

Dark Matter viewed as cosmic gluonic background

(Cosmogenic implications of de Sitter, Anti de Sitter and Poincaré symmetries)

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J.-P.G., *Mass in de Sitter and Anti-de Sitter Universes with Regard to Dark Matter. Universe 2020, 6, 66;*

G. Cohen-Tannoudji (U. Paris Saclay & CEA) and J.-P. G.

Cold Dark Matter : A Gluonic Bose-Einstein Condensate in Anti-de Sitter Space Time

Universe 2021,7,402 & arXiv :2111.01130v2 [gr-qc] and references therein

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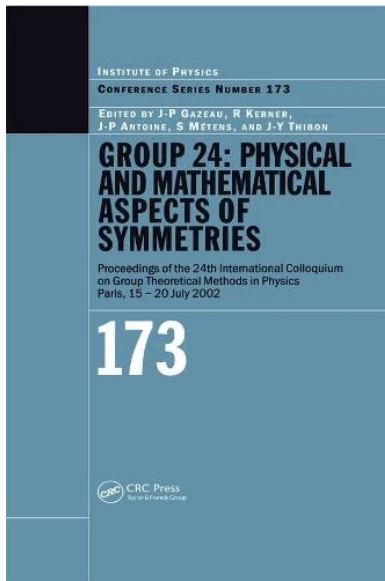
PERSONAL REMEMBRANCE OF MEETINGS WITH RICHARD



GROUP 4 NIJMENGEN 1975 : was our first participation in ICGTMP meetings.
We did not speak to each other



GROUP 10 CANTERBURY 1981 : we start to speak to each other and enjoy jokes !



GROUP 24 PARIS 2002 : we organised it!



Ubu Roi (Joan Miró, 1944)

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Preamble : three motivating quotes

- **T.D. Newton & E.P. Wigner**, *Localized States for Elementary Systems*, *Rev. Mod. Phys.* **21** 400 (1949) :

The concept of an "elementary system" requires that all states of the system be obtainable from the relativistic transforms of any state by superpositions. In other words, there must be no relativistically invariant distinction between the various states of the system which would allow for the principle of superposition. This condition is often referred to as irreducibility condition ...

The concept of an elementary system (...) is a description of a set of states which forms, in mathematical language, an irreducible representation space for the inhomogeneous Lorentz ($\simeq$ Poincaré) group

- **C. Fronsdal**, *Elementary Particles in a Curved Space*, *Rev. Mod. Phys.* **37** 221 (1965) :

A physical theory that treats spacetime as Minkowskian flat must be obtainable as a well-defined limit of a more general physical theory, for which the assumption of flatness is not essential.

- **A.D. Sakharov**, *Vacuum quantum fluctuations in curved space and the theory of gravitation*, *Dok. Akad. Nauk. SSSR* **177** 70 (1967) [*Sov. Phys. Dokl.* **12** 1040 (1968)], quoted by Adler (*Rev. Mod. Phys.* **54** (1982)) :

The presence of the action

$$S_{\text{grav}} = \frac{1}{16\pi G} \int d^4x \sqrt{-g} (R - 2\Lambda) . \quad (1)$$

leads to a metrical elasticity of space, i.e., to generalized forces which oppose the curving of space. Here we consider the hypothesis which identifies the action (1) with the change in the action of quantum fluctuations of the vacuum if space is curved.

1 Three maximal symmetries, Poincaré, dS, AdS

2 Dark matter from QCD : A relic of Quark Period

3 Complements : facts of Λ CDM standard model

Einstein Equations in GR : three (equivalent or not ?) standpoints

► Standpoint 1

$$\underbrace{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}}_{\text{geometrical content}} = \underbrace{-\kappa T_{\mu\nu} - \Lambda g_{\mu\nu}}_{\text{matter content}}, \quad \kappa = \frac{8\pi G}{c^4}.$$

Here, the fundamental state that contains the maximum number of symmetries is the Minkowskian geometry, and the cosmological term $\Lambda g_{\mu\nu}$ may be interpreted as an **extra** pressure, named **world matter** by de Sitter in his debate with Einstein :

$$\Lambda > 0 \sim \text{"dark energy"} \quad \Lambda < 0 \sim \text{"dark matter"}?$$

► Standpoint 2

$$\underbrace{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu}}_{\text{geometrical content}} = \underbrace{-\kappa T_{\mu\nu}}_{\text{matter content}}.$$

Here, the fundamental states that contain the maximum number of symmetries are the de-Sitter (dS) ($\Lambda \equiv \Lambda_{\text{dS}} > 0$) and the Anti-de-Sitter (AdS) ($\Lambda \equiv \Lambda_{\text{AdS}} < 0$) geometries.

