Modified Gravity II: Fairy Godmother or Untamed Beast?

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Outline

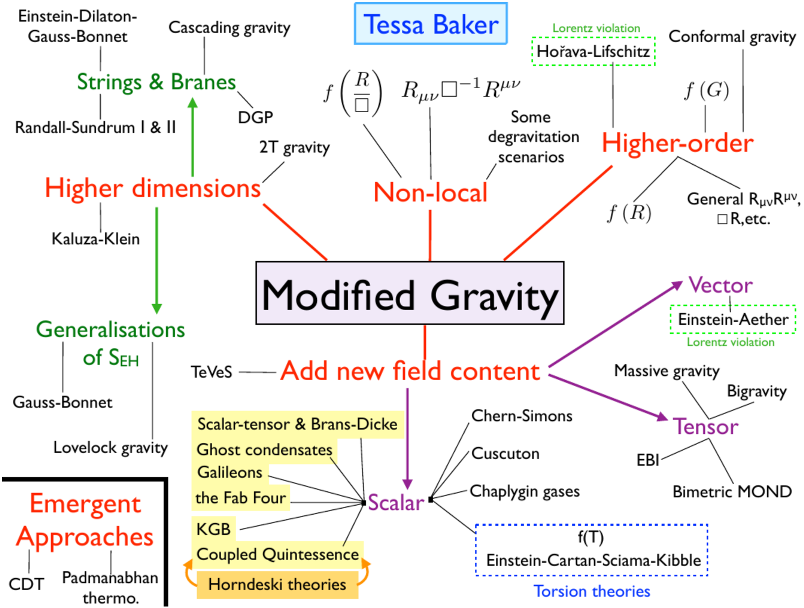
1. **Part II: Basic elements of MOG**

Minimal conditions for alternative theories

Modified gravity theories must obey certain restrictions in order to be a reasonable alternative to General Relativity:

1. ***Action principle***
2. ***Relativistic Invariance***
3. ***Equivalence Principle***
4. ***Causality***
5. ***Positive energy***
6. ***Observational tests***

How many theories?-[T.Baker's map]



1. **Higher derivatives:** f (R) gravity has fourth order time derivatives of metric in the equations of motion but do not propagate the Ostrogradsky ghost.
2. **Extra degrees of freedom:** Metric+ some degrees of freedom[scalar fields, vector or tensor fields]. Ex.STG, Brans-Dicke.
3. **Others:**Non-locals models, brane-world models, Lorentz violation, Teleparallel models, Breaking Diff. (Hovara-gravity) etc.

Consistencies

1. MOGs introduce new degree of freedoms such as scalar, vector, and tensor fields. They also introduce several **types of instabilities**:

$$\mathcal{L} = \mathcal{K}_t \dot{\theta}^2 - \mathcal{K}_x [\nabla\theta]^2 - \mathcal{M}^2\theta^2$$

   - The tachyonic instability: $\mathcal{M}^2 < 0$.
   - Gradient instability: $\mathcal{K}_x < 0$.
   - Ghost instability: $\mathcal{K}_t < 0$.

2. **Strong coupling problem**(EFT): Nonlinear interaction terms, $\mathcal{L}_{NL} = \Lambda_\star^{-1}\Box\theta[\partial\theta]^2$ are important for $\Lambda \leq \Lambda_\star$.[Lack of UV-completion]

Case I: Scalar-Tensor-I

1. **Scalar-tensor theories**: GR+ an extra scalar degree of freedom:

$$16\pi G_N \mathcal{L}_{STG} = F(\Phi)R - Z(\Phi)\nabla_\mu\Phi\nabla^\mu\Phi - U(\Phi) + \mathcal{L}_m[g_{\mu\nu}, \psi_m]$$

2. Jordan frame - Gravity sector modified BUT matter sector minimally coupled to $g_{\mu\nu}$.
3. Dynamically evolving scalar field with kinetic term $(\nabla\Phi)^2 \equiv g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi$ and potential $V(\Phi) = U(\Phi)$.

Scalar-Tensor-II

1. Varying the action wrt $g_{\mu\nu}$ and Φ:

$$F(\Phi)G_{\mu\nu} = 8\pi G_N T_{\mu\nu} + [\nabla_\mu \nabla_\nu - g_{\mu\nu}\Box]F(\Phi) - \frac{1}{2}g_{\mu\nu}U(\Phi) + Z(\Phi)[\nabla_\mu \Phi \nabla_\nu \Phi - \frac{1}{2}g_{\mu\nu}\nabla_\kappa \Phi \nabla^\kappa \Phi]$$

$$Z(\Phi)\Box\Phi = U_{,\Phi} - Z_{,\Phi}\nabla_\kappa \Phi \nabla^\kappa \Phi - F_{,\Phi}R$$

2. Trace of FE yields:

$$F(\Phi)R = 8\pi G_N T - 3\Box F(\Phi) - 2U(\Phi) + Z[\nabla\Phi]^2$$

3. Modifications sourced by $Z(\Phi)$, $F(\Phi)$ and $T = g_{\mu\nu}T^{\mu\nu} = \rho - 3p$
4. Brans-Dicke model is recovered by taking $F(\Phi) = \Phi$ and $Z(\Phi) = \omega/\Phi$.

