

Quantum Chromodynamics (QCD)

— Still an Enigma in Many Ways

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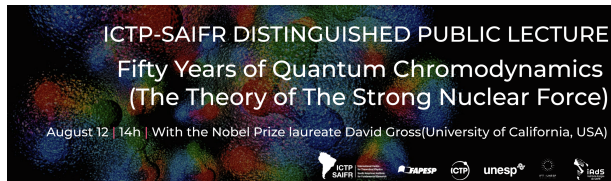
Quantum chromodynamics (Q C D)

- The theory of the strong force, a quantum field theory
- SU(3) gauge theory
- Conserved charge: color
- Degrees of freedom: quarks (spin 1/2) and gluons (spin 1)

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4}G_{\mu\nu}^a(x)G^{a\mu\nu}(x) + \bar{q}(x) \left[i\gamma^\mu (\partial_\mu - gT^a A_\mu^a) - m \right] q(x)$$

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - gf^{abc}A_\mu^b A_\nu^c$$

2023 – QCD's 50th anniversary



50 Years of Quantum Chromodynamics

#

Franz Gross (Jefferson Lab and William-Mary Coll.), Eberhard Klempt (Bonn U., HISKP), Stanley J. Brodsky (SLAC), Andrzej J. Buras (TUM-IAS, Munich), Volker D. Burkert (Jefferson Lab) et al. (Dec 21, 2022)

Published in: *Eur.Phys.J.C* 83 (2023) 1125 • e-Print: [2212.11107](#) [hep-ph]

50 Years of Quantum Chromodynamics

11 – 15 de set. de 2023
Luskin Conference Center
Fuso horário US/Pacific

Quarks and gluons?

Hadron spectroscopy: substructure, Gell-Mann (quarks) and about the same time Zweig (aces)

Quote from Leutwyler (in 50 years of QCD):

... but gave rise to a fundamental puzzle: why do the constituents not show up in experiment? For this reason, the existence of the quarks was considered doubtful: “Such particles [quarks] presumably are not real but we may use them in our field theory anyway ...” [19]. Quarks were treated like the veal used to prepare a pheasant in the royal french cuisine: the pheasant was baked between two slices of veal, which were then discarded (or left for the less royal members of the court). Conceptually, this was a shaky cuisine.

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Moreover, the mood toward a quantum field theory description of the strong interaction was not that great (S-matrix, bootstrap, ...), but the skepticism dissipated rapidly.

How skepticism dissipated:

- Asymptotic freedom (perturbation theory)

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- Discovery of J/ψ and ψ' (november revolution)

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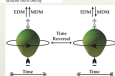
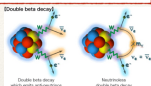
- Asymptotic freedom (perturbation theory)
- Discovery of J/ψ and ψ' (november revolution)
- Discovery of gluons (jets)

How skepticism dissipated:

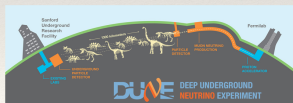
- Asymptotic freedom (perturbation theory)
- Discovery of J/ψ and ψ' (november revolution)
- Discovery of gluons (jets)
- Lattice QCD (nonperturbative computations)

Who cares?

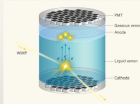
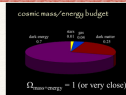
Why is the universe composed of matter? (and not anti-matter)



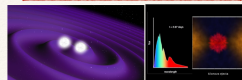
What are the properties of neutrinos?



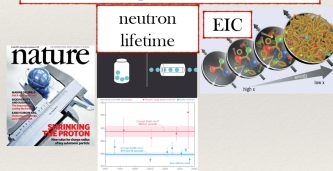
Does dark matter interact with matter? (beyond gravitationally)



What are the properties of dense nuclear matter?



What are the properties of the proton?



