Modified GravityIII: Fairy Godmother or Sleepy Beast?

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Outline

1. **Part III-Screening mechanism**
2. **Part IIIA-Chameleon mechanism**
3. **Part IIIB-Vainshtein mechanism**

Screening mechanism

1. If this scalar degree of freedom couples to matter, it is subject to stringent constraints from the Solar System tests.
2. Screening mechanisms provide a way to evade the SS tests and they are included in many well-studied modified gravity models.
3. We will explore only two types of screening mechanisms: **Chameleon and Vainshtein**.
4. The model should suppress the fifth force at the solar system and introduces modification at the large scale of universe. Explain the current acceleration of the universe.

Screening mechanism: General traits-I

- Consider a **scalar-tensor** model with a **general kinetic term** and **potential term**

$$\mathcal{L} = \phi R - \omega(\phi)\nabla_\mu\phi\nabla^\mu\phi - U(\phi) + K[(\partial\phi)^2, (\partial^2\phi)] + \mathcal{L}_m[g_{\mu\nu}, \psi_m]$$

- **Chameleon mechanism**: $U(\phi)$ is introduced. $M^{\phi}_{\rm eff}$ depends on its background density. We can have large $M^{\phi}_{\rm eff}$ in high density environments and small $M^{\phi}_{\rm eff}$ in low density env. [f(R)-gravity, STG]

- **Vainshtein mechanism** : Nonlinear K-terms effectively suppress the coupling between the scalar field and matter. The Vainshtein mechanism relies on the nonlinearity of the second derivative of $(\partial^2\phi)^p$.

Screening mechanism: General traits-II

How the SM operates to suppress the fifth force for a spherically symmetric object with a mass M and size R?

- **Thin-shell screening**: Screening is determined by the gravitational potential of the object $\Psi = GM/R$. Screening is possible if the condition $\Psi > \chi(\psi_{bg})$ holds.

- **Vainshtein mechanism**: Screening operates well when the curvature of the object exceeds the squared of critical mass $\partial^2\Psi \propto GM/R^3 > \Lambda_c^2$.

It would be nice if there were some mechanisms where the scalar's effects are negligible in the solar system but important for cosmology.

Chameleon mechanism-I: Khoury& Weltman-2004

- Chameleon action in the Einstein frame:

$$S = \int d^4x\sqrt{-g^E}\Big(\frac{{}^E R}{2\kappa^2} - \frac{1}{2}(\partial\phi)^2 - V(\phi)\Big) + S_m[A^2(\phi)g^E_{\mu\nu}, \Phi_m]$$

- Matter fields described by Φ_m couple to ϕ through the Jordan frame metric $g^J_{\mu\nu} = A^2(\phi)g^E_{\mu\nu}$.
- Chameleon Lagrangian (in the weak-field limit and for non-relativistic matter):$[\rho = A^3\rho_J \gg P,\;\; v << 1]$.

$$\mathcal{L}_{\rm cham} = -\frac{1}{2}(\partial\phi)^2 - V(\phi) + A(\phi)T$$

- The equation of motion for the scalar field is

$$\Box\phi = V_{,\phi} - A_{,\phi}T \equiv V^{\rm eff}_{,\phi} \;\;\text{with}\;\; T = -\rho$$

Chameleon mechanism-II

- Chameleon field is governed by a density-dependent effective potential

$$V^{\mathrm{eff}} = V(\phi) + A(\phi)\rho$$

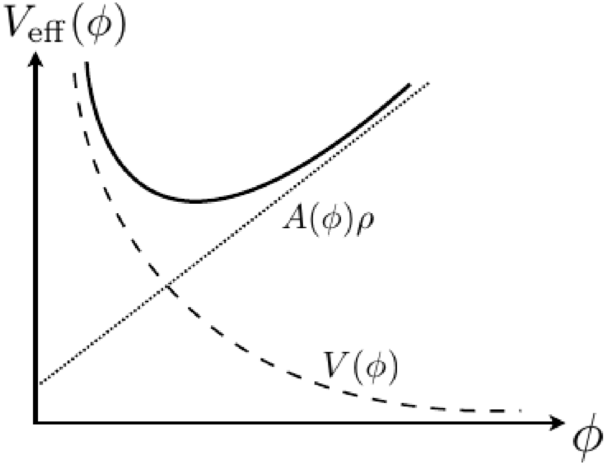
- Effective mass depends on the local matter density.
- For suitably chosen $V(\phi)$, V_{eff} can develop a minimum at ϕ_{min} in the presence of background matter density.
- For $V = \frac{\Lambda^{4+n}}{\phi^n}$ and $A = 1 + \frac{\beta}{M_p}\phi\rho$ the minimum is placed at

$$\phi_{min} = \left(\frac{nM_p\Lambda^{4+n}}{\beta\rho}\right)^{\frac{1}{n+1}}$$