- **Standpoint 3** Actually the split between these two standpoints should not be considered as absolute, since we could as well model situations in a mixed way :

$$\underbrace{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda_L g_{\mu\nu}}_{\text{geometrical content}} = \underbrace{-\kappa T_{\mu\nu} - \Lambda_R g_{\mu\nu}}_{\text{matter content}}.$$

General remarks on the interest of dS/AdS studies

► dS and AdS are *maximally symmetric*

(in a metric space of dimension n , the maximum number of metric preserving symmetries is $n(n+1)/2$, here 10 since $n=4$)

► Their symmetries are one-parameter deformations of Minkowskian symmetry with

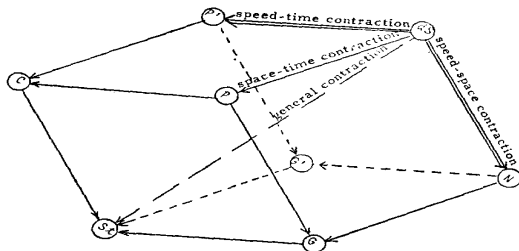
- negative curvature $-\kappa_{\text{dS}} = -\sqrt{|\Lambda_{\text{dS}}|/3}$ ($= -H/c$, H : Hubble parameter)
- positive curvature $\kappa_{\text{AdS}} = \sqrt{|\Lambda_{\text{AdS}}|/3}$

respectively

- As soon as a constant curvature is present, we lose some of our so familiar conservation laws like energy-momentum conservation !
- Then what is the physical meaning of a scattering experiment ("space" in dS is like the sphere $\mathbb{S}^3$, let alone the fact that time is ambiguous) ?
- Which relevant "physical" quantities are going to be considered as (asymptotically ? contractively ?) experimentally available ?¹

1. See (from yesterday) M.V. Takook, J.-P. G., E. Huguet, Asymptotic states and S-matrix operator in de Sitter ambient space formalism, arXiv :2304.04756 [hep-th], and references therein.

- ▶ With the requirements of kinematical rotation, parity, and time-reversal invariance, there exists only one way to “deform” the Poincaré group, namely, in endowing space-time with a certain curvature



Relative-time groups:	(dS) ,	(P') ,	(P) ,	(C) .
Absolute-time groups:	(N) ,	(G') ,	(G) ,	(St) .
Relative-space groups:	(dS) ,	(N) ,	(P) ,	(G) .
Absolute-space groups:	(P') ,	(G') ,	(C) ,	(St) .
Cosmological groups:	(dS) ,	(N) ,	(P) ,	(G') .
Local groups:	(P) ,	(G) ,	(C) ,	(St) .

Remark: The effect of the symmetry S of Eq. (6) is equivalent to a symmetry of the cube with respect to the plane containing the vertices (dS) , (N) , (C) , and (St) .

The eleven kinematics (Bacry & Levy-Leblond, JMP (1968))

Theorem

Under the assumptions that :

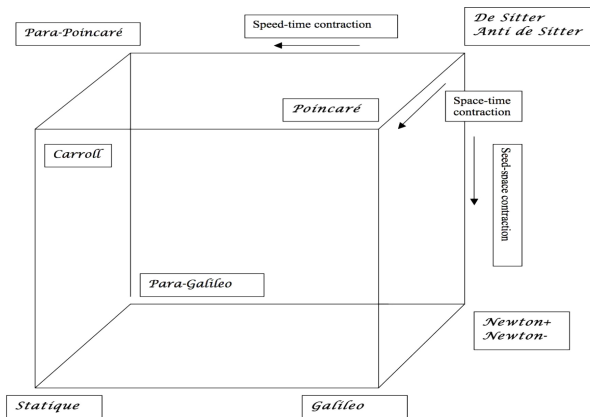
- (1) space is isotropic (rotation invariance) ;*
- (2) parity and time-reversal are automorphisms of the kinematical groups ;*
- (3) inertial transformations in any given direction form a noncompact subgroup,*

then there are eight types of Lie algebras for kinematical groups corresponding to eleven possible kinematics. These algebras are :

- R1 The two de Sitter Lie algebras isomorphic, respectively, to the Lie algebras of $SO(4,1)$ and $SO(3,2)$;*
- R2 The Poincaré Lie algebra ;*
- R3 Two "para-Poincaré" Lie algebras, of which one is isomorphic to the ordinary Poincaré Lie algebra but physically different and the other is the Lie algebra of an inhomogeneous $SO(4)$ group ;*
- R4 The Carroll Lie algebra ;*
- A1 The two "nonrelativistic cosmological" Lie algebras, $Newton_{\pm}$;*
- A2 The Galilei Lie algebra ;*
- A3 The "para-Galilei" Lie algebra ;*
- A4 The "static" Lie algebra.*

While the Lie algebras of class R have no nontrivial central extensions by a one-parameter Lie algebra, those of class A each have one class of such extensions.

