Massive ghosts and stability in higher derivative gravity

Ilya L. Shapiro

Universidade Federal de Juiz de Fora, Minas Gerais, Brazil

Partially supported by: CAPES, CNPq, FAPEMIG, ICTP

Black Holes - Ubu - ES, April, 2015

Contents

- Why should we quantize gravity.
Semiclassical approach and higher derivatives.

- Power counting. Quantum GR or Higher derivative QG?
Ghosts in renormalizable and superrenormalizable QG.

- Why people do not like ghosts? Do they pose a danger?

- Gravitational waves and stability of classical solutions.

- Final word will be said by tachyons.

General Relativity

(GR) is a complete theory of classical gravitational phenomena.

The most important solutions of GR have specific symmetries.

1) Spherically-symmetric solution. Stars ... Black holes.
2) Isotropic and homogeneous metric. Universe.

In both cases there are singularities, curvature and density of matter become infinite, hence GR is not valid at all scales.

Dimensional consideration: Planck scale,

length $l_P = G^{1/2}\,\hbar^{1/2}\,c^{-3/2} \approx 1.4 \cdot 10^{-33}\,cm;$

time $t_P = G^{1/2}\,\hbar^{1/2}\,c^{-5/2} \approx 0.7 \cdot 10^{-43}\,sec;$

mass $M_P = G^{-1/2}\,\hbar^{1/2}\,c^{1/2} \approx 0.2 \cdot 10^{-5}g \approx 10^{19}\,GeV\,.$

Three choice for Quantum Gravity (QG)

One may suppose that the fundamental units indicate to the presence of fundamental physics at the Planck scale.

We can classify the possible approaches into three distinct groups. Namely, we can

● Quantize both gravity and matter fields. This is the most fundamental approach and the main subject of this talk.

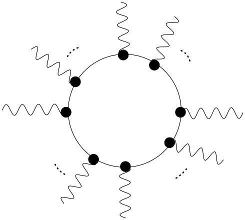
●● Quantize only matter fields on classical curved background (semiclassical approach).
QFT and Curved space-time are well-established notions, which passed many experimental/observational tests.

●●● Quantize something else. E.g., in case of (super)string theory both matter and gravity are induced.

●● **The renormalizable QFT in curved space requires introducing a generalized action of gravity (external field).**
The theory is renormalizable, but only with certain higher derivatives terms in the vacuum action.

Introduction: *Buchbinder, Odintsov & I.Sh. Effective Action in Quantum Gravity (IOPP - 1992);*
I.Sh., Class.Quant.Grav. Topical review (2008), arXiv:0801.0216.

Relevant diagrams for the vacuum sector



Possible covariant counterterms have the structure of

$$S_{vac} = S_{EH} + S_{HD}$$

General considerations about higher derivatives:

- **One should definitely quantize both matter and gravity, for otherwise the QG theory would not be complete.**

- **The diagrams with matter internal lines in a complete QG are exactly the same as in a semiclassical theory.**

- **This means one can not quantize metric without higher derivative terms in a consistent way, since these terms are produced already in the semiclassical theory.**

- **Indeed, most of the achievements in curved-space QFT are related to the renormalization of higher derivative vacuum terms, including Hawking radiation, Starobinsky inflation and others.**

Quantum Gravity (QG)

starts from some covariant action of gravity,

$$S = \int d^4x \sqrt{-g}\, \mathcal{L}(g_{\mu\nu})\,.$$

$\mathcal{L}(g_{\mu\nu})$ **can be of GR,** $\mathcal{L}(g_{\mu\nu}) = -\kappa^{-2}(R + 2\Lambda)$ **or some other.**

Gauge transformation $x'^{\mu} = x^{\mu} + \xi^{\mu}$**. The metric transforms as**

$$\delta g_{\mu\nu} = g'_{\mu\nu}(x) - g_{\mu\nu}(x) = -\nabla_\mu \xi_\nu - \nabla_\nu \xi_\mu\,.$$

In the case of $\quad g_{\mu\nu}(x) = \eta_{\mu\nu} + \kappa h_{\mu\nu}(x)\,,$

$$\delta h_{\mu\nu} = -\frac{1}{\kappa}\left(\partial_\mu \xi_\nu + \partial_\nu \xi_\mu\right) - h_{\mu\alpha}\partial_\nu \xi^\alpha - h_{\nu\alpha}\partial_\mu \xi^\alpha - \xi^\alpha \partial_\alpha h_{\mu\nu} = R_{\mu\nu\,,\,\alpha}\xi^\alpha\,.$$

The gauge invariance of the action means

$$\frac{\delta S}{\delta h_{\mu\nu}} \cdot R_{\mu\nu\,,\,\alpha} \cdot \xi^\alpha = 0\,.$$

One can prove that the same is true for the Effective Action.