Light-hadron masses

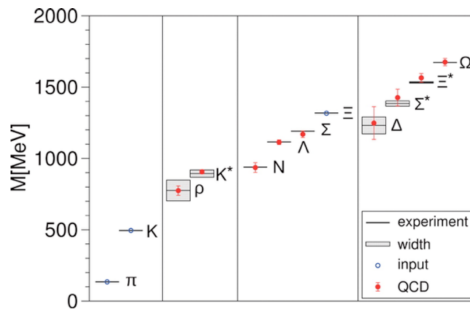
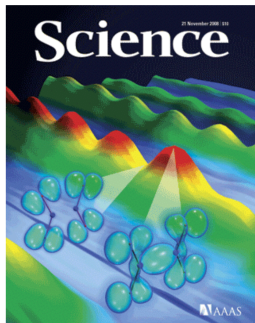
Science

2008

Ab Initio Determination of Light Hadron Masses

S. Dür, Z. Fodor, J. Frison, C. Hoelbling, R. Hoffmann, S. D. Katz, S. Krieg, T. Kurth, L. Lellouch, T. Lippert, K. K. Szabo and G. Vulvert

Science **322** (5905), 1224-1227.
DOI: 10.1126/science.1163233



Hadron-mass differences

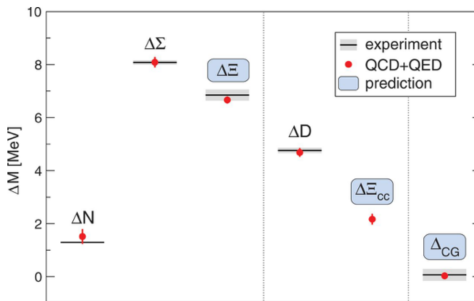
Science

2015

Ab initio calculation of the neutron-proton mass difference

Sz. Borsanyi, S. Durr, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, L. Lellouch, T. Lippert, A. Portelli, K. K. Szabo and B. C. Toth

Science **347** (6229), 1452-1455.
DOI: 10.1126/science.1257050



Nucleon weak axial charge

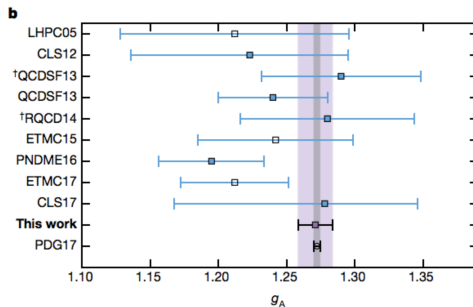


2018

A per-cent-level determination of the nucleon axial coupling from quantum chromodynamics

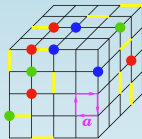
C. C. Chang, A. N. Nicholson, E. Rinaldi, E. Berkowitz, N. Garron, D. A. Brantley, H. Monge-Camacho, C. J. Monahan, C. Bouchard, M. A. Clark, B. Joó, T. Kurth, K. Orginos, P. Vranas & A. Walker-Loud

Nature **558**, 91–94 (2018)



Lattice QCD

Lattice QCD



The **lattice** formulation maps QCD into a (classical) **statistical mechanical model**, which may be studied from first principles by Monte Carlo simulation
⇒ Different approach to QFT, direct access to (representative) gauge-field configurations

Current importance for predictions and data analyses in Particle Physics and Nuclear Physics ⇒ `hep-lat` section, 36th Annual Lattice Symposium, ...)

Possibility to understand **qualitative features of QCD**, e.g. color confinement **nonperturbatively**

One of the challenges in the simulations is to consider **large volumes** to reach the **IR limit**

Lattice hadron mass computation

$h(x)$: hadron interpolating field e.g. $\pi^+(x) = \bar{d}(x)\gamma_5 u(x)$

$$\langle h(x)h^\dagger(x+T) \rangle = \frac{\int [\mathcal{D}\psi\bar{\psi}A_\mu] h(x)h^\dagger(x+T) e^{-\int d^4x \mathcal{L}_{\text{QCD}}}}{\int [\mathcal{D}\psi\bar{\psi}A_\mu] e^{-\int d^4x \mathcal{L}_{\text{QCD}}}}$$

$$\lim_{T \rightarrow \infty} \langle h(x)h^\dagger(x+T) \rangle \sim e^{-M_h T}$$

