Relativistic Corrections to the Growth of Structure in Modified Gravity

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Based on https://arxiv.org/abs/2006.11019

In collaboration with Kazuya Koyama and David Wands

Outline

1. Motivation
2. N-Body Gauge
3. Modified Gravity
4. Results
5. Conclusions/Way forward

Motivation – Current Landscape

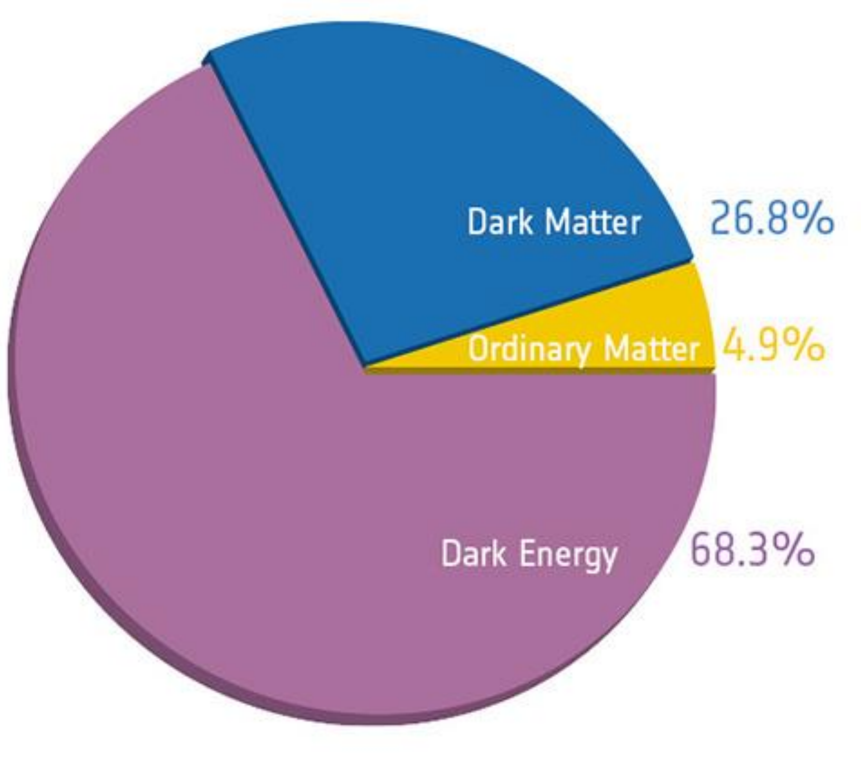
$$H^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda c^2}{3} - \frac{Kc^2}{a^2},$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) + \frac{\Lambda c^2}{3}$$

$$G_{\mu\nu} + \Lambda\, g_{\mu\nu} = \frac{8\pi G}{c^4} T^{(m)}_{\mu\nu}$$

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

$$T_{\mu\nu} = \left(\rho + \frac{P}{c^2}\right) u_\mu u_\nu + P g_{\mu\nu}$$

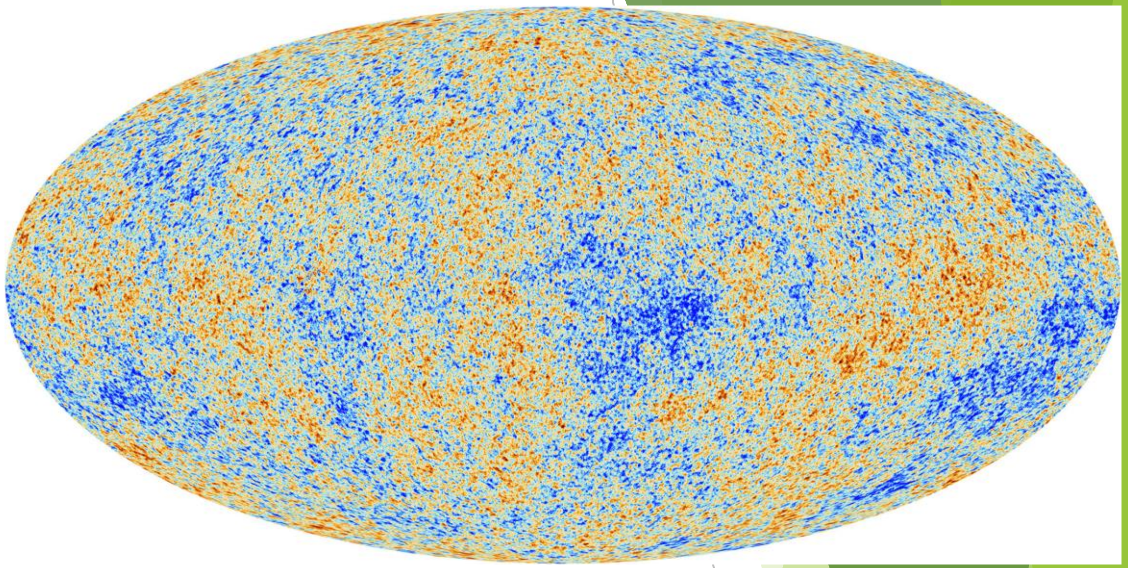


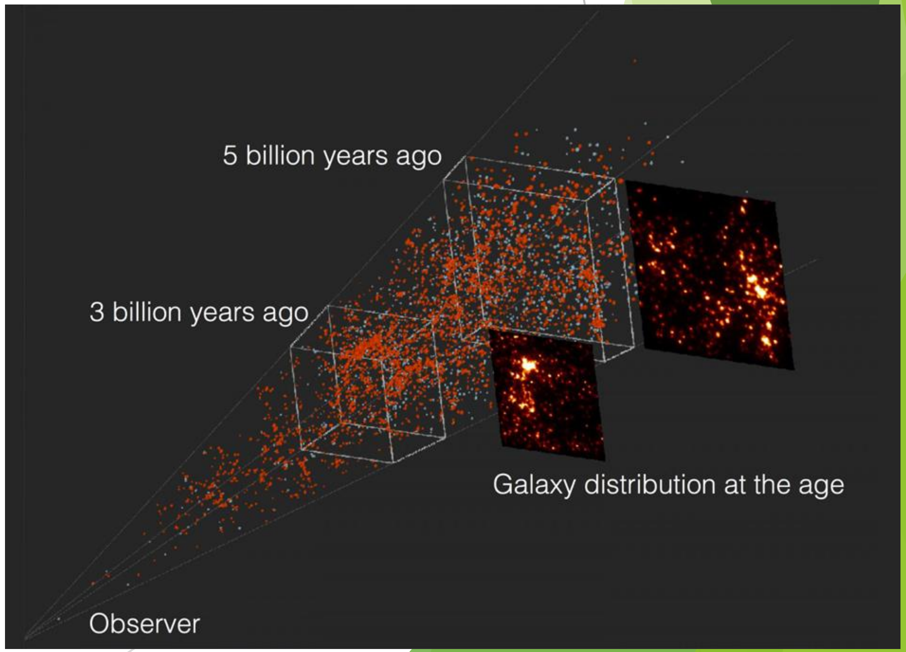
Motivation – Current Landscape

Linear perturbation

$$G_{\mu\nu} \longrightarrow \bar{G}_{\mu\nu} + \delta G_{\mu\nu}$$
$$T_{\mu\nu} \longrightarrow \bar{T}_{\mu\nu} + \delta T_{\mu\nu}$$

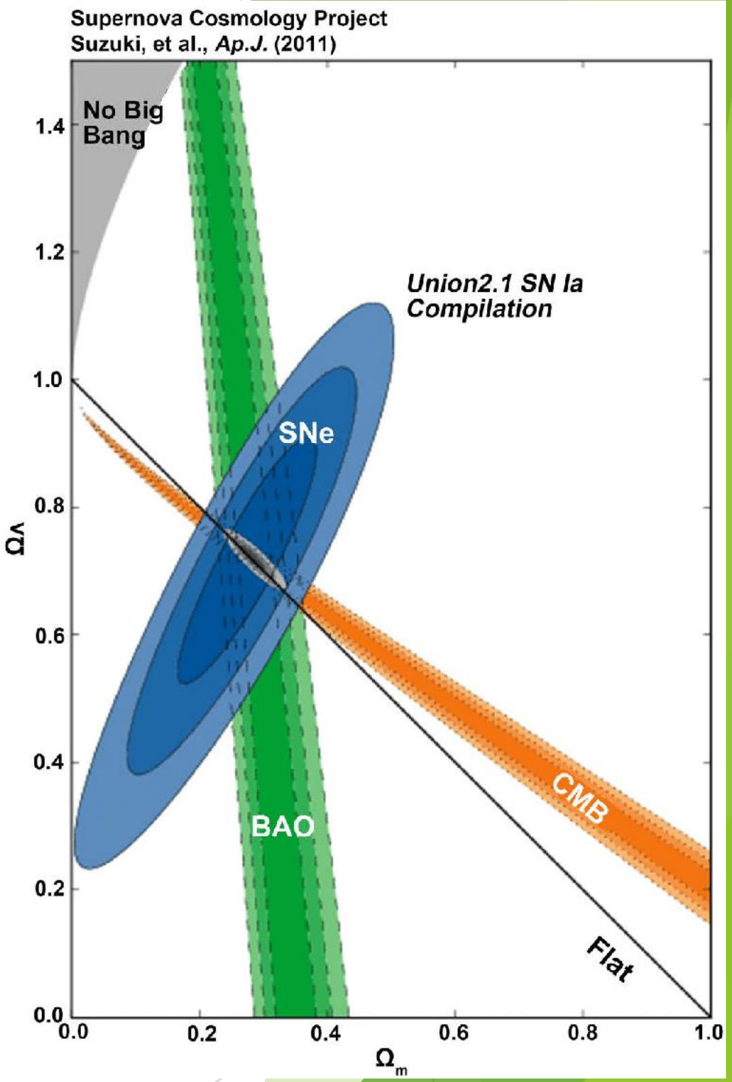
Background





Motivation – Current Landscape

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T^{(m)}_{\mu\nu}$$



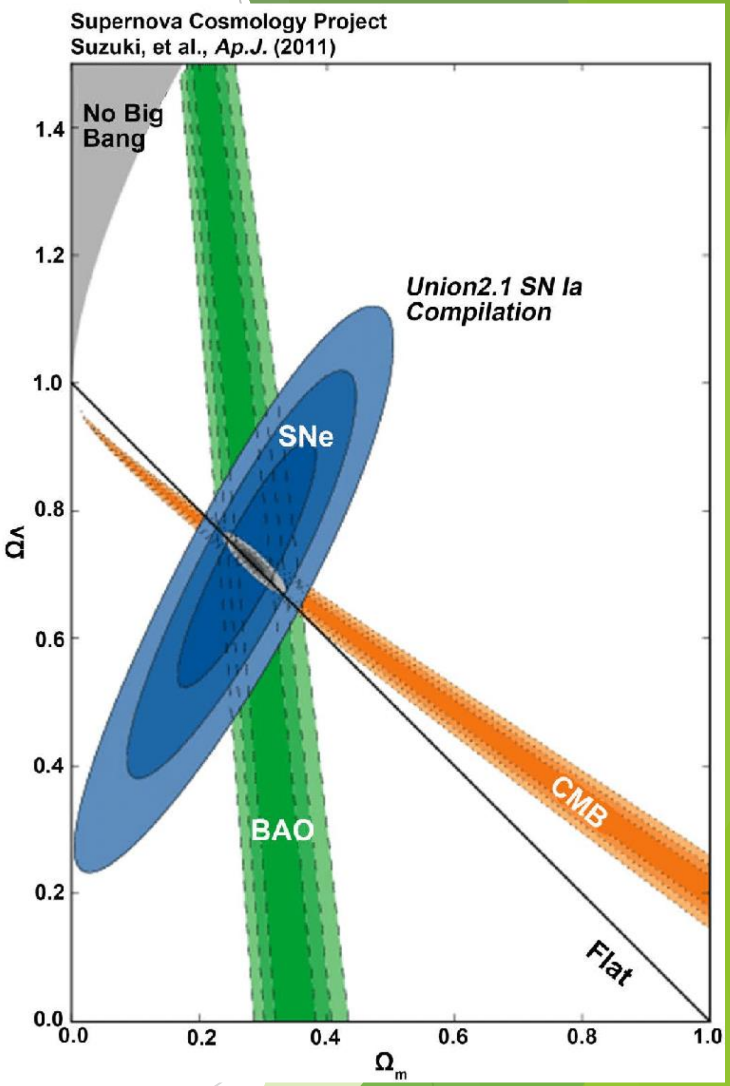
Motivation – Current Landscape

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T^{(m)}_{\mu\nu}$$

Cosmological Constant?