Let us use power counting.

As the first example consider quantum GR.

$$S = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R + 2\Lambda)\,.$$

For the sake of simplicity we consider only vertices with maximal K_ν. Then we have $r_l = K_\nu = 2$ and, combining

$$D + d = \sum_{l_{int}}(4 - r_l) - 4n + 4 + \sum_\nu K_\nu$$

with

$$l_{int} = p + n - 1$$

we arrive at the estimate ($D = 0$ means log. divergences)

$$D + d = 2 + 2p\,.$$

This output means that quantum GR is not renormalizable and we can look for some other starting point.

Perhaps, the most natural is HDQG.

Reason: we need HD's anyway for quantum matter field.

Already known action: $S_{gravity} = S_{EH} + S_{HD}$

where

$$S_{EH} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,\{R + 2\Lambda\}$$

and S_{HD} **include higher derivative terms**

$$S_{HD} = -\int d^4x\sqrt{-g}\left\{\frac{1}{2\lambda}C^2 + \frac{1}{\rho}E + \tau\Box R + \frac{\omega}{3\lambda}R^2\right\},$$

$$C^2(4) = R^2_{\mu\nu\alpha\beta} - 2R^2_{\alpha\beta} + 1/3\,R^2,$$

$$E = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4\,R_{\alpha\beta}R^{\alpha\beta} + R^2.$$

K. Stelle, Phys. Rev. D (1977).

Propagators and vertices in HDQG are not like in quantum GR.
Propagators of metric and ghosts behave like $\mathcal{O}(k^{-4})$ and we have K_4, K_2, K_0 vertices. The superficial degree of divergence

$$D + d = 4 - 2K_2 - 2K_0 .$$

This theory is definitely renormalizable. Dimensions of counterterms are 4, 2, 0.

Well, there is a price to pay: Massive ghosts

$$G_{\mathrm{spin}-2}(k) \sim \frac{1}{m^2}\left(\frac{1}{k^2} - \frac{1}{m^2 + k^2}\right), \quad m \propto M_P .$$

The tree-level spectrum includes massless graviton and massive spin-2 "ghost" with negative kinetic energy and huge mass.

The main point of this talk is a new proposal concerning ghosts and related difficulty of QG.

Including even more derivatives was initially thought to move massive pole to even higher mass scale,

$$S = S_{EH} + \int d^4x \sqrt{-g} \left\{ a_1 R^2_{\mu\nu\alpha\beta} + a_2 R^2_{\mu\nu} + a_3 R^2 + \dots \right.$$

$$\left. + c_1 R_{\mu\nu\alpha\beta} \Box^k R^{\mu\nu\alpha\beta} + c_2 R_{\mu\nu} \Box^k R^{\mu\nu} + c_3 R \Box^k R + b_{1,2,..} R^{k+1}_{\dots} \right\}.$$

Simple analysis shows this theory is superrenormalizable, but the massive ghosts are still here. For the case of real poles:

$$G_2(k) = \frac{A_0}{k^2} + \frac{A_1}{k^2 + m_1^2} + \frac{A_2}{k^2 + m_2^2} + \cdots + \frac{A_{N+1}}{k^2 + m_{N+1}^2}$$

For any sequence $0 < m_1^2 < m_2^2 < m_3^2 < \cdots < m_{N+1}^2$, **the signs of the corresponding terms alternate,** $A_j \cdot A_{j+1} < 0$.

Asorey, Lopez & I. Sh., hep-th/9610006; IJMPhA (1997).

$$D + d = 4 + k(1 - p).$$

Once again: what is bad in the higher-derivative gravity?

For the linearized gravity

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}\,, \qquad (1)$$

we meet

$$G_{\rm spin-2}(k) \sim \frac{1}{m^2}\left(\frac{1}{k^2} - \frac{1}{m^2+k^2}\right), \quad m \propto M_P\,.$$

Tree-level spectrum includes massless graviton and massive spin-2 "ghost" with negative kinetic energy and huge mass.

- **Interaction between ghost and gravitons may violate energy conservation in the massless sector** *(M.J.G. Veltman, 1963).*

- **In classical systems higher derivatives generate exploding instabilities at the non-linear level** *(M.V. Ostrogradsky, 1850).*

- **Without ghost one violates unitarity of the S -matrix.**

There were several attempts to solve the HD ghost problem.

Stelle, Salam & Strathdee, Tomboulis,
Antonidis & Tomboulis, Johnston, Hawking, ...

In what follows we suggest a new approach which is much simpler and is probably working.

Assumption we made to condemn higher derivative theory:

● One can draw conclusions using linearized gravity approximation. S-matrix of gravitons is the main object.