Quark gluon confinement

- Unsolved problem

Quark gluon confinement

- Unsolved problem
- Lattice QCD: falsified flux-tube paradigm

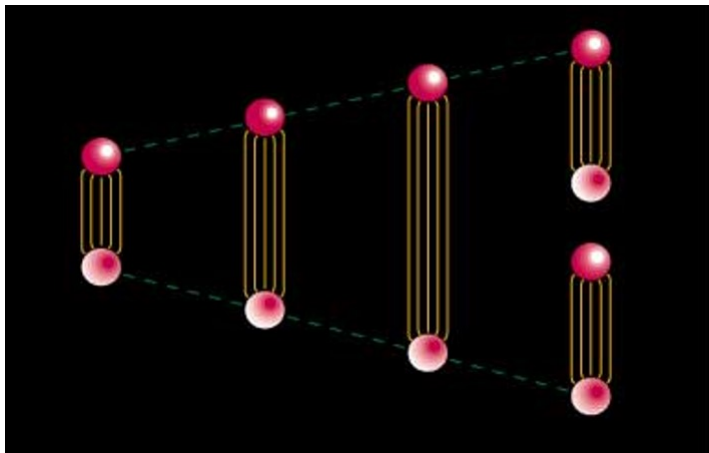
Quark gluon confinement

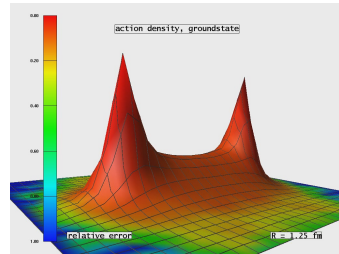
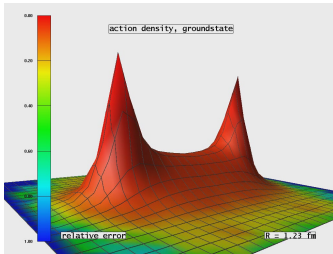
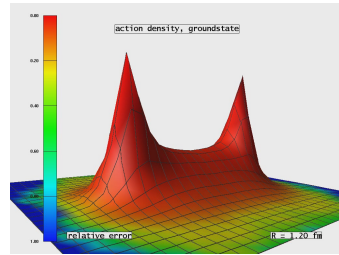
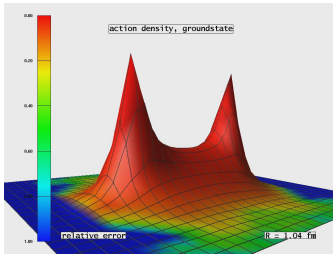
- Unsolved problem
- Lattice QCD: falsified flux-tube paradigm
- Lattice QCD: might have identified its origin

Quark gluon confinement

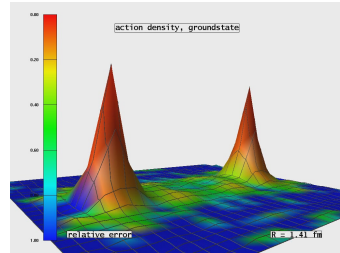
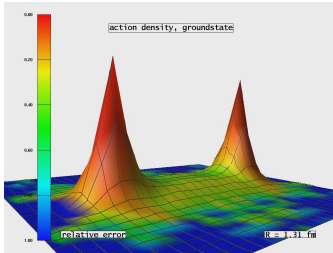
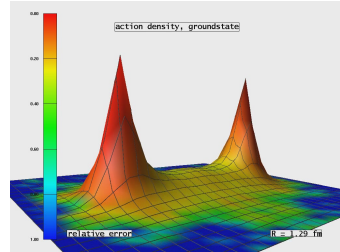
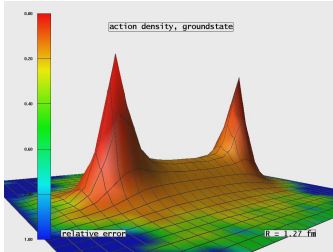
- Unsolved problem
- Lattice QCD: falsified flux-tube paradigm
- Lattice QCD: might have identified its origin
- Lattice QCD & modeling: spectral representation

Flux tubes break





Figs. from G. Bali et al.



Figs. from G. Bali et al.

Breaking scales

Lattice results - static sources*

- Energy stored: $E_\sigma = 1.1 \text{ GeV/fm}$
- Breaking distance r_c :
 - $m_\pi = 280 \text{ MeV}$: $r_c = 1.2 \text{ fm}$
 - $m_K = 460 \text{ MeV}$: $r_c = 1.3 \text{ fm}$

Nonstatic sources:

$$m_\pi = 140 \text{ MeV} \rightarrow r_c = \frac{280 \text{ MeV}}{1.1 \text{ GeV/fm}} < 1/3 \text{ fm}$$

Breaking occurs within a hadron

(No flux tubes in QCD?!)