Modified Gravity?

Dark Energy exotic fluid?



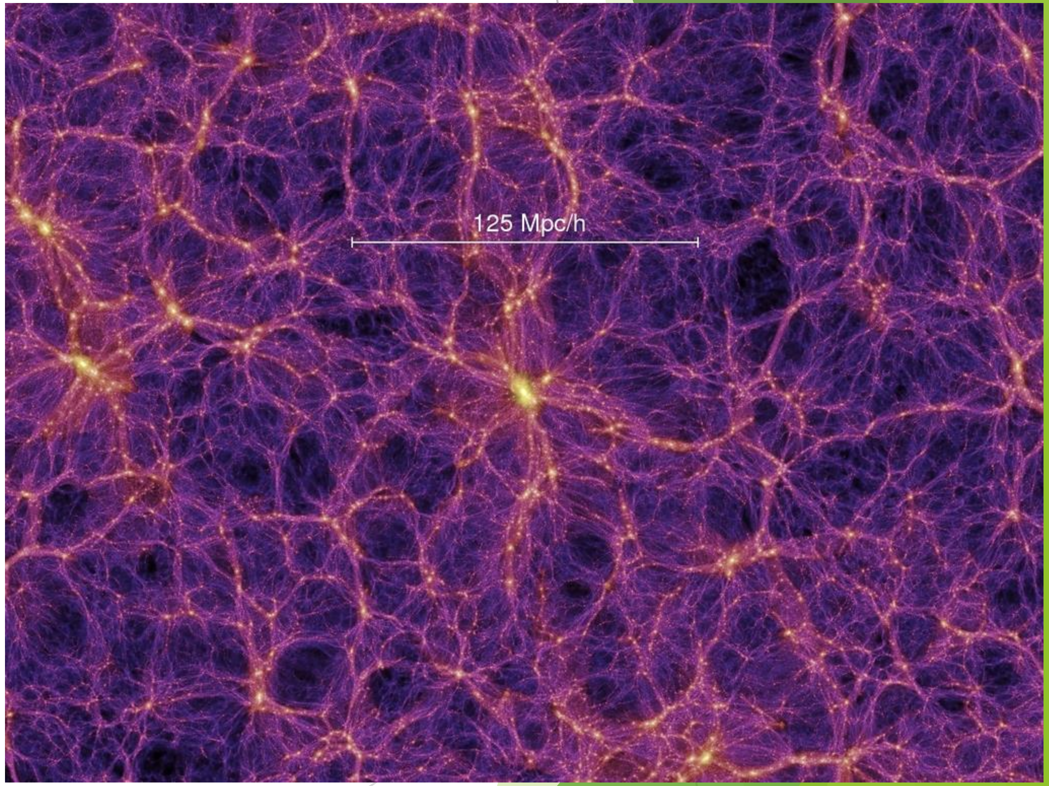
Motivation – Simulations

Motivation – Simulations

Cold Dark Matter simulations:

- Most codes that simulate the structure growth are Newtonian.
- Good approximation for late times and sub-horizon scales.
- System of equations:

$$\nabla^2\Phi_{\rm N} = 4\pi G_N a^2 \delta\rho_m,$$
$$\ddot{\mathbf{x}} + 2H\dot{\mathbf{x}} = -\nabla\Phi.$$



Millennium simulation

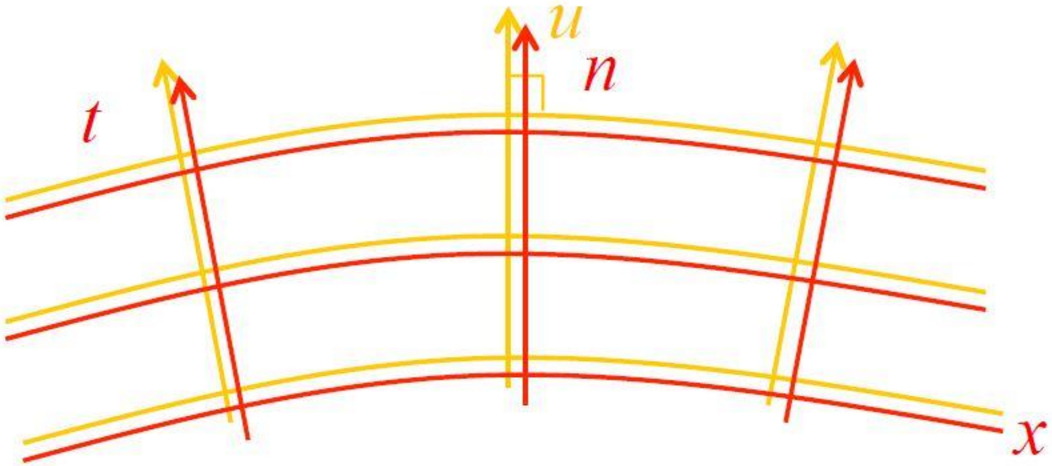
Motivation – Simulations

- Questions:

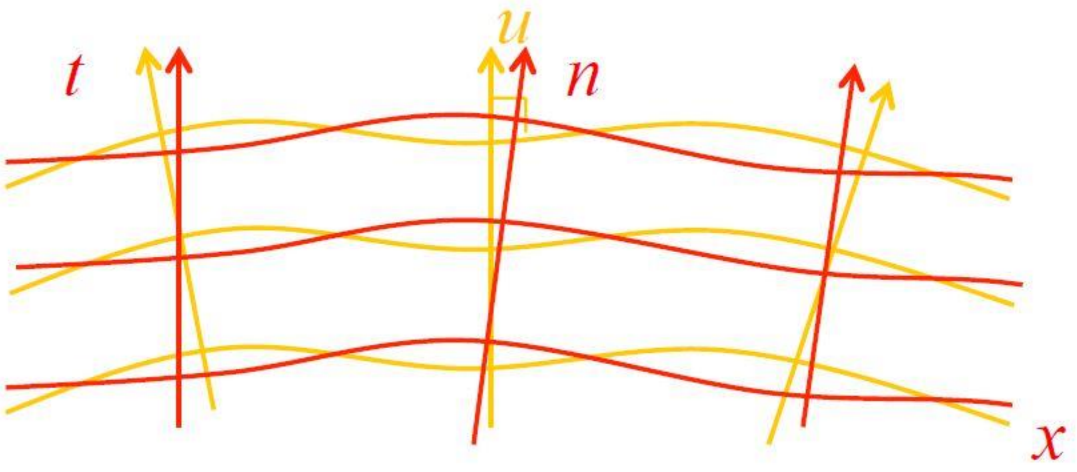
1. What GR coordinate system N-Body simulations live?
2. How/When should we initiate our simulations?
3. How to produce mock catalogues from simulations including GR effects?

Motivation – Simulations (1)

1. What GR coordinate system N-Body simulations live?



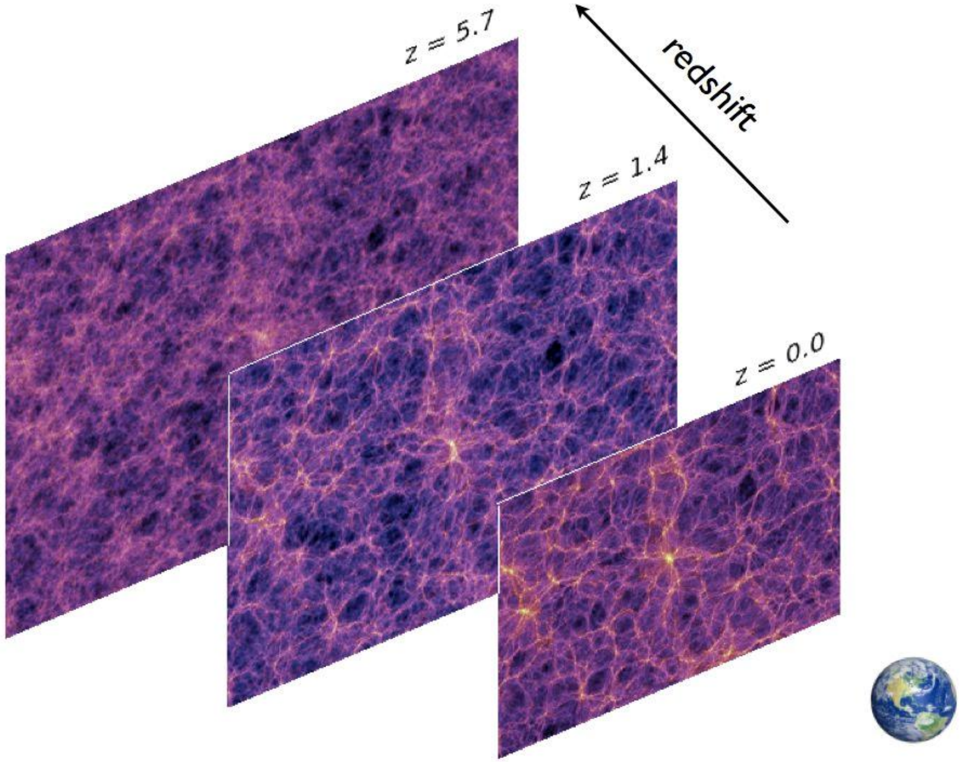
FLRW Cosmology
Background



Inhomogeneous perturbed cosmology
Arbitrary gauge

Motivation – Simulations (1)

1. What GR coordinate system N-Body simulations live?



$$\mathrm{d}s^2 = ??$$

Motivation – Simulations (2)

2. How/When should we initiate our simulations?

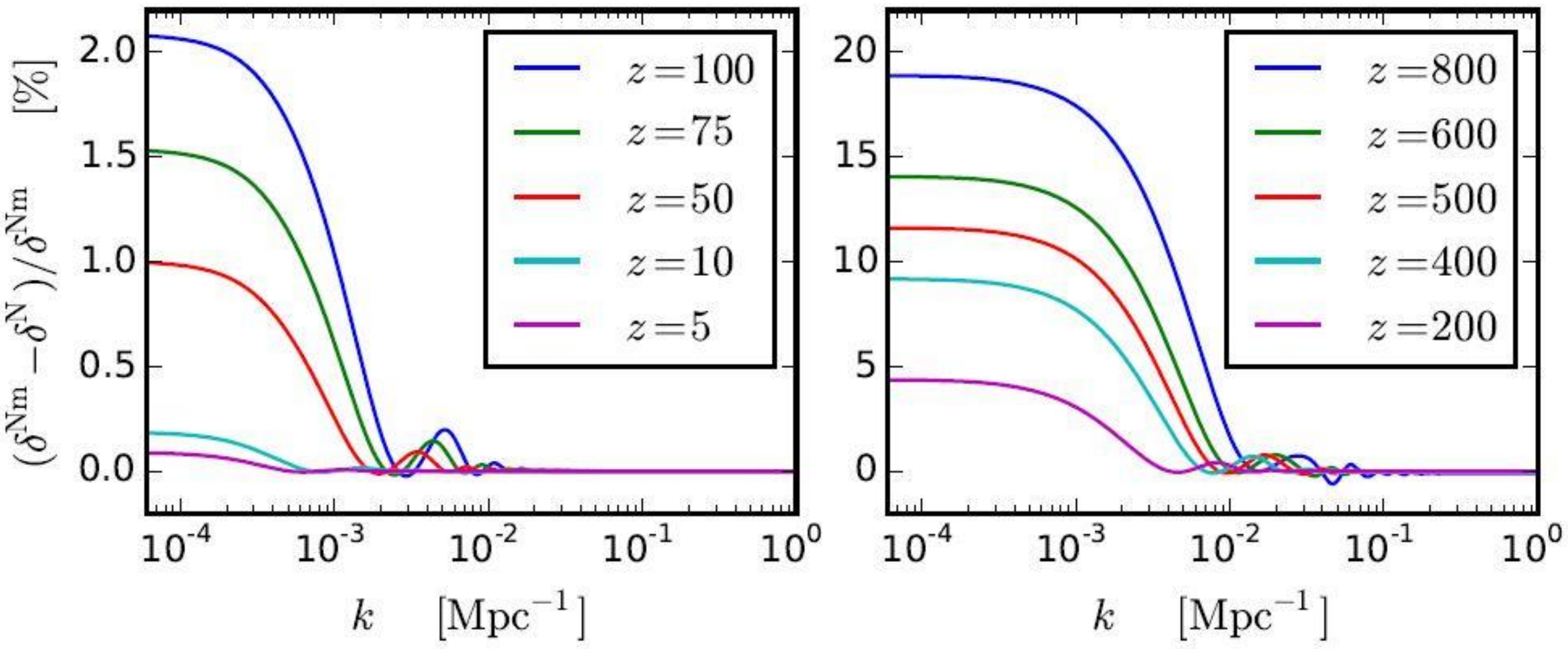
- Unavoidable tension in setting Initial Conditions:

1. We aim to minimize non-linear corrections at early stages.
Initiate simulations at higher redshifts

2. At higher redshifts, the bigger the relativistic effects of photons, neutrinos are, as well as more relativistic space-time itself is.