●● Ostrogradsky instabilities or Veltman scattering are relevant independent on the energy scale, in all cases they produce run-away solutions and "Universe explodes".

There is a simple way to check all these assumptions at once.

Take higher derivative theory of gravity and verify the stability with respect to the linear perturbations on some, physically interesting, dynamical background.

For the sake of generality, consider not only classical HD terms, but also take into account the semiclassical corrections, derived by integrating conformal anomaly.

$$\langle T_\mu^\mu \rangle = \left\{ \beta_1 C^2 + \beta_2 E + a' \Box R \right\} , \quad \textbf{where}$$

$$\begin{pmatrix} \omega \\ -b \\ c \end{pmatrix} = \begin{pmatrix} \beta_1 \\ -\beta_2 \\ \beta_3 \end{pmatrix} = \frac{1}{360(4\pi)^2} \begin{pmatrix} 3N_0 + 18N_{1/2} + 36N_1 \\ N_0 + 11N_{1/2} + 62N_1 \\ 2N_0 + 12N_{1/2} - 36N_1 \end{pmatrix}$$

N_0 **conformal real scalars**, $N_{1/2}$ **Dirac spinors**, N_1 **vectors**.

One can use $\langle T_\mu^\mu \rangle$ to find the finite part of 1-loop EA

$$\frac{2}{\sqrt{-g}}\, g_{\mu\nu} \frac{\delta\, \Gamma_{ind}}{\delta g_{\mu\nu}} = \langle T_\mu^\mu \rangle = \frac{1}{(4\pi)^2} \left(\omega C^2 + bE + c \Box R \right) .$$

Solution *(Riegert, Fradkin & Tseytlin, PLB-1984).*

Useful notation: $\Delta = \Box^2 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu - \frac{2}{3} R\Box + \frac{1}{3}(\nabla^\mu R)\nabla_\mu .$

Anomaly-induced effective action of vacuum

$$\Gamma_{ind} = S_c[g_{\mu\nu}] - \frac{3c+2b}{36(4\pi)^2} \int d^4x \sqrt{-g(x)}\, R^2(x)$$

$$+ \frac{\omega}{4} \int_x \int_y C^2(x)\, G(x,y)\, \big(E - \frac{2}{3}\Box R\big)_y$$

$$+ \frac{b}{8} \int_x \int_y \big(E - \frac{2}{3}\Box R\big)_x\, G(x,y)\, \big(E - \frac{2}{3}\Box R\big)_y\,,$$

where $\int_x = \int d^4x \sqrt{-g}\,, \qquad \Delta_4\, G(x,y) = \delta(x,y).$

One can rewrite this expression using auxiliary scalars,

$$\Gamma_{ind} \,=\, S_c[g_{\mu\nu}] - \frac{3c+2b}{36(4\pi)^2} \int_x R^2(x) + \int_x \Big\{ \frac{1}{2}\,\varphi\Delta_4\varphi - \frac{1}{2}\,\psi\Delta_4\psi$$

$$+ \frac{a}{8\pi\sqrt{-b}}\,\psi\, C^2 + \varphi\, \Big[\frac{\sqrt{-b}}{8\pi}\, \big(E - \frac{2}{3}\,\Box R\big) - \frac{a}{8\pi\sqrt{-b}}\, C^2 \Big] \Big\}\,.$$

where $S_c[\bar{g}_{\mu\nu}] = S_c[g_{\mu\nu}]$ **is an integration constant.**

The (Modified) Starobinsky Model is based on the action

$$S_{total} = -\frac{M_P^2}{16\pi}\int d^4x\sqrt{-g}\,(R+2\Lambda) + S_{matter} + S_{vac} + \bar{\Gamma}_{ind}\,.$$

Equation of motion:

$$\frac{\ddddot{a}}{a} + \frac{3\dot{a}\dddot{a}}{a^2} + \frac{\ddot{a}^2}{a^2} - \left(5+\frac{4b}{c}\right)\frac{\ddot{a}\dot{a}^2}{a^3} - \frac{M_P^2}{8\pi c}\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} - \frac{2\Lambda}{3}\right) = 0,$$

where $k=0$, Λ **is the cosmological constant.**

Particular solutions *(Starobinsky, PLB-1980)*

$$a(t) = a_o\,\exp(Ht)\,,\quad H = \frac{M_P}{\sqrt{-32\pi b}}\left(1 \pm \sqrt{1+\frac{64\pi b}{3}\,\frac{\Lambda}{M_P^2}}\right)^{1/2}.$$

Perturbations of the conformal factor:

$$\sigma(t) \to \sigma(t) + y(t).$$

Stable inflation $c > 0 \iff N_1 < \frac{1}{3}N_{1/2} + \frac{1}{18}N_0\,,$

Simple test of the Modified Starobinsky Model.