Important gluon configurations

Center vortices (associated with the phases of the center of $SU(3)$ gauge group) seem to be responsible for most of the nonperturbative long-range physics*.

Removing them:

- Pure Yang-Mills: no flux tubes
- Dynamical quarks:
 - no hadron mass generation
 - reflection positivity violation

Difficult to model these configurations

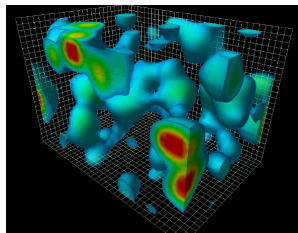
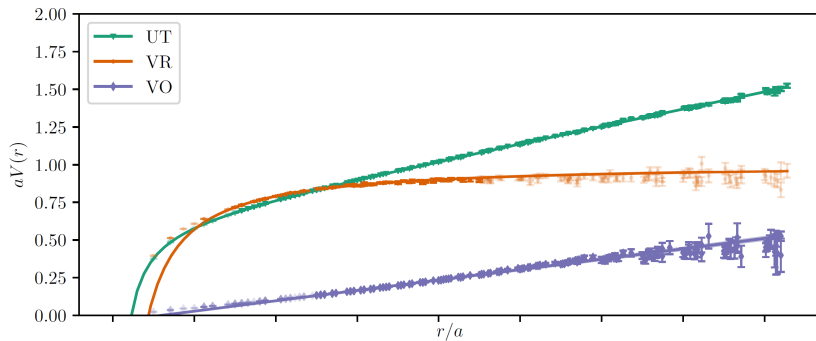


Fig. from D. Leinweber et al.

*'t Hooft, Greensite, Faber, Langfeld, Olejník, Oxman, Reinhardt, Rosa Jr., ...

No vortices, no flux tubes*

Pure Yang-Mills



J. C. Biddle, W. Kamleh, D. B. Leinweber, PRD 106, 054505

Continuum: models

— Positivity violation

Example: scalar field $\varphi(x)$

$$-i\langle 0|T[\varphi(x)\varphi(y)]|0\rangle = \Delta(x-y) = \int \frac{d^4x}{(2\pi)^4} e^{ip\cdot(x-y)} \Delta(p)$$

— Spectral representation

$$\Delta(p) = \int_0^\infty d\sigma^2 \frac{\rho(\sigma^2)}{p^2 - \sigma^2 + i\epsilon}$$

$$\rho(p^2) = (2\pi)^4 \sum_n \delta^{(4)}(p - p_n) |\langle 0|\varphi(0)|n\rangle|^2, \quad P^\mu|n\rangle = p_n^\mu|n\rangle, \quad p_n^0 > 0$$

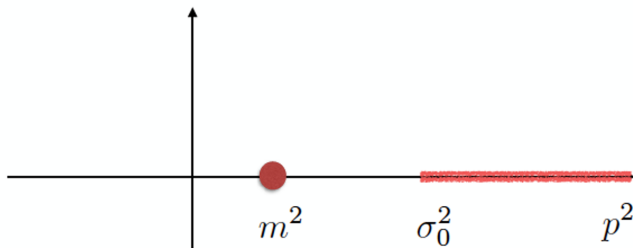
— Positivity (probability interpretation)

$$\rho(p^2) \geq 0$$

QFT theorem

— Herglotz property

The positivity of the spectral function implies that propagators have no zeros or poles off the real axis



Lattice & Continuum methods

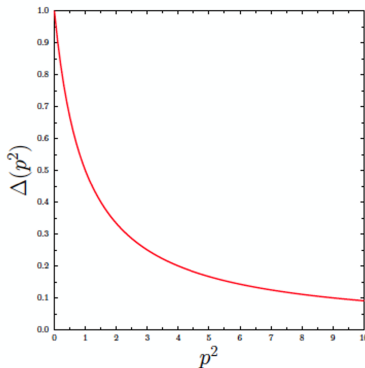
— Euclidean space

- Schwinger functions

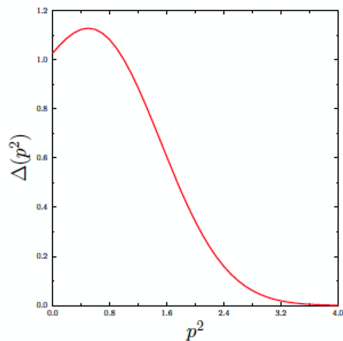
$$\Delta(p^2) = \int_0^\infty d\sigma^2 \frac{\rho(\sigma^2)}{p^2 + \sigma^2}$$

