0} - 1\,, & \text{in general,} \\ \gamma = \begin{cases} \frac{1+\omega_0}{2+\omega_0}\,, & \text{if } m_\varphi^2 \ell^2 \lesssim O(v^1), \\ 1\,, & \text{if } e^{-m_\varphi \ell} \lesssim O(v^1). \end{cases} \end{cases}$$

$$\eta = \frac{1 - \frac{4\pi}{\kappa}\frac{\varphi}{U}}{1 + \frac{4\pi}{\kappa}\frac{\varphi}{U}}$$

The slip is not a constant in general

Results: Brans-Dicke with a potential

$$\gamma_\Sigma = \begin{cases} \frac{\kappa}{4\pi\varphi_0} - 1\,, & \text{in general,} \\ \gamma = \begin{cases} \frac{1+\omega_0}{2+\omega_0}\,, & \text{if } m_\varphi^2 \ell^2 \lesssim O(v^1), \\ 1\,, & \text{if } e^{-m_\varphi \ell} \lesssim O(v^1). \end{cases} \end{cases}$$

$$\eta = \frac{1 - \frac{4\pi}{\kappa}\frac{\varphi}{U}}{1 + \frac{4\pi}{\kappa}\frac{\varphi}{U}}$$

The slip is not a constant in general

Intermediate mass regime:

Due to the lack of a Newtonian limit, the mass needs to be defined by some alternative way.

Using the mass definition of [Alsin, Berti, Will, Zaglauer, PRD 2012], we reproduce their results,

$$\gamma_\Sigma = \eta(r_\oplus).$$

γ_Σ is found to coincide with η in a given radius (Earth's orbit radius): it is a constant.

Results: Brans-Dicke with a potential

$$\gamma = \begin{cases} \frac{\kappa}{4\pi\varphi_0} - 1\,, & \text{in general,} \\ \ldots \end{cases}$$

$$\eta = \frac{1 - \frac{4\pi}{\kappa}\frac{\varphi}{U}}{\ldots}$$

[Alsin, Berti, Will, Zaglauer, PRD 2012] uses a modified PPN approach, where U is a Yukawa potential. — In some sense they use a PPY formalism.

For this result, there is no need to do such modification, one simply needs to use γ_Σ.

...eneral

Inter...

Due t...

Using the mass definition of [Alsin, Berti, Will, Zaglauer, PRD 2012], we reproduce their results,

$$\gamma_\Sigma = \eta(r_\oplus).$$

γ_Σ is found to coincide with η in a given radius (Earth's orbit radius): it is a constant.

Results: metric and Palatini $f(R)$

For small velocities, $f(R)$ is assumed to be approximated by $f(R) = f_1 R + f_2 R^2 + O(R^3)$.

Metric $f(R)$

$$\gamma_\Sigma = \begin{cases} \frac{\kappa}{4\pi f_1} - 1\,, & \text{in general,} \\ \gamma = \begin{cases} \frac{1}{2}, & \text{if } \frac{f_1 \ell^2}{6 f_2} \lesssim O(v^1), \\ 1\,, & \text{if } e^{-\sqrt{\frac{f_1}{6 f_2}}\ell} \lesssim O(v^1). \end{cases} \end{cases}$$

[see also Berry, Gair, PRD 2011]

Palatini $f(R)$

$$\gamma_\Sigma = \gamma = 1$$

[Toniato, Rodrigues, Wojnar, PRD 2020]

In the parameter space with a Newtonian limit, there are only two values for γ.

Hence, a better motivation for [Collet et al Science 2018] for detecting arbitrary γ values comes from scalar-tensor gravity, not from $f(R)$.

Results: Horndeski

We consider the complete Horndeski action, and *approximate* all the potentials with polinomial expansions.

Constant

$$\gamma_\Sigma = \begin{cases} \dfrac{\kappa}{4\pi\, G_{4(0,0)}} - 1\,, & \text{in general,} \\ \gamma = \begin{cases} \dfrac{W - G^2_{4(1,0)}}{W + G^2_{4(1,0)}}, & \text{if } m_\varphi^2 \ell^2 \lesssim O(v^1), \\ 1\,, & \text{if } e^{-m_\varphi \ell} \lesssim O(v^1). \end{cases} \end{cases}$$

For the small and large mass limits, we agree with [Hohmann PRD 2015, Shaoqi, Yungui EPJC 2018].

For the intermediary mass case, we do not agree. Both these works use $\eta = \gamma$, which is wrong.

What could go wrong by using $\gamma_{PPN} \equiv \dfrac{\phi}{\psi} = \eta$ when "γ_{PPN}" does not coincide with the true γ from PPN?

Negligible error example: Cassini bounds on ω_0 for the intermediate mass Brans-Dicke case. In [Perivolaropoulos PRD 2010] it was used η in place of γ. But it turned out that, *for a certain mass definition*, the results are correct within a good approximation [Alsin, Berti, Will, Zaglauer, PRD 2012].