Pelinson, I.Sh., Takakura, NPB; NPB (PS) - 2003.

Consider late Universe, $k = 0, \; H_0 = \sqrt{\Lambda/3}$.

Only photon is active, $N_0 = 0\,, \quad N_{1/2} = 0\,, \quad N_1 = 1\,.$

Graviton typical energy is $H_0 \approx 10^{-42}\, GeV$, $\Longrightarrow$ **all massive particles (even neutrino)** $m_\nu \geq 10^{-12}\, GeV$ **decouple from gravity.** $c < 0 \quad \Longrightarrow$ **today inflation is unstable.**

Stability for the small $H = H_0$ **case:** $H \to H_0 + const \cdot e^{\lambda t}$

$$\lambda^3 + 7H_0\lambda^2 + \left[\frac{(3c-b)4{H_0}^2}{c} - \frac{M_P^2}{8\pi c}\right]\lambda - \frac{32\pi b{H_0}^3 + M_P^2 H_0}{2\pi c} = 0\,.$$

The solutions are $\lambda_1 = -4H_0\,, \quad \lambda_{2/3} = -\frac{3}{2}\,H_0 \pm \frac{M_P}{\sqrt{8\pi|c|}}\,i\,.$

Stability, regardless of higher derivatives!

Consider unstable inflation, matter (or radiation) dominated Universe and assume that the Universe is close to the classical FRW solution. The equation is

$$\frac{\ddddot{a}}{a} + \frac{3\dot{a}\dddot{a}}{a^2} + \frac{\ddot{a}^2}{a^2} - \left(5 + \frac{4b}{c}\right)\frac{\ddot{a}\dot{a}^2}{a^3}$$

$$-\frac{M_P^2}{8\pi c}\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} - \frac{2\Lambda}{3}\right) = -\frac{1}{3c}\,\rho_{matter}\,,$$

The terms of the first line, of quantum origin, behave like $1/t^4$.

The second line terms, of classical origin, behave like $1/t^2$.

After certain time the "quantum" terms become negligible.

Conclusion: For the dynamics of conformal factor, classical solutions are very good low-energy approximations in the theory with quantum corrections and/or higher derivatives.

Stability & Gravitational Waves

As far as classical action and quantum, anomaly-induced term, both have higher derivatives, an important question is whether the stability of classical solutions in cosmology holds or not.

Consider small perturbation

$$g_{\mu\nu} = g^0_{\mu\nu} + h_{\mu\nu}\,, \quad h_{\mu\nu} = \delta\, g_{\mu\nu}\,,$$

where $g^0_{\mu\nu} = \{1,\ -\delta_{ij}\, a^2(t)\}$, $\mu = 0,1,2,3$ **and** $i = 1,2,3$.

$$h_{\mu\nu}(t,\vec{r}) = \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{r}\cdot\vec{k}}\, h_{\mu\nu}(t,\vec{k})\,.$$

Using the conditions $\partial_i h^{ij} = 0$ **and** $h_{ii} = 0$, **together with the synchronous coordinate condition** $h_{\mu 0} = 0$, **we arrive at the equation for the tensor mode**

Fabris, Pelinson and I.Sh., (2001);
Fabris, Pelinson, Salles and I.Sh., (2011).

Using the conditions $\partial_i h^{ij} = 0$ and $h_{ii} = 0$, together with the synchronous coordinate condition $h_{\mu 0} = 0$, we arrive at the equation for the tensor mode in the classical HD theory.

F. Salles and I.Sh., PRD (2014).

$$\frac{1}{3}\ \overset{....}{h}\ + 2H\ \dddot{h} + \Big(H^2 + \frac{M_P{}^2}{32\pi a_1}\Big)\ddot{h} + \frac{2}{3}\Big(\frac{1}{4}\ \frac{\nabla^4 h}{a^4} - \frac{\nabla^2 \ddot{h}}{a^2} - H\ \frac{\nabla^2 \dot{h}}{a^2}\Big)$$

$$-\Big[H\dot{H} + \ddot{H} + 6H^3 - \frac{3M_P{}^2 H}{32\pi a_1}\Big]\ \dot{h}\ - \Big[\frac{M_P{}^2}{32\pi a_1} - \frac{4}{3}\,(\dot{H} + 2H^2)\Big]\ \frac{\nabla^2 h}{a^2}$$

$$-\Big[\Big(24\dot{H}H^2 + 12\dot{H}^2 + 16H\ddot{H} + \frac{8}{3}\ \dddot{H}\ \Big) - \frac{M_P{}^2}{16\pi a_1}(2\dot{H} + 3H^2)\Big]\,h\ = \ 0\,.$$

It looks much simpler than Eqs. with semiclassical corrections,

Fabris, Pelinson and I.Sh., hep-th/0009197;
Fabris, Pelinson, Salles and I.Sh., arXiv:1112.5202.

