Modifief Gravity I: Review of RG

Martín G Richarte[♣]

[♣]PPGCosmo-UFES.AW19.

Outline

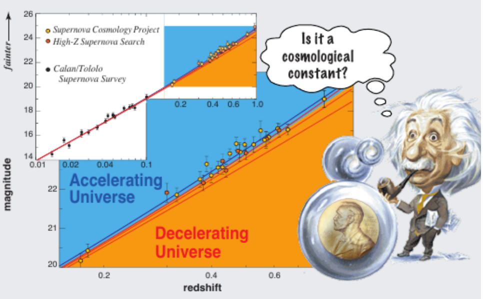
Isn't GR enough?

1. Insight into a fundamental law of nature in environments inaccessible here on earth.
   - Distance: Planck $10^{-30}m$ to GPc $10^{25}m$
   - Density: critical $10^{-27}\text{kg}m^{-3}$ to BH $10^{9}\text{kg}\ m^{-3}$
2. GR in 4D spacetime is a remarkably beautiful, and comparatively minimal metric theory, it is not the only one. We should test it! GR is UV incomplete.
3. It is comparatively ill-tested beyond solar and stellar system scales.
4. It is the only fundamental field yet to be quantized.
5. Cosmological acceleration could suggest something may be awry on cosmic scales.
6. Alternative gravity theories are still compatible with the recent constraints coming from the mergering of BH-BH, BH-NS, NS-NS and the emission of GWs.

1. Acceleration equation for GR plus homogeneous & isotropic universe:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}(\rho + 3P) + \frac{\Lambda}{3}$$

2. Late time acceleration indicates a new cosmic scale physics
   - Is $\Lambda > 0$?
   - Is there a new type of matter with $P_{DE} < -\frac{\rho_{DE}}{3}$? (Violation of SEC)
   - Should be GR modified to explain this phenomena?

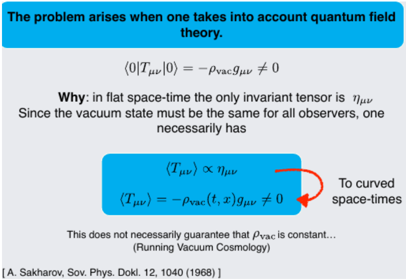


Acceleration of the universe-DE

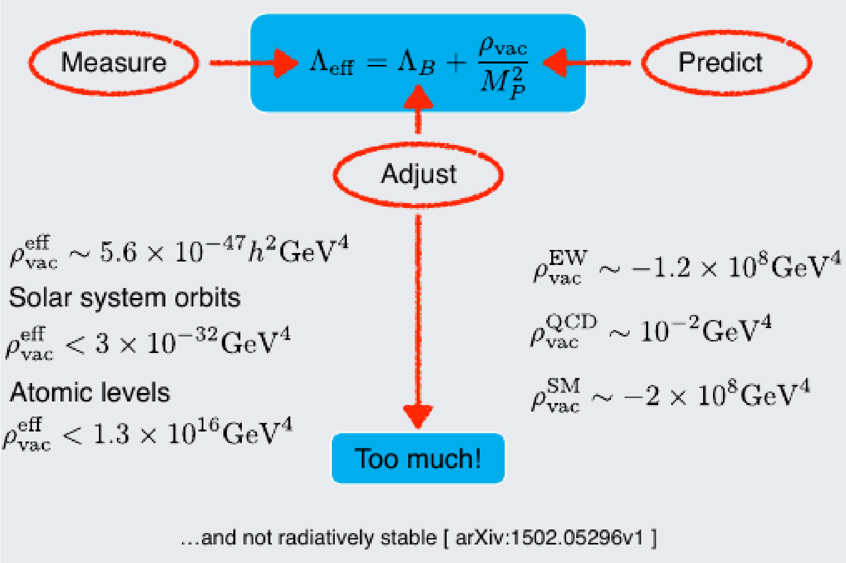
1. The infamous CCP: Cosmological constant fine-tuning issue:

$$\rho_{\Lambda}^{obs} = \frac{\Lambda}{8\pi G} \simeq \Lambda M_P^2 \simeq 10^{-27} Gev^2 \simeq [10^{-3} eV]^2$$

$$\rho^{vac} \simeq \Lambda M_P^4 \simeq 10^{76} Gev^2$$



2. Cosmological constant behaves as vacuum energy which according to the strong equivalence principle gravitates.



Measure
$\Lambda_{\rm eff} = \Lambda_B + \frac{\rho_{\rm vac}}{M_P^2}$
Predict
Adjust
$\rho_{\rm vac}^{\rm eff} \sim 5.6 \times 10^{-47} h^2 \mathrm{GeV}^4$
Solar system orbits
$\rho_{\rm vac}^{\rm eff} < 3 \times 10^{-32} \mathrm{GeV}^4$
Atomic levels
$\rho_{\rm vac}^{\rm eff} < 1.3 \times 10^{16} \mathrm{GeV}^4$
Too much!
$\rho_{\rm vac}^{\rm EW} \sim -1.2 \times 10^{8} \mathrm{GeV}^4$
$\rho_{\rm vac}^{\rm QCD} \sim 10^{-2} \mathrm{GeV}^4$
$\rho_{\rm vac}^{\rm SM} \sim -2 \times 10^{8} \mathrm{GeV}^4$
...and not radiatively stable [arXiv:1502.05296v1]

We do not know how to do this calculation…

What does the space of solutions look like?

Second face of Lovelock's theorem:
any extension of GR is not going to look like GR…

We do not know how quantum vacuum gravitates…

$$\frac{\Lambda}{M_P^2} \sim 10^{-120} \quad \text{but} \quad \frac{\Lambda}{H_0^2} \sim 1$$

Cosmological scales

1. Cosmological scales -a perfect scenario to test gravity beyond the solar system, galaxy, etc.
2. Gravity is the only effect determining cosmological dynamics. So, it should be simple?
3. Simpler (in theory) to model than gravity high density and small scales such a BH and its enviroment.
4. IR modifications can give rise to distinct cosmic expansion history or match ΛCDM (and have other distinguishing effects).

Cornerstone Principles

1. Equivalence Principle (EP): The laws of physics should not depend on the observer's state of motion.
2. No gravity = Special Relativity
3. Different EPs: With gravity, several choices as to how strictly you want to enforce the EP:
   - Weak Equivalence Principle (WEP)
   - Strong Equivalence Principle (SEP)

Weak Equivalence Principle-I

1. **WEP**: All uncharged, freely falling test particles follow the same trajectories, once an initial position and velocity have been prescribed, independent of composition.
2. The two different definitions of mass (i.e. inertial, gravitational) are equivalent; bodies of different constitution feel the same acceleration.
3. No need for different mass-definitions like Newton:
   - (a) force acting on a body depends on its gravitational mass
   - (b) but the reaction on this force depends on the bodys inertial mass.
4. The test mass cancels on LHS and RHS of N2L

$$m_i a = \frac{GMm_g}{r^2} \to a = \frac{GM}{r^2}$$

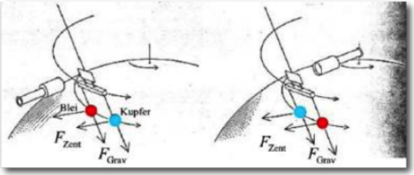
5. Weak = test body's self-gravity and back-reaction on gravitational field are negligible.

Weak Equivalence Principle-II

1. **In GR, all bodies feel the same acceleration because their motion is determined by the very same space-time around them.**
2. WEP test traditionally interpreted in Newtonian terms as specified by the Eotvos is parameter [comparing acceleration of test bodies with different composition]

$$\eta = 2\frac{|a_1 - a_2|}{a_1 + a_2} = 2\frac{|(\frac{m_g}{m_i})_1 - (\frac{m_g}{m_i})_2|}{(\frac{m_g}{m_i})_1 + (\frac{m_g}{m_i})_2} < 4 \times 10^{-13}.$$