Motivation – Simulations (2)

2. How/When should we initiate our simulations?



1606.05588

C. Fidler, et al

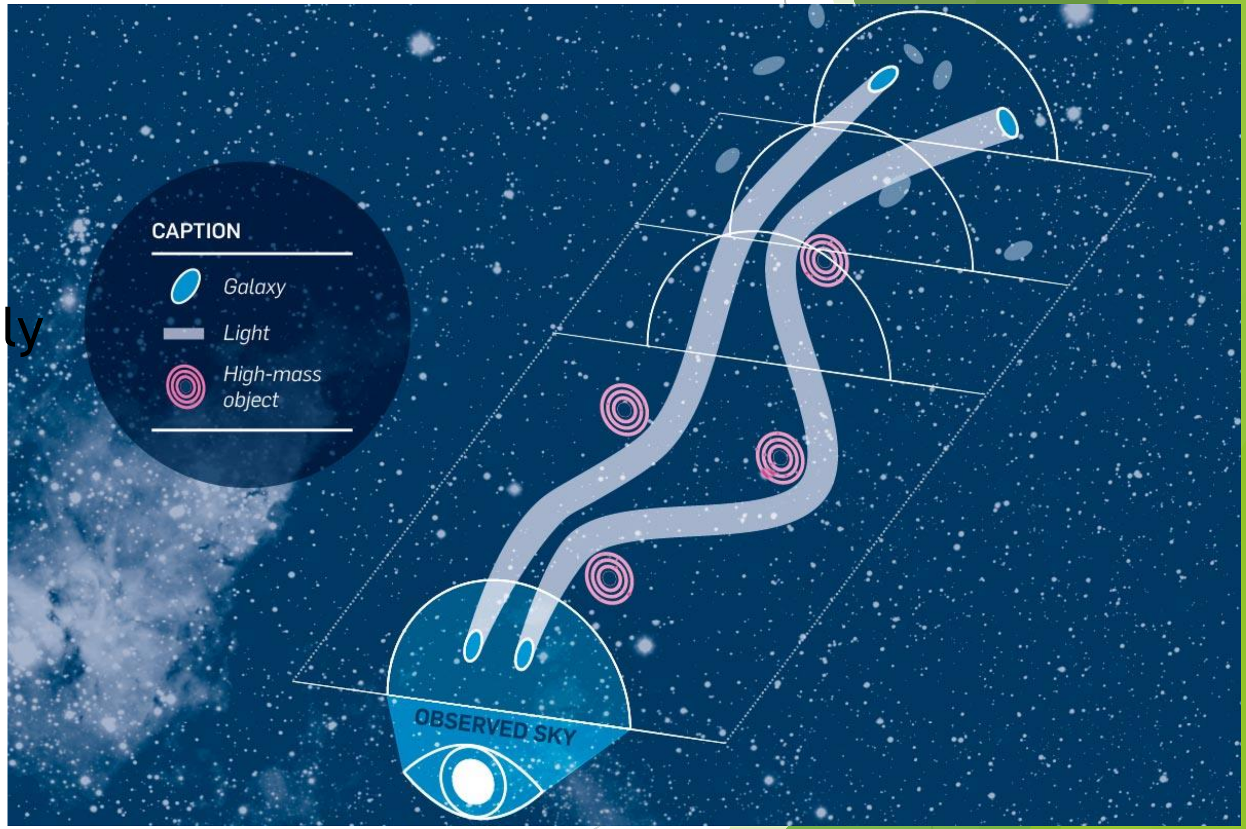
Relative difference, in %, between the relativistic δ^{Nm} and Newtonian δ^{N}, for a simulation initiated at different redshifts.

Motivation – Simulations (3)

3. How to produce mock catalogues from simulations including GR effects?

- Astronomical observations are almost exclusively based on electromagnetic signals.

- How to reconstruct the photon path backwards in time if we don't know the gauge in Newtonian simulations?



https://scitechdaily.com/researchers-try-to-weigh-the-universe-find-standard-model-of-cosmology-may-be-wrong/

Motivation – Simulations

Simulations:

- The problems presented so far share the same roots: relativistic effects and the gauge problem.

- In this talk we will present “natural embeddings” for the initial condition and space-time that have a direct relation to Newtonian simulations.

Newtonian/Relativistic Equations

- N-Body gauge in GR.
- For this we will need to set some conventions and definitions:

Newtonian/Relativistic Equations

- N-Body gauge in GR.
- For this we will need to set some conventions and definitions:

$$g_{00} = -a^2 (1 + 2A) \, ,$$
$$g_{0i} = a^2 \, \mathrm{i}\hat{k}_i B \, ,$$
$$g_{ij} = a^2 \left[\delta_{ij} (1 + 2H_{\mathrm{L}}) + 2 \left(\delta_{ij}/3 - \hat{k}_i\hat{k}_j\right) H_{\mathrm{T}}\right]$$

Perturbed FLRW metric in arbitrary gauge

$$T^0{}_0 = -\sum_\alpha (\rho_\alpha + \delta\rho_\alpha) = -\sum_\alpha \rho_\alpha (1 + \delta_\alpha) \equiv -\rho (1 + \delta) \, ,$$
$$T^i{}_0 = \sum_\alpha (\rho_\alpha + p_\alpha) \, \mathrm{i}\hat{k}^i v_\alpha \equiv (\rho + p) \, \mathrm{i}\hat{k}^i v \, ,$$
$$T^i{}_j = \sum_\alpha (p_\alpha + \delta p_\alpha)\delta^i_j + \frac{3}{2}(\rho_\alpha + p_\alpha) \left(\delta^i_j/3 - \hat{k}^i\hat{k}_j\right) \sigma_\alpha$$
$$\equiv (p + \delta p)\delta^i_j + \frac{3}{2}(\rho + p) \left(\delta^i_j/3 - \hat{k}^i\hat{k}_j\right) \sigma \, ,$$

Matter energy-momentum tensor

Newtonian/Relativistic Equations

- Putting them together: Einstein's Equations

$$4\pi G a^2 \left[\bar{\rho}\delta + 3\mathcal{H}\left(\bar{\rho}+\bar{p}\right)k^{-1}\left(v-B\right)\right] = k^2\Phi\,,$$

$$k^2\left(A + H_\mathrm{L} + \frac{1}{3}H_\mathrm{T}\right) - \left[\partial_\tau + 2\mathcal{H}\right]\left(\dot{H}_\mathrm{T} - kB\right) = -12\pi G a^2\left(\bar{\rho}+\bar{p}\right)\sigma\,,$$

$$4\pi G a^2\left(\bar{\rho}+\bar{p}\right)k^{-1}\left(v-B\right) = \mathcal{H}A - \dot{H}_\mathrm{L} - \frac{1}{3}\dot{H}_\mathrm{T}\,,$$

$$\left(\partial_\tau + 4\mathcal{H}\right)\left(\bar{\rho}+\bar{p}\right)k^{-1}\left(v-B\right) = \delta p - \left(\bar{\rho}+\bar{p}\right)\sigma + \left(\bar{\rho}+\bar{p}\right)A.$$

Newtonian/Relativistic Equations

- Continuity and Euler equations:

Relativistic

$$\dot{\delta}_m + kv_m = -3\dot{H}_{\mathrm{L}},$$
$$[\partial_\tau + \mathcal{H}]\, v_m = -k\,(\Phi + \gamma)$$

$$k^2\gamma = -(\partial_\tau + \mathcal{H})\dot{H}_T + 12\pi G a^2\,(\bar{\rho} + \bar{p})\,\sigma.$$
$$\Phi = H_{\mathrm{L}} + \frac{1}{3}H_{\mathrm{T}} + \mathcal{H}k^{-1}\left(B - k^{-1}\dot{H}_{\mathrm{T}}\right)$$

Newtonian

$$\dot{\delta}_m^{\mathrm{N}} + kv_m^{\mathrm{N}} = 0\,,$$
$$(\partial_\tau + \mathcal{H})\, v_m^{\mathrm{N}} = -k\Phi^{\mathrm{N}}\,,$$
$$k^2\Phi^{\mathrm{N}} = 4\pi G a^2 \rho_m \delta_m^{\mathrm{N}}$$

Newtonian/Relativistic Equations

Why not Newtonian gauge?

$$k^2\Phi^{\mathrm{Newt(P)}} = 4\Pi G_N a^2\left[\delta\rho_m + 3\mathcal{H}(\rho+p)\frac{v_m}{k}\right],$$
$$\dot{\delta}_m + kv_m = -3\dot{\Phi}^{\mathrm{Newt(P)}}$$

Newtonian

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Always present!
Relevant near horizon!

Newtonian

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$$k^2\Phi^{\mathrm{Newt(P)}} = 4\Pi G_N a^2 \left[\delta\rho_m + 3\mathcal{H}(\rho+p)\frac{v_m}{k}\right],$$

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No connection whatsoever
with a relativistic fluid description!

Always present!
Relevant near
horizon!

More on this later!!

Newtonian

$$\dot{\delta}_m^{\mathrm{N}} + k v_m^{\mathrm{N}} = 0\,,$$

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$$k^2\Phi^{\mathrm{N}} = 4\pi G a^2 \rho_m \delta_m^{\mathrm{N}}$$

Newtonian/Relativistic Equations

- So, if we can find a gauge in which the GR equations match the Newtonian equations, we can make Newtonian simulations solve for the full relativistic equations.

- Our task: get rid of γ!!!! in $\left[\partial_\tau + \mathcal{H}\right] v_{\mathrm{m}} = -k(\Phi + \gamma)\,,$

- How?

$$k^2\gamma = -(\partial_\tau + \mathcal{H})\dot{H}_T + 12\pi G a^2\,(\bar{\rho} + \bar{p})\,\sigma.$$

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A bit trickier to be 0, let's take a closer look.