The eleven kinematics (Bacry & Levy-Leblond, JMP (1968))

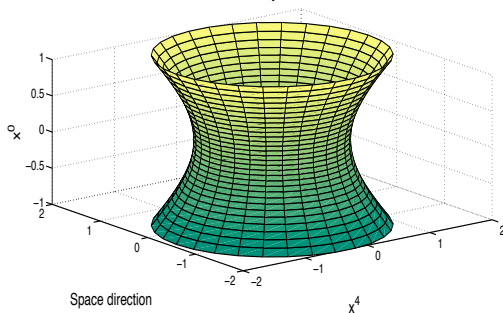


The symmetries

- ▶ Respective invariance (in the relativity or kinematical sense) groups :
 - de Sitter group is the ten-parameter $SO_0(1, 4)$ (or its universal covering $Sp(2, 2)$)
 - Anti de Sitter group is the ten-parameter $SO_0(2, 3)$ (or its two-fold covering $Sp(4, \mathbb{R})$)
- ▶ Both are deformations of the proper orthochronous Poincaré group $\mathbb{R}^{1,3} \rtimes SO_0(1, 3)$ (or $\mathbb{R}^{1,3} \rtimes SL(2, \mathbb{C})$), the kinematical group of Minkowski spacetime

- ▶ de Sitter space viewed as a hyperboloid embedded in a five-dimensional Minkowski space with metric $\eta_{\alpha\beta} = \text{diag}(1, -1, -1, -1, -1)$ (but keep in mind that all points are physically equivalent)

de Sitter space-time



- ▶ Anti de Sitter space viewed as a one-sheeted hyperboloid embedded in another five-dimensional space with metric $\eta_{\alpha\beta} = \text{diag}(1, -1, -1, -1, 1)$ (here too all points are physically equivalent) :

anti de Sitter space-time



Compared classifications of Poincaré, dS and AdS UIR's for quantum elementary systems

- In a given unitary irreducible representation (UIR) of dS and AdS groups, ($\sim$ elementary system in Wigner's sense) their respective generators map to self-adjoint operators in Hilbert spaces of spinor-tensor valued fields on dS and AdS respectively :

$$K_{\alpha\beta} \mapsto L_{\alpha\beta} = M_{\alpha\beta} + S_{\alpha\beta} ,$$

with orbital part $M_{\alpha\beta} = -i(x_\alpha \partial_\beta - x_\beta \partial_\alpha)$ and spinorial part $S_{\alpha\beta}$ acting on the field components.

- The physically relevant UIR's of the Poincaré, dS and AdS groups are denoted by $\mathcal{P}^>(m, s)$ (" $>$ " for positive energies), $U_{\text{dS}}(\varsigma_{\text{dS}}, s)$, and $U_{\text{AdS}}(\varsigma_{\text{AdS}}, s)$, respectively.
- These UIR's are specified by the spectral values $\langle \cdot \rangle$ of their quadratic and quartic Casimir operators.
- The latter define two invariants, the most basic ones being predicted by the relativity principle, namely proper mass m for Poincaré and ς_{dS} , ς_{AdS} for dS and AdS respectively, and spin s for the three cases².

2. Enayati, J.-P. G., Pejhan, Wang, *The de Sitter Group and its Representations, An Introduction to Elementary Systems and Modeling the Dark Energy Universe, Synthesis Lectures on Mathematics & Statistics ; Springer (2022) ; 2201.11457 [math-ph]*



Two basic invariants for Poincaré, dS, and AdS

- For Poincaré the Casimir operators are fixed as

$$Q_{\text{Poincaré}}^{(1)} = P^\mu P_\mu = P^{02} - \mathbf{P}^2 = m^2 c^2,$$

$$Q_{\text{Poincaré}}^{(2)} = W^\mu W_\mu = -m^2 c^2 s(s+1) \hbar^2, \quad W_\mu := \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} J^{\nu\rho} P^\sigma.$$

- For de Sitter,

$$Q_{\text{dS}}^{(1)} = -\frac{1}{2} L_{\alpha\beta} L^{\alpha\beta} = \varsigma_{\text{dS}}^2 - \left(s - \frac{1}{2}\right)^2 + 2 \equiv \langle Q_{\text{dS}}^{(1)} \rangle,$$

$$Q_{\text{dS}}^{(2)} = -W_\alpha W^\alpha = \left(\varsigma_{\text{dS}}^2 + \frac{1}{4}\right) s(s+1), \quad W_\alpha := -\frac{1}{8} \epsilon_{\alpha\beta\gamma\delta\eta} L^{\beta\gamma} L^{\delta\eta}.$$

- For Anti-de Sitter,

$$Q_{\text{AdS}}^{(1)} = -\frac{1}{2} L_{\alpha\beta} L^{\alpha\beta} = \varsigma_{\text{AdS}}(\varsigma_{\text{AdS}} - 3) + s(s+1) \equiv \langle Q_{\text{AdS}}^{(1)} \rangle,$$

$$Q_{\text{AdS}}^{(2)} = -W_\alpha W^\alpha = -(\varsigma_{\text{AdS}} - 1)(\varsigma_{\text{AdS}} - 2)s(s+1), \quad W_\alpha := -\frac{1}{8} \epsilon_{\alpha\beta\gamma\delta\eta} L^{\beta\gamma} L^{\delta\eta}.$$

