Distinguishing Primordial Universe Scenarios with Observations

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Outline of the talk

- Introduction: Standard Model
- Testing Primordial Universe Scenarios (Theory)
- Testing Primordial Universe Scenarios (Observation)
- Mimicking Starobinsky Inflation
- Overview

Primordial Universe

- Planck $\oplus$ WMAP agree on the 6−parameter ΛCDM model
- CMB is the farthest observable
- Inflation is the most popular primordial universe scenario
- Competitive alternatives: Bounce models, pre-big-bang, ekpyrotic, string-gas cosmology, etc...

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scalar spectral index (n_s) tensor-to-scalar ratio (r) non-gaussianity (f_{NL})

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- Current status: there is no reason to privilege one scenario over the others
- Prove by contradiction to distinguish primordial scenarios: a constructive method

Standard Model

- Spacetime geometry:

$$ds^2 = a^2(t)\left[d\eta^2 - \frac{dr^2}{1-kr^2} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right]$$

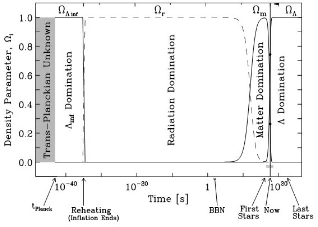
- Dynamics equations

$$H^2 \equiv \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\sum_i \rho_i - \frac{k}{a^2} \qquad\qquad \frac{\ddot a}{a} = -\frac{4\pi G}{3}\sum_i (\rho_i + 3p_i)$$

if $\rho + 3p > 0$ the expansion is decelerated $(\ddot a < 0)$

General Properties

- homogeneous and isotropic > 200 Mpc
- exist for at least 13,7 billions of years
- has been expanding: denser and hotter in the past
- 3 eras: radiation / dust / dark energy
- flat spatial sections ($\Omega_k < 0.005$) / Is it (in)finite?



$$\rho_{rad} \backsim a^{-4} \quad / \quad \rho_{mat} \backsim a^{-3}$$

Thermal History (Microphysics)

- time-energy relation $\rho_{rad} \backsim T^4 \quad \Rightarrow \quad$ in radiation era: $T \backsim \frac{1}{a} \backsim \frac{1}{\sqrt{t}} \qquad t = \frac{1}{T^2}$
- Below a few $MeV's$: *photons, baryons, electrons, neutrinos, dark sector*

$T \backsim$ [$\backsim$ 3 billions of years] galaxies formation

$T \backsim 0.25\, eV$ $\left[10^{12}\, sec\right]$ hydrogen recombination CMB decoupling

$T \backsim 1\, eV$ $\left[10^{11} sec\right]$ radiation-dust era transition

$T \backsim 50\, KeV$ [300 sec] light elements formation

$T \backsim 0.5\, MeV$ [1 sec] Freeze-out $e^- + e^+ \leftrightarrows \gamma$

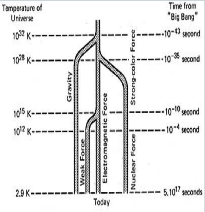
$T \backsim 1\, MeV$ [0.2 sec] neutrino decoupling
$T_\nu \approx 1.401 T_\gamma \quad (p/n$ free-out)

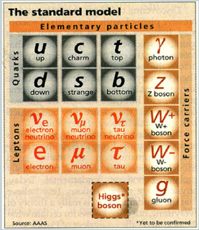
$T \backsim 200\, MeV$ $\left[10^{-5} sec\right]$ formation of baryons
quarks and gluons confinement

$T \backsim 100\, GeV - 10\, TeV$ $\left[10^{-10} - 10^{-14} sec\right]$
particle standard model is still valid

$T \backsim 10\, TeV - 10^{19} GeV$ $\left[10^{-14} - 10^{-44} sec\right]$
beyond particle SM but still classical gravity

$T > 10^{19} GeV$ what's come next?