Sufficiently late times is zero

Newtonian/Relativistic Equations

Comoving Curvature Perturbation:

$$\zeta = H_{\mathrm{L}} + \frac{1}{3}H_{\mathrm{T}} - \frac{\dot{a}}{a}\frac{v - B}{k}$$

Newtonian/Relativistic Equations

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- Trace part of spatial metric

Newtonian/Relativistic Equations

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- Trace part of spatial metric

$$\rho_{\rm count} = \frac{1}{a^3}\sum m\delta_{\rm D}^{(3)}(\mathbf{x} - \mathbf{x}_p)$$

$$\rho_{\rm rel} = (1 - H_{\rm L})\rho_{\rm count}$$

$$H_{\rm L} = 0,$$

$$\rho_{\rm count} = \rho_{\rm rel}$$

Newtonian/Relativistic Equations

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- Peculiar velocity of particles and shift perturbation
- Comoving gauge: $v = B$
- Temporal slicing is fixed, the constant-time hypersurfaces orthogonal to the 4-velocity of the total matter content

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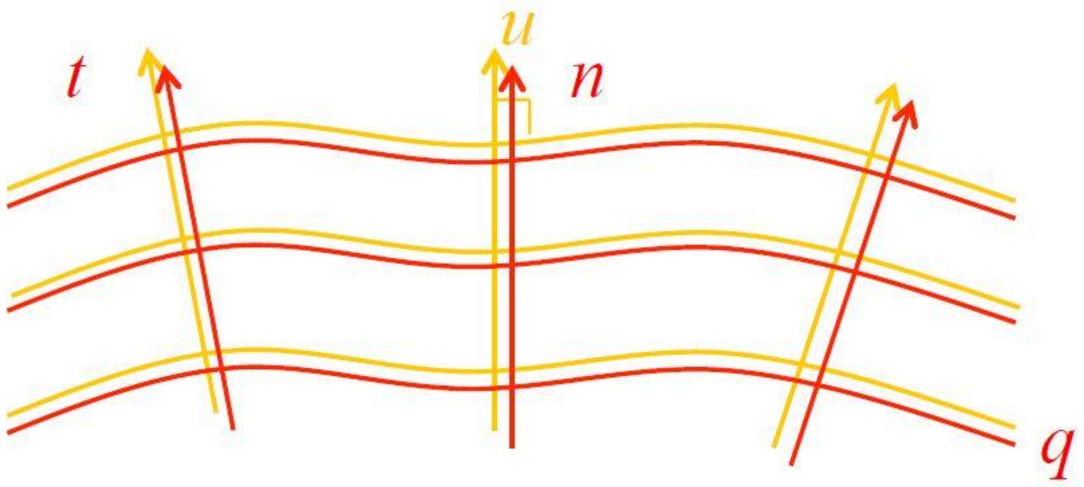

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$$H_{\rm L} = 0,$$

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Newtonian/Relativistic Equations

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- Now, the question:

Can the comoving curvature be constant?

Newtonian/Relativistic Equations

$$\zeta = \frac{1}{3} H_{\mathrm{T}}$$

- Now, the question:

Can the comoving curvature be constant?

- The answer:

Yes! But, how?

Newtonian/Relativistic Equations

- Let's remember our Einstein Equations:

$$4\pi G a^2 \left(\bar{\rho} + \bar{p}\right) k^{-1} \left(v - B\right) = \mathcal{H} A - \dot{H}_{\rm L} - \frac{1}{3}\dot{H}_{\rm T}\,,$$
$$\left(\partial_\tau + 4\mathcal{H}\right) \left(\bar{\rho} + \bar{p}\right) k^{-1} \left(v - B\right) = \delta p - \left(\bar{\rho} + \bar{p}\right) \sigma + \left(\bar{\rho} + \bar{p}\right) A$$

$$H_{\rm L} = 0 \quad + \quad B = v$$

$$0 = \mathcal{H} A - \frac{1}{3}\dot{H}_{\rm T}\,,$$
$$0 = \delta p - \left(\bar{\rho} + \bar{p}\right) \sigma + \left(\bar{\rho} + \bar{p}\right) A$$

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⇒

$$(\bar{\rho} + \bar{p})\,A = \delta p - (\bar{\rho} + \bar{p})\,\sigma$$

Newtonian/Relativistic Equations

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$$0 = \mathcal{H}A - \frac{1}{3}\dot{H}_{\mathrm{T}}\,,$$
$$0 = \delta p - (\bar{\rho} + \bar{p})\,\sigma + (\bar{\rho} + \bar{p})\,A$$

$$\Rightarrow \quad (\bar{\rho} + \bar{p})\,A = \delta p - (\bar{\rho} + \bar{p})\,\sigma$$

$$\Downarrow$$

$$\dot{H}_{\mathrm{T}} = 3\frac{\mathcal{H}}{\bar{\rho} + \bar{p}}\left[-\delta p + (\bar{\rho} + \bar{p})\sigma\right]$$

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⇒

$$(\bar{\rho} + \bar{p})\,A = \delta p - (\bar{\rho} + \bar{p})\,\sigma$$

⇓

$$\dot{H}_{\mathrm{T}} = 3\frac{\mathcal{H}}{\bar{\rho} + \bar{p}}\Big[-\delta p + (\bar{\rho} + \bar{p})\sigma\Big]$$

⇐

$$\delta p = 0 \quad + \quad \sigma = 0$$

Sufficiently late times

Just as we wanted! $\dot{H}_T = 0$

Newtonian/Relativistic Equations

- Therefore, we found a gauge in which in the absence of pressure perturbations and anisotropic stress the relativistic equations match the Newtonian ones.

Newtonian/Relativistic Equations

- Therefore, we found a gauge in which in the absence of pressure perturbations and anisotropic stress the relativistic equations match the Newtonian ones.

- This gauge is called the N-Body gauge:

1. Spatial threading: $H_{\mathrm{L}} = 0$
2. Temporal slicing: $v = B$

Newtonian/Relativistic Equations

- Now, how can we introduce the relativistic effects?

Newtonian/Relativistic Equations

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- Remember Continuity + Euler Equations:

$$
\begin{aligned}
&\dot{\delta}_m^{\rm Nb} + k v_m^{\rm Nb} = 0, \\
&(\partial_\tau + \mathcal{H}) v_m^{\rm Nb} = -k \left(\Phi + \gamma^{\rm Nb} \right), \\
&k^2 \gamma^{\rm Nb} = -(\partial_\tau + \mathcal{H}) \dot{H}_{\rm T}^{\rm Nb} + 12\pi G a^2 (\rho + p)\, \sigma, \\
&k^2 \Phi = 4\pi G a^2 \sum_\alpha \delta\rho_\alpha^{\rm Nb}
\end{aligned}
$$

Newtonian/Relativistic Equations

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$$k^2 \Phi = 4\pi G a^2 \sum_\alpha \delta\rho_\alpha^{\mathrm{Nb}}$$

$$\ddot{\delta}^{\mathrm{Nb}} m + \mathcal{H}\dot{\delta}_m^{\mathrm{Nb}} - 4\pi G a^2 \rho_m \delta_m^{\mathrm{Nb}} = 4\pi G a^2 \delta\rho_{\mathrm{GR}}$$

Newtonian/Relativistic Equations

- Newtonian equation:

$$\ddot{\delta}_m + \mathcal{H}\dot{\delta}_m - 4\pi G a^2 \rho_m \delta_m = 0$$

- Newtonian + GR correction equation:

$$\ddot{\delta}^{\mathrm{Nb}} m + \mathcal{H}\dot{\delta}_m^{\mathrm{Nb}} - 4\pi G a^2 \rho_m \delta_m^{\mathrm{Nb}} = 4\pi G a^2 \delta\rho_{\mathrm{GR}}$$

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Source term

Newtonian/Relativistic Equations

$$\ddot{\delta}^{\rm Nb}m + \mathcal{H}\dot{\delta}_m^{\rm Nb} - 4\pi G a^2 \rho_m \delta_m^{\rm Nb} = 4\pi G a^2 \delta\rho_{\rm GR}$$

Master Equation

Newtonian/Relativistic Equations

$$\ddot{\delta}^{\rm Nb} m + \mathcal{H}\dot{\delta}_m^{\rm Nb} - 4\pi G a^2 \rho_m \delta_m^{\rm Nb} = 4\pi G a^2 \delta\rho_{\rm GR}$$

Master Equation

- Source term:

$$\delta\rho_{\rm GR} = \delta\rho_\gamma^{\rm Nb} + \delta\rho_\nu^{\rm Nb} + \delta\rho_{\rm DE}^{\rm Nb} + \delta\rho_{\rm metric}^{\rm Nb},$$
$$k^2\gamma = 4\pi G a^2 \delta\rho_{\rm metric}$$

Newtonian/Relativistic Equations

$$\ddot{\delta}^{\mathrm{Nb}} m + \mathcal{H}\dot{\delta}_m^{\mathrm{Nb}} - 4\pi G a^2 \rho_m \delta_m^{\mathrm{Nb}} = 4\pi G a^2 \delta\rho_{\mathrm{GR}}$$

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$$k^2\gamma = 4\pi G a^2 \delta\rho_{\mathrm{metric}}$$

- Gauge transformation:

$$\delta\rho_\alpha^{\mathrm{Nb}} = \delta\rho_\alpha^{\mathrm{S/P}} + 3\mathcal{H}\left(1 + w_\alpha\right)\delta\rho_\alpha^{\mathrm{S/P}} \frac{\theta_{\mathrm{tot}}^{\mathrm{S/P}}}{k^2}$$

Newtonian/Relativistic Equations

$$\ddot{\delta}^{\rm Nb}m + \mathcal{H}\dot{\delta}_m^{\rm Nb} - 4\pi G a^2 \rho_m \delta_m^{\rm Nb} = 4\pi G a^2 \delta\rho_{\rm GR}$$

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- Gauge transformation:

$$\delta\rho_\alpha^{\rm Nb} = \delta\rho_\alpha^{\rm S/P} + 3\mathcal{H}\left(1 + w_\alpha\right)\delta\rho_\alpha^{\rm S/P}\frac{\theta_{\rm tot}^{\rm S/P}}{k^2}$$

- Relativistic potential:

$$k^2\gamma = -(\partial_\tau + \mathcal{H})\dot{H}_T + 12\pi G a^2\left(\bar{\rho} + \bar{p}\right)\sigma.$$

Newtonian/Relativistic Equations

$$\ddot{\delta}^{\rm Nb}m + \mathcal{H}\dot{\delta}_m^{\rm Nb} - 4\pi G a^2 \rho_m \delta_m^{\rm Nb} = 4\pi G a^2 \delta\rho_{\rm GR}$$

Master Equation

- Source term:

$$\delta\rho_{\rm GR} = \delta\rho_\gamma^{\rm Nb} + \delta\rho_\nu^{\rm Nb} + \delta\rho_{\rm DE}^{\rm Nb} + \delta\rho_{\rm metric}^{\rm Nb},$$
$$k^2\gamma = 4\pi G a^2 \delta\rho_{\rm metric}$$

- Gauge transformation:

$$\delta\rho_\alpha^{\rm Nb} = \delta\rho_\alpha^{\rm S/P} + 3\mathcal{H}\left(1+w_\alpha\right)\delta\rho_\alpha^{\rm S/P}\frac{\theta_{\rm tot}^{\rm S/P}}{k^2}$$

- Relativistic potential:

$$k^2\gamma = -(\partial_\tau + \mathcal{H})\dot{H}_T + 12\pi G a^2\left(\bar{\rho}+\bar{p}\right)\sigma.$$

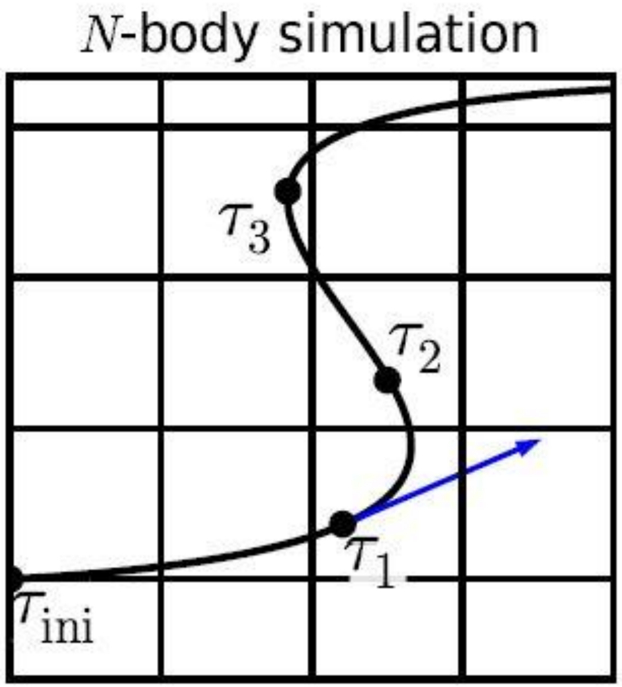
- Traceless term of spatial metric:

$$\dot{H}_{\rm T}^{\rm Nb} = 3\frac{\mathcal{H}}{\rho+p}\left[\left(\rho+p\right)\sigma - \delta p^{\rm S/P} + p'\frac{\theta_{\rm tot}^{\rm S/P}}{k^2}\right]$$