Conformal Transformation Rule-I

1. Conformal transformation of the metric allows a different representation of the theory: $\Omega : (\mathcal{M}, g_{\mu\nu}) \to (\hat{\mathcal{M}}, \hat{g}_{\mu\nu})$

$$g_{\mu\nu} = \Omega^2(\Phi)\hat{g}_{\mu\nu}, \quad \Omega^2(\Phi) > 0$$

2. Angles between vectors are unchanged.
3. Is the above definition arbitrary or not?
4. It does not alter the casual structure of spacetime

Conformal Transformation Rule-II

1. Prove the following identities $[\hat{\nabla}_\beta \omega = \hat{\nabla}_\beta \ln \Omega]$:
2. $g^{\mu\nu} = \Omega^{-2} \hat{g}^{\mu\nu}$ and $g = \Omega^8 \hat{g}$.
3. $\Gamma^\alpha_{\beta\gamma} = \hat{\Gamma}^\alpha_{\beta\gamma} + \Omega^{-1}[\delta^\alpha_\beta \hat{\nabla}_\gamma \Omega + \delta^\alpha_\gamma \hat{\nabla}_\gamma \Omega - \hat{g}_{\beta\gamma} \hat{\nabla}^\alpha \Omega]$
4. $R = \Omega^{-2}[\hat{R} + 6\hat{\Box}\omega - 6\hat{\nabla}^\kappa \omega \hat{\nabla}_\kappa \omega]$

Scalar-Tensor-III

1. Use the C.T $\hat{g}_{\mu\nu,\star} = A_\star^2 g_{\mu\nu}$ with $A_\star^2 = F$
2. Redefine scalar field to write the action in simpler form in terms of ψ:

$$\kappa \frac{d\Psi}{d\Phi} = \frac{Z}{F} + \frac{3}{2}\left(\frac{\partial \ln F}{\partial \Phi}\right)^{\frac{3}{2}}$$

3. Potential is $2\kappa V(\Psi) = U/F^2$.
4. The Langrangian in the Einstein frame is

$$\mathcal{L}_{RG+S} = \frac{1}{2\kappa}\hat{R} - \frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu\Psi\partial_\nu\Psi - V(\Psi) + \hat{\mathcal{L}}_m[\hat{g}_{\mu\nu}, \Psi, \Theta_m]$$

5. The Langragian for the matter sector in the Einstein frame is

$$\hat{\mathcal{L}}_m[\hat{g}_{\mu\nu}, \Psi, \Theta_m] = \frac{\mathcal{L}_m[F^{-1}\hat{g}_{\mu\nu}, \Theta_m]}{F^2(\Phi)}$$

1. EOM for the scalar field couple to the trace of the stress-energy tensor.

$$\hat{\Box}\Psi = V_{,\Psi} + \alpha[\Psi]\hat{T} = V^{eff}_{,\Psi}, \quad \alpha[\Psi] := \frac{1}{2}\frac{d\ln F}{d\Psi}$$

2. In the Jordan frame TEM is conserved in STG, $\nabla_\mu T^{\mu\nu} = 0$. In the Einstein frame, $\hat{\nabla}_\mu \hat{T}^{\mu\nu} = \alpha[\Psi]\hat{T}\partial^\nu\Psi$.
3. Consequence: In Einstein frame particle masses depend on Ψ[5th force].

Scalar-Tensor-V

1. Solar system constraints on STG are obatined due to the coupling $\alpha[\Psi]$ in Einstein frame:

$$|\gamma_{PPN} - 1| = \frac{2\alpha^2}{1+\alpha^2} < 10^{-5} \quad \text{Cassini}$$

$$|\beta_{PPN} - 1| = \frac{\alpha^2}{\sqrt{2}(1+\alpha^2)^2}\alpha' < 10^{-5} \quad \text{Cassini} + \text{LLR}$$

2. Good bounds on $\alpha < 10^{-5}$ but not on α' provided we set $V = 0$ [massless limit]. The fifth force associated with the scalar field has infinite range.

Case II:Metric $f(R)$-I

1. The Lagrangian in the Jordan's frame

$$\mathcal{L}_{grav+m} = \frac{1}{16\pi G_N} f(R) + \mathcal{L}_m[g_{\mu\nu}, \Phi_m]$$

2. Modified Einstein's Equations:

$$f_{,R} R_{\mu\nu} - \nabla_\mu \nabla_\nu f_{,R} + g_{\mu\nu}[\Box f_{,R} - \frac{1}{2} g_{\mu\nu} f(R)] = 8\pi G_N T_{\mu\nu}$$

3. A class of $f(R)$-theories can be constrained using several cosmological data and solar system test.

Several important facts:Metric $f(R)$-II

1. f(R) gravity is a fourth order: hard to find the appropriate boundary condition for a given problem [IVP]. Instability.
2. Solar System constraints fairly complicated.
3. Lack of Birkhoff's theorem: spherically symmetric vacuum solution is no longer unique.
4. EFE reduces to the familiar 2nd OPDE GR+Λ iff $f(R) = R - 2\Lambda$.

