Renormalization Group and Cosmology

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Contents:

- **Renormalization group (RG) in curved spacetime**
- **Cosmological Constant at classical level**
- **Renormalization Group Running of CC**
- **Covariance and Physical Renormalization Group for CC.**
- **Applications to Cosmology: possible models**

Our last preprint:

Jhonny A.A. Ruiz, Tibério P. Netto, J.C. Fabris and I. Sh., Primordial universe with the running cosmological constant, arXiv:1911.06315.

Observation about general structure of renormalization in curved space.

Starting from the first paper

R. Utiyama & B.S. DeWitt, J. Math. Phys. 3 (1962) 608.

we know that the divergences and counterterms in QFT in curved space-time satisfy two conditions:

- **They are covariant if the regularization is consistent with covariance.**

- **They are local functionals of the metric.**

See the book

I.Buchbinder, S. Odintsov & I.Sh., Effective Action in Quantum Gravity (IOPP, 1992).

for introduction and recent papers

I.Sh. Class.Quant.Grav. (2008 - Topical review). arXiv: 0801.0216.

P. Lavrov & I.Sh., Phys.Rev. D81 (2010) 044026.

for a more simple consideration and more rigid proof.

Within the renormalizable theory of matter fields on classical curved background one can expect running of the essential physical parameters. **RG equations for CC and G:**

$$(4\pi)^2\,\mu\frac{d\,\rho_\Lambda^{vac}}{d\mu} = (4\pi)^2\,\mu\,\frac{d}{d\mu}\left(\frac{\Lambda_{vac}}{8\pi G_{vac}}\right) = \frac{N_s m_s^4}{2} - 2N_f m_f^4\,.$$

$$(4\pi)^2\,\mu\,\frac{d}{d\mu}\left(\frac{1}{16\pi G_{vac}}\right) = \frac{N_s m_s^2}{2}\left(\xi - \frac{1}{6}\right) + \frac{N_f m_f^2}{3}\,.$$

It is not clear how these equations can be used in cosmology, where the typical energies are very small.

However, even the running in UV means the ρ_Λ^{vac} can not be much smaller then the fourth power of the typical mass of the theory.

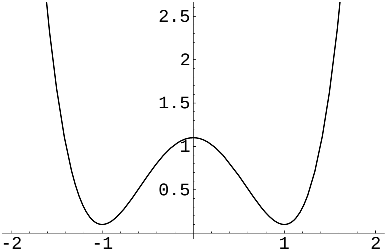
Consequence: the natural value from the MSM perspective is

$\rho_\Lambda^{vac} \sim M_F^4 \sim 10^8\,GeV^4$ **while** $\rho_\Lambda^{obs} = \rho_\Lambda^{vac} + \rho_\Lambda^{ind} \sim 10^{-47}\,GeV^4.$

Induced CC from SSB in the Standard Model.

In the stable point of the Higgs potential $V = -m^2\phi^2 + f\phi^4$ we meet $\Lambda_{ind} = \langle V \rangle \approx 10^8\, GeV^4$ – same order of magnitude as Λ_{vac} !

This is induced CC, similar to the one found by Zeldovich (1968).



The observed CC is a sum $\rho_\Lambda^{obs} = \rho_\Lambda^{vac} + \rho_\Lambda^{ind}$. Since ρ_Λ^{vac} is an independent parameter, the renormalization condition is

$$\rho_\Lambda^{vac}(\mu_c) = \rho_\Lambda^{obs} - \rho_\Lambda^{ind}(\mu_c)\,.$$

Here μ_c is the energy scale where ρ_Λ^{obs} is "measured".

Finally, the main CC relation is

$$\rho_\Lambda^{obs} = \rho_\Lambda^{vac}(\mu_c) + \rho_\Lambda^{ind}(\mu_c)\,.$$

The ρ_Λ^{obs} which is likely observed in SN-Ia, LSS and CMB is

$$\rho_\Lambda^{obs}(\mu_c) \;\approx\; 0.7\,\rho_c^0 \;\propto\; 10^{-47}\,GeV^4.$$

The CC Problem is that the magnitudes of $\rho_\Lambda^{vac}(\mu_c)$ and $\rho_\Lambda^{ind}(\mu_c)$ are a huge **55** orders of magnitude greater than the sum!