Qualitative results were achieved by using

I. Analytical methods. We can approximately treat all coefficients as constants. There is a mathematically consistent way to check when (and whether) it works. With the Wolfram's Mathematica software, manipulating our equation is not so difficult, in fact.

II. Numerical methods. CMBEasy software or Wolfram's Mathematica 9 can be applied to gravitational waves and provide the results which are perfectly consistent with the output of method I.

Net Result: The stability does not actually depend on quantum corrections. It is completely defined by the sign of the classical coefficient a_1 of the Weyl-squared term. The sign of this term defines whether graviton or ghost has positive kinetic energy!

We can distinguish the **three** cases. First two are:

- **The coefficient of the Weyl-squared term is** $a_1 < 0$ **Then**

$$G_{\rm spin-2}(k) \sim \frac{1}{m^2}\left(\frac{1}{k^2} - \frac{1}{m^2+k^2}\right), \quad m \propto M_P\,,$$

there are no growing modes up to the Planck scale, $\vec{k}^2 \approx M_P^2$.

For the dS background this is in a perfect agreement with
Starobinsky, Let.Astr.Journ. (in Russian) (1983);
Hawking, Hertog and Real, PRD (2001).

- **The coefficient** $a_1 > 0$ **or** $a_1 > 0\,,\ G \to -G$.

$$G_{\rm spin-2}(k) \sim \frac{1}{m^2}\left(-\frac{1}{k^2} + \frac{1}{m^2+k^2}\right), \quad m \propto M_P\,.$$

and there are rapidly growing modes at any scale.

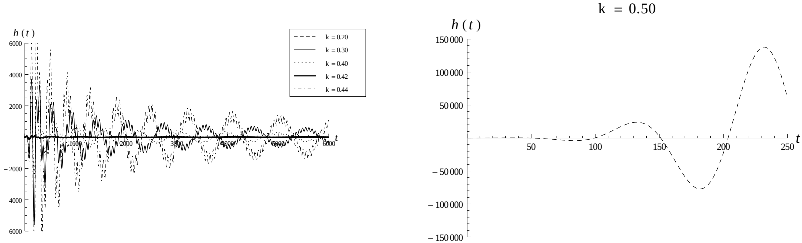


Illustration. Radiation-dominated Universe. There are no growing modes until the frequency k **achieves the value** ≈ 0.5 **in Planck units. Starting from this value, we can observe the massive ghost making its destructive work.**

So, where is the ghost??

In fact, the result is natural. The anomaly-induced quantum correction is $\mathcal{O}(R^3_{....})$ and $\mathcal{O}(R^4_{....})$, Until the energy is not of the Planck order of magnitude, these corrections can not compete with classical $\mathcal{O}(R^2_{....})$ - terms.

For $a_1 < 0$ there are no growing tensor modes in the higher derivative gravity on cosmological backgrounds.

Massive ghosts are present only in the vacuum state. We just do not observe them "alive" until the typical energy scale is below the Planck mass.

- **All in all, massive ghosts do not pose real danger below the Planck scale. Above M_P we need new ideas to fight ghosts.**

Can we check other metric backgrounds?

- **Black hole solutions: conflicting results.**

B. Whitt, Phys. Rev. D32 (1985) 379.
Yu.S. Myung, Phys. Rev. D88 (2013) 024039, arXiv:1306.3725.

It is not clear yet to which extent the results depend on the choice of the boundary conditions, on the frequency of initial seeds of perturbations etc.

The problem is rich in astrophysical applications, see, e.g.,

R.A. Konoplya, A. Zhidenko, Rev.Mod.Phys. (2011); 1102.4014,

- **General curved background by using normal coordinates**

F. Salles and I.Sh., Phys. Rev. (2014); **further work in progress.**

Now we consider the third case.

Giulia Cusin, Filipe de O. Salles, I.Sh., arXive:1503.08059.

- **The coefficient of the Weyl-squared term is** $a_1 > 0$ **Then**

$$G_{\text{spin}-2}(k) \sim -\frac{1}{m^2}\left(\frac{1}{k^2 - m^2} - \frac{1}{k^2}\right), \quad m \propto M_P\,,$$

a graviton plus a ghost-tachyon with the Planck-scale mass.

In this case there are growing modes with all frequencies!

Why is that? What is the notion of tachyon?