Proper mass versus “at rest” energy in de Sitter : Garidi mass

- ▶ While **proper** mass $\equiv$ **at rest** energy $\equiv$ energy spectrum **infimum** of self-adjoint time translation generator P_0 in Minkowski, these two quantities come apart in de Sitterian/Anti-de Sitterian geometry
- ▶ They have to be devised from a flat-limit viewpoint, i.e. from the study of the contraction limit $\Lambda \rightarrow 0$ of these representations
- ▶ In this respect, a mass formula for dS has been established by Garidi (2003)³ :

$$m_{\text{dS}}^2 := \frac{\hbar^2 \Lambda_{\text{dS}}}{3c^2} (\langle Q_{\text{dS}}^{(1)} \rangle - 2) = \frac{\hbar^2 \Lambda_{\text{dS}}}{3c^2} \left(\varsigma_{\text{dS}}^2 + \left(s - \frac{1}{2} \right)^2 \right) .$$

Proper mass versus “at rest” energy in de Sitter : contraction dS $\rightarrow$ Poincaré

- This definition should be understood through the contraction limit of representations :

$$\text{dS UIR} \longrightarrow \text{Poincaré UIR} .$$

- More precisely, with

$$\Lambda_{\text{dS}} \rightarrow 0 \quad \varsigma_{\text{dS}} \rightarrow \infty , \quad \text{while fixing} \quad \varsigma_{\text{dS}} \hbar \sqrt{\Lambda_{\text{dS}}} / \sqrt{3} c = m_{\text{Poincaré}} \equiv m .$$

we have

$$U_{\text{dS}}(\varsigma_{\text{dS}}, s) \xrightarrow[\substack{\Lambda_{\text{dS}} \rightarrow 0, |\varsigma_{\text{dS}}| \rightarrow \infty \\ |\varsigma_{\text{dS}}| \sqrt{\Lambda_{\text{dS}}} / \sqrt{3} = \frac{mc}{\hbar}}]{c_{>} \mathcal{P}^{>}(m, s) \oplus c_{<} \mathcal{P}^{<}(m, s)} .$$

- Note the breaking of dS irreducibility into a direct sum of two Poincaré UIR's with positive and negative energy respectively.
- To some extent the choice of the factors $c_{<}$, $c_{>}$, is left to a **local tangent** observer. The latter will naturally fix one of these factors to 1 and so the other one is forced to vanish.
- This crucial dS feature originates from the dS group symmetry mapping any point $(x^0, P) \in H_{\text{dS}}$ into its mirror image $(x^0, -P) \in H_{\text{dS}}$ with respect to the x^0 -axis.
- Under such a symmetry the four dS generators L_{0a} , $a = 1, 2, 3, 4$, (and particularly L_{04} which contracts to energy operator!) transform into their respective opposite $-L_{0a}$, whereas the six L_{ab} 's remain unchanged.

Proper mass versus at rest energy in de Anti de Sitter

- Concerning AdS a mass formula (JPG&Novello 2007)⁴ similar to that one for dS exists :

$$m_{\text{AdS}}^2 = \frac{\hbar^2 |\Lambda_{\text{AdS}}|}{3c^2} \left(\langle \mathcal{Q}_{\text{AdS}}^{(1)} \rangle - \langle \mathcal{Q}_{\text{AdS}}^{(1)} | \varsigma_{\text{AdS}} = s+1 \rangle \right)$$

$$= \frac{\hbar^2 |\Lambda_{\text{AdS}}|}{3c^2} \left[\left(\varsigma_{\text{AdS}} - \frac{3}{2} \right)^2 - \left(s - \frac{1}{2} \right)^2 \right] .$$

- One here deals with the AdS group representations $U_{\text{AdS}}(\varsigma_{\text{AdS}}, s)$ with $\varsigma_{\text{AdS}} \geq s+1$ (discrete series and its lowest limit), and their contraction limit holds with no ambiguity

$$U_{\text{AdS}}(\varsigma_{\text{AdS}}, s) \xrightarrow[\varsigma_{\text{AdS}} \sqrt{|\Lambda_{\text{AdS}}|/3} = \frac{mc}{\hbar}]{\Lambda_{\text{AdS}} \rightarrow 0, \varsigma_{\text{AdS}} \rightarrow \infty} \mathcal{P}^>(m, s) .$$

- Now, contraction formulae for both dS and AdS give us the freedom to write

$$m_{\text{dS}} = m_{\text{AdS}} = m ,$$

- This agrees with the Einstein position that the proper mass of an elementary system should be independent of the geometry of space-time, or equivalently it should not exist any difference between inertial and gravitational mass.

4. J.-P. G. and M. Novello, J. Phys. A : Math. Theor., 41, 304008 (2008); Int. J. Mod. Phys. A, 26, 3697 (2011).

Rest energy of a free particle in AdS versus dS and Poincaré

- Each **Anti-deSitterian quantum elementary system** (in the Wigner sense) has a discrete energy spectrum bounded below by its **rest energy**

$$E_{\text{AdS}}^{\text{rest}} = \left[m^2 c^4 + \hbar^2 c^2 \frac{|\Lambda_{\text{AdS}}|}{3} \left(s - \frac{1}{2} \right)^2 \right]^{1/2} + \frac{3}{2} \hbar \sqrt{\frac{|\Lambda_{\text{AdS}}|}{3}} c ,$$