Obviously, these two huge terms do cancel.

"Why they cancel so nicely" is the CC Problem (Weinberg, 1989).

The origin of the problem is the difference between the M_F scale of ρ_Λ^{ind} and ρ_Λ^{vac} and the μ_c scale of ρ_Λ^{obs}.

We can see that the CC Problem is nothing else but a sort of hierarchy problem, perhaps the most difficult one.

At low energies the quantum effects of some kind may produce a screening of the observable CC.

Some realizations of this idea:

1) **IR effects of quantum gravity. Qualitative discussion -** *Polyakov, 1982, 2001.*

2) **Attempt to support this idea by direct calculations on fixed** dS **background** – *Tsamis & Woodard et al, 1995-2010.*

3) **More real thing: IR quantum effects of the conformal factor in** $4d$ – *Antoniadis and Mottola, 1992.*

4) **Using the assumed non-Gaussian UV fixed point in Quantum Gravity, assymptotic safety** – *Reuter, Percacci et al, from 2000.*

5) **Driving induced CC between the GUT scale** M_X **and the cosmic scale** μ_c **by the quantum effects of GUT's.** – *I.Sh., 1994; Jackiw et al, 2005.*

Much less radical approach is to ask whether the cosmological constant can vary due to the RG running in the far (deep) IR

At high energies scalar m_s and fermion m_f produce running:

$$(4\pi)^2 \, \mu \, \frac{d\rho_\Lambda}{d\mu} \, = \, \frac{m_s^4}{2} - 2m_f^4 + \dots . \qquad (1)$$

To use this RG in cosmology, we have to answer two questions:

- **What is** μ?
- •• **At which energy scale Eq.** (1) **can be used?**

The answer to • is almost obvious: in the late Universe $\mu \sim H$.
The answer to •• is not that simple.

If applied to the late Universe, (1) **results in too fast running of CC, breaking the standard cosmological model.**

This does not happen, because in QFT there is a phenomenon called decoupling.

For simplicity, consider a fermion loop effect in QED.

In the UV, the mass of quantum fermion is negligible, this simplifies the form factor, and we arrive at

$$\tilde{\beta}\, F^{\mu\nu}\, \ln\Big(\frac{\Box}{\mu^2}\Big)\, F_{\mu\nu}\,.$$

The momentum-subtraction β-function

$$\beta_e^1 \,=\, \frac{e^3}{6a^3\,(4\pi)^2}\,\Big[20a^3 - 48a + 3(a^2-4)^2\,\ln\Big(\frac{2+a}{2-a}\Big)\Big]\,,$$

$$a^2 = \frac{4\Box}{\Box - 4m^2}\,.$$

Special cases:

UV limit $\quad p^2 \gg m^2 \;\Longrightarrow\; \beta_e^{1\;UV} = \dfrac{4\,e^3}{3\,(4\pi)^2} + \mathcal{O}\Big(\dfrac{m^2}{p^2}\Big)\,.$

IR limit $\quad p^2 \ll m^2 \;\Longrightarrow\; \beta_e^{1\;IR} = \dfrac{e^3}{(4\pi)^2}\cdot\dfrac{4\,p^2}{15\,m^2} + \mathcal{O}\Big(\dfrac{p^4}{m^4}\Big)\,.$

This is the standard form of the Appelquist and Carazzone decoupling theorem (PRD, 1977).

In the gravitational sector we meet Appelquist and Carazzone - like decoupling, but only in the higher derivative sectors.
In the perturbative approach, with $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, we do not see running for the cosmological and inverse Newton constants.
Why do we get $\beta_\Lambda = \beta_{1/G} = 0$ **?**

Momentum subtraction running corresponds to the insertion of, e.g., $\ln(\Box/\mu^2)$ **formfactors into effective action.**

Say, in QED: $$-\frac{e^2}{4}F_{\mu\nu}F^{\mu\nu} + \frac{e^4}{3(4\pi)^2}\,F_{\mu\nu}\,\ln\Big(-\frac{\Box}{\mu^2}\Big)\,F^{\mu\nu}\,.$$

Similarly, one can insert formfactors into

$$C_{\mu\nu\alpha\beta}\,\ln\Big(-\frac{\Box}{\mu^2}\Big)\,C_{\mu\nu\alpha\beta}\,.$$

However, such insertion is impossible for Λ and for $1/G$, because $\Box\Lambda \equiv 0$ and $\Box R$ is a full derivative.