Non-negligible problems: i) A non-constant η used as γ imply that light deflection and time delay in Brans-Dicke have an impact parameter dependence different from GR, but they are precisely the same.

ii) Some models are considered as wrong because the numerical value of η is too far from the γ bounds, but these bounds are in general irrelevant for η. Recall that $\gamma_\Sigma = \alpha_e + \gamma_e - 1$ and $\eta = \gamma_e / \alpha_e$.

Observational consequences I

0. Tests like Collett et al 2018 are more naturally motivated by scalar-tensor gravity, not $f(R)$.

1. We extended and clarified the use of γ_Σ in a PPN framework.
   (the notation and general use is ours, but it appeared as an $f(R)$ effective γ in [Berry, Gair, PRD 2011]).

2. γ_Σ is a constant for Horndeski (and it needs not to be so for other modified gravity theories).

3. Since it is a constant, it fully parameterizes light bending and the time delay (similarly to γ).
   Moreover, with d as the impact parameter, $\delta\theta \propto d^{-1}$ and $\delta t \propto (\ln d)^{-1}$.

4. Since it is clear that there is a well defined Newtonian limit in the solar system, distant galaxies are the natural playgrounds to test γ_Σ.

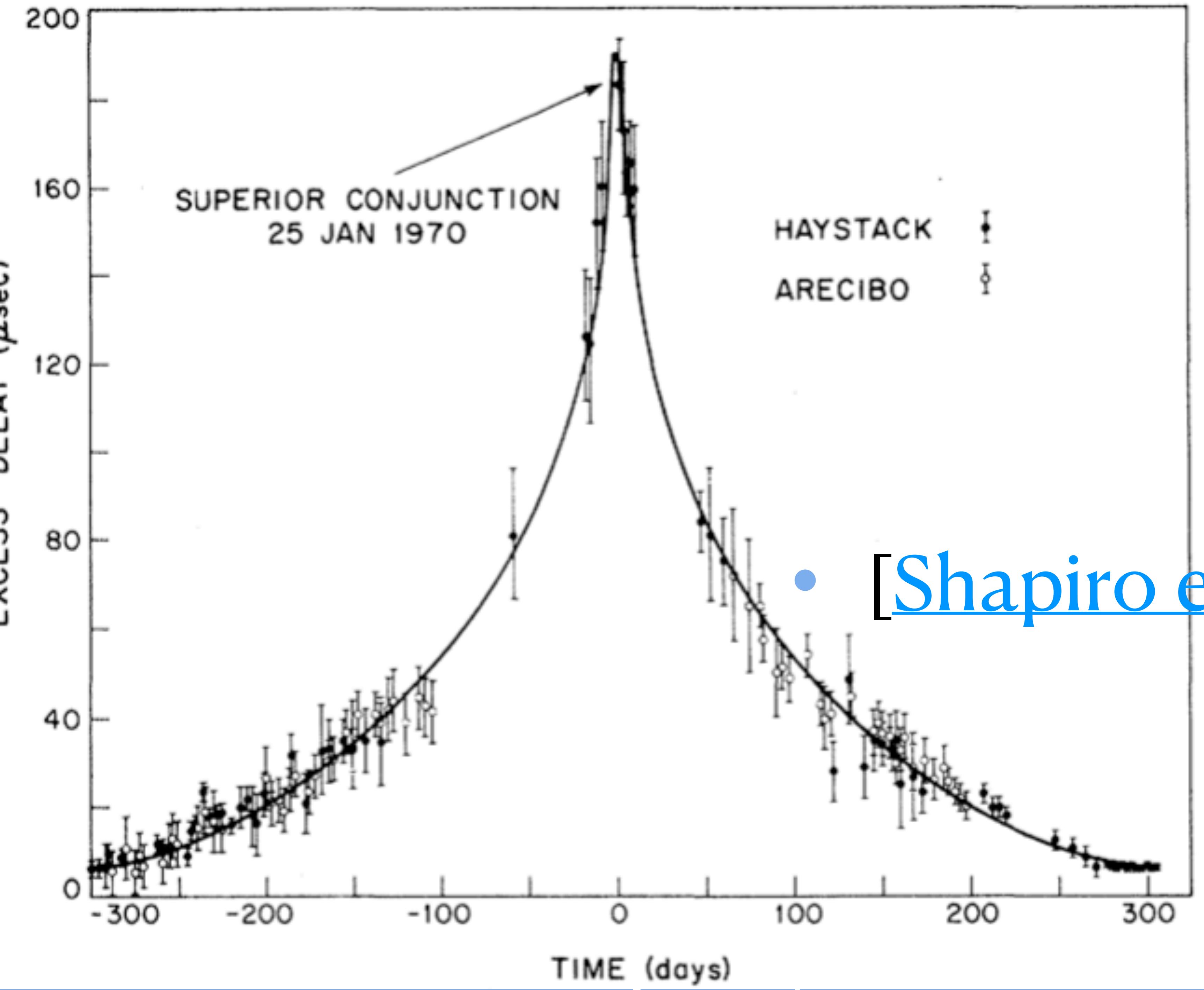
5. Even if γ is measured to be ~ 1 in the solar system, γ and γ_Σ may be significantly different in distant galaxies due to cosmological evolution.
   (Cosmological evolution of PPN parameters requires PPN Cosmology [Clifton, Sanghai PRL 2019]).