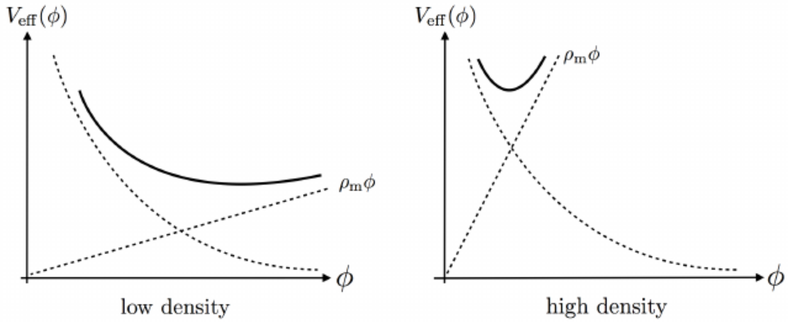
- The mass of small fluctuations around the minimum is

$$m_\phi^2 := V^{eff}(\phi_{min}) = n(n+1)\Lambda^{4+n}\frac{1}{\phi_{min}^{(n+2)}} \propto \rho^{\frac{n+2}{n+1}}$$

Chameleon mechanism: V_{eff} depends on local matter density



Chameleon mechanism: Low and High density



1. Low ρ implies small m_ϕ^2. Light chameleon implies the fifth force is long-range interaction [$\lambda_c \propto m_\phi^{-1}$] and could be observable.
2. High ρ implies large m_ϕ^2. Heavy chameleon implies the fifth force is short-range interaction [$\lambda_c \propto m_\phi^{-1}$] and is unobservable.

Chameleon mechanism: Constraints

1. Constraints come from lab. tests of the inverse square law. For a local density $\rho_{lab} \simeq 10^{-3} g\mathrm{cm}^{-3}$, there is an upper limit of on the fifth-force range assuming gravitational strength coupling:

$$m_{\phi}^{-1}(\rho_{\mathrm{lab}}) \lesssim 40\mu\mathrm{m}$$

2. For $n \ll 1$ and $\rho_{\mathrm{cosmo}} = 10^{-30} g\mathrm{cm}^{-3}$ the Compton wavelength of the chameleon field is

$$m_{\phi}^{-1}(\rho_{\mathrm{cosmo}}) \lesssim \mathrm{Mpc} \simeq 10^{28}\mu\mathrm{m}$$

3. For SS (Bg+DM), $\rho_{\mathrm{solar-system}} = 10^{-25} g\mathrm{cm}^{-3}$, λ_c^{cham} is larger than the size of the solar system ($\simeq 10^{-4} pc$):

$$m_{\phi}^{-1}(\rho_{\mathrm{solar-sytem}}) \lesssim 4.8\mathrm{pc}$$

4. But chameleon can hide itself, by suppressing its coupling to massive objects, via a thin-shell effect.

How the chameleon screening mechanism operates? Spherically symmetric source and thin-shell effect

- We consider a SS object with radius R, homogeneous density $\rho_{\rm in}$ and total mass M .
- This object is immersed in a homogeneous medium with density $\rho_{\rm out}$.
- The density profile is