Newtonian/Relativistic Equations

$$\ddot{\delta}_m + \mathcal{H}\dot{\delta}_m - 4\pi G a^2 \rho_m \delta_m = 0$$

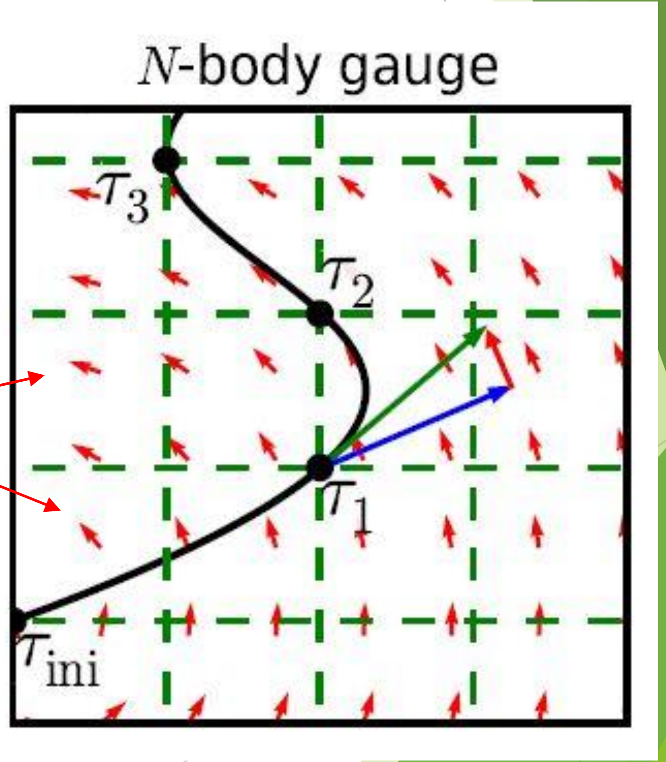
$$\ddot{\delta}^{\mathrm{Nb}} m + \mathcal{H}\dot{\delta}_m^{\mathrm{Nb}} - 4\pi G a^2 \rho_m \delta_m^{\mathrm{Nb}} = 4\pi G a^2 \delta\rho_{\mathrm{GR}}$$



1606.05588

“Fictious force”

Metaphorically described as rotations



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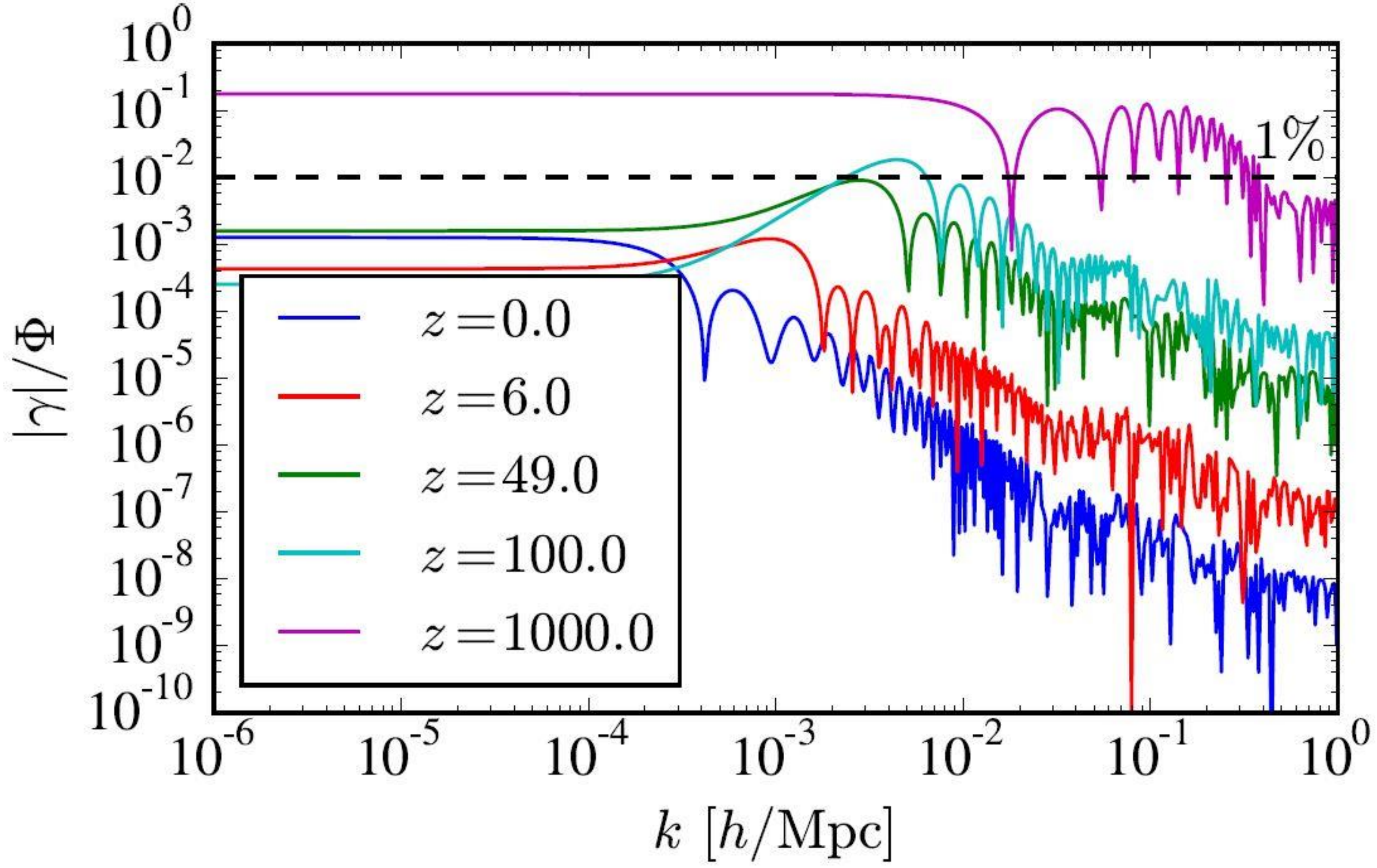
Newtonian/Relativistic Equations



1505.04756

C. Fidler et al.

Modified Gravity

- Our goal is to consistently introduce dark energy perturbations coming from Horndeski theory.

- Our basic assumptions:

1. Bianchi identities hold

2. Conservation of Energy Momentum tensor

- Matter species interact only gravitationally with the DE scalar field!

$$G_{\mu\nu} = 8\pi G \left(T_{\mu\nu} + E_{\mu\nu} \right)$$

1504.04623

K. Koyama

Effective Energy Momentum Tensor: absorbs all contributions coming from modified gravity.

Modified Gravity

- Horndeski Action:

$$S[g_{\mu\nu}, \phi] = \int d^4x \sqrt{-g} \left[\sum_{i=2}^{5} \frac{1}{8\pi G} \mathcal{L}_i[g_{\mu\nu}, \phi] + \mathcal{L}_m[g_{\mu\nu}, \psi_M] \right]$$

$$\begin{aligned}
\mathcal{L}_2 &= G_2(\phi,\ X)\,, \\
\mathcal{L}_3 &= -G_3(\phi,\ X)\Box\phi\,, \\
\mathcal{L}_4 &= G_4(\phi,\ X)R + G_{4X}(\phi,\ X)\left[(\Box\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu}\right]\,, \\
\mathcal{L}_5 &= G_5(\phi,\ X)G_{\mu\nu}\phi^{;\mu\nu} - \frac{1}{6}G_{5X}(\phi,\ X)\left[(\Box\phi)^3 + 2\phi_{;\mu}{}^{\nu}\phi_{;\nu}{}^{\alpha}\phi_{;\alpha}{}^{\mu} - 3\phi_{;\mu\nu}\phi^{;\mu\nu}\Box\phi\right]
\end{aligned}$$

Modified Gravity

- Background equations of motion:

- Matter and Dark Energy interact gravitationally.
- Therefore, the continuity equation for DE must be set as:

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⇒

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$\Rightarrow$

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Dark Energy background
Density and pressure!

Modified Gravity

- Linear perturbation theory: subtract the contributions coming from the perturbed Einstein Tensor

- Synchronous gauge:

$$k^2\eta - \frac{1}{2}\frac{\dot{a}}{a}\dot{h} = 4\pi G a^2 \delta\rho_m + 4\pi G a^2 \delta\rho_{DE},$$

$$k^2\dot{\eta} = 4\pi G a^2 \left(\bar{\rho} + \bar{p}\right)\theta_m + 4\pi G a^2 \left(\bar{\rho}_{DE} + \bar{p}_{DE}\right)\theta_{DE},$$

$$\ddot{h} + 2\frac{\dot{a}}{a}\dot{h} - 2k^2\eta = -8\pi G a^2 \delta p_m - 8\pi G a^2 \delta p_{DE},$$

$$\ddot{h} + 6\ddot{\eta} + 2\frac{\dot{a}}{a}\left(\dot{h} + 6\dot{\eta}\right) - 2k^2\eta = -24\pi G a^2 \left(\bar{\rho} + \bar{p}\right)\sigma_m - 24\pi G a^2 \left(\bar{\rho}_{DE} + \bar{p}_{DE}\right)\sigma_{DE}.$$

Modified Gravity + Relativistic Effects

- We now have all we need to consistently introduce the effects coming from DE perturbations!

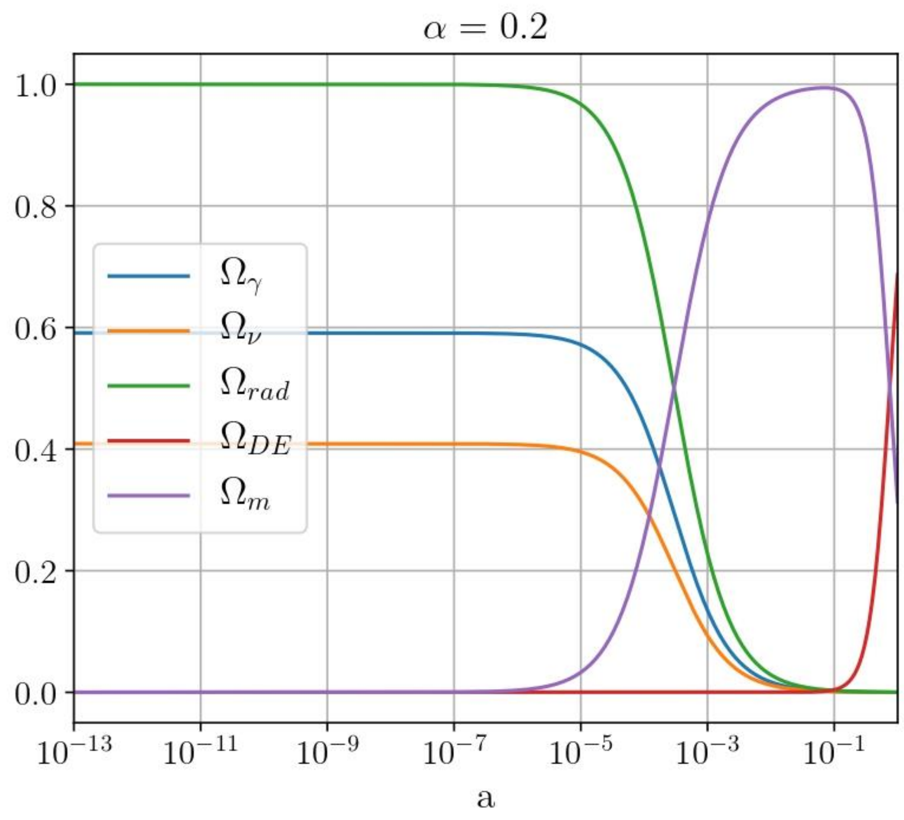
- As an example, we will demonstrate the effect of relativistic species (photons, massive/massless neutrinos and DE) in the matter power spectrum.