- Hence, to the order of $\hbar$, a “massive” AdS elementary system is **a deformation of both a relativistic free particle with rest energy mc^2 and a 3d isotropic quantum harmonic oscillator with ground state energy $3/2 \hbar \sqrt{|\Lambda_{\text{AdS}}|/3} c \equiv 3/2 \hbar \omega_{\text{AdS}}$** ⁵
- In contrast to AdS, energy is ill-defined for dS. However a local tangent observer will naturally choose the invariant with positive sign in :

$$E_{\text{dS}}^{\text{rest}} = \pm \left[m^2 c^4 - \hbar^2 c^2 \frac{\Lambda_{\text{dS}}}{3} \left(s - \frac{1}{2} \right)^2 \right]^{1/2} , \quad m \geq \frac{\hbar}{c} \sqrt{\frac{\Lambda_{\text{dS}}}{3}} \left| s - \frac{1}{2} \right| .$$

- Noticeable simplification in both AdS and dS for **fermions** $s = 1/2$:

$$\begin{aligned} \text{for dS : } E_{\text{dS}}^{\text{rest}} &= mc^2 , \\ \text{for AdS : } E_{\text{AdS}}^{\text{rest}} &= mc^2 + \frac{3}{2} \hbar \omega_{\text{AdS}} . \end{aligned}$$

5. J.-P. G. and J. Renaud, Relativistic harmonic oscillator and space curvature, Phys.Lett. A **179** 67 (1993).
 M. Enayati, J.-P. G., M. del Olmo, H. Pejhan, Anti-de Sitterian “massive” elementary systems and their Minkowskian and Newtonian limits, *in progress*.

At Rest Energy of a free particle in AdS versus dS and Poincaré (continued)

- In the massless case and spin s , we have

$$\text{for dS : } E_{\text{dS}}^{\text{rest}} = \pm i\hbar \sqrt{\frac{\Lambda_{\text{dS}}}{3}} c \left(s - \frac{1}{2} \right) ,$$

$$\text{for AdS : } E_{\text{AdS}}^{\text{rest}} = \hbar \sqrt{\frac{|\Lambda_{\text{AdS}}|}{3}} c (s + 1) .$$

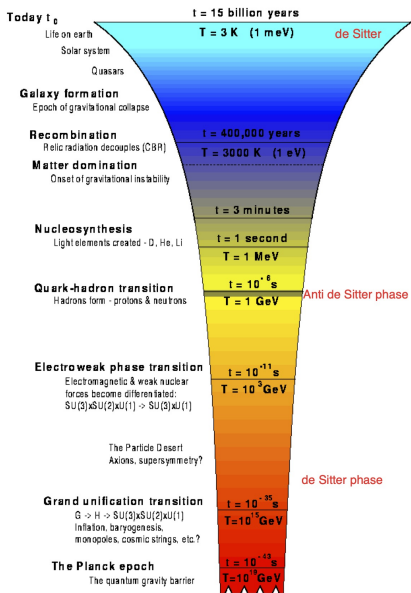
- Therefore, while for dS the energy at rest makes sense only for massless fermionic systems and is just zero, for AdS the energy at rest makes sense for any spin, and in particular for **spin 1 massless bosons** we get

$$E_{\text{AdS}}^{\text{rest}} = 2\hbar\omega_{\text{AdS}} .$$

and for **scalar massless bosons**

$$E_{\text{AdS}}^{\text{rest}} = \hbar\omega_{\text{AdS}} .$$

Cosmology chronology : (effective) de Sitter and Anti de Sitter phases



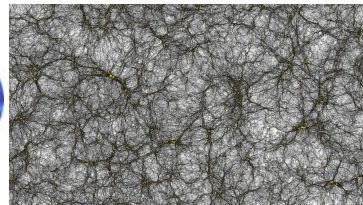
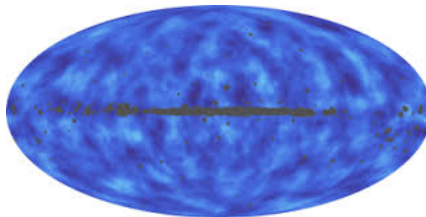
Some (observational) facts about dark matter

- ▶ According to Planck 2015 analysis⁶ of CMB power spectrum, our Universe is spatially flat, accelerating, and composed of 5% baryonic matter, 27% cold dark matter (CDM, non baryonic) and 68% dark energy (Λ)⁷
- ▶ (Cold) dark matter is observed by its gravitational influence on luminous, baryonic matter
- ▶ The dark matter mass halo and the total stellar mass are coupled through a function that varies smoothly with mass (with controversial exception(s)...)
- ▶ Up to now (?), all hypothetical particle models (WIMP, Axions, Neutrinos ...) failed direct or indirect detection tests
- ▶ Similarly, alternative theories (e.g. MOND) to dark matter are not fully successful (?) in explaining clusters and the observed pattern in the CMB

6. Planck 2018 Results ; VI. Cosmological parameters *A&A* **2020**, A6, 641. [arXivhttps://arxiv.org/abs/1807.06209](https://arxiv.org/abs/1807.06209)

7. The Search for Dark Matter L. Baudis, *European Review* **26** 70-81 (2017)

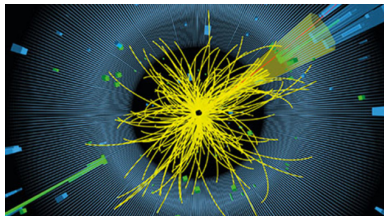
Lensing Potential and Virtual Universe



Largest virtual universe ever simulated : Quantum cosmogony is a gedanken experiment that consists in bringing back into the tableau the visible matter that has been removed as a foreground possibly interfering with the CMB. A section of it on the right.