Further discussion:
Ed. Gorbar & I.Sh., JHEP (2003); I.Sh., Cl.Quant.Grav. (2008).

Is it true that physical $\beta_\Lambda = \beta_{1/G} = 0$ **?**

Probably not. Perhaps the linearized gravity approach is simply not an appropriate tool for the CC and Einstein terms.

Let us use the covariance arguments. The EA can not include odd terms in metric derivatives. In the cosmological setting this means no $\mathcal{O}(H)$ **and also no** $\mathcal{O}(H^3)$ **terms, etc. Hence**

$$\rho_\Lambda(H) = \frac{\Lambda(H)}{16\pi G(H)} = \rho_\Lambda(H_0) + \frac{3\nu}{8\pi}\left(H^2 - H_0^2\right), \qquad \nu = const\,.$$

Then the conservation law for $G(H;\nu)$ **gives**

$$G(H;\nu) = \frac{G_0}{1+\nu\,\ln\left(H^2/H_0^2\right)}, \quad \textbf{where} \quad G(H_0) = G_0 = \frac{1}{M_P^2}\,.$$

Here we used the identification

$\mu \sim H$ **in the cosmological setting.**

The same $\rho_\Lambda(\mu)$ follows from the assumption of the Appelquist and Carazzone - like decoupling for CC.

I.Sh., J.Solà, JHEP (2002);
A.Babic, B.Guberina, R.Horvat, H.Štefančić, PRD 65 (2002);
I.Sh., J.Solà, C.España-Bonet, P.Ruiz-Lapuente, PLB 574 (2003).

For a single particle $\beta_\Lambda^{MS}(m) \sim m^4$,

hence the quadratic decoupling gives

$$\beta_\Lambda^{IR}(m) = \frac{\mu^2}{m^2}\,\beta_\Lambda^{MS}(m) \sim \mu^2 m^2 .$$

The total beta-function will be given by algebraic sum

$$\beta_\Lambda^{IR} = \sum k_i \mu^2 m_i^2 = \sigma M^2\,\mu^2 \propto \frac{3\nu}{8\pi}\,M_P^2\,H^2 .$$

This leads to the same result in the cosmological setting,

$$\rho_\Lambda(H) = \rho_\Lambda(H_0) + \frac{3\nu}{8\pi}\,M_p^2\,\big(H^2 - H_0^2\big) .$$

All in all, it is not a surprise that the eq.

$$G(\mu) = \frac{G_0}{1 + \nu \, \ln \left(\mu^2/\mu_0^2\right)}\,.$$

emerges in different approaches to renorm. group in gravity:

- **Higher derivative quantum gravity.**

A. Salam & J. Strathdee, PRD (1978);
E.S. Fradkin & A. Tseytlin, NPB (1982).

- **Non-perturbative quantum gravity with (hipothetic) UV-stable fixed point.**

A. Bonanno & M. Reuter, PRD (2002).

- **Semiclassical gravity.**

B.L. Nelson & P. Panangaden, PRD (1982).

So, we arrived at the two relations:

$$\rho_\Lambda(H) \,=\, \rho_\Lambda(H_0) \,+\, \frac{3\nu}{8\pi}\, M_p^2 \left(\mu^2 - \mu_0^2\right) \tag{1}$$

and

$$G(\mu) = \frac{G_0}{1+\nu\,\ln\left(\mu^2/\mu_0^2\right)}\,. \tag{2}$$

Remember the standard identification

$\mu \,\sim\, H$ **in the cosmological setting.**

A. Babic, B. Guberina, R. Horvat, H. Štefančić, PRD (2005).

Cosmological models based on the assumption of the standard AC-like decoupling for the cosmological constant:

- **Models with (1) and energy matter-vacuum exchange:**

I.Sh., J.Solà, Nucl.Phys. (PS), IRGA-2003;
I.Sh., J.Solà, C.España-Bonet, P.Ruiz-Lapuente, PLB (2003).