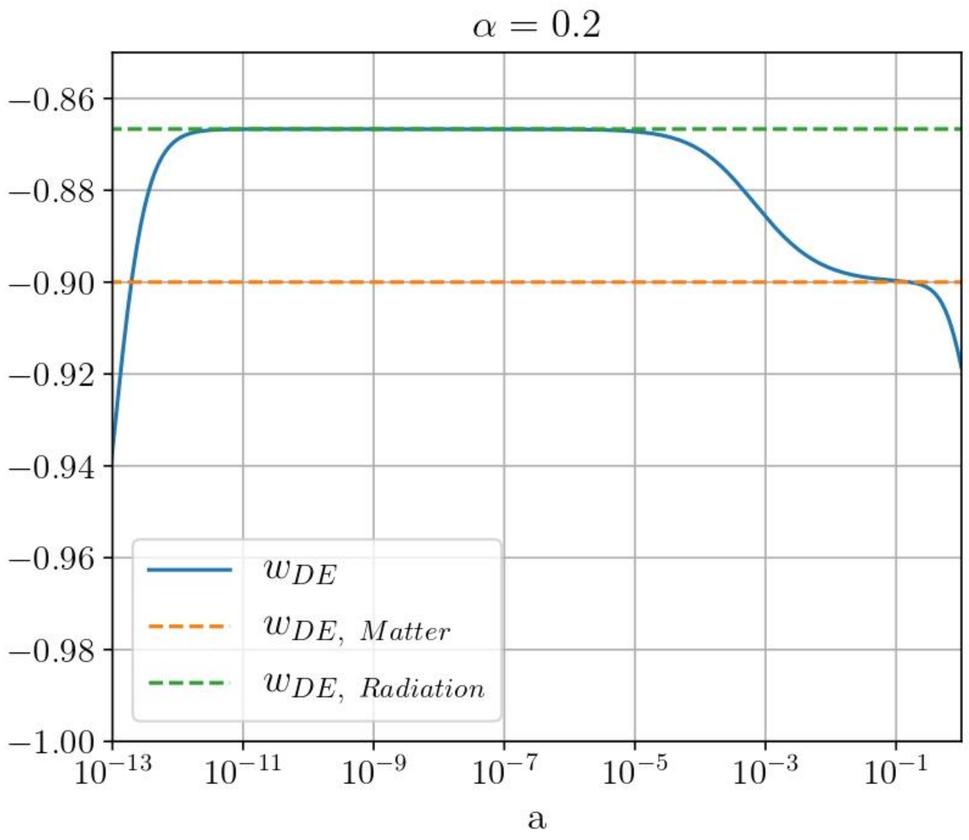
- K-essence model:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R + p(\phi, X)\right] + S_M,$$

$$p(\phi, X) = \frac{V_0}{\phi^\alpha}\left(-X + X^2\right)$$

Modified Gravity + Relativistic Effects





Massless neutrinos background

$$w_\phi = \frac{(1 + w_m)\,\alpha}{2} - 1$$

Modified Gravity + Relativistic Effects

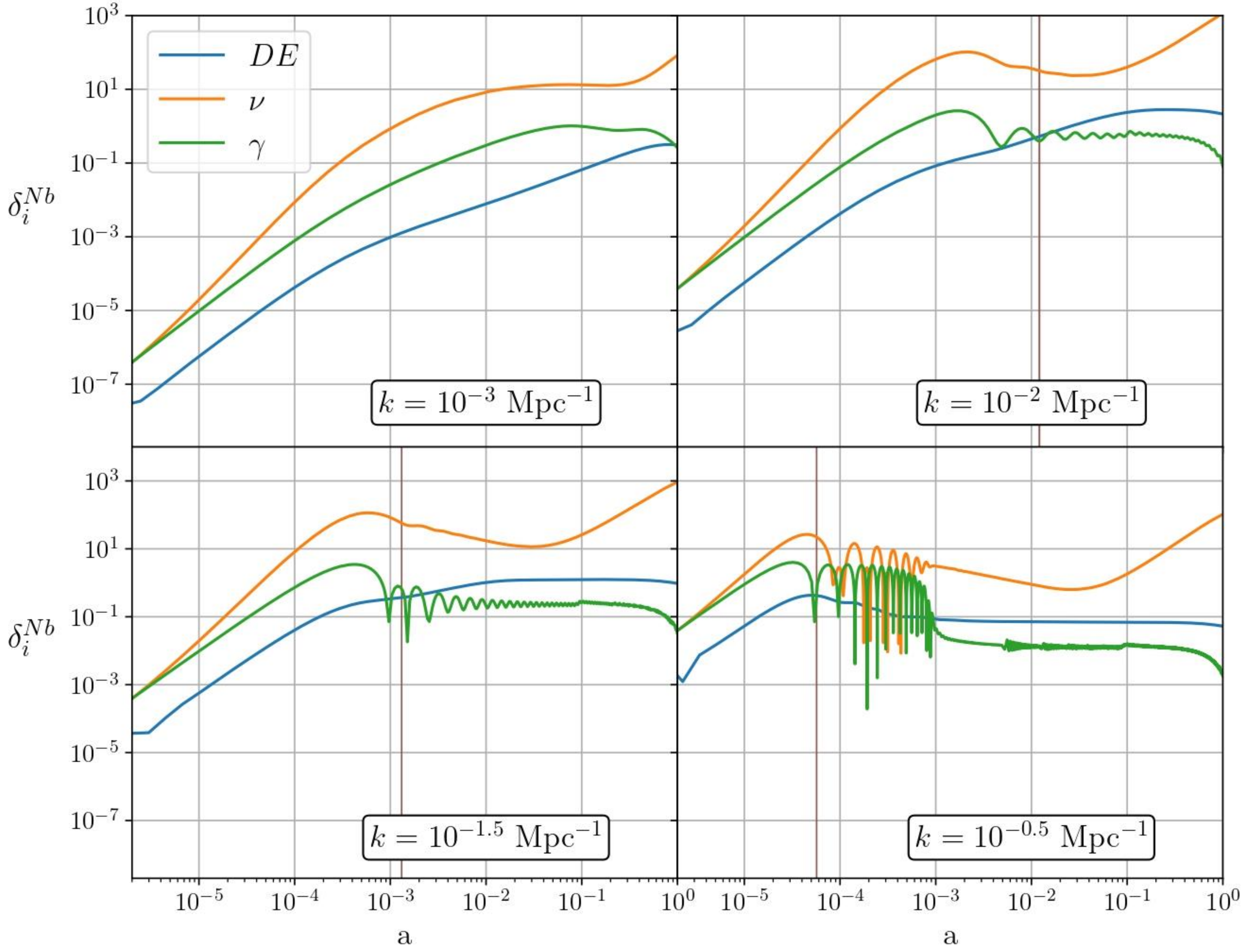
- Density perturbations in k-essence:

$$\delta\rho_{\text{k-ess.}} = -\frac{1}{3}H\Big\{aV_X\left[\alpha_{\rm K}H^2 + 9(p_m+\rho_m)\right] + 6V_XH' + \alpha_{\rm K}HV_X'\Big\},$$

$$\delta p_{\text{k-ess.}} = -\frac{2aH^3\lambda_6}{3\alpha_{\rm K}}V_X - \frac{1}{3}H^2\lambda_2V_X',$$