Eotvos' experiment



- measurement using torsion balance
- two test-masses of different composition with equal weight $\rightarrow$ equal gravitational mass
- a net torque will show up if the equivalence principle is violated
- in term of the Eötvös-parameter

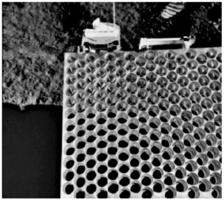
$$\eta = 2\frac{|a_1 - a_2|}{|a_1 + a_2|} \approx 10^{-9}$$

Experiments to test WEP

1. Lunar Laser Ranging (LLR):By using laser shootings to the Moon, one can accurately measure the distance from Earth and to detect possible shifts (initially suggested by Newton).If the distance Earth-Moon is known with 2cm accuracy, one can compare the measured orbit with the theoretical orbit. In this way they have been able to validate the EP with a relative accuracy better than 10^{-12}.[arXiv:1203.2150]
2. MICROSCOPE: Test of equivalence principle with an accuracy of $\eta = 10^{-15}$[Phys. Rev. Lett. 119, 231101 (2017)].
3. STEP: To investigate the Equivalence of inertia and passive gravitational mass. Two masses are in free fall in an orbit around the Earth, and if there is a violation of the EP they tend to follow slightly different orbits. The orbiting masses fall all the way around the Earth and never strike the ground, so that any small difference in the rate of fall can build a large displacement. An accuracy of $\eta = 10^{-18}$ [Phys. Rev. Lett. 117, 071103, 2016]

LLR experiment

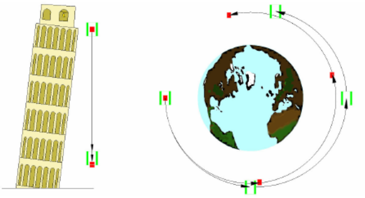
- By using laser shootings to the Moon, one can accurately measure the distance from Earth and to detect possible shifts (initially suggested by Newton).
- During the 1st lunar landing (1969 - Apollo 11) a reflector was deposited on the Moon. In total 5 reflectors, deposited by robotized Soviet missions and by US astronauts (1969-73).
- If the distance Earth-Moon is known with 2cm accuracy, one can compare the measured orbit with the theoretical orbit. In this way they have been able to validate the EP with a relative accuracy better than 10^{-12}.
- After 2006, a new station of laser telemetry in construction in New Mexico (Ranging Laser APOLLO) will provide Earth-Moon distances with an accuracy of a few millimetres. The integration of the signal on several years will enable a relative precision of 10^{-13} for the test of the principle of equivalence.



STEP experiment

- STEP is a joint European-U.S. space program to investigate the Equivalence of inertia and passive gravitational mass. *The STEP experiment is conceptually a modern version of Galileo's Free-Fall Experiment.*
- STEP will advance the sensitivity of EP tests by 5 or 6 orders of magnitude, into regions where the principle may break down.
- A violation of EP at any level would have significant consequences for modern gravitational theory.

- In STEP, the masses are in free fall in an orbit around the Earth, and if there is a violation of the EP they tend to follow slightly different orbits. The orbiting masses fall all the way around the Earth and never strike the ground, so that any small difference in the rate of fall can build a large displacement.
- SQUID magnetometer circuits are used to measure displacements as small as 10^{-13} cm.



Strong Equivalence Principles

1. Definition SEP 1: The WEP is valid for massive gravitating objects as well as test particles, and in all freely falling frames one recovers (locally, and up to tidal gravitational forces) the same special relativistic physics, independent of position or velocity.
2. Definition SEP 2: *Locally and at any point of spacetime physics is that of special relativity (Lorentz invariant) and is not affected by the presence of a gravitational field*
3. We can quantify the notion of locally: the spatial size D of the local inertial frame is much smaller that the typical lengthscale L of the gravitational field, $R_{\mu\nu\kappa\epsilon} \propto 1/L^2$. Hence, in a region $D \ll L$ the field is almost homogeneous and can be cancelled by inertial forces.
4. The SEP implies the WEP: the behavior of freely falling test-bodies in a gravitational field is locally (i.e. in a local inertial frame) indistinguishable from that of a free body. Since the latter is universal, also freely falling bodies should behave in a universal way.

