High energy collisions near black holes

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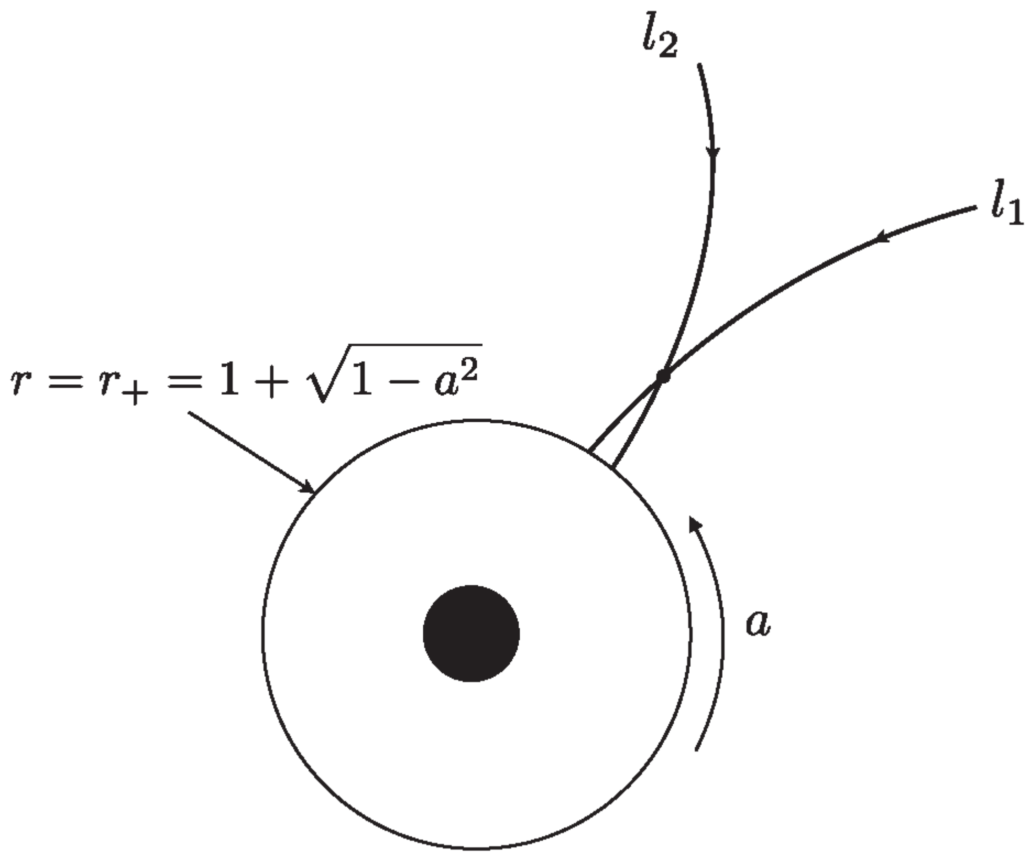
M. Banados, J. Silk, and S. M. West PRL 2009



$$-2(1 + \sqrt{1 + a}) < l < 2(1 + \sqrt{1 - a})$$

Both particles experience blue shift, centre of mass frame is in free fall.

Let two particles collide in the vicinity of a rotating black hole. Then, under certain conditions, an indefinitely growth of the energy in the center of mass frame $E_{c.m}$ becomes possible. This effect was found for (i) extremal horizons and (ii) free particles.
Some objections against the BSW effect were connected with failure of the factors (i), (ii), or both.
However, the effect was found for nonextremal horizons as well (Grib and Pavlov 2010).
BSW
effect arises due to the presence of the horizon as such, no matter how its explicit metric looks like (OZ)
Also, the presence of a force can be, in principle, compatible with the BSW effect. (Tanatarov and OZ) Moreover, sometimes it leads to another version of this effect that is absent without a force (Schwarzschild)

Kinematic explanation

$$E^2_{c.m.} = -(p_1^\mu + p_2^\mu)(p_{1\mu} + p_{2\mu}) = m_1^2 + m_2^2 - 2m_1 m_2 u_1^\mu u_{2\mu}.$$

$$\gamma = -u_1^\mu u_{2\mu} = \frac{1}{\sqrt{1-w^2}}$$

BSW effect occurs if $w \to 1$ w is relative velocity

$$w^2 = 1 - \frac{(1-v_1^2)(1-v_2^2)}{[1-v_1 v_2(\vec{n}_1\vec{n}_2)]^2}$$

$$\vec{v}_1 = v_1 \vec{n}_1$$

The most interesting case: $\boldsymbol{v_1 < 1,}$ $v_2 \to 1$

Collision of rapid particle with slow target

Relative velocity close to c

Kinematics. Acceleration versus decceleration

Naïve expectation: to achieve large $E_{c.m.}$

we must have large velocities and individual energies.

No! The condition of criticality selects slow particle among all possible ones

$$X = E - \omega L = \frac{mN}{\sqrt{1-v^2}},$$

Usual particle rapid, v→1
Critical particle has v<1

Collision between rapid and slow particles

"Almost" any particle is rapid (usual one)

Special subset of slow particles is responsible for large energy in CM frame

Strong gravity ensures BSW effect since it almost "halts" this kind of particles.

Extremal versus nonextremal

Problems with attaining extremality, a=0,998 (Thorne)

Jacobson et al, Berti at al: difficulties in realization

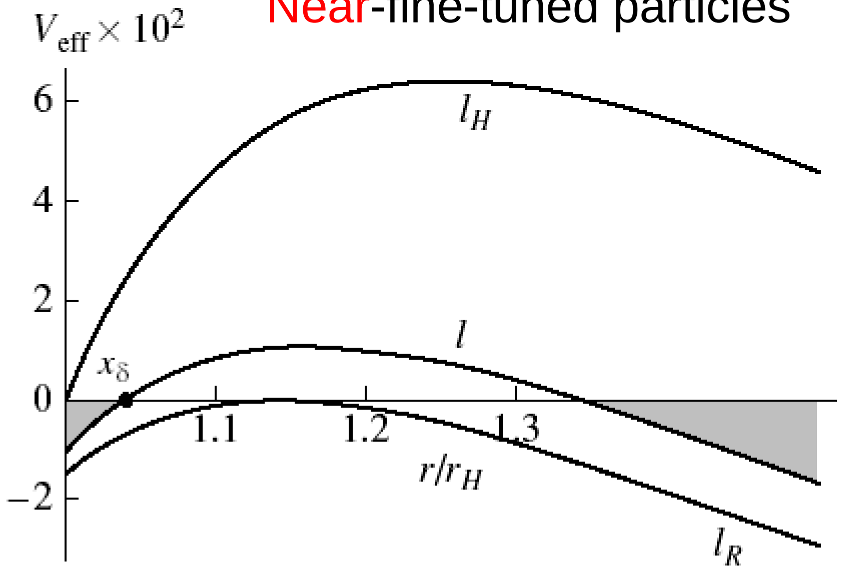
Grib and Pavlov: nonextremal Kerr

Extremal case: collision near horizon

$$E_{c.m.} \approx \frac{m}{\sqrt{\delta}} \sqrt{\frac{2(L_H - L_2)}{1-\sqrt{1-a^2}}} \qquad L_1 = L_{(H)} - \delta$$

$$L_{(H)} = \frac{E}{\omega_H}$$

Near-fine-tuned particles



The effective potential for $A = 0.95$ and $l_R \approx 2.45$, $l = 2.5$, $l_H \approx 2.76$. Allowed zones for $l = 2.5$ are shown by the gray color.

Grib and Pavlov 2010

Extremality is NOT necessary condition for BSW effect

$$l_1 = l_H - \delta \qquad X_H = O(\delta)$$

$$E_{\mathrm{c.m.}} \approx \frac{m}{\sqrt{\delta}} \sqrt{\frac{2(l_H - l_2)}{1 - \sqrt{1 - A^2}}} \qquad E_{c.m.} = O(\delta)$$

Four-velocity

$$u_i^\mu = \frac{l^\mu}{2\alpha_i} + \beta_i N^\mu + s_i^\mu, s_i^\mu = A_i a^\mu + B_i b^\mu$$

Case 1
always

$$-(u_1 u_2) = \frac{1}{2}(\frac{\beta_1}{\alpha_2} + \frac{\beta_2}{\alpha_1}) - (s_1 s_2).$$

$\alpha = 0$

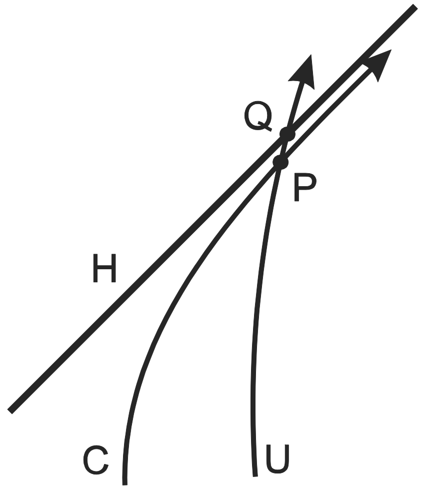
For head-on collision means that particle cannot cross horizon

$$E_{c.m.}^2 = m_1^2 + m_2^2 - 2m_1 m_2 (u_1 u_2)$$

Case 2
Special condition

$$E_{c.m.}^2 = m_1^2 + m_2^2 + m_1 m_2 [\frac{\beta_1}{\alpha_2} + \frac{\beta_2}{\alpha_1} - 2(s_1 s_2)].$$

Geometrically



C is critical particle

U is usual particle

Proper time grows unbound (T. Jacobson, Grib and Pavlov, O. Z.)

$$\tau \sim \left|\ln X\right|$$

Kinematic censorship

The presence of force

Key feature of BSW: existence of critical trajectories
Simplest case
Pure rotating black hole
Critical condition:

$$E - \omega_H L = 0$$

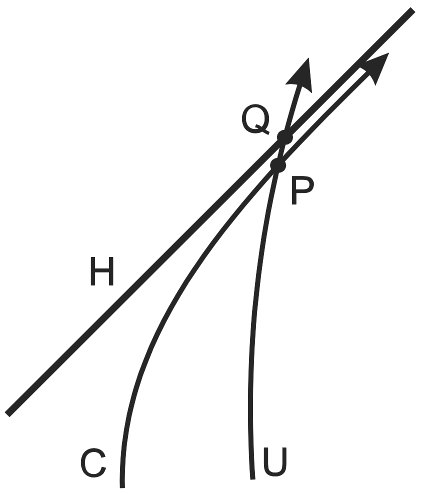
Force due to Coulomb potential:

$$E - \omega_H L - q\varphi_H = 0$$

Force does not destroy effect but changes critical condition

Choice of frame ZAMO (Bardeen et al) is convenient frame
But TWO kinds of such a frame

Usual particle crosses horizon, we choose OZAMO (orbital)



Fine-tuned particle does NOT cross horizon

Relevant frame is FZAMO (free falling)

Acceleration of particles by nonrotating charged black holes

Role of rotation $L_1 = \frac{E_1}{\omega_H}$ If $\omega_H \to 0.$ $L_1 \to \infty$