Case II:Metric $f(R)$-III

- $f(R)$ can explain the **accelerating expansion** if $\nabla_\mu f_R = 0$.

$$G_{\mu\nu} = -\frac{1}{2} g_{\mu\nu}[R - \frac{f}{f_{,R}}] = -g_{\mu\nu}\lambda(R)$$

- The trace of the above equation, $f_{,R}R - 2f(R) = 0$, has a nonzero positive root $R_0 = 4\lambda(R_0) \equiv 4\Lambda_0$ [Geometric Dark energy],

- Trace of the MEF yields

$$(R + 3\Box)f_{,R} - 2f = T$$

- This gives a wave equation for $f_{,R}$:

$$f(R): \quad \Box f_{,R} = \frac{T}{3} + \frac{2f - Rf_{,R}}{3} \qquad GR: R = 4\Lambda - T$$

- $f_{,R}$ is a dynamical DOF . Initial data require $g_{\mu\nu}$ and $\dot{g}_{\mu\nu}$, but also $f_{,R}$ and $\dot{f}_{,R}$.
- There is no extra degree of freedom in GR.

Is it possible to link STG with $f(R)$-models?-I

1. The Lagrangian for $f(R)$-gravity model can be written as

$$\mathcal{L}_{f+m} = [\frac{1}{2\kappa^2} f_R R - U] + \mathcal{L}_m[g_{\mu\nu}, \psi_m], \quad U \equiv \frac{f_R R - f}{2\kappa^2}$$

2. Apply a CF: i-$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ and ii-$R = \Omega^2[\hat{R} - 6\hat{\Box}\omega - 6\hat{g}^{\mu\nu}\omega_{,\mu}\omega_{,\nu}]$:

$$\mathcal{L}_{f+m}\sqrt{-g} = \sqrt{-\hat{g}}\Big[\frac{\Omega^{-2} f_R}{2\kappa^2}\Big(\hat{R} - 6\hat{\Box}\omega - 6\hat{g}^{\mu\nu}\omega_{,\mu}\omega_{,\nu}\Big) - U\Omega^{-4}\Big] + \sqrt{-\hat{g}}\Omega^{-4}\mathcal{L}_m[\Omega^{-2}\hat{g}_{\mu\nu}, \psi_m], \quad V \equiv \Omega^{-4} U$$

3. Select $\Omega^2 = f_{,R}$ and $\kappa\varphi = \sqrt{3/2}\ln f_{,R}$:

$$\hat{\mathcal{L}}_{f+m} = \frac{1}{2\kappa^2}\hat{R} - \frac{1}{2}\hat{g}^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu} - V(\varphi) + \Omega^{-4}\mathcal{L}_m[e^{-\kappa\sqrt{\frac{2}{3}}\varphi}\hat{g}_{\mu\nu}, \psi_m]$$

Is it possible to link STG with $f(R)$-models?-II

1. In the Einstein frame the scalar field φ has a coupling Q with non-relativistic matter given by $e^{-\kappa\sqrt{\frac{2}{3}}\varphi} = e^{2Q\varphi}$ with $Q = -\kappa/\sqrt{6}$.
2. In the case of $V = 0$, the local gravity constraints are not satisfied because the field extra field has $Q = \mathcal{O}(1)$.
3. The local gravity test can be avoided through a screeninig mechanism by selecting the form of $f(R)$.

1. Modify gravity at large scales but not at small scales!!!
2. Geodesic equation after a CF [$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$]:

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\kappa}\frac{dx^\nu}{d\tau}\frac{dx^\kappa}{d\tau} + \left(\delta^\mu_\kappa[\ln\Omega]_{,\nu} + \delta^\mu_\nu[\ln\Omega]_{,\kappa}\right)\frac{dx^\nu}{d\tau}\frac{dx^\kappa}{d\tau} - g_{\kappa\nu}[\ln\Omega]_{,\mu}\frac{dx^\kappa}{d\tau}\frac{dx^\nu}{d\tau} = 0$$

3. The coupling to matter produces a fifth force: Violation of WEP (UFF)

$$m\ddot{x}^i = -m\partial^i\Psi - Q\partial^i\ln\Omega(\varphi)$$

Negative aspects about MOG

1. Which side of the Einstein equation is modified, or which term in action is modified?[criterion is in fact ambiguous].
2. Initial value formulation-Hamiltonian unbounded below (generic for higher derivative theories)
3. Instabilities: ghost, tachyonic, adiabatic
4. Singularities in formation of neutron stars
5. Superluminal effects
6. Strong coupling (loops)
7. Inapplicability of macroscopic field equations