Consider general second-order action of a free field $h(x) = h(t, \vec{r})$

$$S(h) = \frac{s_1}{2}\int d^4x\,\{\dot{h}^2 - (\nabla h)^2 - s_2 m^2 h^2\}\,.$$

$s_{1,2} = \pm 1$ **for different types of fields.**

Perform the Fourier transform in the space variables,

$$h(t,\vec{r}) \,=\, \frac{1}{(2\pi)^3}\int d^3k\, e^{i\vec{k}\cdot\vec{r}}\, h(t,\vec{k})\,,$$

and consider the dynamics of each $h \equiv h(t,\vec{k})$ **separately.**

$$S_{\vec{k}}(h) \,=\, \frac{s_1}{2}\int dt\,\big\{\dot{h}^2 - k^2 h - s_2 m^2 h^2\big\} \,=\, \frac{s_1}{2}\int dt\,\big\{\dot{h}^2 - m_k^2 h^2\big\}\,,$$

where

$$k^2 = \vec{k}\cdot\vec{k}\,, \qquad m_k^2 = s_2 m^2 + k^2\,.$$

- **Normal healthy field corresponds to** $s_1 = s_2 = 1.$

Kinetic energy is positive. The minimal action can be achieved for a static configuration. The equation of motion is of the oscillatory type,

$$\ddot{h} + m_k^2 h \,=\, 0\,.$$

with the usual periodic solution.

- **Massive ghost has** $s_1 = -1, \quad s_2 = 1$.

It is not a tachyon, because $m_k^2 = s_2 m^2 + k^2 \geq 0$.

Kinetic energy is negative, but one can postulate zero variation of the action and arrive at the normal oscillatory equation.

A particle with negative kinetic energy has the tendency to achieve a maximal speed, but a free particle can not accelerate, for this would violate energy conservation.

A free ghost does not produce any harm to the environment, being isolated from it.

However, if we admit an interaction with healthy fields, the tendency of a ghost is to accelerate and transmit a positive energy difference to these healthy fields.

Tachyon has $s_2 = -1$. For relatively small momenta $m_k^2 < 0$ in Eq. the equation of motion is

$$\ddot{h} - \omega^2 h \,=\, 0\,, \qquad \omega^2 = \left|m_k^2\right|\,, \qquad m_k^2 = s_2 m^2 + k^2\,.$$

If the particle moves faster than light the solution is of the oscillatory kind, indicating that such a motion is "natural" for this kind of particle (Sudarshan et al, 1962 - ...).

But for a smaller velocities the equation is anti-oscillatory, with exponential-type unstable solutions,

$$h \,=\, h_1 e^{\omega t} \,+\, h_2 e^{-\omega t}\,. \qquad (*)$$

For a field interacting with an external gravitational background, it is possible that the same wave changes from being a normal healthy state to a tachyonic one.

In principle, this situation may produce very strong effects at both quantum and classical levels.

D. Vanzella et all, 2010-2015.

In the fourth-order gravity the equations for the metric perturbations in flat space are

$$\ddot{\ddot{h}} + 2k^2\ddot{h} + k^4 h - \frac{1}{32\pi G a_1}\left(\ddot{h} + k^2 h\right) = 0\,.$$

It proves useful to introduce a new notation

$$\frac{1}{32\pi G a_1} = -\,s_2\,m^2\,, \quad \textbf{where} \quad s_2 = -\mathrm{sign}\,a_1 \quad \text{and} \quad m^2 > 0\,.$$

Then

$$\left(\frac{\partial^2}{\partial t^2} + \vec{k}^2\right)\left(\frac{\partial^2}{\partial t^2} + m_k^2\right)h = 0\,, \quad \textbf{where} \quad m_k^2 = k^2 + s_2 m^2\,.$$

The general formula for the frequencies is

$$\omega_{1,2} \approx \pm\, i\left(k^2\right)^{1/2} \quad \textbf{and} \quad \omega_{3,4} \approx \pm\left(-\,m_k^2\right)^{-1/2},$$

For a negative a_1 there are only imaginary frequencies and hence oscillator-type solutions (ghost case).

On the contrary, for a positive a_1 the roots $\omega_{3,4}$ are real, since in this case $-m_k^2 > 0$ for sufficiently small k^2 (tachyon).

The main difference between ghosts and tachyons is that a ghost may cause instabilities only when it couples to some healthy fields or to the background, while with tachyons there is no such a protection.

The situation with ghosts can be kept under control in the effective field theory framework, and in general when the intensity of the background fields is low and involved energies insufficient to generate a ghost from vacuum.

On the contrary, no low-energy protection can be expected in the theory with tachyons, because they produce instabilities independently on their interaction to normal particles or on the intensity of the background.

In other words, for tachyons the exponential behavior occurs at all frequencies, and not only above the Planck threshold.