- If $\rho(\sigma^2) \geq 0$, then:

$$\frac{d\Delta(p^2)}{dp^2} = - \int_0^\infty d\sigma^2 \frac{\rho(\sigma^2)}{(p^2 + \sigma^2)^2} \leq 0$$



If $\Delta(p^2)$ looks like this



then $\rho(\sigma^2) \leq 0$ for some σ^2

Positivity violation, not a physical particle

First lattice result

RAPID COMMUNICATIONS

PHYSICAL REVIEW D 67, 091502(R) (2003)

$SU(2)$ Landau gluon propagator on a 140^3 lattice

Attilio Cucchieri* and Tereza Mendes†

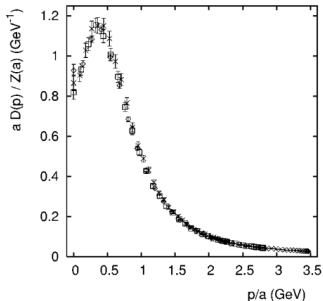
Instituto de Física de São Carlos, Universidade de São Paulo, C.P. 369, 13560-970 São Carlos, SP, Brazil

Andre R. Taurines‡

*Instituto de Física, Universidade Federal do Rio Grande do Sul, Av. Bento Gonçalves, 9500, Campus do Vale,
91501-970 Porto Alegre, RS, Brazil*

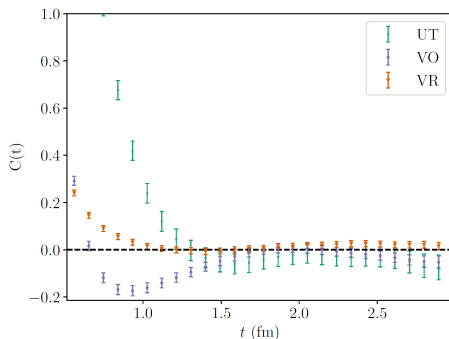
(Received 27 February 2003; published 19 May 2003)

We present a numerical study of the gluon propagator in the lattice Landau gauge for three-dimensional pure $SU(2)$ lattice gauge theory at couplings $\beta=4.2, 5.0, 6.0$ and for lattice volumes $V=40^3, 80^3, 140^3$. In the limit of large V we observe a decreasing gluon propagator for momenta smaller than $p_{dec}=350^{+100}_{-50}$ MeV. Data are well fitted by Gribov-like formulas and seem to indicate an infrared critical exponent κ slightly above 0.6, in agreement with recent analytic results.



Latest lattice result*

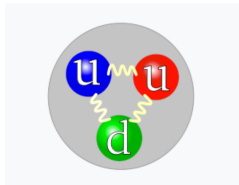
$$C(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dp_0 \int_0^{\infty} d\sigma^2 \frac{\rho(\sigma^2)}{p_0^2 + m^2} e^{-ip_0 t} = \int_0^{\infty} d\sigma e^{-\sigma t} \rho(\sigma^2)$$



*J. C. Biddle, W. Kamleh, D. B. Leinweber, PRD 107, 094507(2023)

Emergence of visible matter

What is the origin of the mass of protons and neutrons (nucleons)?



