Introduction to 2D dilaton gravity

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Contents

- Why dilaton?
- General model and particular cases
- Even more general: Poisson sigma models
- All classical solutions
- AdS_2/CFT_1

Review: Grumiller, Kummer and D.V, Phys.Rept. (2002).

Support: CNPq, FAPESP

Why dilaton?

The Einstein-Hilbert action in 2D

$$\int d^2x\sqrt{-g}R$$

has vanishing local variations and thus describes no dynamics.
A remedy: include a scalar field X. For example:

$$S = \int d^2x\sqrt{-g}\,X(R-2\Lambda).$$

This is the first 2D dilaton gravity - the Jackiw-Teitelboim (JT) model (1984). Classical solutions to this model are locally dS or AdS spacetimes.

Let us add to the action a kinetic term for X and a potential [Banks, ..., I. Shapiro, ..., around 1990-1992]:

$$L = \int d^2x \sqrt{-g}\left[\tfrac{1}{2}RX - \tfrac{1}{2}U(X)(\nabla X)^2 + V(X)\right]$$

Important particular cases:
Spherical reduction from D dimensions:

$$V = -(D-2)(D-3)X^{\frac{D-4}{D-2}} \qquad U = -\frac{D-3}{(D-2)X}.$$

String gravity (CGHS):

$$V = -2X \qquad U = -\frac{1}{X}.$$

First order formulation

The action above is equivalent to

$$L = \int [X^a De_a + Xd\omega + \epsilon \mathcal{V}(X_a X^a, X)],$$

where ω is the spin-connection, e_a is zweibein, $a = \pm$, $De_a = de_a + a\omega \wedge e_a$, ϵ is the volume 2-form, X^a are two auxiliary fields that generate torsion constraints.

$$\mathcal{V} = \frac{1}{2}U(X)X^a X_a + V(X)$$

To prove the equivalence: solve the torsion constraints to express ω through e. The resulting action will depend on the metric rather than on the zweibein.

Poisson sigma models

These are 2D sigma models with a target space being a Poisson manifold with local coordinates X^I (scalars from the 2D point of view), gauge fields A_I (one-forms from the 2D point of view) and a Poisson tensor $P^{IJ}(X)$ satisfying the Jacobi identity

$$P^{IJ}\partial_J P^{KL} + \mathrm{cycl}(IKL) = 0$$

The action [Schaller & Strobl, 1994]

$$L = \int \left[dX^I \wedge A_I + \tfrac{1}{2} P^{IJ} A_J \wedge A_I \right]$$

is invariant under the gauge transformations

$$\begin{aligned} \delta X^I &= P^{IJ}\lambda_J \\ \delta A_I &= -d\lambda_I - (\partial_I P^{JK})\lambda_K A_J \end{aligned}$$

With the choice $X^I = (X, X^a)$, $A_I = (\omega, e_a)$ and

$$P^{ab} = \mathcal{V}\epsilon^{ab}, \qquad P^{aX} = X^b \epsilon_b{}^a$$

one recovers 2D dilaton gravities in the 1st order formalism. The gauge parameters λ^a correspond to diffeomorphisms, while λ^X describes local Lorentz rotations.

All classical solutions

[Bergamin, Grumiller, Kummer, DV, 2005]
Consider a 1st order Euclidean dilaton gravity in complex variables

$$\int \left[\bar{Y} De + Y \bar{D}\bar{e} + Xd\omega + \epsilon\mathcal{V}(2\bar{Y}Y, X) \right]$$

where

$$Y = (X^1 + iX^2)/\sqrt{2}, \qquad e = (e^1 + ie^2)/\sqrt{2}$$

Though the bar means a complex conjugation, it is convenient to view all fields as independent complex variables.
Equations of motion:

$$dX - i\bar{Y}e + iY\bar{e} = 0, \qquad DY + ie\mathcal{V} = 0$$

$$d\omega + \epsilon\frac{\partial\mathcal{V}}{\partial X} = 0, \qquad De + \epsilon\frac{\partial\mathcal{V}}{\partial Y} = 0$$

By combining equations on the 1st line, we obtain

$$d(\bar{Y}Y) + \mathcal{V}dX = 0$$

that is integrated to

$$dC = 0, \qquad C = w(X) + e^{Q(X)}\bar{Y}Y$$

with

$$Q(X) = \int^X U(z)dz, \qquad w(X) = \int^X e^{Q(z)}V(z)$$

This allows to express $\bar{Y}Y$ through X.
The existence and the form of absolutely conserved quantity follows also from the Poisson sigma model arguments.