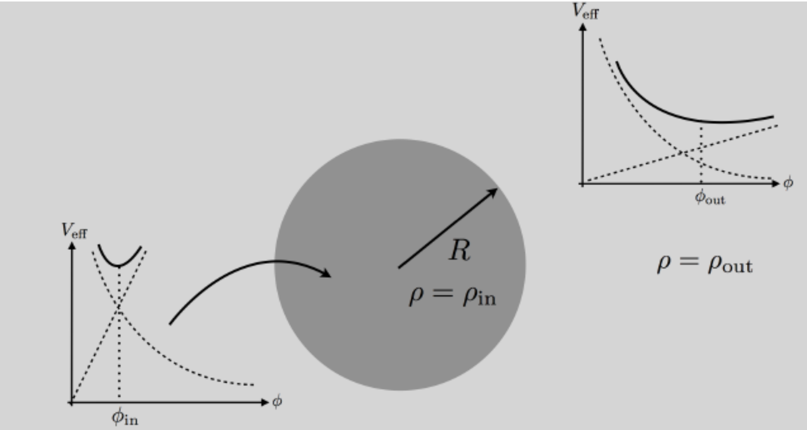
$$\rho(r) = \begin{cases} \rho_{in} & r \leq R \\ \rho_{out} & r \geq R \end{cases}$$

- Near the object, the gravitational field is weak and the e.o.m. for the scalar field is

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = V_{,\phi} + \frac{\beta\rho}{M_p}.$$

- Boundary condition: i-$d\phi/dr \to 0$ at $r = 0$ and ii-$\phi \to \phi_{out}$ for $r \to \infty$.

Sketch of the set-up for thin-shell calculation.



How the chameleon screening mechanism operates?-I

- For a large body the field approaches the minimum of its effective potential deep in the interior:

$$\phi \simeq \phi_{in}, \quad r < R$$

.

- Outside the object, but still within an ambient Compton wavelength away ($r < m_\phi^{-1}$), the field profile is:

$$\phi \simeq \phi_{out} + \frac{C}{r}, \quad R < r < m_\phi^{-1}$$

.

- The exterior solution becomes

$$\phi \simeq \phi_{out} - \frac{R(\phi_{out} - \phi_{in})}{r}, \quad R < r < m_\phi^{-1}$$

.

- Since $\nabla^2\phi = 0$ both inside and outside the source, the body acts as a conducting sphere. Any chameleon charge is confined to a thin shell of thickness ΔR near the surface.

How the chameleon screening mechanism operates?-II

- The surface charge density $\sigma = \beta\rho_{in}\Delta R/M_p$ supports the discontinuity in field gradients:

$$\left[\frac{d\phi}{dr}\right]_{r=R_+} = \sigma$$

.

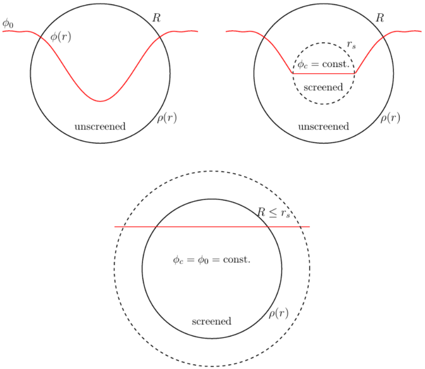
- Since $\nabla^2\phi = 0$ both inside and outside the source, the body acts as a conducting sphere. Any chameleon charge is confined to a thin shell of thickness ΔR near the surface.
- We can solve for the shell thickness under certain condition [HW]:

$$\frac{\Delta R}{R} = \frac{\Delta\phi}{6\beta M_p \Phi_N(R)}, \quad \Phi_N(R) = \frac{M}{8\pi M_p^2 R}$$

- An object is therefore said to be screened if $\Delta R \ll R$. The approximations breaks down if $\Delta R \gg R$ and the object is said to be unscreened.

How the chameleon screening mechanism operates?-III

- Only the mass from a thin shell (with thickness ΔR) of this spherical object contributes the fifth force while the rest gets screened.



How the chameleon screening mechanism operates?-IV

- GR was founded on the **equivalence principle**: The motion of test-bodies is independent of their structure and composition. Any kind of **violation of EP** can be detected by looking at the Newtonian limit of the theory.

- Let us consider a particle with mass m. In the Newtonian limit the Newton equation is

$$m\ddot{x} = -m\nabla\Phi^{N}_{ext} - \beta\nabla\phi_{ext}$$

- Chameleon-like theories do not satisfy the equivalence principle because $m \neq \beta$ unless the screening radius is zero or equal to the radius of the object [i.e. unless the object is fully screened or fully unscreened].