Einstein Equivalence Principles

1. The Einstein equivalence principle (EEP) is a more powerful and far-reaching concept:
2. WEP is valid.
3. The outcome of any local non-gravitational experiment is independent of the velocity of the freely-falling reference frame in which it is performed - local Lorentz invariance (LLI).
4. The outcome of any local non-gravitational experiment is independent of where and when in the universe it is performed - Local Position Invariance (LPI).
5. Violated by some theories in which vector or tensor fields introduce a preferred frame.
6. Most theories considered satisfy this principle.
7. **EEP**:Laws of physics are the same for any observer following a test body free-falling observer.

All the equivalence principles

1. Newton's equivalence principle (NEP): In the Newtonian limit, the inertial and gravitational masses of a body are equals.
2. Weak Equivalence Principle (WEP)
    - The local effects of motion in a curved spacetime (gravitation) are indistinguishable from those of an accelerated observer in flat spacetime, without exception.
    - Test particles with negligible self-gravity behave, in a gravitational field, independently of their properties. ii-[Empiric law of universality of free fall-a lively relic of Galilei's grand achievements]
    - Verify that the equivalence of gravitational mass and inertial mass.
3. The above principles presented so far all refer only to the mechanical behaviour of particles. The following statement covers instead all non-gravitational physics.
4. Strong equivalence principle (SEP):
    - The gravitational motion of a small test body depends only on its initial position in spacetime and velocity, and not on its consitution. (WEP)
    - -The outcome of any local experiment (gravitational or not) in a freely falling laboratory is independent of the velocity of the

Lessons from the equivalence principle

1. The principle of equivalence has great power.
2. One can generalise all the spacial relativistic laws of physics to curved space-time, and the curvature not needs to be small.
3. The laws of physics, written in component form, change on passage from flat space-time to curved space-time by a mere replacement of all commas by semicolons.
4. In SR $\partial_\mu T^{\mu\nu} = 0$ and in GR $\nabla_\mu T^{\mu\nu} = 0$.
5. In a freely falling system, the connection coefficients vanish $\Gamma^{\mu}_{\nu\kappa} = 0$, at the origin of the freely falling system.

Summary

- GR is based on two important principles:
  1. Mach's principle: The presence of matter curves the geometry of spacetime.
  2. Equivalence principle: Locally a free-falling observer and an inertial observer are indistinguishable.
- What are the physical consequences of these statements?
  1. Gravity is a local condition of spacetime
  2. Gravity sees all (including vacuum energy)
  3. In Newtonnian gravity m_I and m_G happen to be the same, in GR it is a founding principle

1. Use an action to describe gravity in 4D space-time

$$S = \int dtL = \int d^4x\sqrt{-g}\mathcal{L}$$

- L is called Langrangian [units of energy]
- $\mathcal{L}$ is called Langrangian density [units of energy density]
- $g = det(g_{\mu\nu})$. It is needed to make L transforms as a scalar under coordinate transformation

2. Gravity and matter components:

$$\mathcal{L} = \mathcal{L}_{grav}[g_{\mu\nu}(x), \partial_\kappa g_{\mu\nu}(x), ...] + \mathcal{L}_m[g_{\mu\nu}(x), \Psi]$$

- $\mathcal{L}_m$: Matter Lagrangian density
- Ψ: Matter fields, coupled to gravity through $g_{\mu\nu}$. The coupling could be linear/nonlinear. Model standard of particle and any other fields.