Computers gave an answer to the question



Nucleon mass comes from
quarks and gluons

Yet, we are not satisfied

We want to know more:

How did it happen?*

*F. Wilczek, *The lightness of being: Mass, ether, and the unification of forces*
(Basic Books, 2008)

Back ~ 40 years

- $|h(\mathbf{p})\rangle$: hadron state*, $p = (E_h(\mathbf{p}), \mathbf{p})$
- $\langle h(\mathbf{p})|T^{\mu\nu}(x)|h(\mathbf{p})\rangle = p^\mu p^\nu / E_h(\mathbf{p})$, $T^{\mu\nu}(x)$: en.-mom. tensor
- $\langle h(\mathbf{p})|T^\mu_\mu(x)|h(\mathbf{p})\rangle = p^\mu p_\mu / E_h(\mathbf{p}) = m_h^2 / E_h(\mathbf{p})$
- Take $m_{\text{light}} = 0$ and $m_{\text{heavy}} = \infty$ in QCD Lagrangian:

Classical action is scale invariant: $x^\mu \rightarrow \lambda x^\mu$

Conserved current: $\partial_\mu J_D^\mu(x) = 0$ where $J_D^\mu(x) = x_\nu T^{\mu\nu}(x)$

Since $\partial_\mu T^{\mu\nu}(x) = 0 \rightarrow \partial_\mu J_D^\mu(x) = 0 \rightarrow T^\mu_\mu(x) = 0 \Rightarrow m_h = 0$

*Normalized such that expectation value of T^{00} gives the hadron energy

Back ~ 40 years - cont'd

- Quantum action **IS NOT** scale invariant
- Needs regularization, introduces length scale

$$\alpha_s = g^2/4\pi \xrightarrow{\text{reg.}} \alpha_s(\mu)$$

$$T^\mu_\mu(x) = \frac{\beta(\alpha_s)}{2\alpha_s} G^a_{\mu\nu}(x) G^{a\mu\nu}(x)$$

This is the trace anomaly

Trace anomaly - Nucleon mass

$$m_N = \frac{\beta(g)}{2g} \langle N | G_{\mu\nu}^a G^{a\mu\nu} | N \rangle + \sum_{l=u,d,s} \langle N | m_l (1 + \gamma_{m_l}) \bar{q}_l q_l | N \rangle$$

$\Downarrow$
 $\simeq 880 \text{ MeV}$

$\Downarrow$
 $\simeq 60 \text{ MeV } (\sigma\text{-term})$

$$\beta(g) \simeq -b \frac{g^3}{16\pi^2}, \quad b = 11 - \frac{2n_l}{3} \quad (\text{heavy quarks integrated out})$$

Experimental access to $\langle N | G_{\mu\nu}^a G^{a\mu\nu} | N \rangle$?

Inequality (almost saturated, chromomagnetic part suppressed)*:

$$-\frac{1}{2} \langle N | g^2 G_{\mu\nu}^a(x) G^{a\mu\nu}(x) | N \rangle = \langle N | [(g\mathbf{E}^a)^2 - (g\mathbf{B}^a)^2] | N \rangle \leq \langle N | (g\mathbf{E}^a)^2 | N \rangle$$

Shifman, Vainshtein & Zakharov (1978)

*Sibirtsev & Voloshin (2005)

How about the pion?

When $m_{\text{light}} = 0 \rightarrow m_{\pi} = 0$

(pion is a Goldstone boson)

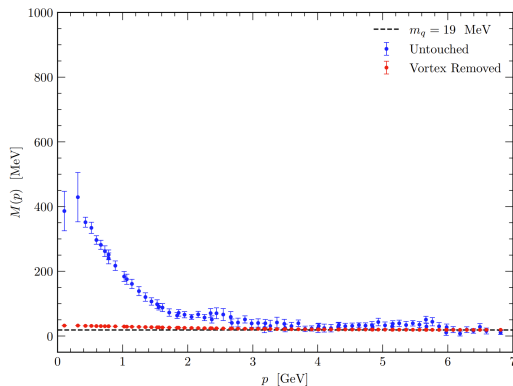
$$\frac{\beta(\alpha_s)}{2\alpha_s} \langle N(\mathbf{p}) | G_{\mu\nu}^a(x) G^{a\mu\nu}(x) | N(\mathbf{p}) \rangle = \frac{m_N^2}{E_N(\mathbf{p})} \neq 0$$

$$\frac{\beta(\alpha_s)}{2\alpha_s} \langle \pi(\mathbf{p}) | G_{\mu\nu}^a(x) G^{a\mu\nu}(x) | \pi(\mathbf{p}) \rangle = \frac{m_{\pi}^2}{E_N(\mathbf{p})} = 0$$

How does this happen?