Non-linearity in the kinetic operator-I

- Galileon's lagrangian:

$$\mathcal{L}_g = -3(\partial\pi)^2 - \frac{1}{\Lambda_s^3}\Box\pi(\partial\pi)^2 - \frac{g}{M_p}\pi\rho_{\rm m}$$

- Equations of motion are second-order:

$$3\Box\pi + \frac{1}{\Lambda_s^3}\left[(\Box\pi)^2 - \partial_\mu\partial_\nu\pi\partial^\mu\partial^\nu\pi\right] = \frac{g}{2M_p}\rho_{\rm m}$$

- They enjoy the galilean shift symmetry: $\pi \to \pi + c$ [HW]
- We must consider the galileon model as an EFT valid for scales $\Lambda \leq \Lambda_s$.[NR]

$$\mathcal{L}_g = -3(\partial\pi)^2 - \frac{1}{\Lambda_s^3}\Box\pi(\partial\pi)^2 + \sum_{n\geq 3}\frac{c_n}{\Lambda_s^{3n-4}}(\Box\pi)^n - \frac{g}{M_p}\pi\rho_{\rm m}$$

- Vainshtein mechanism: $\Lambda_s^{-3}\Box\pi(\partial\pi)^2 \gg (\partial\pi)^2$ near massive objects: $\Box\pi \gg \Lambda_s^3$.

Non-linearity in the kinetic operator-II

- There is a regime where **the galileon term can dominate** while all other operators are negligible-[Two relevant parameters]
- The parameter that controls [at the classical level] non-linearity is $\alpha_{cl} = \Box\pi/\Lambda_s^3$.
- The parameter that controls the relevance of quantum corrections is $\alpha_q = \partial^2/\Lambda_s^2$
- There are situations where classical non-linearities are important $\alpha_{cl} \gg 1$ while quantum corrections remain under control $\alpha_q \ll 1$.

Solution around spherically-symmetric source-I

- To illustrate the Vainshtein screening for a static point source of mass M we consider $T = -M\delta^3(x)$.
- For $\bar{E} = \nabla\pi$ the EOMs become

$$\nabla.\left(6\bar{E} + \hat{r}\frac{4}{\Lambda_s^3}\frac{E^2}{r}\right) = \frac{gM}{M_p}\delta^3(x)$$

- Integrating over a sphere centered at the origin yields

$$\left(6E + \frac{4}{\Lambda_s^3}\frac{E^2}{r}\right) = \frac{gM}{4\pi r^2 M_p}$$

- The nontrivial profile is crucial to the Vainshtein effect:

$$E_{\pm} = \frac{\Lambda_s^3}{r}\left(\pm\sqrt{9r^4 + 4rr_v^3} - 3r^2\right)$$

Solution around spherically-symmetric source-II

- We introduced the **Vainshtein radius** r_v:

$$r_v \equiv \left(\frac{gM}{4\pi M_p}\right)^{\frac{1}{3}} = \left(g r_s L^2\right)^{\frac{1}{3}}, \quad r_s \equiv \frac{M}{4\pi M_p^2}, \quad \Lambda_s \equiv (L^{-2} M_p)^{1/3}$$

- We work with the $+$ branch only because the $-$ branch has unstable perturbations.
- **Far from the source** $[r \gg r_v]$, the solution approximates a $1/r^2$ profile:

$$E(r \gg r_v) = \frac{3r\Lambda_s^3}{4}\left(\sqrt{1 + \frac{4r_v^3}{9r^3}} - 1\right) \simeq \frac{g}{3}\frac{M}{8\pi r^2 M_p}$$

- The fifth force of the Galileon is $F_\pi = g|\nabla\pi| \simeq g^2 F_g/3$ for $r \gg r_v$
- For $M \gg M_p$ and $\Lambda_s^{-1} \ll r_v$ both expansion parameters are small:

$$\alpha_{cl} \propto \left(\frac{r_v}{r}\right)^3 \ll 1, \quad \alpha_q \propto 1/[r\Lambda_s]^2 \ll 1$$

Solution around spherically-symmetric source-III

1. **Close to the source**[$r \ll r_v$], the solution reduces to $1/r^{1/2}$ profile:

$$E(r \ll r_v) \simeq \frac{\Lambda_s^3}{2}\frac{r_v^{3/2}}{\sqrt{r}}$$

2. The galileon force is suppressed compared to gravity for $r \ll r_v$:

$$\frac{F_\pi}{F_{gravity}} \simeq \left(\frac{r}{r_v}\right)^{\frac{3}{2}} \ll 1$$