Testing Primordial Universe (Theory)

Shortcomings of the Standard Model

- physical mechanism to produce the expansion
- matter / anti-matter asymmetry
- cosmic singularity in a finite past time

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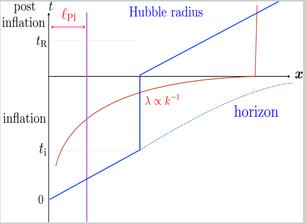
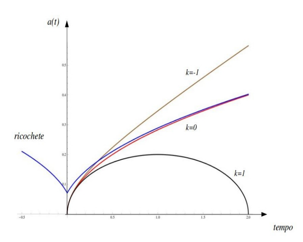
- initial condition for structure formation

Inflation	Bounce
accelerated expansion phase	contraction → expansion





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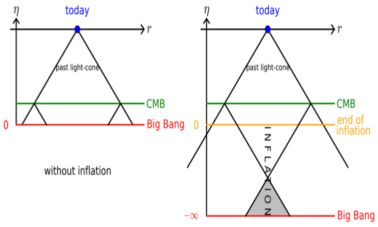
$$\Delta r = \int \frac{dt}{a} \Rightarrow \quad \frac{R_{LSS}}{R_H} \sim 10^2$$

Inflation

accelerated expansion phase

$$t_0 = 0 \quad \text{but} \quad a \simeq e^{Ht} \quad \rightarrow \eta_0 = -\infty$$



Bounce

contraction → expansion

$$t_0 = -\infty$$

"All the time of eternity""

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$$\frac{\mathrm{d}}{\mathrm{d}t}\|\Omega - 1\| = -\frac{2\ddot{a}}{\dot{a}^3}$$

Inflation	Bounce
accelerated expansion phase	contraction $\rightarrow$ expansion
$\ddot{a} > 0 \quad \text{and} \quad \dot{a} > 0$	$\ddot{a} < 0 \quad \text{but} \quad \dot{a} < 0$

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Inflation	Bounce
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Quantum Vacuum Fluctuations	Quantum Vacuum Fluctuations
physical length much smaller than the curvature scale	Universe is immense with negligible curvature
as in Flat Spacetime	very close to Flat Spacetime

Testing Primordial Universe (Observations)

- While waiting for primordial gravitational waves....
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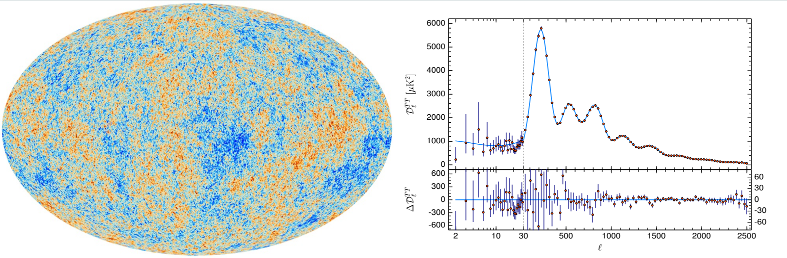


- Three main observables:

scalar spectral index — tensor-to-scalar ratio — non-gaussianity

$$n_s = \frac{\mathrm{d}\log \mathrm{P}_{\mathscr{R}}}{\mathrm{d}\log k} \qquad r = \frac{\mathrm{P}_{\mathscr{R}}}{\mathrm{P}_{\mathscr{T}}} \qquad f_{NL}$$

- but there is also

B-mode polarization, tensor spectral index, running, etc...