Angular momentum versus charge

Reissner-Nordstrom

Pure radial motion

$\omega_H = 0$ and $L_1 = L_2 = 0$

particles charged, nongeodesic motion

$$ds^2 = -dt^2 f + \frac{dr^2}{f} + r^2 d\omega^2. \qquad f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$$

$$m^2 \dot{r}^2 = (E - \frac{qQ}{r})^2 - m^2 f. \qquad mu^0 = m\dot{t} = \frac{1}{f}(E - \frac{qQ}{r}),$$

$$X_i = E_i - \frac{q_i Q}{r}, Z_i = \sqrt{X_i^2 - m^2 f}.$$

$$E_{cm}^2 = m^2{}_1 + m^2{}_2 + 2\frac{X_1 X_2 - Z_1 Z_2}{f}$$

Rotating BH	Static charged BH	1 critical + 1 usual
ω	Q	$E^2_{c.m.} \sim N^{-1}$
L	q	

Motion in Schwarzschild metric under action of force of unspecified nature

$$ds^2 = -N^2 dt^2 + N^{-2} dt^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2)$$

Force depends on r

$$mu^t = \frac{X}{N^2},$$

$$mu^r = \sigma P, \qquad E = |m\beta(r_+)| \cdot \overline{\qquad - m^2 N^2} \geq 0$$

$$X = m\beta(r) + E. \qquad \beta = -\delta \int_r^\infty dr' a(r'), \qquad \delta = \pm 1$$

$$\gamma = \frac{X_1 X_2 - \sigma_1 \sigma_2 P_1 P_2}{m_1 m_2 N^2}.$$

Critical trajectory:

$$X(r_h) = 0 \qquad X = m\beta(r) + E.$$

$$\beta < 0 \qquad E = |m\beta(r_+)|\,.$$

BSW versus Penrose $X > 0$

If $\beta < 0$ No states with E<0, particle cannot be product of decay in Penrose process but can contribute to BSW

If $\beta > 0$ E<0 is possible but it is not critical, X=0 impossible

General approach. FOUR-VELOCITIES AND CLASSIFICATION OF TRAJECTORIES

$$ds^2 = -N^2 dt^2 + g_{\varphi\varphi}(dt - \omega d\varphi)^2 + \frac{dr^2}{A} + g_{\theta\theta} d\theta^2.$$

$$u^\mu = \left(\frac{\mathcal{X}}{N^2}, \frac{\mathcal{L}}{g_\varphi} + \frac{\omega \mathcal{X}}{N^2}, \sigma \frac{\sqrt{A}}{N} P, 0 \right) \qquad \sigma = \pm 1$$

$$P = \sqrt{\mathcal{X}^2 - N^2 \left(1 + \frac{\mathcal{L}^2}{g_{\varphi\varphi}} \right)}, \qquad \mathcal{X} = \varepsilon - \omega \mathcal{L},$$

Near-horizon limit: $N \to 0,\ A \to 0$

$$u^r \approx \frac{\sqrt{A}}{N}$$ usual

If $|u^r|$

changes slower than $\sqrt{A}$ subcritical

but faster than $\dfrac{\sqrt{A}}{N}$

if $|u^r| \sim \sqrt{A}$ critical

if faster than $\sqrt{A}$ ultracritical

$$u^t \approx \frac{1}{N^2} \text{ for usual particles,}$$

$$u^t \approx \frac{u^r}{N\sqrt{A}} \text{ for subcritical ones,}$$

$$u^t \approx \frac{1}{N} \text{ for critical and ultracritical.}$$

Near-horizon expansion of metric

$$N^2 = \kappa_p v^p + o(v^p), \qquad A = A_q v^q + o(v^q),$$

$$\omega = \omega_H + \hat{\omega}_k v^k + \cdots + \hat{\omega}_{l-1} v^{l-1} + \omega_l(\theta) v^l + o(v^l),$$

$$g_a = g_{aH} + g_{a1} v + o(v).$$

$$u^r = (u^r)_c v^c + o(v^c). \qquad v = r - r_h$$

$$\mathcal{X} \approx \mathcal{X}_s v^s,$$

Our goal. From simplest picture of usual and critical particles to pass to general picture characterized by p, q, k, l and force

Forces

$$a_o^{(t)} = (a_o^{(t)})_{n_0} v^{n_0} + o(v^{n_0}),$$
$$a_o^{(r)} = (a_o^{(r)})_{n_1} v^{n_1} + o(v^{n_1}),$$
$$a_o^{(\varphi)} = (a_o^{(\varphi)})_{n_2} v^{n_2} + o(v^{n_2})$$

$$v = r - r_h$$

p, q, k, l and n_0, n_1, n_2

For subcritical particles

$$\mathcal{X} \approx \frac{N}{\sqrt{A}} |u^r| \to s = \frac{p-q}{2} + c$$

For critical and ultracritical particles

$$s = \frac{p}{2}$$

for ultracritical particles, there exists another restriction

$$(u^r)^2 = \frac{A}{N^2}\left(\mathcal{X}^2 - N^2\left(1 + \frac{\mathcal{L}^2}{g_{\varphi\varphi}}\right)\right) \sim v^{2c},$$

where $c > \frac{q}{2}$.

$$\mathcal{X}^2 = N^2\left(1 + \frac{\mathcal{L}^2}{g_{\varphi\varphi}}\right) \text{plus } v^{2c+p-q} \text{ terms.}$$

$$P = \sqrt{\mathcal{X}^2 - N^2\left(1 + \frac{\mathcal{L}^2}{g_{\varphi\varphi}}\right)},$$

$$u_O^\varphi = (u_O^\varphi)_H + (u_O^\varphi)_b v^b + o(v^b).$$

$$\mathcal{L} \approx \sqrt{g_{\varphi\varphi}}(u_O^\varphi)_H + \sqrt{g_{\varphi\varphi}}(u_O^\varphi)_b \nu^b.$$

$$u^t \sim v^{-\beta},$$

$$\tau \sim v^{-\alpha} \qquad \alpha = c - 1$$

Energy of collision

$$E^2{}_{c.m.} = m^2{}_1 + m^2{}_2 + 2m_1 m_2 \gamma$$

$$\gamma = -u_{1\mu} u_2{}^{\mu}$$

$$\gamma = \frac{\mathcal{X}_1 \mathcal{X}_2 - P_1 P_2}{N^2} - \frac{\mathcal{L}_1 \mathcal{L}_2}{g_{\varphi\varphi}}.$$

TABLE I. Characteristics of the near-horizon behavior of u^r, u^t, $\mathcal{X}$, and the proper time τ. Here, the proper time changes as $\tau \sim v^{-\alpha}$. The value $\alpha = 0$ means that the proper time logarithmically diverges $\tau \sim |\ln v|$.

	Type	c	β	s	$\alpha = c - 1$
1	Usual	$\frac{q-p}{2}$	p	0	$\frac{q-p-2}{2}$
2	Subcritical	$\frac{q-p}{2} < c < \frac{q}{2}$	$\frac{p+q}{2} - c$	$\frac{p-q}{2} + c,\ 0 < s < \frac{p}{2}$	$\frac{q-p-2}{2} < \alpha < \frac{q-2}{2}$
3	Critical	$\frac{q}{2}$	$\frac{p}{2}$	$\frac{p}{2}$	$\frac{q-2}{2}$
4	Ultracritical	$c > \frac{q}{2}$	$\frac{p}{2}$	$\frac{p}{2}$	$\alpha > \frac{q-2}{2}$

TABLE II. The possibility of BSW phenomenon for different types of particles. D means that the gamma factor diverges, and R means that it is regular.

	First particle	Second particle	d	γ
1	Usual	Usual	0	R
2	Usual	Subcritical	s_2	D
3	Subcritical	Subcritical	$\|s_1 - s_2\|$	D if $s_1 \neq s_2$ R if $s_1 = s_2$
4	Usual or subcritical	Critical or ultracritical	$p/2 - s_1$	D
5	Critical or ultracritical	Critical or ultracritical	0	R

TABLE III. Behavior of the angular component of acceleration.

	Type of trajectory	n_2
1	Usual	$\frac{q-p}{2} + b - 1$
2	Subcritical	$\frac{q-p}{2} + b - 1 < n_2 < b - 1 + \frac{q}{2}$
3	Critical	$\frac{q}{2} + b - 1$
4	Ultracritical	$n_2 > \frac{q}{2} + b - 1$

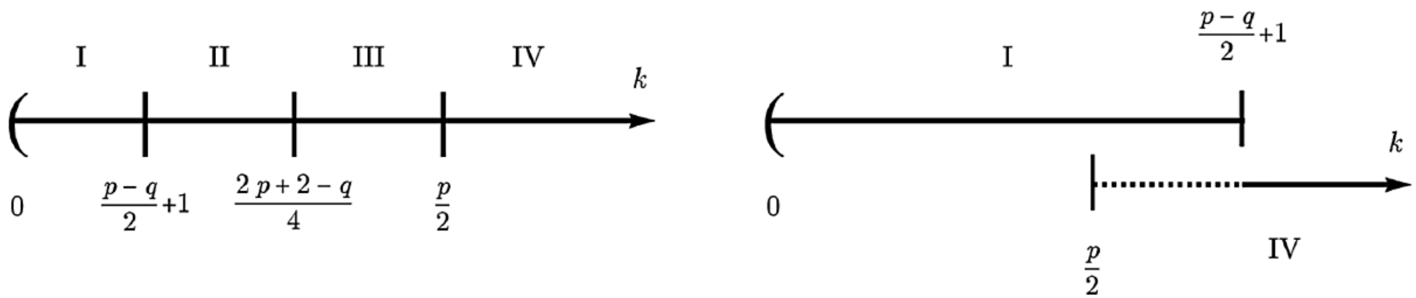


FIG. 1. Relevant regions of k. The left panel corresponds to $q > 2$. The right panel corresponds to $q \leq 2$, where regions II and III do not exist anymore and also points at regions' I and IV overlap. We define points on intersection to belong to region I (for explanation, see Sec. VII E).

$$\omega = \omega_H + \omega_k v^k + \ldots$$

TABLE IV. Classification of near-horizon trajectories for different k regions for $q > 2$ (ultraextremal horizon). The fourth solution in (64) is not presented in this table. Definitions of different k regions are given in Fig. 1.

	k Region	n_1 Range	c	Type of trajectory
		Stationary metric		
1	I	For any type of trajectory, n_1 is negative (forces diverge)		
2	II	$2k + \frac{q-2-2p}{2} < n_1 < \frac{q-p}{2} - 1 + k$	$n_1 + 1 + p/2 - k$	Subcritical
		$n_1 = \frac{q-p}{2} - 1 + k$	$q/2$	Critical
		$n_1 = \frac{q-p}{2} - 1 + k$ and (28)	Any $c > \frac{q}{2}$	Ultracritical
3	III	$\max(0, \frac{q-2-2p}{2}) < n_1 \leq 2k + \frac{q-2-2p}{2}$	$\frac{2n_1+2+q}{4}$	Subcritical
		$2k + \frac{q-2-2p}{2} < n_1 < \frac{q-p}{2} - 1 + k$	$n_1 + 1 + p/2 - k$	Subcritical
		$n_1 = \frac{q-p}{2} - 1 + k$	$q/2$	Critical
		$n_1 = \frac{q-p}{2} - 1 + k$ and (28)	Any $c > \frac{q}{2}$	Ultracritical
4	IV	$\max(0, \frac{q-2-2p}{2}) < n_1 < \frac{q-2}{2}$	$\frac{2n_1+2+q}{4}$	Subcritical
		$n_1 = \frac{q-2}{2}$	$q/2$	Critical
		$n_1 = \frac{q-2}{2}$ and (28)	Any $c > \frac{q}{2}$	Ultracritical
		Static metric		
5	$k = 0$	Same results as in IV for stationary metric		

TABLE V. Classification of near-horizon trajectories for different k regions for $q = 2$ (extemal horizon). Regions II and III in this case are absent, so they are not presented in this table. The fourth solution in (64) is also not presented in this table. Definitions of different k regions are given in Fig. 1.

	k Region	n_1 Range	c	Type of trajectory
		Stationary metric		
1	I	For any type of trajectory, n_1 is negative (forces diverge)		
2	IV	$n_1 = 0$	1	Critical
		$n_1 = 0$ and (28)	Any $c > 1$	Ultracritical
		Static metric		
3	$k = 0$	Same results as in IV for stationary metric		

TABLE VI. Classification of near-horizon trajectories for different k regions for $q < 2$ (nonextremal horizon). Regions II and III in this case are absent, so they are not presented in this table. The fourth solution in (64) is also not presented in this table. Definitions of different k regions are given in Fig. 1.

	k Region	n_1 Range	c	Type of trajectory
		Stationary metric		
1	I and IV	For any type of trajectory, n_1 is negative (forces diverge)		
		Static metric		
2	$k = 0$	For any type of trajectory, n_1 is negative (forces diverge)		

TABLE VII. Possible k regions depending on p and q. Definitions of different k regions are given in Fig. 1.