- • **Models with (1), (2) and without matter-vacuum exchange:**

I.Sh., J.Solà, H.Štefančić, JCAP (2005).

- **Models with constant $G \equiv G_0$ and permitted energy exchange between vacuum and matter sectors.**

For the equation of state $P = \alpha\rho$ the solution is analytical,

$$\rho(z;\nu) \,=\, \rho_0\,(1+z)^r\,,$$

$$\rho^\Lambda(z;\nu) \,=\, \rho_0^\Lambda \,+\, \frac{\nu}{1-\nu}\,[\,\rho(z;\nu)-\rho_0\,]\,,$$

The limits from density perturbations / LSS data: $|\nu| < 10^{-6}$.

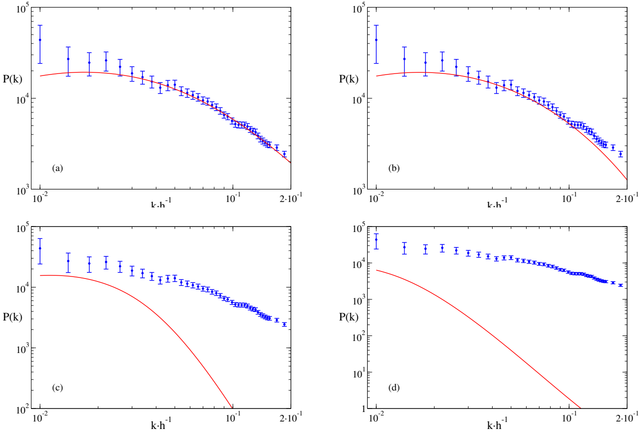
Analog models :
R.Opher & A.Pelinson, PRD (2004) **(and problem detected)**;
P. Wand & X.H. Meng, Cl.Q.Gr. 22 (2005).

Direct analysis of cosmic perturbations:
J. Fabris, I.Sh., J. Solà, JCAP 0702 (2007).

For the Harrison-Zeldovich initial spectrum, the power spectrum today is obtained by integrating the eqs. for perturbations.

Initial data based on $w(z)$ **from** *J.M. Bardeen et al, Astr.J. (1986).*

Results of numerical analysis for the ● model:



The ν-dependent power spectrum vs the LSS data from the 2dfFGRS. The ordinate axis represents $P(k) = |\delta_m(k)|^2$ where $\delta_m(k)$ is the solution at $z = 0$. $\nu = 10^{-8},\ 10^{-6},\ 10^{-4},\ 10^{-3}$.
In all cases $\Omega_B^0, \Omega_{DM}^0, \Omega_\Lambda^0 = 0.04, 0.21, 0.75$.

●● **Models with variable $G = G(H)$ but without energy exchange between vacuum and matter sectors.**

Theoretically this looks much better!

$$\rho_\Lambda(H) \,=\, \rho_\Lambda(H_0) \,+\, \frac{3\nu}{8\pi}\,M_p^2\left(H^2 - H_0^2\right).$$

By using the energy-momentum tensor conservation we find

$$G(H;\nu) = \frac{G_0}{1 + \nu\,\ln\left(H^2/H_0^2\right)}\,, \quad \textbf{where} \quad G(H_0) = \frac{1}{M_P^2}\,.$$

These relations exactly correspond to the RG approach discussed above, with $\mu = H$.

I.Sh., J.Solà, H.Štefančić, JCAP (2005).

The limits on ν from density perturbations, etc.

J.Grande, J.Solà, J.Fabris & I.Sh., Cl. Q. Grav. 27 (2010) .

An important general result is: In the models with variable Λ and G in which matter is covariantly conserved, the solutions of perturbation equations *do not* depend on the wavenumber k.

As a consequence we meet relatively weak modifications of the spectrum compared to ΛCDM.

The bound $\nu < 10^{-3}$ comes just from the "F-test". It is related only to the modification of the function $H(z)$.