$$\theta_{\text{k-ess.}} = -k^2V_X.$$

$$V_X = a\frac{\delta\phi}{\phi'}$$

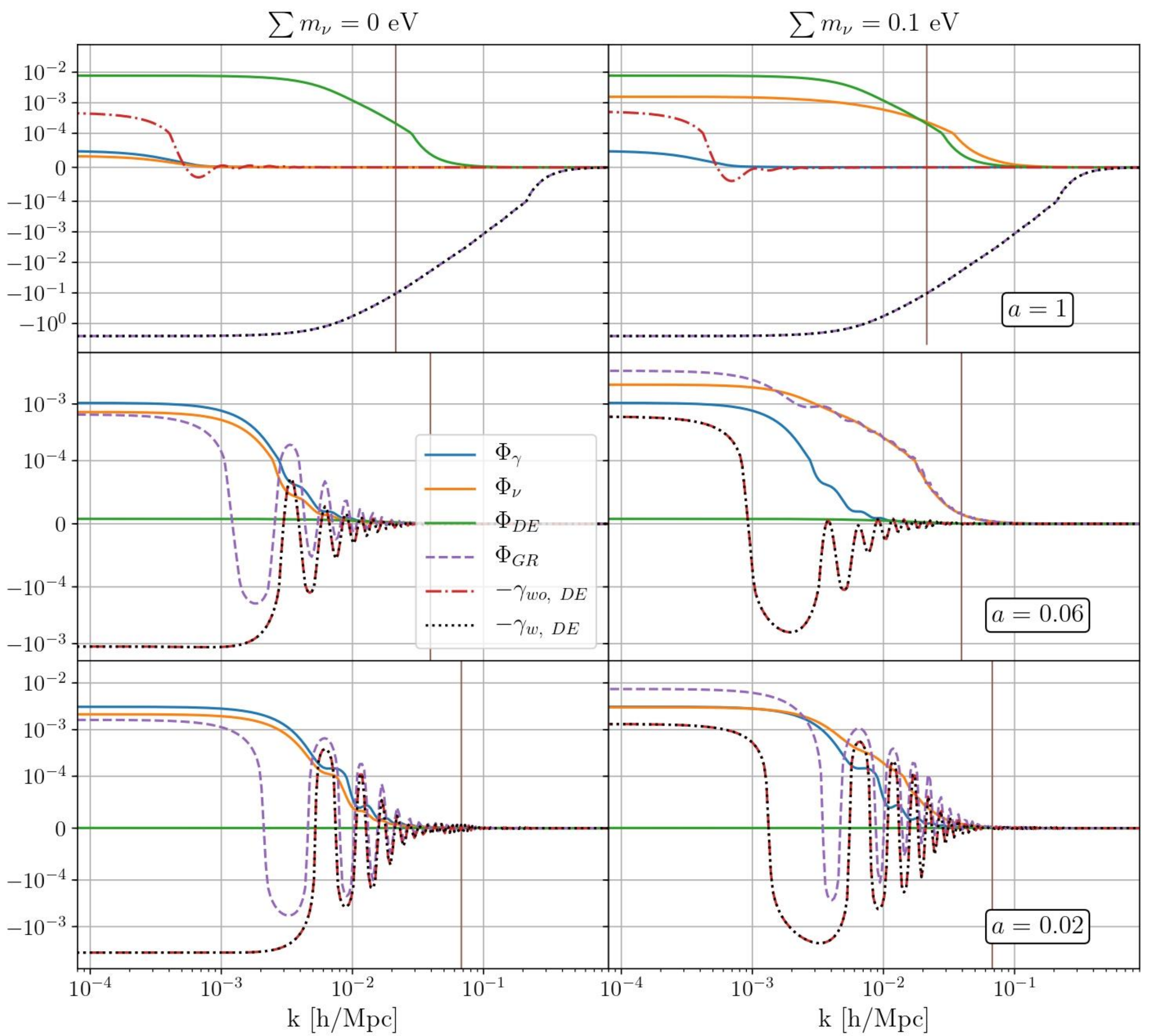


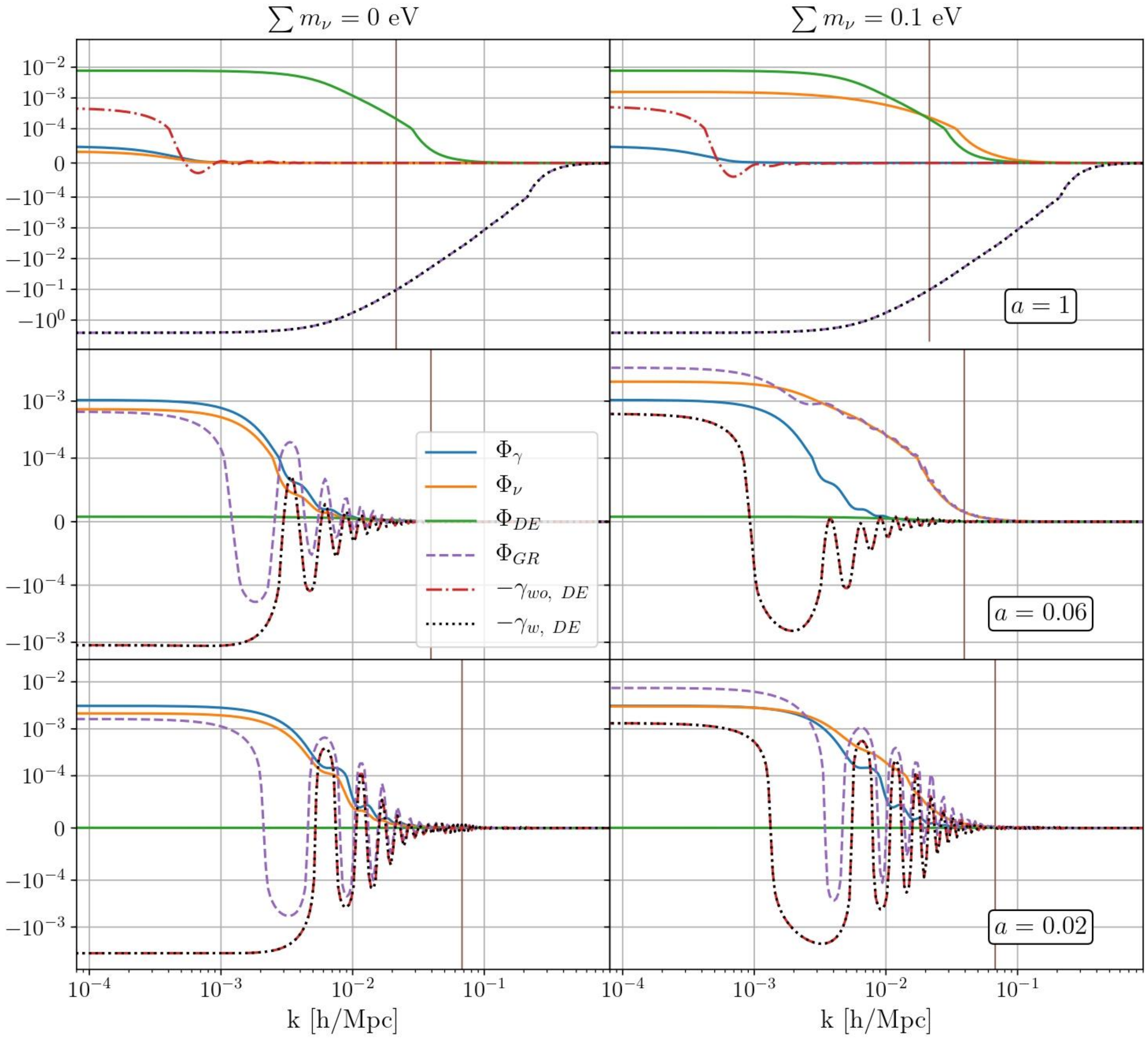
Modified Gravity + Relativistic Effects

- "Potential" for each relativistic spec

$$k^2\Phi_\alpha = 4\pi G a^2 \delta\rho_\alpha^{\rm Nb}$$

$$k^2\gamma = -(\partial_\tau + \mathcal{H})\dot{H}_T + 12\pi G a^2\left(\bar{\rho}+\bar{p}\right)\sigma.$$





$\sum m_\nu = 0$ eV
$\sum m_\nu = 0.1$ eV
Φ_γ
Φ_ν
Φ_{DE}
Φ_{GR}
$-\gamma_{wo,\ DE}$
$-\gamma_{w,\ DE}$
$a = 1$
$a = 0.06$
$a = 0.02$
k [h/Mpc]
k [h/Mpc]

Modified Gravity + Relativistic Effects

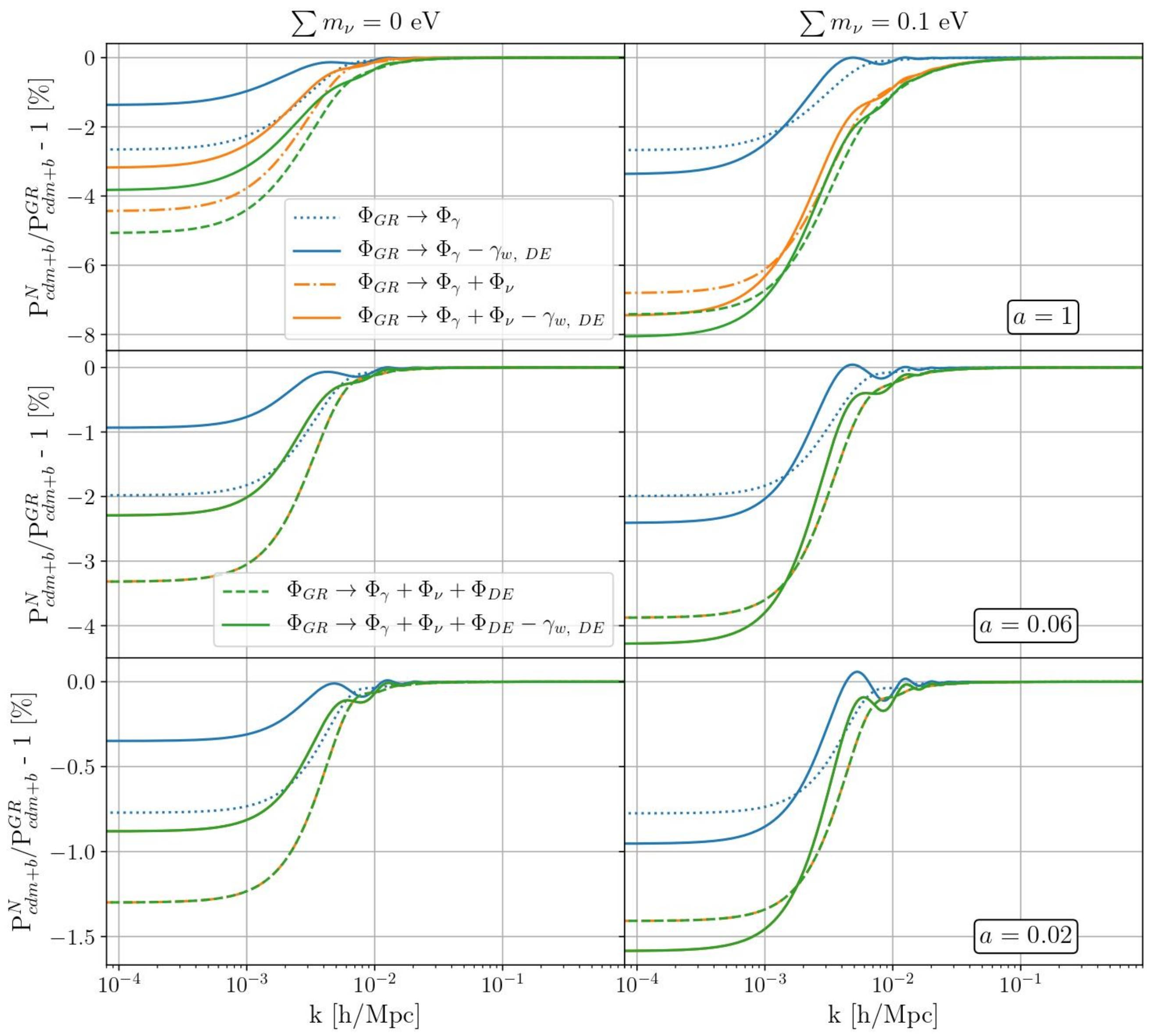
- Matter power spectrum:

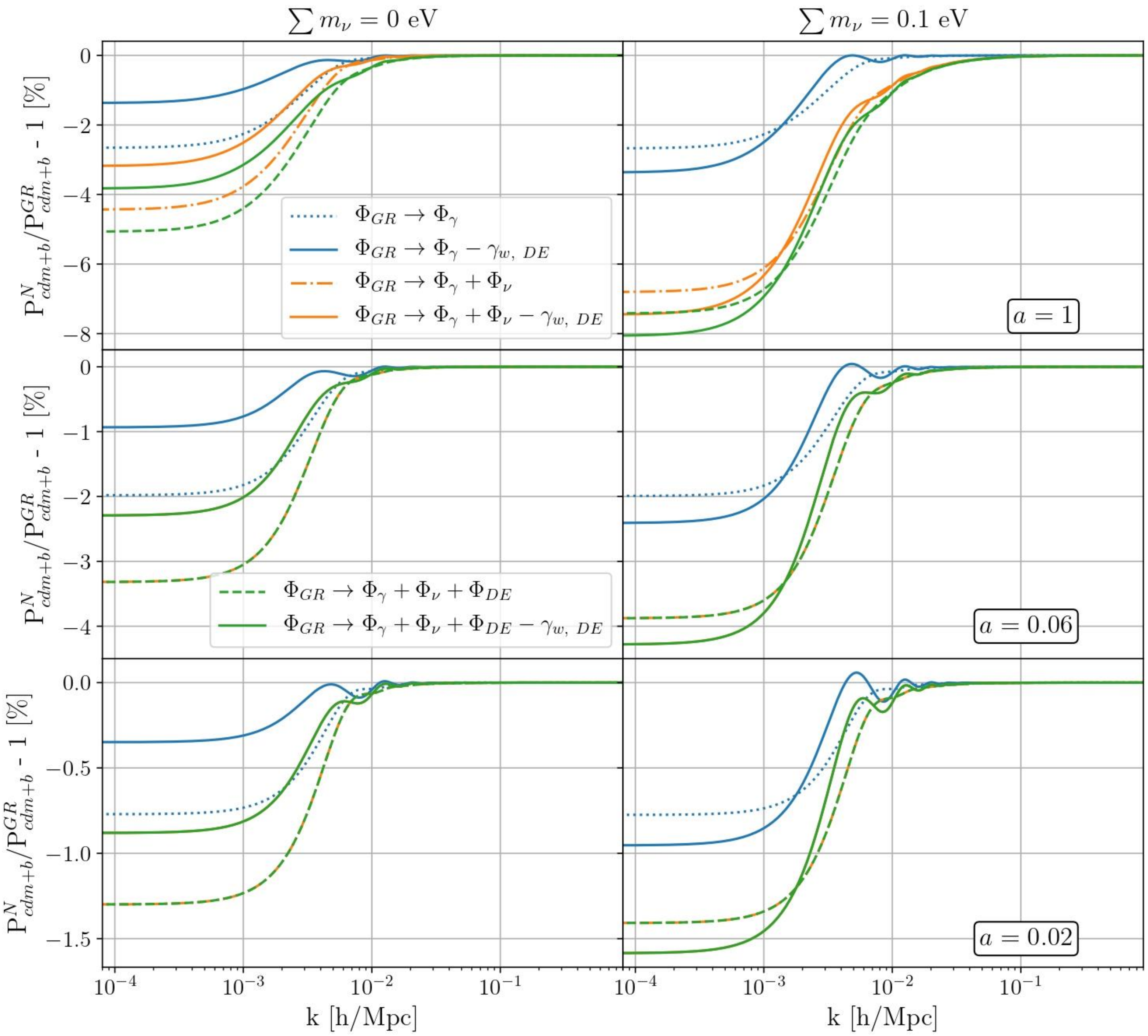
- P^{GR} solution of:

$$\ddot{\delta}^{\rm Nb} m + \mathcal{H}\dot{\delta}_m^{\rm Nb} - 4\pi G a^2 \rho_m \delta_m^{\rm Nb} = 4\pi G a^2 \delta\rho_{\rm GR}$$

- P^N solution of:

$$\ddot{\delta}_m + \mathcal{H}\dot{\delta}_m - 4\pi G a^2 \rho_m \delta_m = 0$$





$\sum m_\nu = 0$ eV
$\sum m_\nu = 0.1$ eV
$\mathrm{P}^N_{cdm+b}/\mathrm{P}^{GR}_{cdm+b}$ - 1 [%]
$\Phi_{GR} \to \Phi_\gamma$
$\Phi_{GR} \to \Phi_\gamma - \gamma_{w,\ DE}$
$\Phi_{GR} \to \Phi_\gamma + \Phi_\nu$
$\Phi_{GR} \to \Phi_\gamma + \Phi_\nu - \gamma_{w,\ DE}$
$\Phi_{GR} \to \Phi_\gamma + \Phi_\nu + \Phi_{DE}$
$\Phi_{GR} \to \Phi_\gamma + \Phi_\nu + \Phi_{DE} - \gamma_{w,\ DE}$
$a = 1$
$a = 0.06$
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k [h/Mpc]

Non-linear scales

- Gauge considerations:

1. The comoving synchronous gauge used in hi_class, as well as the comoving gauge temporal slicing, break down at non-linear scales.