Condition for q	Possible k regions
$q \leq 2$	I and IV
$2 < q < p + 2$	I, II, III, and IV
$p + 2 \leq q < 2p + 2$	II, III, and IV
$q \geq 2p + 2$	III and IV

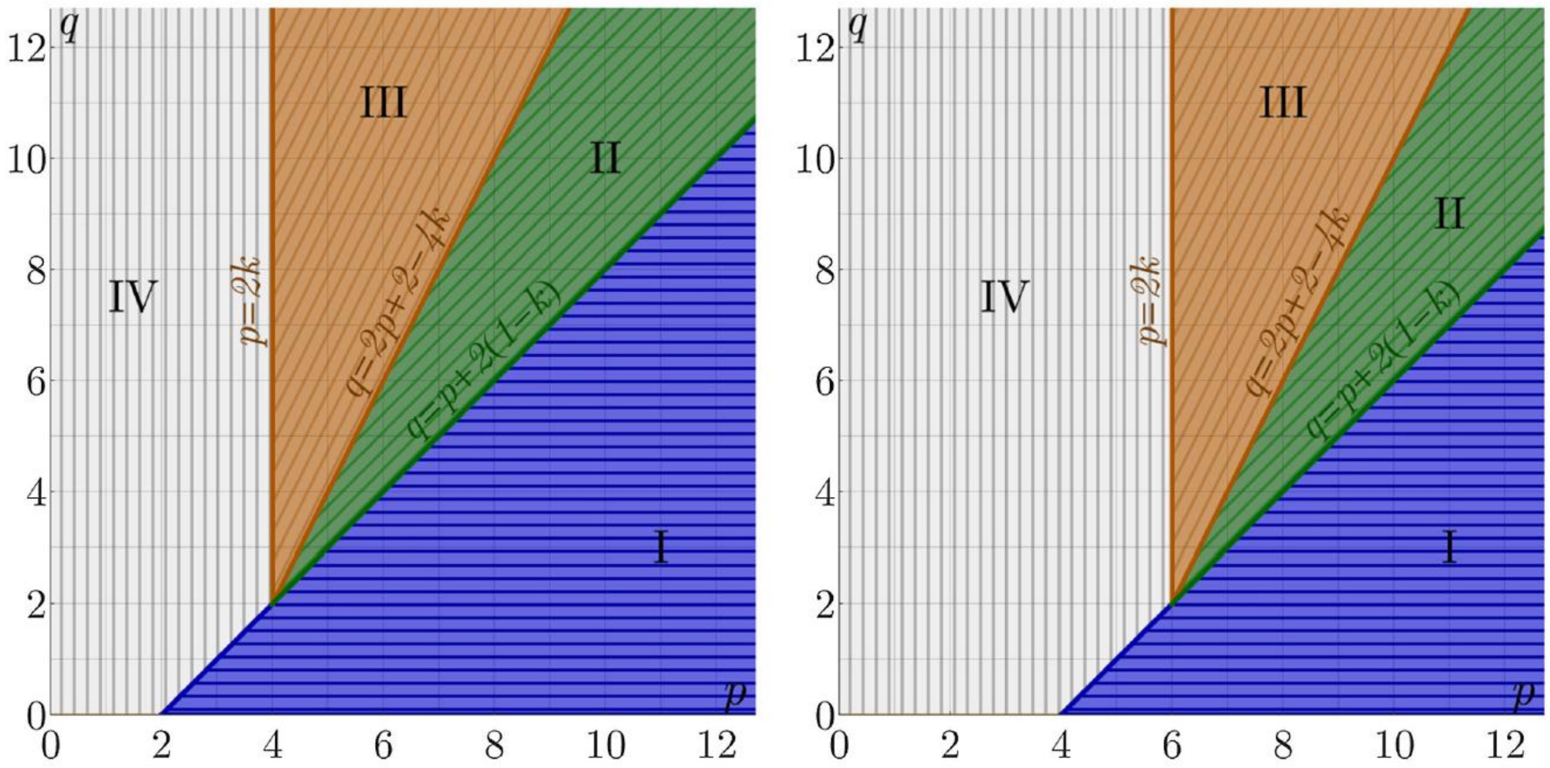


FIG. 2. Plot showing in which region lies k for all types of horizons, depending on p and q. The left panel is drawn for $k = 2$, and the right panel for $k = 3$. Different k regions are represented with different colors: IV (gray), III (orange), II (green), I (blue).

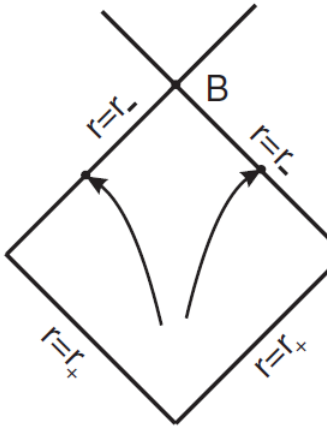
TABLE VIII. Classification of cases when forces are finite for different types of horizons and trajectories.

	Type of horizon	Type of trajectory	Region of k
1	Nonextremal	All types	Absent
2	Extremal	Subcritical	Absent
		Critical	$k \geq p/2$ or $k = 0$
		Ultracritical	
3	Ultraextremal	Subcritical	$k \geq \frac{p-q}{2} + 1$ or $k = 0$
		Critical	
		Ultracritical	

PARTICLES WITH FINITE PROPER TIME: KINEMATIC CENSORSHIP PRESERVED

Extremal BH: $$\tau \sim |\ln v|$$

More tricky case: collision near inner nonextremal horizon:



Objections against BSW effects
(Jacobson and Sotiriou, Berti et al

1) No extremal BHs in nature, maximum a =0.998 (Thorne)
2) Infinite proper time
3) Backreaction due to gravitational radiation

Now: arbitrary p, q + force

All trajectories with a finite proper time and infinite E_cm either experience infinite force, or the horizon
fails to be regular.

Near-horizon properties of trajectories with finite force relevant for Bañados-Silk-West effect

Forward-in-time condition $\frac{dt}{d\tau} > 0 \quad \chi > 0 \quad r > r_h$

$$\chi = \varepsilon - \omega L$$

BSW 1 particle fine-tuned $\chi(r_h) = 0$

Force should not spoil particle and make it “bad”

Bad fine-tuned particle: $\frac{d\chi}{dr}(r_h) < 0$

Free particle: No Penrose effect for a fine-tuned one

$$\chi = \varepsilon - \omega L \qquad \omega'(r) < 0$$

Kerr, Kerr-Newman

Penrose process: particle with negative energy. Then, $\chi > 0$

requires L<0. But this entails

$$\frac{d\chi}{dr} < 0$$

Bad fine-particle!

Fine-tuned particle: L > 0, E>0
Thus no BSW with fine-tuned having E<0

Also, fine-tuned particle cannot appear as result of Penrose process near horizon

Near-horizon expansion of metric

$$N^2 = \kappa_p v^p + o(v^p), \qquad A = A_q v^q + o(v^q),$$

$$\omega = \omega_H + \hat{\omega}_k v^k + \cdots + \hat{\omega}_{l-1} v^{l-1} + \omega_l(\theta) v^l + o(v^l),$$

$$g_a = g_{aH} + g_{a1} v + o(v).$$

$$u^r = (u^r)_c v^c + o(v^c). \qquad v = r - r_h$$

$$\mathcal{X} \approx \mathcal{X}_s v^s,$$

$$a^{(r)}{}_0 \approx \left(a^{(r)}{}_0\right)_H v^{n_1} \qquad a^{(\phi)}{}_0 \approx \left(a^{(\phi)}{}_0\right)_H v^{n_2}$$

$$v = r - r_h \qquad \chi \approx X_s v^s \qquad \omega \approx \omega_H + \omega_k v^k$$

$$\chi \approx X_s v^s$$

$$\frac{d\chi}{dr} = C_1 v^{n_1+p-q/2-s} + C_2 v^{n_2+p-c-s} - L_H k \omega_k v^{k-1}$$

TABLE I. Table showing which conditions have to hold for different terms to be dominant in the expression for $\frac{dX}{dr}$ and under which conditions this quantity is positive. Here, the first term means the term with the radial acceleration in Eq. (12), the second one corresponds to coupling between the angular acceleration and angular momentum there, and the third term does not contain acceleration and arises entirely due to the angular momentum. We denoted $b = \frac{\kappa_p}{s}\left[\frac{(a_0^{(r)})_{n_1}}{\sqrt{A_q}} + \frac{L_H(a_o^{(\phi)})_{n_2}}{\sqrt{g_{\phi H}}(u^r)_c}\right]$.

Dominant terms	Restrictions on n_1, n_2, k	Conditions for $X_s > 0$	Conditions for real X_s
First	$n_1 = 2s + \frac{q}{2} - p - 1$ $n_2 > 2s + c - p - 1$ $k > s$	...	$(a_0^{(r)})_{n_1} > 0$
Second	$n_1 > 2s + \frac{q}{2} - p - 1$ $n_2 = 2s + c - p - 1$ $k > s$	...	$\frac{L_H(a_o^{(\phi)})_{n_2}}{(u^r)_c} > 0$
Third	$n_1 > 2s + \frac{q}{2} - p - 1$ $n_2 > 2s + c - p - 1$ $k = s$	$L_H\omega_k < 0$	...
First and second	$n_1 = 2s + \frac{q}{2} - p - 1$ $n_2 = 2s + c - p - 1$ $k > s$	...	$\frac{(a_0^{(r)})_{n_1}}{\sqrt{A_q}} + \frac{L_H(a_o^{(\phi)})_{n_2}}{\sqrt{g_{\phi H}}(u^r)_c} > 0$
First and third	$n_1 = 2s + \frac{q}{2} - p - 1$ $n_2 > 2s + c - p - 1$ $k = s$	$\frac{(a_0^{(r)})_{n_1}}{\sqrt{A_q}} > 0$ or if $\frac{(a_0^{(r)})_{n_1}}{\sqrt{A_q}} \leq 0$ and $L_H\omega_k < 0$	...
Second and third	$n_1 > 2s + \frac{q}{2} - p - 1$ $n_2 = 2s + c - p - 1$ $k = s$	$\frac{L_H(a_o^{(\phi)})_{n_2}}{\sqrt{g_{\phi H}}(u^r)_c} > 0$ or if $\frac{L_H(a_o^{(\phi)})_{n_2}}{\sqrt{g_{\phi H}}(u^r)_c} \leq 0$ and $L_H\omega_k < 0$	...
First, second, and third	$n_1 = 2s + \frac{q}{2} - p - 1$ $n_2 = 2s + c - p - 1$ $k = s$	$b > 0$ or if $b \leq 0$ and $L_H\omega_k < 0$	...