Einstein field equations-I

1. Einstein-Hilbert action

$$\mathcal{L}_{grav} = \frac{1}{16\pi G}[R - 2\Lambda]$$

2. Einstein's Field equations obtained by varying the with respect to metric

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

3. $G_{\mu\nu}$ is the Einstein tensor and is a function of $g_{\mu\nu}(x),\ \partial_\kappa g_{\mu\nu}(x),\ \partial_\vartheta \partial_\kappa g_{\mu\nu}(x)$:

$$G_{\mu\nu} := R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R,$$

where $R = g^{\mu\nu} R_{\mu\nu}$ and the energy momemtum tensor is

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\partial[\sqrt{-g}\mathcal{L}_m]}{\partial g^{\mu\nu}}.$$

Theoretical consistency: EFE

- In 4 dimensions, consider $L = L(g, \partial g, \partial\partial g)$. Then Lovelock's theorem in D = 4 states that GR with cosmological constant is the unique metric theory given by the Lagrangian:

$$\mathcal{L}_{grav} = \frac{1}{16\pi G}[R - 2\Lambda]$$

  with the following properties:
  - Equations of motion of 2nd order (Ostrogradski no–go theorem 1850: theories with $\mathcal{L} \supset \partial^n g_{\mu\nu}/\partial t$, $n > 2$ are unstable).
  - It corresponds to a symmetric two-tensor, $G_{\mu\nu} + \Lambda g_{\mu\nu}$.
  - It admits Bianchi identities, $\nabla_\mu[G^{\mu\nu} + \Lambda g^{\mu\nu}] = 0$.
- Under these hypotheses GR is the unique massless-tensorial 4 dimensional theory of gravity.

1. We can describe the motion of some objects moving through spacetime as some trajectories.
2. We use a continuous parameter λ that describes every point on the trajectory.
3. In a curved space time, we have the coordinates x_μ and we describe the trajectory using four expressions $x_\mu(\lambda)$
4. The trajectory of any particle is known as its world line and the proper-time for an observer moving along their world-line is defined as: $d\tau = \sqrt{g_{\mu\nu}[x^\kappa(\lambda)]dx^\mu(\lambda)dx^\nu(\lambda)}$.
5. Particles move on geodesics and satisfy the geodesic equation:

$$\ddot{x}^\mu + \Gamma^\mu_{\nu\kappa}\dot{x}^\nu\dot{x}^\kappa = 0, \quad \dot{x}^\nu \equiv \frac{dx^\nu}{d\lambda}$$

$$\Gamma^\mu_{\nu\theta} \equiv \frac{1}{2}g^{\mu\kappa}[g_{\nu\kappa,\theta} + g_{\kappa\theta,\nu} - g_{\nu\theta,\kappa}]$$

Post-Newtonian approximation-I

1. Newtonian Gravity:$[v(t) \ll c, \Phi_N \simeq v^2 \ll c^2]$ $\nabla^2\Phi_N = -4\pi G_N\rho$ and $\ddot{x}^i = -\partial^i\Phi_N$. Keep terms only at order $\mathcal{O}(1) = c^{-2}$
2. Post-Newtonian gravity: We keep the next-to-leading-order terms in v/c, i.e., $\mathcal{O}(n) = c^{-2n}$ with $n > 1$.
3. Solar system objects have $(v/c)^2 \simeq 10^{-5/-10}$ and so post-Newtonian effects are sub-dominant to the Newtonian behaviour by at least three orders-of-magnitude.
4. The TEM of a fluid is $T^{\mu\nu} = \rho[1 + \Pi + P/\rho]u^\mu u^\nu + Pg^{\mu\nu}$ and the perturbed metric can be expanded as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$.
5. Let us find the EFE at 1PN:
   - $T^{\mu\nu} = \rho u^\mu u^\nu$ and $u^\nu = (1, 0)$ so $T^{0i} = T^{ij} = 0$
   - $ds^2 = (-1 + h_{00})dt^2 + [\delta_{ij} + h_{ij}]dx^i dx^j$ with $h_{00} = 2\Phi$ and $h_{ij} = 2\Psi\delta_{ij}$
   - The $0 - 0$ component of EFE leads $\nabla^2\Phi = -4\pi G_N\rho$ and the $i - j$ component leads $\nabla^2\Psi = -4\pi G_N\rho$.
   - Geodesic equation at 1PN leads:$\ddot{x}^i = -\partial^i\Phi$. At this level of approx. the second potential does not appear (Ψ)