Our interpretation of the complex topology of the spacetime of the dark universe : a **web of dark filaments** that are tensionless dark strings freely moving in a **void space** (the white regions in the figure) **with negative curvature** (i.e. de Sitter) related to the (positive) cosmological constant, whereas the spacetime **inside the filaments** has a **positive curvature** (i.e. Anti de Sitter) related to an effective negative cosmological constant

Quark-Gluon Plasma : experimental evidence



From Strong interactions News
Protons probe quark-gluon plasma at CMS 13 January 2017

- ▶ Theories predicting the existence of quark-gluon plasma were developed in the late 1970s and early 1980s (Satz, Rafelsky, Kapusta, Müller, Letessier...)
- ▶ Quark-gluon plasma was detected for the first time at CERN (2000)
- ▶ Lead and gold nuclei have been used for collisions yielding QGP at CERN SPS and BNL RHIC, respectively
- ▶ The current estimate of the hadronization temperature for light quarks is $T_{cf} = 156.5 \pm 1.5 \text{ MeV} \approx 1.8 \times 10^{12} \text{ K}$ (“chemical freeze-out temperature”).⁸

8. Pasechnik, R.; Šumbera, M. Phenomenological Review on Quark-Gluon Plasma : Concepts vs. Observations. *Universe* **2017**, *3*, 7.

Andronic, A.; Braun-Munzinger, P.; Redlich, K.; Stachel, J. Decoding the phase structure of QCD via particle production at high energy. *Nature* **2018**, *561*, 321–330. doi :10.1038/s41586-018-0491-6.

Cold dark matter $\sim$ CGB : A parallel between dark matter, CMB, and CNB :

- **CMB** $\rightarrow$ photon decoupling, i.e. photons started to travel freely through space rather than constantly being scattered by electrons and protons in plasma (**QED effect**) : **we see those photons !**
- **CNB** $\rightarrow$ is the relic of the neutrino decoupling when the rate of weak interactions between neutrinos and other forms of matter dropped below the rate of expansion of the universe, which produced a cosmic neutrino background of freely streaming neutrinos (**electroweak effect**) : **so far indirect evidence, e.g. Big Bang nucleosynthesis predictions of the helium abundance, CMB anisotropies ...**
- **CGB ?** Our scenario : Dark matter $\rightarrow$ gluonic component of the quark epoch (quark-gluon plasma) which freely subsists after hadronization within an effective AdS environment (QCD effect) : **we do not observe those gluons but we see their gravitational effects and dark matter could be as well named cosmic gluonic background (CGB)...**

Cold dark matter : a QCD effect

- As a matter of fact the contribution of the so-called di-gluons through what is called by Adler (1982)⁹ the *gluon pairing amplitude* to the **QCD trace anomaly** reads

$$\langle T^\mu_\mu \rangle_0 = -\frac{1}{8} [11N_c - 2N_f] \left\langle \frac{\alpha_s}{\pi} (F^a_{\mu\nu} F^{a\mu\nu})^r \right\rangle_0 \propto \Lambda_{\text{AdS}}$$

- N_c is the number (=3) of colors, and N_f the effective number of quark flavors (~ 3). As asserted by G. Cohen-Tannoudji¹⁰ : *The minus sign in the right hand side shows that when the factor $(11N_c - 2N_f)$ is positive, all the QCD condensates contribute negatively to the energy density, which means that the QCD world-matter is globally an AdS world-matter (dominance of an AdS world-matter over a smaller dS world-matter), and so*
- the bosonic (gluon) loops, proportional to N_c , contribute to the AdS world matter
 - the fermionic (quark) loops, proportional to N_f , contribute to the normal dS world matter
- Compare the ratio $\frac{11}{2} \frac{N_c}{N_f} \sim 5.5$ with the estimate [dark matter]/[visible matter] $\sim 27/5 = 5.4$.

9. S.L. Adler, Einstein gravity as a symmetry breaking effect in quantum field theory, *Review of Modern Physics* **1982**, 54, 729.