R. Opher & A. Pelinson, astro-ph/0703779.
J.Grande, R.Opher, A.Pelinson, J.Solà, JCAP 0712 (2007).

One can obtain the same restriction for ν also from the primordial nucleosynthesis (BBN).

Is the first-type model completely useless?

The answer to this question is negative. The model can be applied to the early Universe, where the creation of particles is not suppressed by the mass/energy threshold.

Jhonny A.A. Ruiz, Tibério P. Netto, J.C. Fabris and I. Sh., Primordial universe with the running cosmological constant, arXiv:1911.06315.

We assume that the matter contents of the Universe consist from baryons and DM. DM is made of a kind of the GUT's remnants and hence has masses that are much greater than the value of H.

Thus, the DM can be regarded to decouple, in the sense that DM particles are not created from the vacuum. Thus, an appropriate description of DM is an ideal gas of massive particles adiabatically expanding and becoming less relativistic with time.

To describe such a gas, we shall use the simple and convenient Reduced Relativistic Gas (RRG) model, which was originally developed by Sakharov in the classical paper

A.D. Sakharov, Soviet Physics JETP 22 (1966) 241

and recently found new applications,

G.B. Peixoto, I.Sh., F. Sobreira, gr-qc/0412050; MPLA 20 (2005).

J. Fabris, I.Sh., F. Sobreira, arXiv:0806.1969, JCAP (2009).

J. Fabris, I.Sh, A.Toribio, arXiv:1105.2275, PRD 85 (2012).

J. Fabris, A.Toribio, W. Zimdahl, I.Sh., EPhJC, arXiv:1312.1937.

S.C. dos Reis, I.Sh., EPJC, arXiv:1712.03066.

G. Pordeus-da-Silva, R.C. Batista, L.G. Medeiros, JCAP (2019), arXiv:1904.0990.

Due to its simplicity, RRG is useful for the running CC models.

The main equations for the background are RRG's EoS,

$$P_{dm} = \frac{\rho_{dm}}{3}\Big[1 - \Big(\frac{mc^2}{\varepsilon}\Big)^2\Big]^2 = \frac{\rho_{dm}}{3}\Big(1 - \frac{\rho_d^2}{\rho_{dm}^2}\Big). \tag{1}$$

Here

$$\rho_{dm} = \frac{n\,mc^2}{\sqrt{1-\beta^2}} \quad \textbf{and} \quad \rho_d = n\,mc^2.$$

Then we have the main equation for the "running"

$$\rho_\Lambda(H) = \rho_\Lambda(H_0) + \frac{3\nu}{8\pi}\,M_p^2\,\big(H^2 - H_0^2\big) \tag{2}$$

and the definition of vacuum-BM energy exchange,

$$\rho_r' - \frac{3(1+w)}{1+z}\,\rho_r = -\rho_\Lambda', \qquad \textbf{where} \qquad w \approx \frac{1}{3}, \tag{3}$$

because at the reheating epoch BM was ultra-relativistic.

The Hubble parameter is given by the Friedman equation

$$H^2(z) = \frac{8\pi G}{3}\left[\rho_\Lambda(z) + \rho_r(z) + \rho_{dm}(z)\right].$$

The solution of the full system of equations leads to

$$\rho'_\Lambda = \frac{\nu}{1-\nu}(\rho'_r + \rho'_{dm}). \qquad \textbf{and} \qquad \rho'_r - \frac{\zeta}{1+z}\rho_r = -\nu\rho_{dm},$$

where $\zeta = 3(1+w)(1-\nu)$, **and finally gives**

$$\Omega_r(z) = C_0(1+z)^\zeta - \frac{\nu\Omega^0_{dm}(1+z)^3}{\sqrt{1+b^2}}\left[\sqrt{1+b^2(1+z)^2} + \frac{\zeta}{3-\zeta}\,{}_2F_1(\alpha,\beta;\gamma;Z)\right],$$

with

$$C_0 = \Omega^0_r + \frac{\nu\Omega^0_{dm}}{\sqrt{1+b^2}}\left[\sqrt{1+b^2} + \frac{\zeta}{3-\zeta}\,{}_2F_1(\alpha,\beta;\gamma;-b^2)\right].$$