Similarly,

$$d(e/Y) = dX \wedge (e/Y)U(X),$$

which yields

$$d(e^{-Q}/Y) = 0$$

So that

$$e^{-Q}/Y = df$$

with some complex zero-form f. That is it,

$$e = Ye^{Q}df, \qquad \bar{e} = i\frac{dX}{Y} + \bar{Y}e^{Q}df$$
$$\omega = \mathcal{V}e^{Q}df - i\frac{dY}{Y}$$

By imposing the reality condition one obtains a general classical solution depending of 3 arbitrary functions (gauge degrees of freedom) and one integration constant C (mass of the solution).

AdS/CFT

According to the AdS/CFT conjecture gravity theories in asymptotically AdS spaces correspond to conformal theories at the asymptotic boundary. To implement this conjecture in 2D at the classical level, one has to

- define asymptotic conditions asymptotic symmetry algebras
- define asymptotic degrees of freedom
- compute the action for these degrees of freedom

Let us see, how the first step can be done [Grumiller, Salzer, DV, 2015].

Let us take the JT gravity, denote by ρ the radial coordinate, s.t. $\rho \to \infty$ corresponds to the asymptotic region of AdS_2. Let φ be a second coordinate. We require that at the asymptotics all fields behave as listed below plus subleading terms

$$\begin{aligned} X^0 &= X_R e^{\rho} - X_L e^{-\rho} & \qquad e_{\varphi 0} &= \tfrac{1}{2} e^{\rho} - \tfrac{1}{2} M e^{-\rho} \\ X^1 &= X^1(\varphi) & e_{\rho 1} &= 1 \\ X &= X_R e^{\rho} + X_L e^{-\rho} & \omega_{\varphi} &= -\tfrac{1}{2} e^{\rho} - \tfrac{1}{2} M e^{-\rho} \end{aligned}$$

Other fields vanish. X_R, X_L and M are some functions of φ. This comes from partially fixing the gauge, partially solving the equations of motion and by an educated guess. By equations of motion, $X_{R,L}$ and X^1 may be expressed through M.

The **asymptotic symmetry algebra** is generated by bulk gauge transformations that

- Preserve the form of asymptotic conditions
- Change M.

Parameters of these transformations depend on a single arbitrary function $\lambda(\varphi)$

$$\begin{aligned}\lambda^0 &= \tfrac{1}{2}\lambda e^{\rho} - \tfrac{1}{2}(M\lambda + 2\lambda'')e^{-\rho} \\ \lambda^1 &= -\lambda' \\ \lambda^X &= -\tfrac{1}{2}\lambda e^{\rho} - \tfrac{1}{2}(M\lambda + 2\lambda'')e^{-\rho}\end{aligned}$$

M is transformed as

$$\delta M = -M'\lambda - 2M\lambda' - 2\lambda'''$$

This is the Virasoro algebra with a non-zero central charge.

Q & A

Q: How can one make sure that M represents a true asymptotic degree of freedom?
A: It is enough to compute canonical boundary charges that have to depend on asymptotic degrees of freedom and be finite.

Q: How can one obtain an action for asymptotic degrees of freedom?
A: By substituting the solutions in JT action with a suitable boundary term. Depending on details, the resulting model may be a de Alfaro-Fubini-Furlan conformal quantum mechanics or a Schwarzian action [Maldacena et al, 2016].

Further developments

- By imposing looser asymptotic conditions in the JT model one can get larger asymptotic symmetry algebras, as, e.g., the loop algebra of $\mathfrak{sl}(2)$, Virasoro plus an $\mathfrak{u}(1)_k$, see [Grumiller et al, 2017].
- There are higher spin and supersymmetric extensions.
- Extensions to generic dilaton gravities: work in progress.