TABLE II. Table showing how n_1 and n_2 are related for different types of particles and different dominant terms.

	Subcritical	Critical	Ultracritical
Third term	n_1 and n_2 are not related	n_1 and n_2 are not related	n_1 and n_2 are not related
First and third term	n_1 and n_2 are not related	$n_2 > n_1$	$n_2 > n_1$
Second and third term	$n_1 > n_2$	$n_1 > n_2$	n_1 and n_2 are not related
First, second and third term	$n_1 > n_2$	$n_1 = n_2$	$n_2 > n_1$

ANALYSIS FOR DIFFERENT TYPES OF HORIZONS

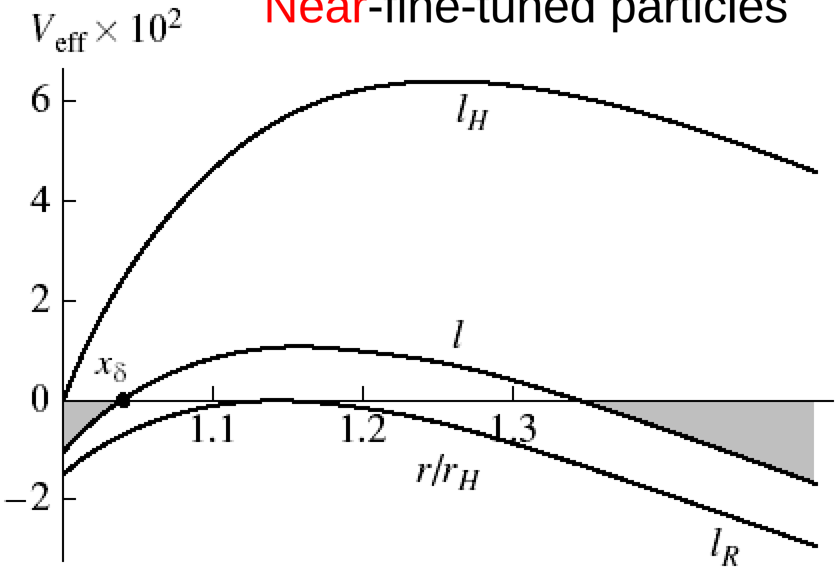
Nonextremal horizon

$k=s<p/2$

Extremal horizon

$q=2$, $p>2$ or $p=2$

Near-fine-tuned particles



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Grib and Pavlov 2010

Extremality is NOT necessary condition for BSW effect

$$l_1 = l_H - \delta \qquad X_H = O(\delta)$$

$$E_{\rm c.m.} \approx \frac{m}{\sqrt{\delta}} \sqrt{\frac{2(l_H - l_2)}{1-\sqrt{1-A^2}}} \qquad E_{c.m.} = O(\delta)$$

Generalization

$$\mathcal{X} = \delta + X_s v^s + o(v^s), \qquad s > 0,$$

TABLE I. Table showing classification of different near-fine-tuned particles.

Condition	Type of particle with nonzero $\delta \ll 1$	Abbreviation
$s < p/2$	Near subcritical	NSC
$s = p/2$	Near critical	NC
$s = p/2$ and (24)	Near ultracritical	NUC
$s > p/2$	Near overcritical	NOC

New type of particles: near overcritical

ABLE II. Table showing behavior of γ factor for $v_c \sim v_{e,t}$ in a case when first particle is fine-tuned and second ear-fine-tuned. Here d is defined by relation $\gamma \sim v_c^{-d}$.

First particle	Second particle	d
U or SC	NSC	$\lvert s_1 - s_2\rvert$ or $\lvert s_1 - r_2\rvert$ if $\delta_2 = -X_s^{(2)} v_c^s + B_r^{(2)}$
C or UC	NSC	$\frac{p}{2} - s_2$ or $\frac{p}{2} - r_2$ if $\delta_2 = -X_s^{(2)} v_c^s + B_r^{(2)} v_c^r$
U or SC	NC, NUC, or NOC	$\frac{p}{2} - s_1$
C or UC	NC, NUC, or NOC	0

ABLE III. Table showing behavior of γ factor for $v_c \sim v_{e,t}$ in a case when both particles are near-fine-tuned. He is defined by relation $\gamma \sim v_c^{-d}$.

First particle	Second particle	d
NSC	NSC	$\lvert s_1 - s_2\rvert$
		$\lvert r_1 - s_2\rvert$ if $\delta_1 = -X_s^{(1)} v_c^s + B_r^{(1)} v_c^r$
		$\lvert s_1 - r_2\rvert$ if $\delta_2 = -X_s^{(2)} v_c^s + B_r^{(2)} v_c^r$
		$\lvert r_1 - r_2\rvert$ if $\delta_{1,2} = -X_s^{(1,2)} v_c^s + B_r^{(1,2)}$
NSC	NC, NUC, or NOC	$\frac{p}{2} - s_1$
NC, NUC, or NOC	NC, NUC, or NOC	0

ABLE IV. Classification of near-horizon trajectories for different k regions for $q > 2$ (ultraextremal horizon
ne fourth solution in (108) is not presented in this table.

k region	n_1 range	s	Type of trajectory
	Stationary metric		
I	For any type of trajectory n_1 is negative (forces diverge)		
II and III	$\max\left(0, \frac{q-p}{2} - 1\right) < n_1 \leq k + \frac{q-p}{2} - 1$	First and second in (108)	NSC
	$n_1 = k + \frac{q-p}{2} - 1$	Second in (108)	NC and NOC
	$n_1 = k + \frac{q-p}{2} - 1$ and (24)	Second in (108)	NUC
IV	$\max\left(0, \frac{q-p}{2} - 1\right) < n_1 < \frac{q-2}{2}$	First in (108)	NSC
	$n_1 = \frac{q-2}{2}$	First in (108)	NC
	$n_1 = \frac{q-2}{2}$ and (24)	First in (108)	NUC
	$\frac{q-2}{2} < n_1 \leq k + \frac{q-p}{2} - 1$	First and second in (108)	NOC
	Static metric		
$k = 0$	Same results as in IV for stationary metric		NSC, NC, and NU
	$n_1 > \frac{q-2}{2}$	Third in (108)	NOC

E V. Classification of near-horizon trajectories for different k regions for $q = 2$ (ultraextremal horizon). The f
is not presented in this table.

k region	n_1 range	s	T
	Stationary metric		
I	For any type of trajectory n_1 is negative (forces diverge)		
IV	$n_1 = 0$	First in (108)	
	$n_1 = 0$ and (24)	First in (108)	
	$0 \leq n_1 \leq k + \frac{q-p}{2} - 1$	First and second in (108)	
	Static metric		
$k = 0$	Same results as in IV for stationary metric		
	$n_1 > 0$	Third in (108)	

LE VI. Classification of near-horizon trajectories for different k regions for $q < 2$ (ultraextremal horizon). The
is not presented in this table.

k region	n_1 range	s
	Stationary metric	
I		For any type of trajectory n_1 is negative (forces diverge)
IV	$0 < n_1 \leq k + \frac{q-p}{2} - 1$	First and second in (108)
	Static metric	
$k = 0$	$0 < n_1$	Third in (108)

TABLE VII. Classification of cases when forces are finite for different types of horizons and trajectories.

	Type of horizon	Type of trajectory	Region of k
1	Nonextremal	NOC	IV or static
2	Extremal	NC, NUC, and NOC	IV or static
3	Ultraextremal	NSC, NC, NUC, and NOC	II, III, IV, or static

TABLE VII. Classification of cases when forces are finite for different types of horizons and trajectories.

	Type of horizon	Type of trajectory	Region of k
1	Nonextremal	NOC	IV or static
2	Extremal	NC, NUC, and NOC	IV or static
3	Ultraextremal	NSC, NC, NUC, and NOC	II, III, IV, or static

Table showing for which particles and which ranges of their motion the BSW phenomenon is possible if forc
es are finite. This table describes all new possible cases when both particles are near-fine-tuned or one is fine-tun
cle is near-fine-tuned. In near-horizon collisions, near-fine-tuned particles with $v_c \ll v_{t,e}$ behave similar to us
ear-fined particles are similar to fine-tuned ones. The last column displays the equation number describing a co

rizon	First particle's type and range			Second particle's type and range		
l	NOC	$v_c \ll v_t^{(1)}$ U	$\delta_1 \gg v_c^{p/2}$	NOC	$v_c \sim v_t^{(2)}$	$\delta_2 \sim v_c^{p/}$
	NC, NUC, NOC	$v_c \ll v_{e,t}^{(1)}$ U	$\delta_1 \gg v_c^{p/2}$	NC, NUC, NOC	$v_c \sim v_{e,t}^{(2)}$	$\delta_2 \sim v_c^{p/}$
al	NSC	$v_c \ll v_{e,t}^{(1)}$ U	$\delta_1 \gg v_c^s$	NSC	$v_c \sim v_{e,t}^{(2)}$	$\delta_2 \sim v_c^s$
	NSC	$v_c \sim v_{e,t}^{(1)}$	$\delta_1 \sim v_c^s$	NSC	$v_c \sim v_{e,t}^{(2)}$	$\delta_2 \sim v_c^s$
	NSC	$v_c \gg v_{e,t}^{(1)}$ SC	$\delta_1 \ll v_c^s$	NSC	$v_c \sim v_{e,t}^{(2)}$	$\delta_2 \sim v_c^s$
	NSC	$v_c \ll v_{e,t}^{(1)}$ U	$\delta_1 \gg v_c^s$	NC, NUC, NOC	$v_c \sim v_{e,t}^{(2)}$	$\delta_2 \sim v_c^{p/}$
	NSC	$v_c \sim v_{e,t}^{(1)}$	$\delta_1 \sim v_c^s$	NC, NUC, NOC	$v_c \sim v_{e,t}^{(2)}$	$\delta_2 \sim v_c^{p/}$
	NSC	$v_c \sim v_{e,t}^{(1)}$	$\delta_1 \sim v_c^s$	NC, NUC	$v_c \gg v_{e,t}^{(2)}$ C, UC	$\delta_2 \ll v_c^{p}$
	NSC	$v_c \gg v_{e,t}^{(1)}$ SC	$\delta_1 \ll v_c^s$	NC, NUC, NOC	$v_c \sim v_{e,t}^{(2)}$	$\delta_2 \sim v_c^{p/}$
	NC, NUC, NOC	$v_c \ll v_{e,t}^{(1)}$ U	$\delta_1 \gg v_c^{p/2}$	NC, NUC, NOC	$v_c \sim v_{e,t}^{(2)}$	$\delta_2 \sim v_c^{p/}$

X. Table showing for which particles and which ranges of their motion BSW phenomenon is possible if forces
are finite. This table describes all the cases with the BSW effect when usual, fine-tuned, or near-fined particles
umn indicates the line number in Table II from [13] that describes a corresponding case.