${}_2F_1(\alpha,\beta;\gamma;Z)$ **is the hypergeometric function,**

$$\alpha = -\frac{1}{2}, \qquad \beta = \frac{3-\zeta}{2}, \qquad \gamma = \frac{5-\zeta}{2} \qquad \text{and} \qquad Z = -b^2(1+z)^2.$$

$\Omega_\Lambda(z)$ **is directly obtained by integrating the equation**

$$\Omega_\Lambda(z) = B_0 + \frac{\nu}{1-\nu}\,[\Omega_r(z) + \Omega_{dm}(z)],$$

where

$$B_0 \;=\; \Omega_\Lambda^0 - \frac{\nu}{1-\nu}\,\big(\Omega_r^0 + \Omega_{dm}^0\big).$$

Finally, the Hubble parameter can be found from the Friedmann equation,

$$H(z) \;=\; H_0\sqrt{\Omega_\Lambda(z) + \Omega_r(z) + \Omega_{dm}(z)}.$$

The total effective equation of state can be obtained using the second Friedman equation,

$$-2(1+z)HH' + 3H^2 \;=\; -8\pi G P_t \;\equiv\; -8\pi G\, w_{eff}(z)\rho_t,$$

where

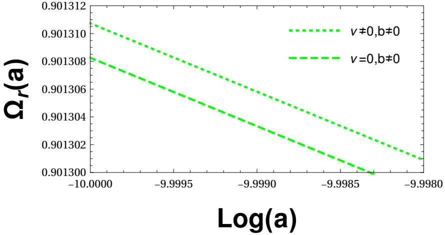
$$\rho_t(z) \equiv \rho_\Lambda(z) + \rho_r(z) + \rho_{dm}(z).$$

Thus,

$$w_{eff}(z) = \frac{2H'}{3H} - 1.$$

All in all, the background cosmological solution depends on the two parameters, namely the running ν and the DM warmness b.

Illustrating plots:



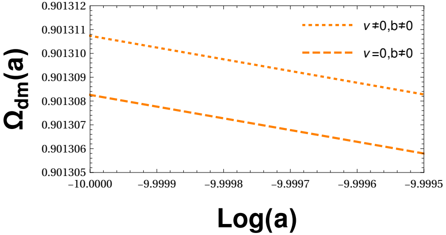


Figure: **The relative densities for radiation and DM component modeled as the RRG. There is a small difference at the primordial epoch in comparison to the Standard ΛCDM Model. This difference does not affect the BBN process.**

Including perturbations

The cosmological perturbations can be analyzed following the approach developed by Fabris et al earlier.

This implies simultaneous perturbations of metric, energy density and the four-velocities in the co-moving coordinates,

$$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}, \quad \rho_i \to \rho_i + \delta\rho_i, \quad U^\alpha \to U^\alpha + \delta U^\alpha, \quad V^\alpha \to V^\alpha + \delta V^\alpha$$

in the synchronous gauge $h_{0\mu} = 0$. **Here** U^α **is the DM velocity and** V^α **is the radiation (baryonic matter, in our case) velocity.**

We use the constraint $\delta U^0 = \delta V^0 = 0$.

The perturbation of the DM pressure should be derived from the equation of state

$$\delta P_{dm} = \frac{\delta\rho_{dm}}{3}\Big[1 - \Big(\frac{mc^2}{\varepsilon}\Big)^2\Big]^2 = \frac{\delta\rho_{dm}(1-r)}{3},$$

meaning that the perturbations satisfy the same EoS as the background quantities.

Conclusions

- The running of the cosmological constant at low energies can not be ruled out in the QFT framework. This situation makes the studies in this area mainly phenomenological.

- There are essentially two different models with the running of the cosmological constant. The first one is based on the vacuum-matter energy exchange, and the second forbids such an exchange and assumes variable Newton constant instead.

- The models of the second kind are somehow less interesting, because their observational consequences are less evident.

- On the other hand, the creation of particles from the vacuum at low energies is strongly restricted from the QFT side.

- We have shown that the models of the first type are still useful, in the description of the early Universe.