No vortices, no hadron mass generation*

Dynamical Fermions

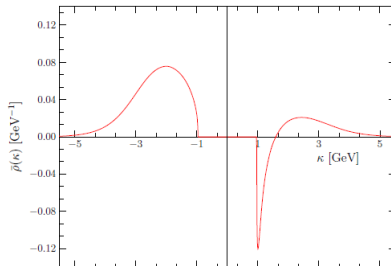


Model study - 2019

Few-Body Syst (2019) 60:49
<https://doi.org/10.1007/s00601-019-1517-9>

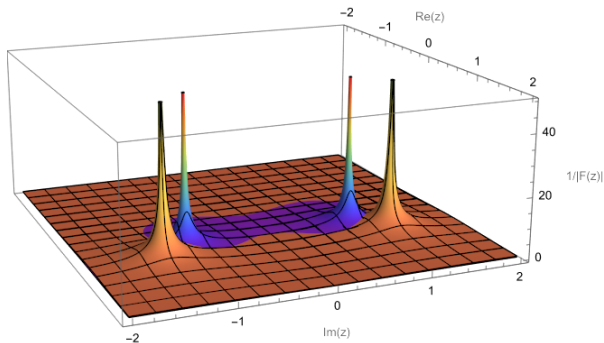
E. L. Solis · C. S. R. Costa · V. V. Luiz · G. Krein 

Quark Propagator in Minkowski Space



Just to mud the water a little bit

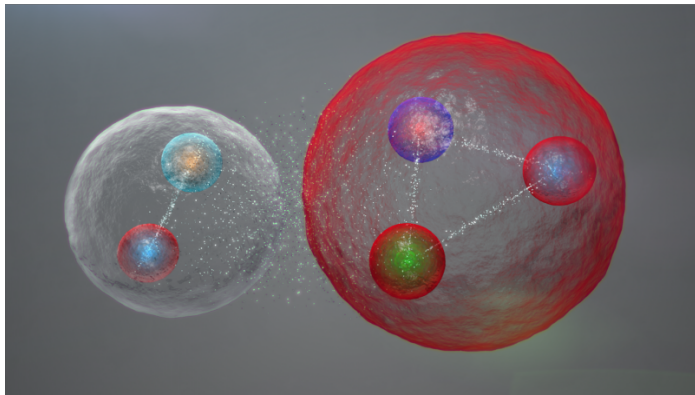
Linear-sigma model: finite temperature*



*A.M.A. Silva, *Complex Fermion Mass Poles in the Linear Sigma Model at Finite Temperature*, MSc IFT (2024)

Experiment: quarkonium-nucleon scattering

Access to trace anomaly matrix element: $\langle N | (gE^a)^2 | N \rangle$
(small QN relative momentum)



Quarkonium: $\underbrace{\phi(s\bar{s}), \eta_c(c\bar{c})}_{\text{light}}, \underbrace{J/\psi(c\bar{c}), \eta_b(b\bar{b}), \Upsilon(b\bar{b})}_{\text{heavy}}$

Gluon mass and center-vortices*

—An analytic computation

Hamiltonian formalism: vacuum state (vortices+monopoles)

$$\Psi(A, \zeta) = \langle A, \eta | \Psi \rangle = \sum_{\{\gamma\}} \psi_{\{\gamma\}} \delta(A - a(\{\gamma\})) \delta(\zeta - (-\Delta)^{-1} \nabla b(\{\gamma\}))$$

- sum is over all vortex networks $\{\gamma\}$ with weight $\psi_{\{\gamma\}}$
- $a(\{\gamma\})$, $b(\{\gamma\})$: vortex color electric and magnetic gauge potentials
- static correlation function:

$$\langle \Psi | \tilde{b}_i(\vec{k}) \tilde{b}_j(\vec{k}') | \Psi \rangle \sim \left(\delta_{ij} - \frac{k_i k_j}{\vec{k}^2 + m^2} \right),$$

- m^2 : **generated mass**, depends on parameters in $\psi_{\{\gamma\}}$, fit lattice data on Wilson loop

* David Rosa Jr., GK, Luis Oxman, Bruno Soares

My bet for the future:

PRX QUANTUM 5, 037001 (2024)