horizon	First particle's type and range			Second particle's type and ran		
l	NC, NUC, NOC	$v_c \ll v_{e,t}^{(1)}$	$\delta_1 \gg v_c^{p/2}$	NC, NUC	$v_c \gg v_{e,t}^{(2)}$	δ_2
		U				
		U			C, UC	
remal	NSC, NC, NUC, NOC	$v_c \ll v_{e,t}^{(1)}$	$\delta_1 \gg v_c^{p/2}$	NSC	$v_c \gg v_{e,t}^{(2)}$	δ
		U				
		U			SC	
	NSC, NC, NUC, NOC	$v_c \gg v_{e,t}^{(1)}$	$\delta_1 \ll v_c^{p/2}$	NSC	$v_c \gg v_{e,t}^{(2)}$	δ
		SC				
		SC			SC	
	NSC	$v_c \ll v_{e,t}^{(1)}$	$\delta_1 \gg v_c^{s}$	NC, NUC	$v_c \gg v_{e,t}^{(2)}$	δ_2
		U				
		U			C, UC	
	NSC	$v_c \gg v_{e,t}^{(1)}$	$\delta_1 \ll v_c^{s}$	NC, NUC	$v_c \gg v_{e,t}^{(2)}$	δ_2
		SC				
		SC			C, UC	

Head-on collisions

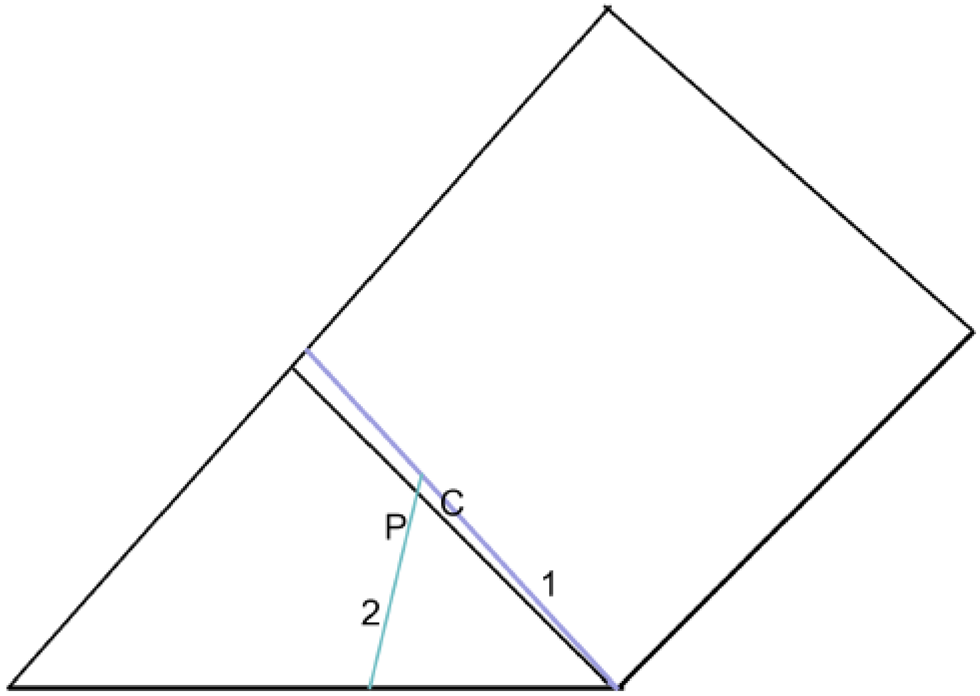


Fig 1. Particle collision in Scenario 1

Particle 1 comes from remote past, particle 2 comes from a white hole crossing the past horizon in point P. Collision occurs near an arbitrary point C of the past horizon.

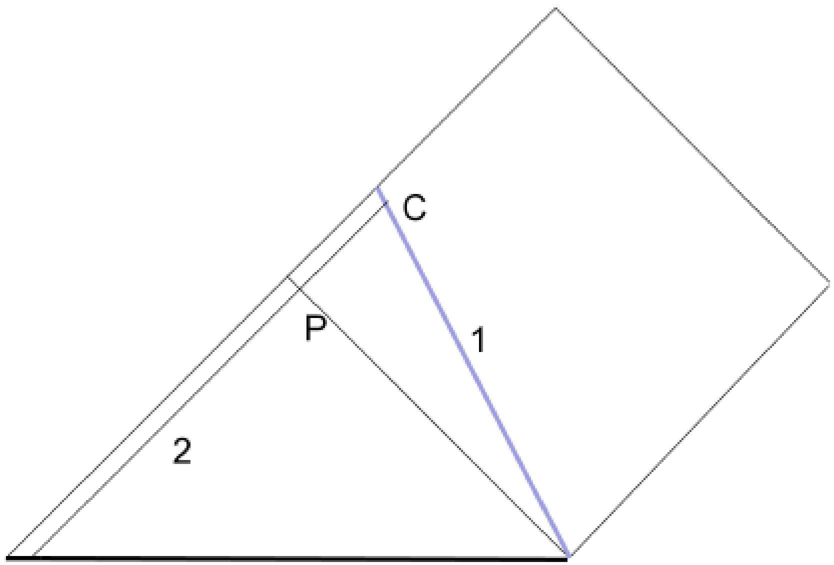


Fig. 2. Particle collisions in Scenario 2

Particle 2 crosses the past horizon in point P near the bifurcation point. Collision occurs near an arbitrary point C of the future horizon.

Head-on collisions

$$\gamma = \frac{X_1 X_2 + P_1 P_2}{N^2}$$

Table 1. Table showing behavior of gamma factor for frontal collision of particles of different types. Here d is defined to be degree of gamma factor's divergence: $\gamma \sim v^{-d}$.

First particle	Second particle	d
Usual	Usual	p
Usual	Subcritical	$p - s_2$
Usual	Critical or ultracritical	$p/2$
Subcritical	Usual	$p - s_1$
Subcritical	Subcritical	$p - s_1 - s_2$
Subcritical	Critical or ultracritical	$p/2 - s_1$
Critical or ultracritical	Usual	$p/2$
Critical or ultracritical	Subcritical	$p/2 - s_2$
Critical or ultracritical	Critical or ultracritical	0

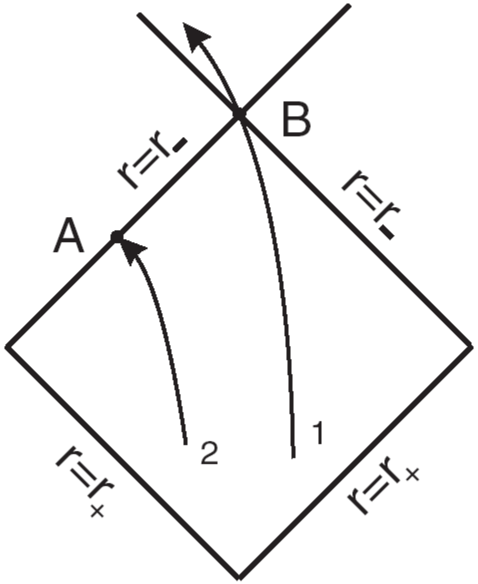
Table 2. Table showing all cases in which the UGEF phenomenon is possible for a nonextremal horizon.

First particle	τ_1	Second particle	BH or WH
Usual	F	Usual, subcritical, critical or ultracritical	WH
Subcritical	F	Usual, subcritical, critical or ultracritical	WH
Critical or ultracricial	F	Usual or subcritical	WH

Notes: In this table the first and the third columns show a type of particles for which BSW phenomenon is possible, the second column shows how a proper time behaves for the first (outgoing) particle (here "*F*" means that a proper time is finite, "*D*" means that it diverges). The last column shows for which spacetimes this phenomenon is possible (here "BH" stands for a black hole, "WH" for a white hole).

Collisions near inner horizon

Again, one of two particle should be critical. Then, the following cases are possible.



Kinematic censorship preserved

Fig. 1. Impossibility of strong version of BSW effect. Critical particle 1 passes through bifurcation point whereas usual one 2 hits left horizon

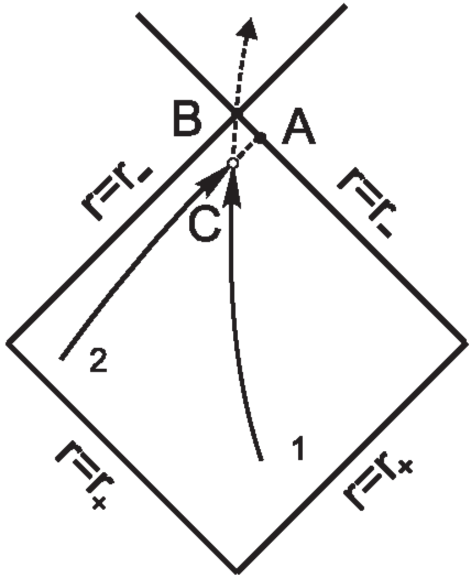
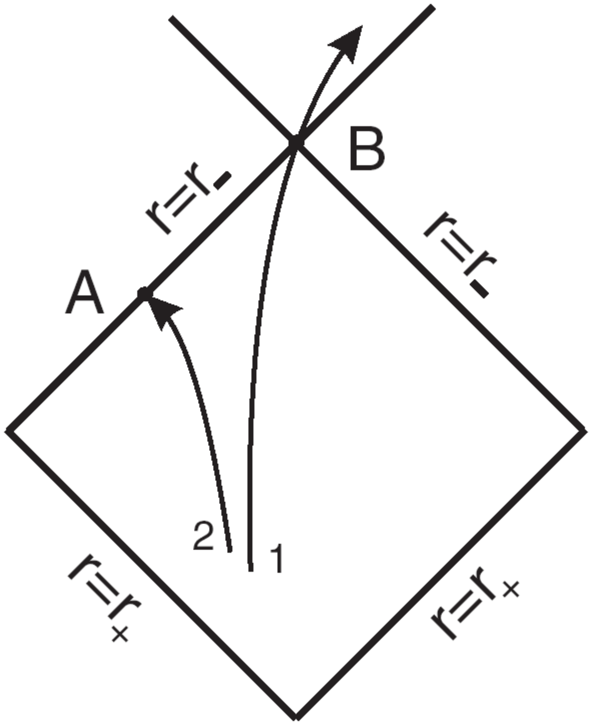


Fig. 2. The weak version of BSW effect. Near-horizon collision between Critical particle 1 and usual one 2.