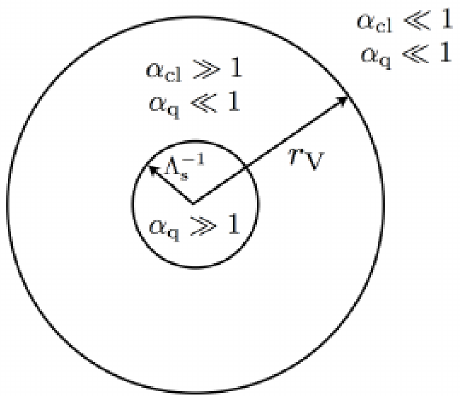
3. The classical non-linearity parameter is of large and the quantum parameter does not change:

$$\alpha_{cl} \propto \left(\frac{r_v}{r}\right)^{3/2} \gg 1, \quad \alpha_q \propto 1/[r\Lambda_s]^2 \ll 1$$

4. At distances $r \gg \Lambda_s^{-1}$ Q.C. are small and classical description is correct.

Solution around spherically-symmetric source-IV

- There is a window, $\Lambda_s^{-1} \ll r \ll r_v$, where classical nonlinearities are important and Q. C. effects are under control.



- A similar mechanism can be found in GR.[Not today!!!]

Table 2. The radii R (physical size), $r_g = 2GM$ (Schwarzschild radius) and r_V (Vainshtein radius) of some typical objects. Unit is meter. For estimation we have used $\beta = 2\sqrt{2}/3$ and $r_c = H_0^{-1} \sim 4000\,\mathrm{Mpc} \sim 1.2 \times 10^{26}$ m.

Object	R	r_g	r_V
Milky Way	$\sim 0.9 \times 10^{21}$	$\sim 2 \times 10^{15}$	$\sim 3 \times 10^{22}$
Sun	$\sim 0.7 \times 10^{9}$	$\sim 3 \times 10^{3}$	$\sim 3.5 \times 10^{18}$
Earth	$\sim 6 \times 10^{6}$	$\sim 9 \times 10^{-3}$	$\sim 5 \times 10^{16}$
Atom	$\sim 5 \times 10^{-11}$	$\sim 1.8 \times 10^{-54}$	$\sim 3 \times 10^{-1}$

Observational constraints on Vainshtein mechanism

1. The strongest bound on the cubic galileon comes from the LLR.
2. Deep inside r_v, the galileon-mediated force is weak and gives a small correction to the Newtonian potential:

$$\frac{\delta\Phi}{\Phi} \simeq \frac{g^2}{2}\left(\frac{r}{r_v}\right)^{3/2} \lesssim 2.4 \times 10^{-11}$$

3. Using the LLR bound we can get a constraint on the scale L. Using that $r_{s,Earth} = 0.886\mathrm{cm}$ and the distance Earth-Moon $r = 3.48 \times 10^{10}\mathrm{cm}$ we arrived at

$$L \gtrsim \frac{H_0^{-1}}{20g^{3/2}} \simeq 140\mathrm{Mpc}g^{-3/2} \text{ where } H_0^{-1} = 3000\mathrm{Mpc} = 10^{28}\mathrm{cm}$$

4. The galileon-mediated force becomes important at late times and on large scales.

- There are several nice references to study modified gravity:

1. Alternative Theories of Gravity. N. Yunes, [arXiv:1304.3473], [arXiv:gr-qc/0403100], [arXiv:1012.3144].
2. Cosmological Tests of Modified Gravity-Kazuya Koyama-[arXiv:1504.04623]
3. Testing General Relativity with Present and Future Astrophysical Observations. E. Berti et al. [arXiv:1501.07274]
4. Modified Gravity and Cosmology-T. Clifton et al. [arXiv:1106.2476]
5. Massive Gravity-C. de Rham, Living Rev. Relativ. 17 (2014) 7, [arXiv:1401.4173].
6. f(R) gravity- A. De Felice, S. Tsujikawa, Living Rev.Relativ. 13 (2010) 3,[arXiv:1002.4928].
7. R. Maartens and K. Koyama, Living Rev. Rel. 13 (2010) 5, [arXiv:1004.3962].
8. C. M. Will, Living Rev. Rel. 17 (2014) 4, [arXiv:1403.7377].