Observational consequences II:
Is it possible to directly test if γ_Σ is a constant?

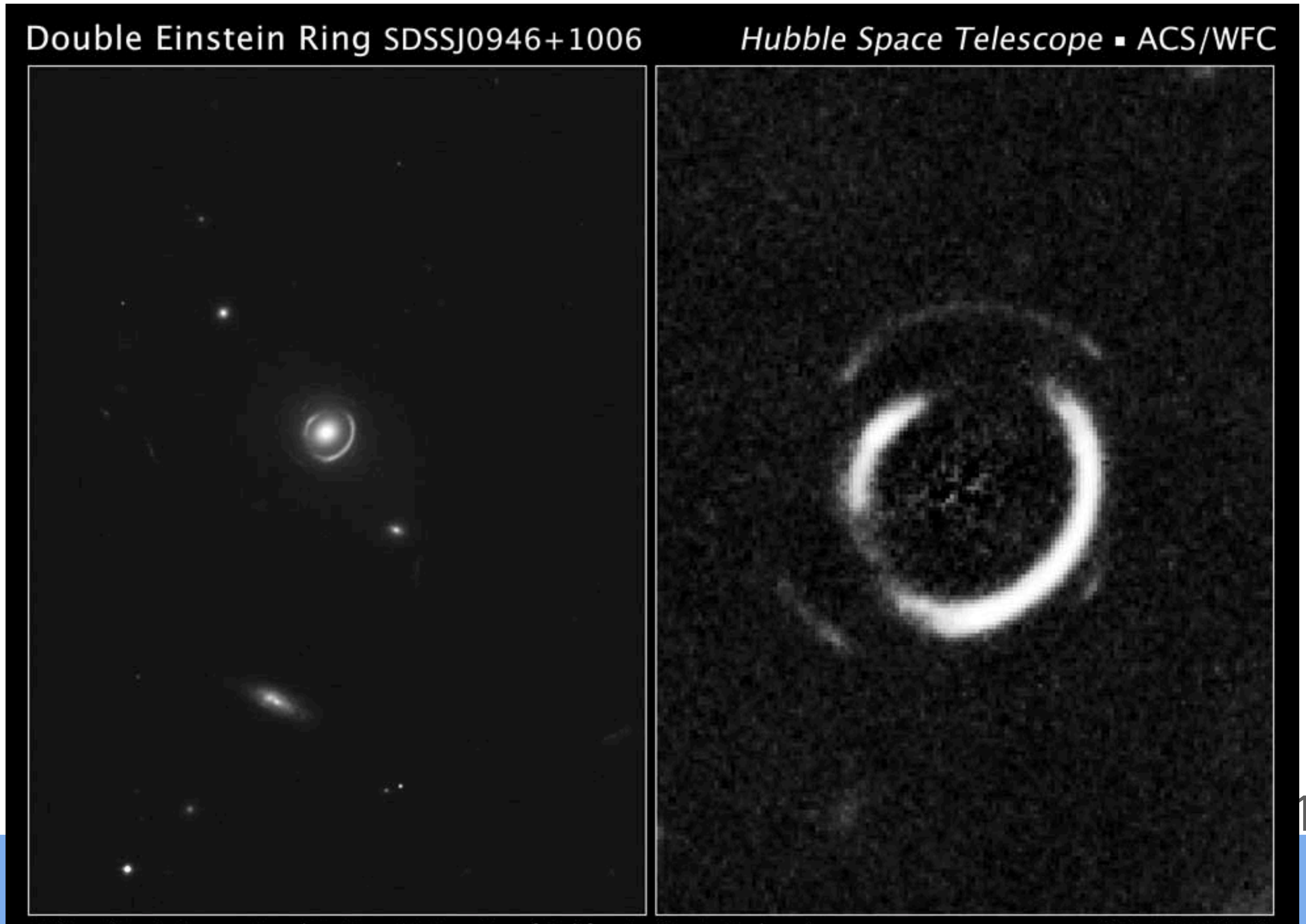
- Yes! Even without providing a mass definition.
- The idea is to test the impact parameter dependence. In principle it is possible, either through time delay or light deflection, but for external galaxies this is not so easy.

Impact parameter dependence in the solar system

A double Einstein Ring system



- [Shapiro et al PRL 1971]



the slip