2. In 1810.10835 a variant of the N-Body gauge was introduced,
in which the spatial threading is the same as the N-Body gauge,
but the temporal slicing is the same as the Poisson gauge. This choice of coordinates is called N-boisson gauge.

$$kB = \dot{H}_T$$

Non-linear scales

- Modified gravity considerations:

1. On the other side of non-linear scales, we have Modified Gravity.
As long as density perturbations are not so small,
we are safe to assume these perturbations will not contribute to the non-linear clustering of structures.

Non-linear scales

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As long as density perturbations are not so small,
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2. But what are small density perturbations in MG?
For k-essence the only parameter controlling the non-linear scales is the speed of sound of such perturbations c_s^2.
For general Horndeski models this is not so trivial!

Non-linear scales

- Modified gravity considerations:

2. But what are small density perturbations in MG?
For k-essence the only parameter controlling the non-linear scales is the speed of sound of such perturbations c_s^2.
For general Horndeski models this is not so trivial!

$$D\left(2-\alpha_{\mathrm{B}}\right)V_X'' + 8aH\lambda_7 V_X' + 2a^2H^2\left[\frac{c_{\mathrm{sN}}^2 k^2}{a^2H^2} - 4\lambda_8\right]V_X = \frac{2c_{\mathrm{sN}}^2}{aH}k^2\eta + \frac{3a}{2HM_*^2}\left[2\lambda_2\delta\rho_{\mathrm{m}} - 3\alpha_{\mathrm{B}}\left(2-\alpha_{\mathrm{B}}\right)\delta p_{\mathrm{m}}\right]$$

Matter density perturbations feed scalar perturbations already at linear order!

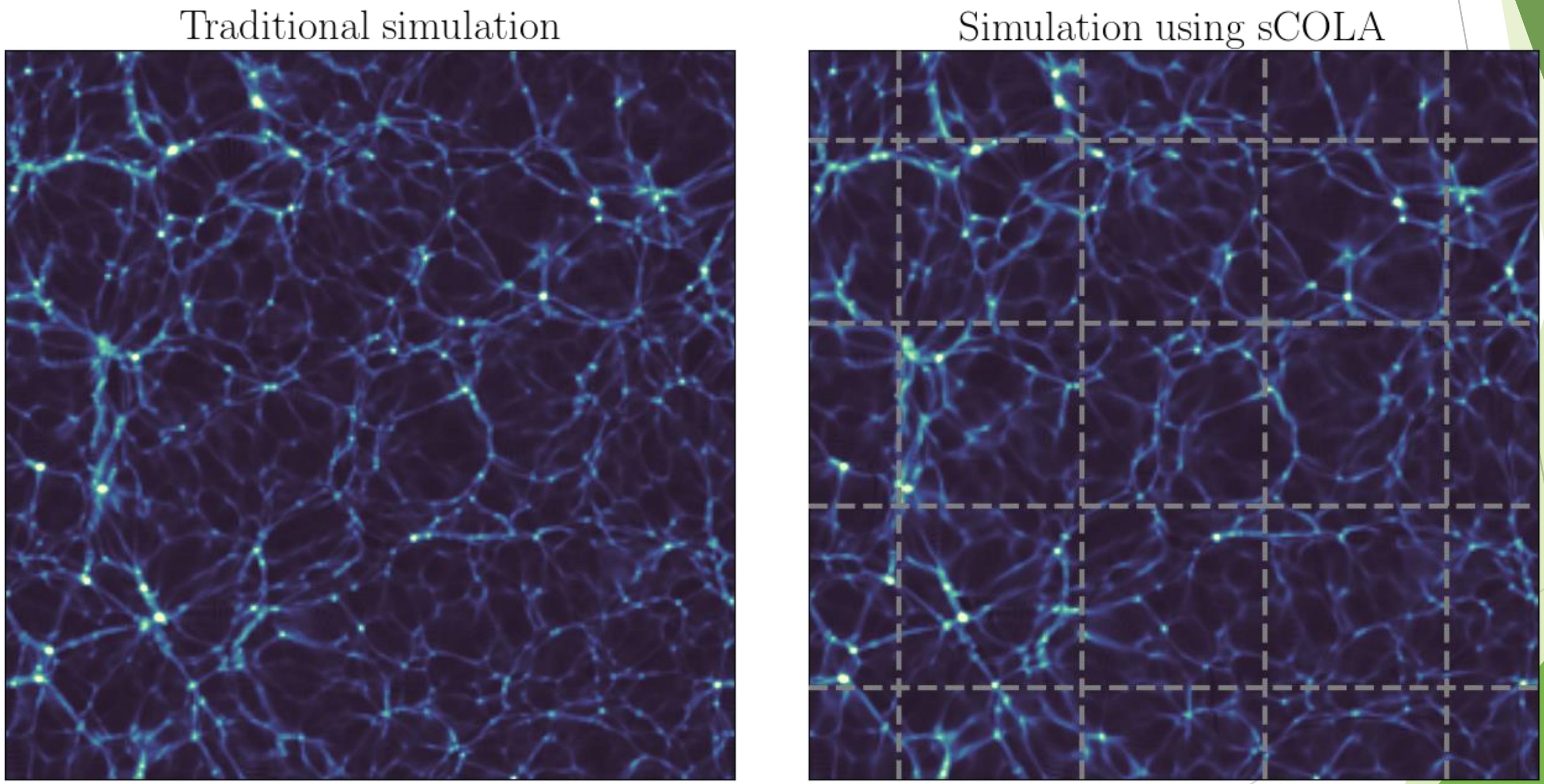
Non-linear scales

- Modified gravity considerations:

3. We’re currently implementing on hi_class the general case of Horndeski theories, and with this we can add consistency checks, that can flag out density perturbations too small.

4. Irrespective of the “smallness” of the density perturbation in MG, we must have a concrete desctription of these theories in the mildly non-linear scales.
In order to do so, one must then rely on the Quasi-static Approximation (QSA) and in an accurate screening mechanism (model dependent).
This is already implemented in approximate methods, such as COLA.

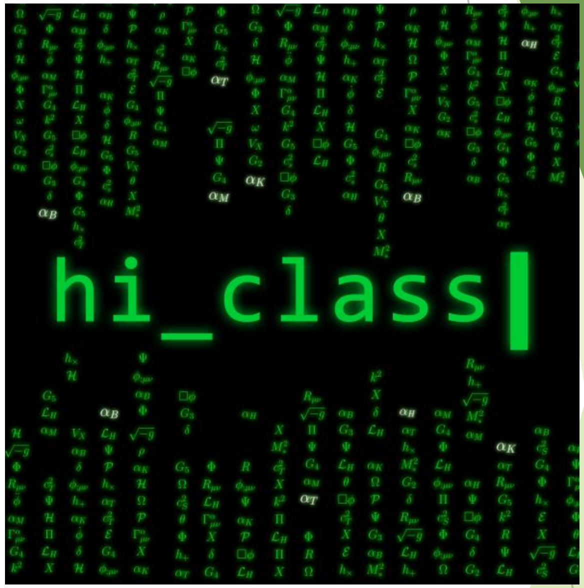
Non-linear scales



http://www.florent-leclercq.eu/index.php

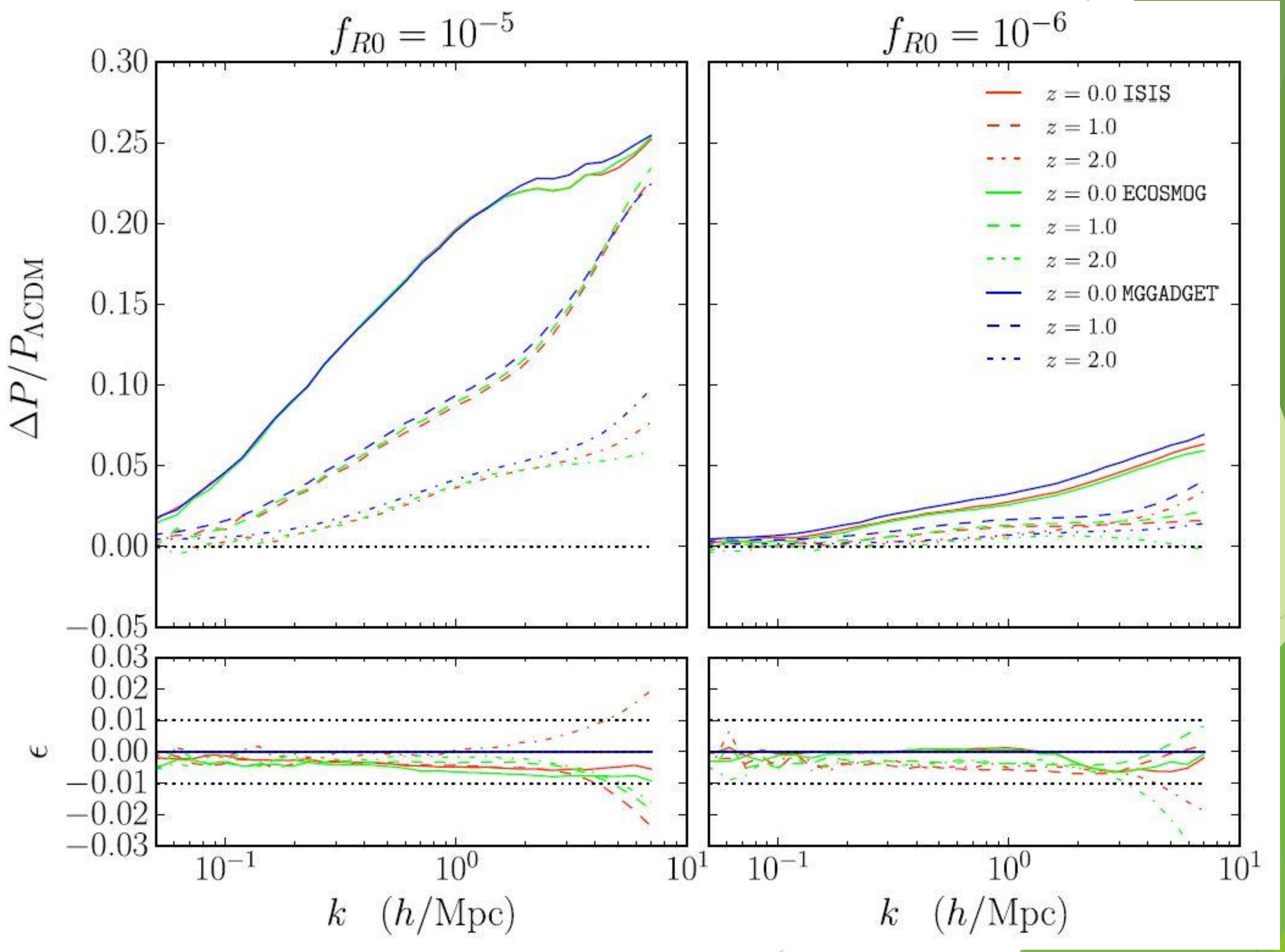
Way forward

- Implement the general case in hi_class



Way forward

- Go to non-linear scales



Way forward

- Investigate relativistic effects on Modified Gravity simulations

Obrigado!