10. G. Cohen-Tannoudji, The de Broglie universal substratum, the Lochak monopoles and the dark universe, *Annales de la Fondation Louis de Broglie* **2019**, 44, 187–209 ; (<https://arxiv.org/abs/1507.00460v10>) ▶

Cold dark matter : Bose-Einstein condensation of (di-)gluons in effective Anti-de Sitter background

- First remind that in an AdS effective background ($\sqrt{|\Lambda_{\text{AdS}}|/3} c \equiv \omega_{\text{AdS}}$:

$$E_{\text{AdS}}^{\text{rest}} = \left[m^2 c^4 + \hbar^2 \omega_{\text{AdS}}^2 \left(s - \frac{1}{2} \right)^2 \right]^{1/2} + \frac{3}{2} \hbar \omega_{\text{AdS}}$$

- As an assembly of N_G non-interacting (i.e. colorless) scalar bosonic di-gluons with individual energies $E_n = E_{\text{AdS}}^{\text{rest}} + n \hbar \omega_{\text{AdS}}$ and degeneracy $g_n = (n+1)(n+3)/2$, those remnant components, analogous to isotropic harmonic oscillators in 3-space, are assumed to form a grand canonical Bose-Einstein ensemble whose the chemical potential μ is, at temperature T , fixed by the requirement that the sum over all occupation probabilities at temperature T yields

$$N_G = \sum_{n=0}^{\infty} \frac{g_n}{\exp \left[\frac{\hbar \omega_{\text{AdS}}}{k_B T} (n + \nu_0 - \mu) \right] - 1}, \quad \nu_0 := \frac{E_{\text{AdS}}^{\text{rest}}}{\hbar \omega_{\text{AdS}}}.$$

Cold dark matter : Bose-Einstein condensation of (di-)gluons in effective Anti-de Sitter background

- The number N_G is very large and so the gas condensates at temperature

$$T_c \approx \frac{\hbar\omega_{\text{AdS}}}{k_B} \left(\frac{N_G}{\zeta(3)} \right)^{1/3} \quad \zeta(3) \approx 1.2 \quad (\text{Riemann zeta function})$$

to become the currently observed dark matter

- The above formula is standard for all isotropic harmonic traps. Actually there is no harmonic trap here, it is the AdS geometry **due to QCD trace anomaly** which originates the harmonic spectrum on the quantum level
- To support this scenario it is known from ultra-cold atoms physics that Bose Einstein condensation can occur in non-condensed matter but also in gas, and that this phenomenon is not linked to *interactions* but rather to the *correlations implied by quantum statistics*.

An estimate

- ▶ Although we do not precisely know at which stage beyond the hadronization phase transition does take place the gluonic Bose Einstein condensation, let us see if our estimate on T_c yields reasonable orders of magnitude
- ▶ Take T_c equal to the current CMB temperature, $T_c = 2.78\text{K}$, and $|\Lambda_{\text{AdS}}| \approx \frac{5.5}{6.5} \times \frac{11}{24} \times \Lambda_{\text{dS}} = 0.39 \times \Lambda_{\text{dS}}$ (an estimate based on the Λ CDM model, see complements), with $\Lambda_{\text{dS}} \equiv \text{present } \Lambda = 1.1 \times 10^{-52}\text{m}^{-2}$. We then get the estimate on the number of di-gluons in the condensate :

$$N_G \approx 5 \times 10^{88}$$

- ▶ This seems reasonable since the gluons are around 10^9 times the number of baryons, and the latter is estimated to be around 10^{80} .

We have tentatively explained dark matter by actually asking a simple question (!) :

what becomes the huge amount of gluons after the transition from QGP period to hadronization ?

Moreover, our approach lies at the crossroads of the two current standard models, the Λ CDM Model of cosmology and the Standard Model of particle physics.

THANK FOR YOUR ATTENTION !

A reminder about the cosmological formalism ($c = 1$) : Robertson metric

- In an **isotropic** and **homogeneous** cosmology, the Einstein's equation reads as

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G_N T_{\mu\nu} + \Lambda g_{\mu\nu}$$

where the stress energy momentum stands for a perfect fluid with density ρ and isotropic pressure P , i.e.,

$$T_{\mu\nu} = -Pg_{\mu\nu} + (P + \rho)u_\mu u_\nu.$$

- Its solution is the Robertson metric

$$ds^2 = dt^2 - R^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right)$$

- $R(t)$: the time-dependent radius of the universe. It is the cosmological scale factor (also noted $a(t)$) which determines proper distances in terms of the comoving coordinates.
- k : curvature index
- r is dimensionless

A reminder about the cosmological formalism ($c = 1$) : Friedmann-Lemaître equations

- Physical quantities R , ρ , and P obey the Friedmann-Lemaître equations of a perfect fluid modelling the material content of the universe.

$$H^2 \equiv \left(\frac{\dot{R}}{R} \right)^2 = \frac{8\pi G_N \rho}{3} - \frac{k}{R^2} + \frac{\Lambda}{3}$$

$$\frac{\ddot{R}}{R} = \frac{\Lambda}{3} - \frac{4\pi G_N}{3} (\rho + 3P)$$

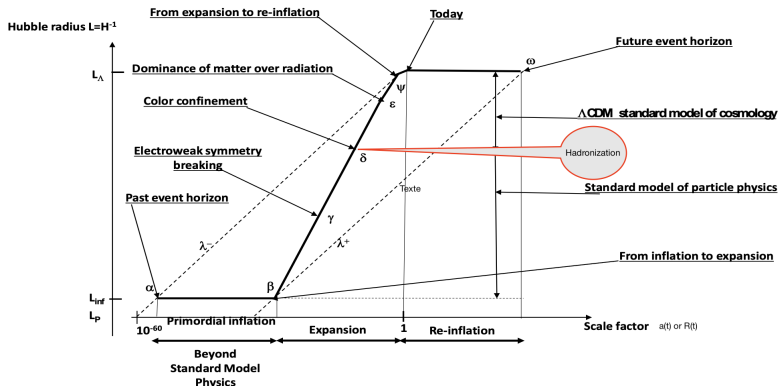
$$\dot{\rho} = -3H(\rho + P) \quad (\text{Conservation of the energy}).$$