Roadmap

Quantum Computing for High-Energy Physics: State of the Art and Challenges

Alberto Di Meglio^{1,*}, Karl Jansen^{2,3,†}, Ivano Tavernelli^{4,‡}, Constantia Alexandrou^{3,5},
Srinivasan Arunachalam⁶, Christian W. Bauer⁷, Kerstin Borras^{8,9}, Stefano Carrazza^{1,10},
Arianna Crippa^{2,11}, Vincent Croft¹², Roland de Putter⁶, Andrea Delgado¹³, Vedran Dunjko¹²,
Daniel J. Egger⁴, Elias Fernández-Combarro¹⁴, Elina Fuchs^{1,15,16}, Lena Funcke¹⁷,
Daniel González-Cuadra^{18,19}, Michele Grossi¹, Jad C. Halimeh^{20,21}, Zoë Holmes²²,
Stefan Kühn², Denis Lacroix²³, Randy Lewis²⁴, Donatella Lucchesi^{1,25},
Miriam Lucio Martinez^{26,27}, Federico Meloni⁸, Antonio Mezzacapo⁶, Simone Montangero^{1,25},
Lento Nagano²⁸, Vincent R. Pascuzzi⁶, Voica Radescu²⁹, Enrique Rico Ortega^{30,31,32,33},
Alessandro Roggero^{34,35}, Julian Schuhmacher⁴, Joao Seixas^{36,37,38}, Pietro Silvi^{1,25},
Panagiotis Spentzouris³⁹, Francesco Tacchino⁴, Kristan Temme⁶, Koji Terashi²⁸, Jordi Tura^{12,40},
Cenk Tüysüz^{2,11}, Sofia Vallecorsa¹, Uwe-Jens Wiese⁴¹, Shinjae Yoo⁴², and Jinglei Zhang^{43,44}

White paper on

Quantum Information Science and Technology for Nuclear Physics

Input into U.S. Long-Range Planning, 2023

Motivation

Why bring quantum information science
to QFT and QCD in particular?

- Offers wider perspective on the quantumness of QFT
- Builds on quantities different from the traditional correlation functions and scattering amplitudes
- Brings in new concepts, such as entanglement and complexity
- Opens new simulation paradigms, as analog and digital quantum computing

Quantum simulation

Prominent QCD phenomena unreachable with
current lattice simulations

- Finite baryon density (sign problem)
- Real time dynamics
- Time-like hadron form factors
- Quark-gluon fragmentation, inelastic processes

Beyond QCD: quantum gravity, condensed matter, ...

High stakes and a lot of hype

Quantum Computing: a future perspective for scientific computing

Quantum Computing

Karl Jansen

Cairns, The XVIth Quark Confinement and
the Hadron Spectrum Conference, 19.8.2024



QC for high energy, nuclear and condensed matter physics

Leaving the era of 1+1 dimensional toy models

Today

- > real time simulation
 - **thermalization**
 - **scattering**
 - **quenching**
- > various **spin models**
- > thermal field theory
- > ...



- > real time simulation
 - string breaking, formation of bound states
- > collisions, **scattering**
- > **non-perturbative** renormalization
- > ...

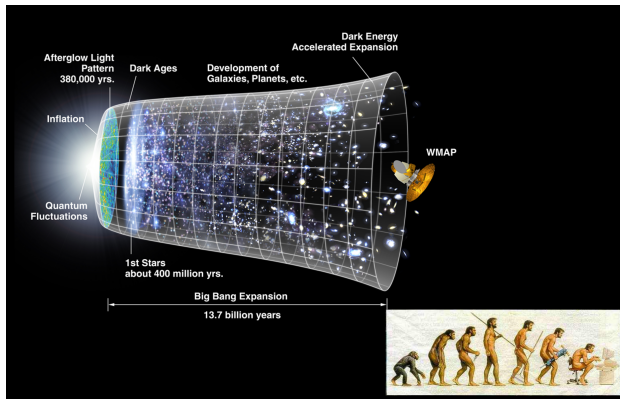


- > simulations of **heavy ion collisions**
- > **quark gluon plasma**
- > **nuclear** physics
- > multi-Higgs models
- > conformal field theories
- > ...

⇒ **Completely new insight in condensed matter and high energy physics!**

Conclusion

QCD is omnipresent in the Universe
and we must understand how it works



Thank you

Funding