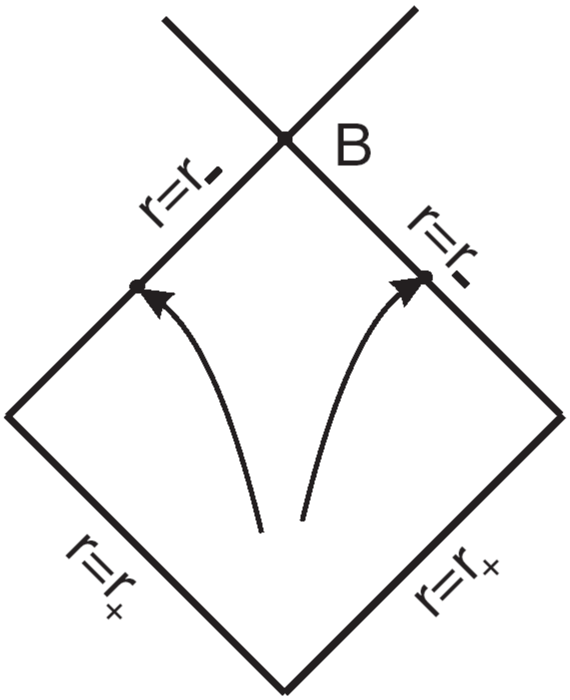
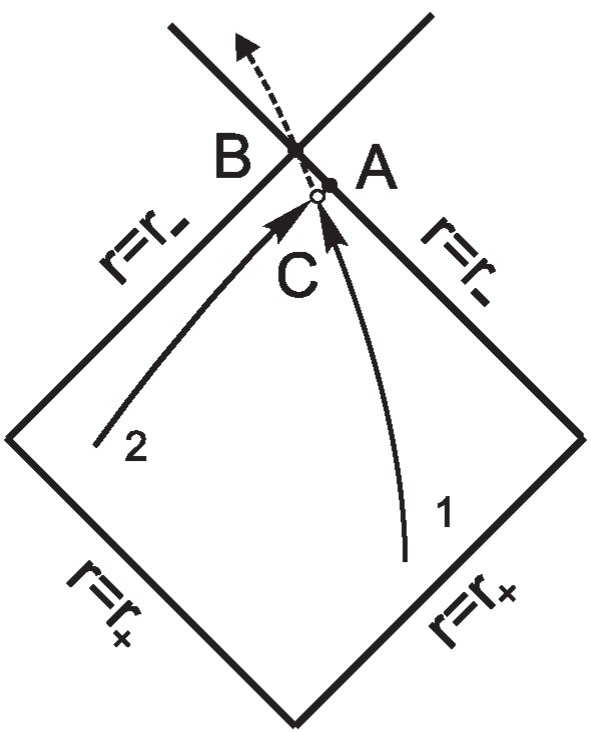


Fig. 4. Impossibility of strong version of PS effect. Two usual particles hit different branches of horizon.

Fig. 3. Impossibility of strong version. Critical particle 1 passes through bifurcation point, whereas
a usual one 2 hits left horizon.

Kinematic censorship



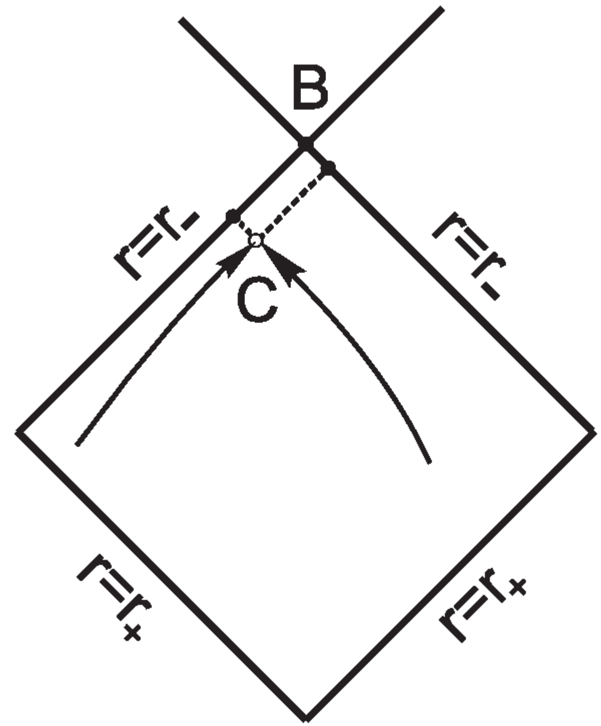


Fig. 5. Weak version of effect. Near-horizon collision between critical particle 1 and usual one 2

Fig. 6. Weak version of effect. Collision between two usual particles near left horizon.

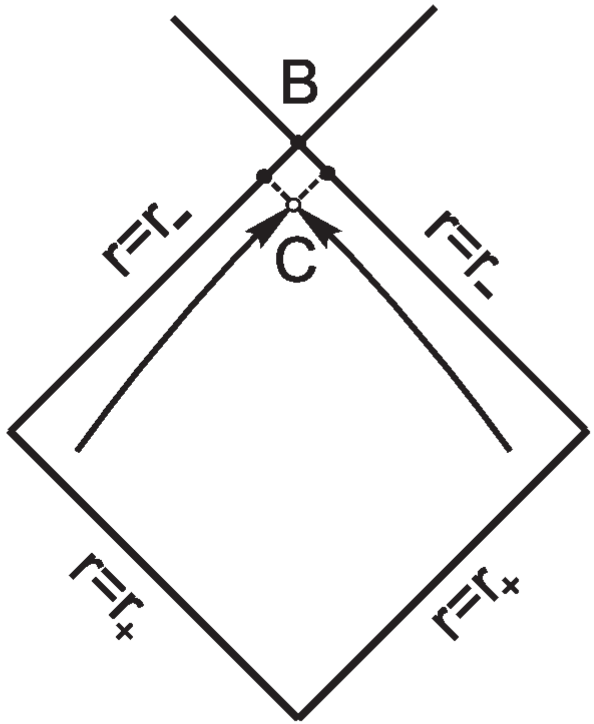


Fig. 7. Weak version of effect. Collision between two usual particles near bifurcation point.

Analog in flat space-time

Minkowski and Milne metrics

$ds^2 = -dt^2 + dx^2.$ $x = \tilde{t}\sinh\tilde{x},$ $t = \tilde{t}\cosh\tilde{x},$

$t < 0, |x| < |t|.$ $ds^2 = -d\tilde{t}^2 + \tilde{t}^2 d\tilde{x}^2.$ $\tilde{t}^2 = t^2 - x^2,$ $\tanh\tilde{x} = \frac{x}{t}.$

right horizon $x = -t$ $\tilde{t} = 0, \tilde{x} = -\infty,$

left one $x = +t$ $\tilde{t} = 0, \tilde{x} = +\infty.$

bifurcation point $x = 0 = t$ $\tilde{t} = 0, |\tilde{x}| < \infty.$

$$X = mu_\mu \xi^\mu \qquad x - Vt = x_0. \qquad \tilde{t} = \frac{X}{\cosh\tilde{x}_0 (V\cosh\tilde{x} - \sinh\tilde{x})} = \frac{x_0}{\sinh\tilde{x} - V\cosh\tilde{x}},$$

$$X = -x_0 E, E = \frac{1}{\sqrt{1-V^2}}.$$

critical if X=0

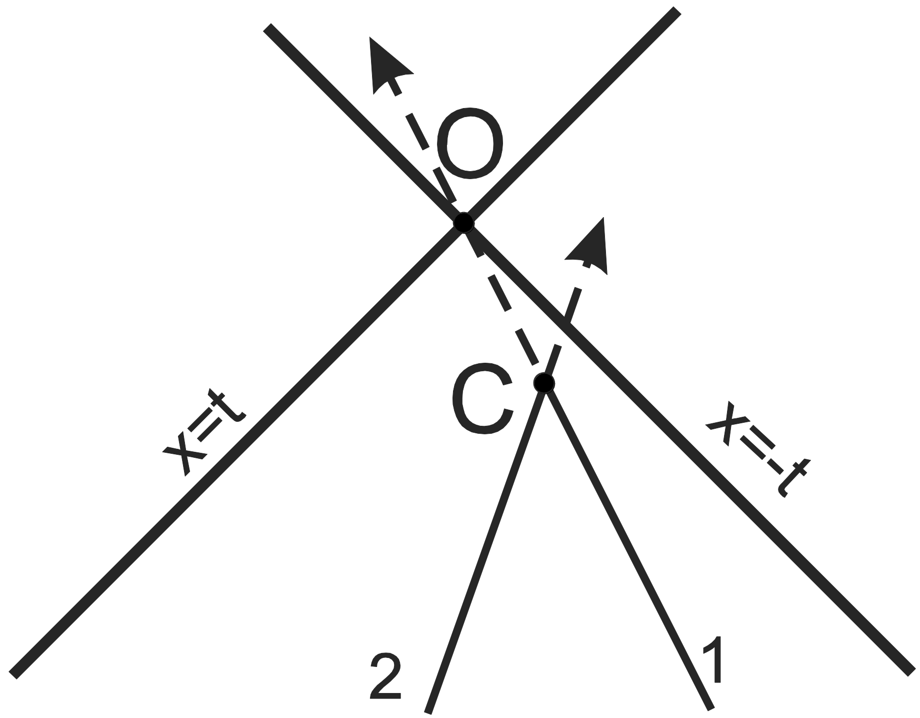
near-critical if X small

usual X=O(1)

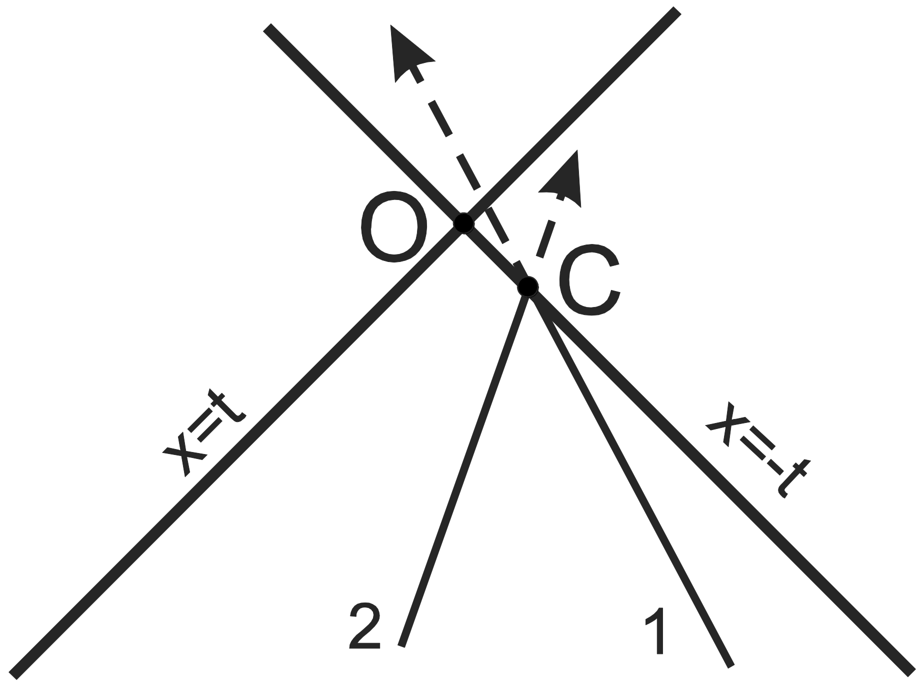
Types of collisions leading to the BSW effect.