- Note that the cosmological term $\Lambda g_{\mu\nu}$ is taken to the right-hand side of the Einstein's equation and may be interpreted as an **extra** pressure, named **world matter** by de Sitter in his debate with Einstein :

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N (P + \rho) u_\mu u_\nu + (\Lambda - 8\pi G_N P) g_{\mu\nu}$$

- According to the sign of this extra pressure one talks of a de Sitter world matter (Λ positive, pressure negative) or an anti-de Sitter world matter (Λ negative, pressure positive).

Cosmology chronology : Hubble radius $L(a) \equiv H^{-1}(a)$ ($c = 1$) is plotted versus the scale factor $a(t) \equiv R(t)$ in logarithmic scale



Flatness sum rule and its consequences

- From the first FL equation at $\Lambda \approx 0$:

$$\frac{k}{R^2} = \frac{8\pi G_N}{3}\rho - H^2 \equiv \frac{8\pi G_N}{3}\rho - \frac{8\pi G_N}{3}\rho_c$$

- The ($\sim$ observed) flatness rule $k = 0$ expresses the vanishing of the spatial curvature :

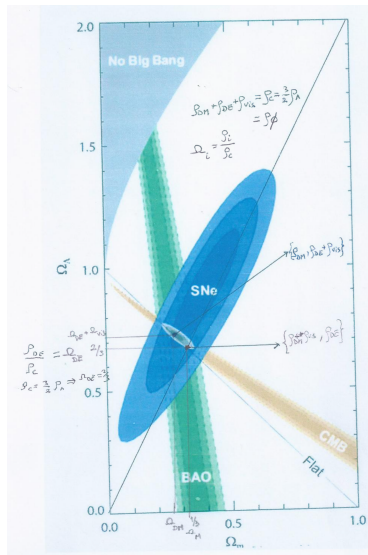
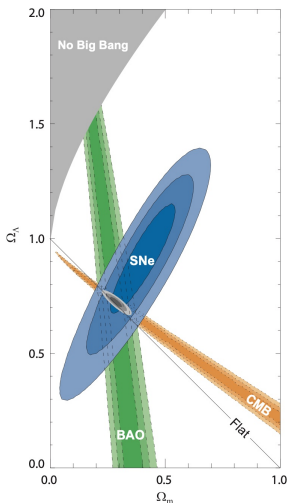
$$\rho - \rho_c \equiv \rho_{\text{vis}} + \rho_{\text{DM}} + \rho_{\text{DE}} - \rho_c = 0,$$

with

$$\rho_{\text{vis}} = \rho_{\text{bar}} + \rho_{\text{rad}}, \quad \rho_{\text{DE}} = \frac{\Lambda}{8\pi G_N}.$$

- Hence the so-called critical density $\rho_c := \frac{3H^2}{8\pi G_N}$ is the energy density at the boundaries in the far past and in the far future of the Hubble horizon in the absence of any “integration constant” Λ and any spatial curvature ($k = 0$)

68.3 %, 95.4 % and 99.7% confidence level contours on $\Omega_\Lambda \equiv \Omega_{DE}$ and Ω_M obtained from CMB, BAO and the Union SN set ($P/\rho \equiv w = -1$)



Inside the "confidence area" of the figure $\Omega_\Lambda \equiv \Omega_{DE}$ versus Ω_M

- From the second FL equation

$$\frac{\ddot{R}}{R} = \frac{\Lambda}{3} - \frac{4\pi G_N}{3} (\rho + 3P) \equiv -\frac{4\pi G_N}{3} (\rho - 2\rho_{DE} + 3P) \equiv -\frac{4\pi G_N}{3} (\rho_{\text{effective}} + 3P)$$

one infers that at the inflexion points $\ddot{R} = 0$ one has the "equation of state" (EoS)

$$w_{\text{inflexion}} \equiv P/\rho_{\text{effective}} = -1/3$$

- Inside the "confidence area" of the figure $\Omega_\Lambda = \rho_{DE}/\rho_c$ versus $\Omega_M = \rho_m/\rho_c$ one finds the points

- $(\Omega_{DM}, \Omega_{DE} + \Omega_{vis})$
- $(\Omega_M = 1/3, \Omega_{DE} = 2/3)$

- The value $\Omega_{DE} = 2/3$ results from our assumption completing the flatness sum rule as which the total energy vanishes (from the Robertson metric) :

$$\rho_{vis} + \rho_{DM} + \rho_{DE} = \rho_c = \frac{3}{2}\rho_{DE}$$