The bad news for those expecting an eternal ΛCDM universe is that massive ghost will eventually become tachyon.

At the end of the ΛCDM universe the energy of the background is very law. At low energies all massive fields decouple and only the quantum effects of virtual photons are relevant.

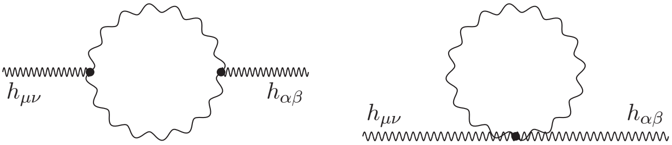


Figure: **Photon loops with two external gravitational lines.**

These quantum effects are pretty well-known,

$$\Gamma_{Weyl-squared} = -\frac{1}{320\pi^2}\int\sqrt{-g}\,C_{\alpha\beta\lambda\tau}\,\log\left(\frac{\Box}{\mu^2}\right)C^{\alpha\beta\lambda\tau}\,.$$

$$\Gamma_{Weyl-squared} = -\frac{1}{320\pi^2}\int\sqrt{-g}\,C_{\alpha\beta\lambda\tau}\,\log\left(\frac{\Box}{\mu^2}\right)C^{\alpha\beta\lambda\tau}\,.$$

It is easy to see that:

- **This expression is time-dependent during the cosmological evolution.**
- **Log. function is slow, hence we can approximately treat** $\log\left(\Box/\mu^2\right)$ **as a slowly varying parameter.**
- **Namely,**

$$g_{\mu\nu} = a^2(\eta)\,\bar{g}_{\mu\nu} = e^{\sigma(\eta)}\,\bar{g}_{\mu\nu} \;\Rightarrow\; \log\left(\frac{\Box}{\mu^2}\right) \propto -2\sigma(\eta) = -2\sigma(t)\,.$$

For any initial value of a_1 (including zero!) we meet

$$a_1^{eff}(t) = a_1 + \frac{1}{160\pi^2}\,\sigma(t)\,.$$

What this means, from the side of Physics?

At the final stage of the ΛCDM universe

$$\sigma(t) = H_0 t\,, \quad H_0 \sim \sqrt{\frac{8\pi \,\times\, 0.7\,\rho_c^0}{3\,M_P^2}} \approx 10^{-42}\,GeV\,.$$

Then the effective coefficient is

$$a_1^{eff}(t) \,=\, a_1 + \frac{1}{160\pi^2}\,\sigma(t) \,=\, a_1 + \frac{1}{160\pi^2}\,H_0 t\,.$$

Earlier or later a_1^{eff} will change sign and become positive.

The moment of this occurrence will be quite remarkable, but nobody will perhaps appreciate this, because all points of the space will explode at once.

The time remaining until tachyonic modes emerge in the spectrum of gravitational waves is

$$t_q \,=\, \frac{160\pi^2\,a_1}{H_0} \,\simeq\, 2.4\cdot 10^{13}\,yr \,=\, 2.4\cdot 10^4\,bi\,,$$

based on the assumption that $a_1(\mathrm{initial}) = 1$.

The remaining time until the gravitational wave explosion is more than one thousand times longer than the time which already passed from the Big Bang.

However, this result is based on the assumption that the starting value is $a_1 = 1$. The dependence is linear, so it is pretty easy to reduce or increase this time interval (only theoretically!!).

Regardless of the quantitative side of this prediction, it looks interesting that our knowledge of quantum corrections to gravity is sufficient to know how the ΛCDM universe will end up.

According to our analysis, there will be an instant gravitational waves explosion due to the tachyonic instabilities.

The explosion intensity will not be unrestricted.

The tachyonic modes will have energy density up to the Planck order of magnitude, since we did not quantize gravity.

Another natural restriction: the instability corresponds to linear perturbations. It might happen that next orders in the perturbative expansion in $h_{\mu\nu}$ will restore the stability.

However, from the practical side both these restrictions do not matter too much.

Even this "restricted" gravitational explosion should be capable to destroy the symmetry (homogeneity and isotropy) of the metric, leading to strong changes of space-time properties.

- **Consider the list of approximations which have been used.**

Heat-up question: "Can we trust logarithmic approximation at the one-loop level?" - Yes.

- **Can we expect qualitative change in the result by taking the higher-order loops into account?**

Formally, higher-loop contributions do not change the sign of the β_1-function (c-theorem), but this is not sufficient to draw conclusions about the (ir)relevance of the higher-loop terms.