Scenario	V_1	V_2	X_1	X_2	β_1	β_2	Location
Aa	$\approx +1$	≈ -1	< 0	> 0	$2\left\|X_1\right\|$	$\sim \tilde{t}_c^2 \ll \beta_1$	Bifurcation point
Ab	≈ -1	$\approx +1$	> 0	< 0	$\sim \tilde{t}_c^2$	$2\left\|X_2\right\| \gg \beta_1$	Bifurcation point
Ba	intermediate	≈ -1	0	> 0	$\sim \left\|\tilde{t}_c\right\|$	$\sim \tilde{t}_c^2 \ll \beta_1$	Bifurcation point
Bb	intermediate	$\approx +1$	0	< 0	$\sim \left\|\tilde{t}_c\right\|$	$2\left\|X_2\right\| \gg \beta_1$	Bifurcation point
Ca	≈ -1	intermediate	0	< 0	$\sim \left\|\tilde{t}_c\right\|$	$2\left\|X_2\right\| \gg \beta_1$	Horizon
Cb	$\approx +1$	intermediate	0	> 0	$\sim \left\|\tilde{t}_c\right\|$	$\sim \tilde{t}_c^2 \ll \beta_1$	Horizon

Similar effects for cosmological horizons



Collision between the critical particle 1 and a usual particle 2 near the horizon



Collision between the near-critical particle 1 and a usual particle 2 on the horizon

Confined Penrose process and black-hole bomb

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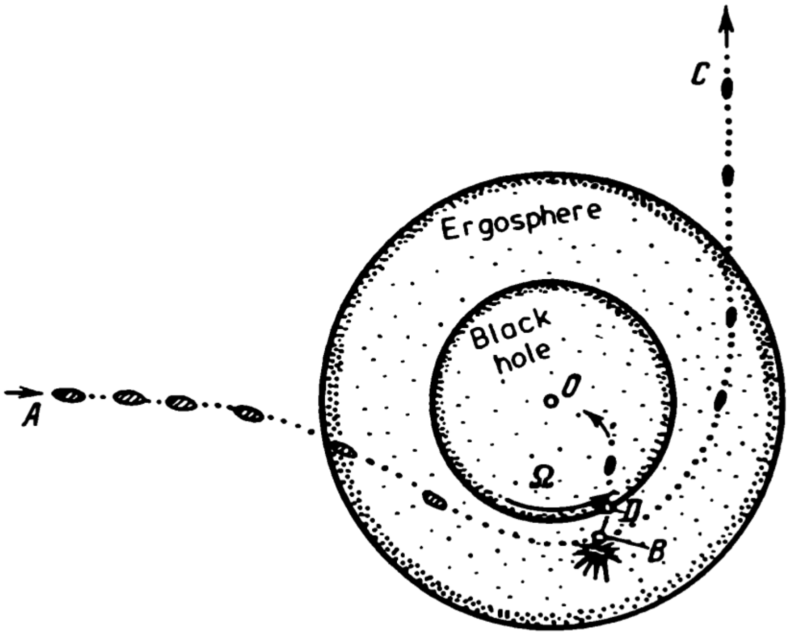
PP in charged nonrotating case, Denardo-Ruffini 1973 results (RN)

$$m\frac{dr}{d\tau} = \sigma P, P = \sqrt{(E - q\varphi)^2 - m^2 N^2}$$

$$Q < M\sqrt{G}$$

$$N^2 = 1 - \frac{2GM}{r} + \frac{Q^2}{r^2}$$

RN black hole



Penrose process

More than 1000 citations   Google: mentiones about 5 960 000 times

Counterpart: amplification of waves from rotating black holes

Zel'dovich explained his reasoning: "A spinning metal sphere emits electromagnetic radiation, and so, similarly, a spinning black hole should emit gravitational waves."

Kip Thorne. Black Holes and Time Warps: Einstein's Outrageous Legacy. 1994. P. 429

Ya. B. Zel’dovich, Generation of waves by a rotating body, JETP Lett. 14, 180 (1971).
Ya. B. Zel’dovich, Amplification of cylindrical electromagnetic waves from a rotating body,
JETP 35, 1085 (1971).

A. A. Starobinskii, Sov. Phys. JETP 37 (1973) 28.
A. A. Starobinskii and S. M. Churilov, Sov. Phys. JETP 38 (1974) 1.

Bomb

Concave mirror around black hole

Zel’dovich 1971
Press and Teukolsky 1972

Convex mirror around black hole

Vilenkin 1978

Amplication of waves

What about particles? Kukubu et al 2021 (with negative result),
No black hole bomb

It does exist in another scenario OZ 2022

PP in charged nonrotating case, Denardo-Ruffini 1973 (RN
Denardo and Treves 1973
Denardo, Hively and Ruffini 1974

$$m\frac{dr}{d\tau}=\sigma P, P=\sqrt{(E-q\varphi)^2-m^2N^2}$$

$$Q\le M\sqrt{G}$$

$$N^2=1-\frac{2GM}{r}+\frac{Q^2}{r^2}$$

RN black hole

$$\varphi=\frac{Q}{r}$$

Boundary of generalized ergoregion: E=0,

$$\left(\frac{qQ}{mr_e}\right)^2=1-\frac{2MG}{r_e}+\frac{Q^2}{r_e^2}$$

Rotating case. Ergoregion: $g_{00}>0$ Properties of geometry

Charged static case: particle's property

$$\frac{dt}{d\tau}=\frac{E-q\varphi}{f}>0$$

$$E-q\varphi>0$$

But E can be negative

Flat space-time limit: $G \to 0$ $r_e \to \frac{qQ}{m}$

No black hole

Gravity disappears but generalized ergoregion remains

N. Dadhich, The Penrose process of energy extraction in electrodynamics. ICTP preprint IC 80/98, 1980.

OZ 2019, 2022 SPP = Penrose process with unbounded efficiency

PP and even SPP

Gravity disappears but generalized ergoregion remains
Simplest collisional process: in flat space-time
Cheshire cat smile

$$ds^2 = -dt^2 f + \frac{dr^2}{f} r^2 d\omega^2 \qquad f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}.$$

Pure radial motion

$$m\dot{t} = \frac{X}{f},$$

$$p^r \equiv m\dot{r} = \sigma P, \qquad P = \sqrt{X^2 - m^2 f}.$$

$$X = E - q\varphi, \qquad \varphi = \frac{Q}{r}.$$

The forward-in-time condition implies $\dot{t} > 0$, whence

$$X > 0$$

$$\varepsilon = \frac{E}{m}, \qquad \tilde{q} = \frac{q}{m}$$

The system can have a turning point r_t, where $P = 0$. Then

$$r_t = \frac{1}{\varepsilon^2 - 1}(\varepsilon\tilde{q}Q - M \pm \sqrt{C}),$$

$$C = (M - \varepsilon\tilde{q}Q)^2 + (1 - \tilde{q}^2)Q^2(\varepsilon^2 - 1)$$

In what follows we will be interested in the case when a particle has the energy $E > m$, so $\varepsilon > 1$.

GENERAL SCENARIO OF DECAY

$$E_0 = E_1 + E_2,$$

$$q_0 = q_1 + q_2,$$

$$p_0^r = p_1^r + p_2^r, \qquad P = \sqrt{X^2 - m^2 f}$$

For given m_i q_i

$$X_1 = \frac{1}{2m_0^2}(X_0 b_1 + P_0 \delta \sqrt{d}), \qquad P_1 = \left| \frac{P_0 b_1 + \delta X_0 \sqrt{d}}{2m_0^2} \right|,$$

$$X_2 = \frac{1}{2m_0^2}(X_0 b_2 - P_0 \delta \sqrt{d}), \qquad P_2 = \left| \frac{P_0 b_2 - \delta X_0 \sqrt{d}}{2m_0^2} \right|.$$

$$b_{1,2} = m_0^2 \pm (m_1^2 \quad \delta = \pm 1$$

where $i = 0,\ 1,\ 2,$

$$d = b_1^2 - 4m_0^2 m_1^2 = b_2^2 - 4m_0^2 m_2^2,$$

$$\delta = \pm 1$$

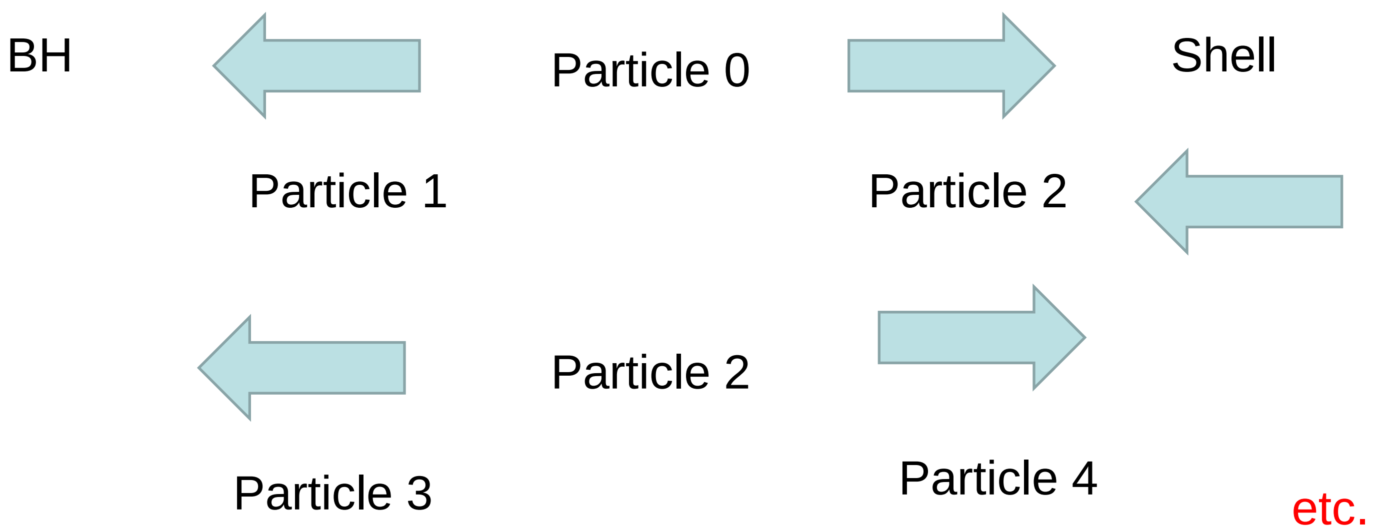
Decay in turning point

$$P_i = 0$$

Consequences: $X_i = m_i \sqrt{f(r_0)}$

$$m_0 = m_1 + m_2$$

Reflecting shell $r_B > r_0$



$$E_{2n+1} = m_{2n+1}\sqrt{f(r_0)} + \frac{q_{2n+1}Q}{r_0},$$

$$E_{2n+2} = m_{2n+2}\sqrt{f(r_0)} + \frac{q_{2n+2}Q}{r_0}.$$

The energy $E_{2n+1} < 0$, provided $q_{2n+1} = -|q_{2n+1}|$, where $|q_{2n+1}| > q^*_{2n+1}$,

$$q^*_{2n+1} = \gamma m_{2n+1}, \qquad \gamma = \frac{r_0}{Q}\sqrt{f(r_0)}.$$

$m_{2n} < m_0$ is finite

Result depends crucially on q_{2n}

If $\lim_{n\to\infty} q_n = q_\infty < \infty$

E in the limit is finite as well.

Scenario 1 Kokubu et al, PRD 2021

Assumption:

$$q_{2n+1} = -(1+\Delta)q^*_{2n+1}, \qquad q_{2n+2} = q_{2n} - q_{2n+1}$$

$$\Delta \text{ is some constant}$$

We also assume

$$m_{2n+1} = \alpha_1 m_{2n}, \qquad m_{2n+2} = \alpha_2 m_{2n},$$

$$\alpha_1 + \alpha_2 = 1$$

$$m_{2n} = \alpha_2^n m_0,$$

$$m_{2n+1} = \alpha_1 \alpha_2^n m_0$$

$$q_{2n+1} = -(1+\Delta)\frac{m_{2n+1}\sqrt{f}}{Q} r_0 = -m_0(1+\Delta)\gamma\alpha_1\alpha_2^n,$$

$$E_{2n+1} = -m_{2n+1}\sqrt{f}\Delta,$$

$$\varepsilon_{2n+1} = -\sqrt{f(r_0)}\Delta.$$

$\frac{q_{2n+1}}{m_{2n+1}} = \text{const}$ ε_{2n+1} const

$$\lim_{n\to\infty} E_{2n} = E_0 + m_0\gamma\Delta\frac{Q}{r_0} = \lim_{\alpha_2\to 0} E_{2n},$$

$$\lim_{n\to\infty} q_{2n} = q_0 + \gamma(1+\Delta) = \lim_{\alpha_2\to 0} q_{2n}.$$

$$\lim_{n\to\infty} m_{2n} = \lim_{\alpha_2\to 0} m_{2n} = 0$$

particles become “photons”.