Post-Newtonian approximation-II

1. What is the new prediction of general relativity and its consequences?
2. In the nonrelativistic equations of motion for particle does not appear Ψ. We must study the motion of relativistic particles (photons).
3. **Bending of the light**: our first prediction of GR that could not be derived from Newtonian physics.
   - If one calculates the motion of photons to Newtonian order the effect of the curved spacetime is to bend their trajectory by an angle

     $$\theta = \frac{4G_N M}{b},$$

     b is the impact parameter and M the mass of the central object.
   - Suppose that we have an alternate theory of gravity with $\Phi = 2\Phi_N$ but $\Psi = 2\gamma_{PPN}\Psi$, being γ_{PPN} the Eddington light-bending parameter.

     $$\theta = \frac{(1+\gamma_{PPN})}{2}\frac{4G_N M}{b},$$

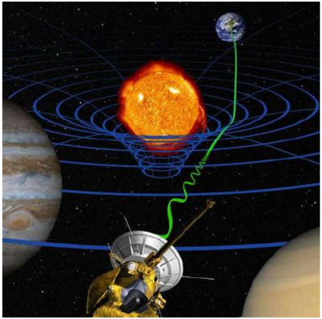
Classical Metric Tests

Deflection of light:

$$\delta\phi_{def} = \left(\frac{1+\gamma}{2}\right)\left(\frac{4GM}{c^2 b}\right)$$

Shapiro Delay:

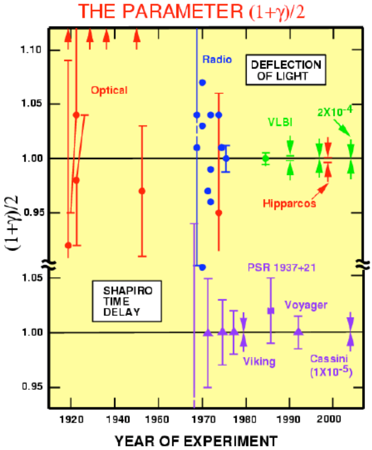
$$\delta t = \left(\frac{1+\gamma}{2}\right)\left(\frac{4GM}{c^3}\right)\left[\log\left(\frac{4r_e r_R}{r_c}\right)+1\right]$$



Mercury Perihelion Precession:

$$\delta\phi_{prec} = \frac{1}{3}(2+2\gamma-\beta)\left(\frac{6\pi GM}{c^2 a(1-\varepsilon^2)}\right)$$

Classical Metric Tests



Deflection of light:

Shapiro Delay:

$$\gamma = 0.99992 \pm 0.00012$$

Mercury Precession:

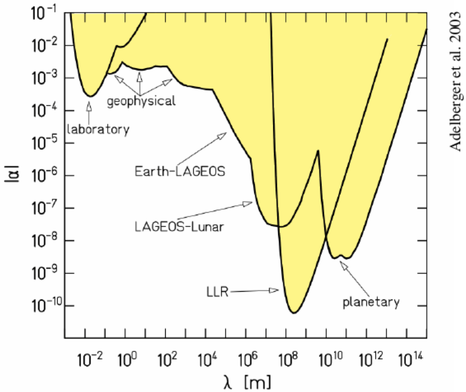
$$\beta = 0.999 \pm 0.003$$

Are we done?

Astrophysical tests of GR

1. Weak Equivalence Principle is tested through different experiments such as LLR, STEP, MICROSCOPE with a precision of $\eta \simeq 10^{-12} - 10^{-15} - 10^{-18}$.
2. The deflection angle of stars due to the Sun $\theta = (0.99992 \pm 0.00023)\theta_{GR}$ where $\theta_{GR} = 1.75''$ is the prediction in GR.
3. The time delation due to the Sun's gravitational field measured by Cassini satellite $\Delta t = (1.00001 \pm 0.00001)\Delta t_{GR}$.
4. The doppler tracking measurements of the Cassini spacecraft gives for the PPN parameter $|\gamma_{PPN} - 1| < (2.1 \pm 2.3) \times 10^{-5}$.

The PPN formalism is not exhaustive



Experimental limits on the parameters of a Yukawa potential

$$V(r) = -G\frac{m_1\, m_2}{r}\left[1 + \alpha\, e^{-r/\lambda}\right]$$