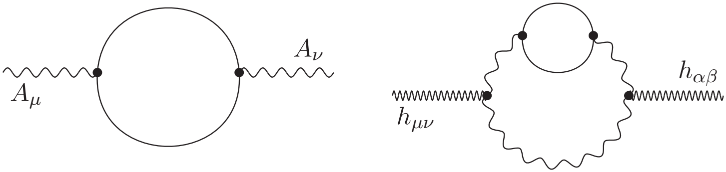
In the UV there will be higher-log. contributions, capable to produce a strong change in the running of a_1. However, the situation at low energies (far IR) is quite different.

Let us remember that second- and higher-loop corrections to the one-photon bubble include a loop of electrons or of other massive charged fermions.

Because of the Appelquist & Carazzone decoupling theorem the contribution of the second loop is suppressed by a factor

$$\left(\frac{H_0}{m_e}\right)^2 \approx 10^{-77}$$

for the dynamics of the conformal factor.



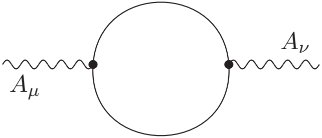
Since this is the part which governs the running of a_1^{eff}, it seems that there are no chances for higher loops to change the result.

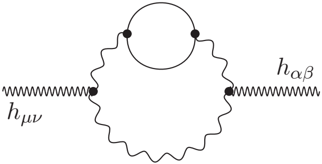
However, our interest is not the dynamics of the conformal factor itself, but its interaction to gravitational waves.

The typical energy of the external particle should be the one of the gravitational wave. Then the relevant ratio which defines the effective cut-off for the higher-loop contributions is

$$\big(\mathcal{E}_{GW}/m_e\big)^2$$

for a tensor mode with energy $\mathcal{E}_{GW} \gg H_0$. For instance, taking $\mathcal{E}_{GW} = 1eV$ we have “only” ten-orders decoupling.





Up to the frequencies of the order of electron mass, the one-loop approximation is completely robust. Only above this threshold there is a small chance of stabilization by higher loops.

What about quantum gravity (QG) effects, which have been neglected so far?

It is not easy to give a definite answer due to the variety of existing models of QG. Let us consider a short list of the possibilities which are better explored.

The standard effective framework for the IR effects of QG assumes that GR is a universal theory of IR quantum gravity.

J. Donoghue - 1994, PRL & PRD, gr-qc/9405057

QG based on GR is non-renormalizable, hence there is no consistent perturbative β-function for the parameter a_1.

One can easily derive the 1-loop logarithmic form factor.

G. 'tHooft and M.Veltman, (1974).

However, it is gauge-fixing dependent and vanish on-shell.

R. Kallosh, O. Tarasov and I.V. Tyutin (1978).

Therefore, no physical correction can be expected. Similar situation holds at higher loops.

- **Higher derivative QG (HDQG)**

The β-function for the parameter a_1 is well-defined, free of ambiguities. According to the well-verified calculations

E.S. Fradkin & A.A. Tseytlin (1982);
I. Avramidi & A.O. Barvinsky (1986);
I. Antoniadis and E. Mottola (1992);
G.B. Peixoto & I.Sh. (2003)

the contribution of HDQG enhance the one of the photon loop by a factor of $10 - 20$ **in the four-derivative QG case.**

Indeed, this statement requires great care. In fact, the IR effects of HDQG are not sufficiently well-explored.

The standard point of view is that the universal IR limit of all these theories is quantum GR. Then we come back to the irrelevant contribution of QG in the IR.

- **In (super)string theory the terms providing the β-function for the parameter a_1 are removed by means of the Zwiebach transformation for the "background" metric.**

B. Zwiebach, (1985).

$$\Gamma = \int d^D x \sqrt{g}\, e^{-2\Phi} \Big\{ -R + 4\alpha' \big(R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 4\, R_{\alpha\beta} R^{\alpha\beta} + R^2 \big) + ... \Big\}.$$

By construction, the β-function for the parameter a_1 is zero.

However, string theory is not supposed to significantly correct QFT results at low and very low energies.

Otherwise we would observe such corrections in precision experiments, e.g., the ones that test QED and Standard Model calculations. Using string theory to evaluate the β_1-function in the far IR is not reasonable from a conceptual point of view.

After all, in known versions of QG & string theories the IR running of universe to the tachyonic end can't be cancelled.

Conclusions

- One should definitely quantize both matter and gravity, for otherwise the QG theory would not be complete. And it can not be done without HD terms.

- For QG with higher derivatives (HDQG) the propagator includes massive nonphysical mode(s) called ghosts.

- These massive ghosts are capable to produce terrible instabilities, but ... for this end there should be at least one such ghost excitation in the initial spectrum.

- At least in the cosmological case, the ghost is not actually generated below Planck scale.

- The final conclusion is that the HDQG may be a perfect candidate to be an effective QG below the Planck scale.

- Finally, we can predict an intensive tachyonic explosion at the end of the ΛCDM universe.