Efficiiency $\eta = \frac{E_{2n}}{E}.$

$$\lim_{n\to\infty} \eta_n = 1 + \gamma\Delta\frac{m_0 Q}{E_0 r_0} = 1 + \frac{m_0\Delta}{E_0}\sqrt{f(r_0)}$$ finite

Scenario 2

OZ PRD 2022

$$q_{2n+1} = q_{2n}\beta_1,$$

$$q_{2n+2} = q_{2n}\beta_2,$$

$$\beta_1 + \beta_2 = 1,$$

$$q_{2n+2} = \beta_2^{n+1} q_0$$

$$q_{2n+1} = \beta_1 \beta_2^n q_0.$$

$$\beta_1 < 0, \qquad \beta_2 > 1.$$

$$E_{2n+1} = m_{2n+1}\left[\sqrt{f(r_0)} - \frac{|\beta_1| q_0 Q}{\alpha_1 r_0 m_0}\left(\frac{\beta_2}{\alpha_2}\right)^n\right]$$

$$E_{2n+2} = m_{2n+2}\left[\sqrt{f(r_0)} + \left(\frac{\beta_2}{\alpha_2}\right)^{n+1} \frac{q_0 Q}{m_0 r_0}\right]$$

$$E_{2n+1} = m_{2n+1}\left[\sqrt{f(r_0)} - \frac{|\beta_1| q_0 Q}{\alpha_1 r_0 m_0}\left(\frac{\beta_2}{\alpha_2}\right)^n\right]$$

If $$\frac{|\beta_1| q_0 Q}{\alpha_1 r_0 m_0} > \sqrt{f(r_0)}$$ Amplification occurs from 1st event

$$\sqrt{f(r_0)} - \frac{|\beta_1| q_0 Q}{\alpha_1 r_0 m_0}\left(\frac{\beta_2}{\alpha_2}\right)^n > 0.$$ Can start from some n

$\beta_2 > 1$ and $\alpha_2 < 1$

$\lim_{n\to\infty} E_{2n}$ and $\lim_{n\to\infty} \eta_n$ diverge

E_n grows exponentially

bomb

ROTATING BLACK HOLES

$$ds^2 = -N^2 dt^2 + g_\phi (d\phi - \omega dt)^2 + \frac{dr^2}{A} + g_\theta d\theta^2$$

$$m\dot{t} = \frac{X}{N^2},$$

$$X = E - \omega L,$$

$$\theta = \frac{\pi}{2}.$$

$$p^r = m\dot{r} = \sigma P$$

$$P = \sqrt{X^2 - \tilde{m}^2 N}$$

$$m\dot{\phi} = \frac{L}{g_\phi} + \frac{\omega X}{N^2}$$

$$\tilde{m}^2 = m^2 + \frac{L^2}{g_\phi}$$

Decay in turning point

$$\tilde{m}_0 = \tilde{m}_1 + \tilde{m}_2$$

$$E_1 = E_0 \frac{b_1}{2m_0^2} - \frac{\sqrt{d}}{2m_0^2} \sqrt{E_0^2 + m_0^2 g_{tt}},$$

$$E_2 = E_0 \frac{b_2}{2m_0^2} + \frac{\sqrt{d}}{2m_0^2} \sqrt{E_0^2 + m_0^2 g_{tt}},$$

The requirement $E_1 < 0$ leads to the condition

$$E_0 < E_0^* \equiv \frac{\sqrt{g_{tt} d}}{2m_1}.$$

Maximum efficiency for particle 2 when it is mssless

Multiple

Penrose process

$$L_{2n+1} = \frac{L_0 b_{2n+1}}{2m_{2n}^2} - \frac{\sqrt{d_{2n+2}}\sqrt{g_\phi}}{2m_{2n}^2}\tilde{m}_{2n},$$

$$L_{2n+2} = \frac{L_{2n} b_{2n+2}}{2m_{2n}^2} + \frac{\sqrt{d_{2n+2}}\sqrt{g_\phi}}{2m_n^2}\tilde{m}_{2n},$$

$$E_{2n+1} = \frac{b_{2n+1}}{2m_{2n}^2}E_{2n} - \frac{\sqrt{d_{2n+2}}}{2m_{2n}^2}\sqrt{E_{2n}^2 + m_{2n}^2 g_{tt}},$$

$$E_{2n+2} = \frac{b_{2n+2}}{2m_{2n}^2}E_{2n} + \frac{\sqrt{d_{2n+2}}}{2m_{2n}^2}\sqrt{E_{2n}^2 + m_{2n}^2 g_{tt}},$$

$$b_{2n+1} = m_{2n}^2 + m_{2n+1}^2 - m_{2n+2}^2,$$

$$b_{2n+2} = m_{2n}^2 + m_{2n+2}^2 - m_{2n+1}^2,$$

$$d_{2n+2} = b_{2n+2}^2 - 4m_{2n}m_{2n+2} = b_{2n+1}^2 - 4m_{2n}^2 m_{2n+1}^2.$$

Possibility of negative energy:

$$E_{2n} < E_{2n}^{*} = \frac{\sqrt{g_{tt}} d_{2n+2}}{2m_{2n+1}}$$

$$E_{2n+2} \approx E_{2n} s_n,$$

$$s_n < 1$$

Growth of energy stops, no BH bomb

Decay in arbitrary point

Wald 1974

$$E_{\max} = \frac{m_2}{m_0}\gamma\left(E_0 + v\sqrt{E_0^2 + m_0^2 g_{tt}}\right)$$

In our case $$v = \frac{\sqrt{d}}{b_2} = \sqrt{1 - \frac{4m_2^2 m_0^2}{b_2^2}}.$$

$$E_{2n+2} \approx s_n E_{2n}$$

$$s_n = \frac{m_{2n+2}}{m_{2n}}\gamma_n(1 + v_n) = \frac{m_{2n+2}}{m_{2n}}\sqrt{\frac{1 + v_n}{1 - v_n}}$$

Amplification $s_n > 1$

$$v_n > \frac{1-\alpha_n^2}{1+\alpha_n^2}, \qquad \alpha_n = \frac{m_{2n+2}}{m_{2n}} < 1.$$

For decay in turning point

No BHB

$$s_n = \frac{b_{2n+2} + \sqrt{b_{2n+1}^2 - 4m_{2n}^2 m_{2n+1}^2}}{2m_{2n}^2} \leq 1$$

Another type of high energy process in charged background.
Role of naked singularity

ENERGY EXTRACTION

Particles 0 and 1 move towards center, particle 2 escapes

$$E_2 = X_2 + \frac{q_2 Q}{r_0} \qquad X_0 = E_0 - \frac{q_0 Q}{r}$$

If $q_2 > 0$ and $r_0 \to 0$

X is bounded. Then, this requires

$$q_0 < 0, \qquad q_2 > 0.$$

A. No turning point

$$E_2 \approx \frac{Q}{r_0} \left[q_2 + \frac{1}{2m_0^2} \left(|q_0| \Delta_- + \sqrt{q_0^2 - m_0^2} \sqrt{D} \right) \right]$$

$$E_2 \to \infty \text{ when } r_0 \to 0$$

$$|q_0| < \frac{\Delta_-}{2m_2}$$

Escapes

$$\Delta_- = m^2{}_0 - m^2{}_1 + m^2{}_2$$

B. Decay in the turning point

$$X_0 = E_0 + \frac{|q_0|Q}{r_0}$$

$$E_2 = \frac{E_0 \Delta_-}{2m_0^2} + \frac{Q}{r_0}\left(q_2 + |q_0|\frac{\Delta_-}{2m_0^2}\right)$$

We want to minimize r_0

$$|\tilde{q}_0| = 1 - \beta, \quad \beta \ll 1 \qquad\qquad r_0 \approx \frac{Q^2\beta}{M + \varepsilon Q}$$

$$\tilde{q} = \frac{q}{m}$$

$$E_2 \approx \frac{(M + \varepsilon Q)}{Q\beta}\left(q_2 + |q_0|\frac{\Delta_-}{2m_0^2}\right)$$

SPP but for charged particles only

ALTERNATIVE TYPE OF SCENARIO

Article 2 after decay moves in the same
direction as
particle 1, so r continues to decrease.
However, immediately
after decay, particle 2 bounces back from
the potential
barrier.

$$E_2 \approx \frac{Q(m_2 + q_2)}{r_0}$$

SPP exists as well

FLAT SPACE-TIME LIMIT

A. No turning point

$$E_2 \approx \frac{Q}{r_0}\left[\frac{|q_0|}{2m_0^2}(\Delta_- + \sqrt{D}) - |q_2|\right]$$

$$|q_2| < \frac{|q_0|}{2m_0^2}(\Delta_- + \sqrt{D}) \qquad r_0 \gg \frac{\sqrt{G}Q}{c^2}$$

If $r_0 \to 0$ q_0 Is fixed, particle 2 falls in center, no escape

However, if $|q_0| = ar_0$ with $\alpha = O(1)$

$$\alpha < Q^{-1}\left(\frac{\Delta_-}{2m_2} - E_0\right)$$ SPP

Alternative scenario

Particle 2 bounces back

$$E_2 \approx \frac{q_2 \Delta_-}{2m_2|q_0|Q}$$

If $q_0 \to 0$, $E_2 \to \infty$.

SPP

Summary

A. Black hole bomb (BHB)

1) The possibility of BHB depends crucially on type of scenario.
2) For some scenarios BHB is possible, even for decay in turning point.
3) If BH is rotating, BHB for neutral particles is impossible for any scenario, if decay occurs in turning point.

observer can unfold in different ways for typical and special scenarios.

4) If we relax the condition of decay in turning point, for some constraints on parameters of debris BHB is possible.

5) Difference between static charged case and rotating one. The effective mass creates a potential barrier in the second case (so L affects dynamics) but is absent in the first one.

Role of p=0 (zero-momentum observers)

6) Crucial point: ergosphere. Horizon is formally is not mandatory.

B. Process near naked singularity

1) SPP without fine-tuning
2) To gain large E, there is no need to have large q.
3) Now the original particle does not need to be ultrarelativistic having any finite value E/m.
4) Any type of the Reissner-Nordström spacetime (black hole, flat space-time as its limit, naked singularity) is pertinent to the SPP

Thank you!

CONCLUSIONS

High energy collisions due to horizon

Role of critical trajectories
Force does NOT spoil BSW effect, critical trajectories survive
Classification of fine-tuned trajectories:
subcritical, critical, and ultracritical ones depending on
their near-horizon behavior
Finiteness of acceleration in OZAMO frame
Kinematic censorship preseved
Further steps: nonequatorial motion, near-fine-tuned particles

Thank you!