Connection with observations[1] : Correlation functions

two point $\langle \zeta^*(\vec{k}_1)\,\zeta(\vec{k}_2)\rangle = \dfrac{(2\pi)^2}{k^3}\delta(\vec{k}_1 - \vec{k}_2)\mathscr{P}_\zeta(k_1) \quad \Rightarrow \quad (n_s\,, r)$

bispectrum $\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle = (2\pi)^7\delta\left(\vec{k}_1 + \vec{k}_2 + \vec{k}_3\right)\dfrac{\mathscr{P}_\zeta^2}{\prod k_i^3}\mathscr{A}\left(\vec{k}_1, \vec{k}_2, \vec{k}_3\right) \quad \Rightarrow \quad f_{NL}$

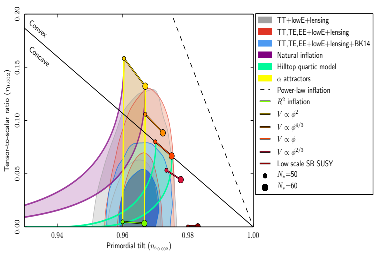
- Power spectrum parametrization

$$\mathrm{P}_{\mathscr{R}} = A_s\left(\frac{k}{k_*}\right)^{n_s-1} \qquad n_s - 1 = \frac{\mathrm{d}\ln \mathrm{P}_{\mathscr{R}}}{\mathrm{d}\ln k}$$

$$\mathrm{P}_{\mathscr{T}} = A_t\left(\frac{k}{k_*}\right)^{n_t} \qquad k_* = 0.05\mathrm{Mpc}^{-1}$$

- Tensor-to-scalar ratio

$$r \equiv \frac{\mathrm{P}_{\mathscr{T}}}{\mathrm{P}_{\mathscr{R}}} \qquad k_* = 0.002\mathrm{Mpc}^{-1}$$



$$n_s = 0.9649 \pm 0.0042\,, \quad r_{0.002} < 0.064$$

[1] Planck 2018 results. X. Constraints on inflation arXiv:1807.06211v1 [astro-ph.CO]
Planck2018 results. VI. Cosmological parameters arXiv:1807.06209v1 [astro-ph.CO]

Testing Primordial Universe (Observations)

- FLRW Background $\mathrm{d}s^2 = a^2(\eta)\left[\mathrm{d}\eta^2 - \gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j\right]$
- Generic scalar perturbations

$$\mathrm{d}s^2 = a^2(\eta)\left[(1+2\phi)\,\mathrm{d}\eta^2 - 2\nabla_i B\,\mathrm{d}x^i\mathrm{d}\eta - \left((1-2\psi)\,\gamma_{ij} + 2\nabla_j\nabla_i E\right)\mathrm{d}x^i\mathrm{d}x^j\right]$$

- Gauge freedom....
- coordinate transformation $x^\alpha \longrightarrow \tilde{x}^\alpha = x^\alpha + \delta x^\alpha$ $\tilde{g}_{\mu\nu} = \frac{\partial x^\alpha}{\partial \tilde{x}^\mu}\frac{\partial x^\beta}{\partial \tilde{x}^\nu} g_{\alpha\beta}$

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- with $\delta x^\alpha = (\delta x^0, \partial^i \xi)$

$$\tilde{\phi} = \phi - \frac{1}{a}\left(a\,\delta x^0\right)', \qquad \tilde{\psi} = \psi + \frac{a'}{a}\delta x^0, \qquad \tilde{B} = B + \delta x^0 - \xi', \qquad \tilde{E} = E - \xi,$$

- matter perturbation

$$\delta\tilde{\rho} = \delta\rho + \rho'\,\delta x^0 \qquad\qquad \delta\tilde{\varphi} = \delta\varphi + \varphi'\,\delta x^0$$

$$\delta\tilde{p} = \delta p + p'\,\delta x^0$$

or

$$T^\mu(\vec{x},\eta)_\nu = T^\mu_0{}_\nu(\eta) + \delta T^\mu{}_\nu(\vec{x},\eta) \qquad\qquad \varphi(\vec{x},\eta) = \varphi_0(\eta) + \delta\varphi(\vec{x},\eta)$$

Testing Primordial Universe (Observations)

- gauge invariant variables

$$\Phi = \phi + \frac{1}{a}\left[(B - E')a\right]' \qquad \Psi = \psi - \frac{a'}{a}(B - E')$$

- spatial curvature perturbation

$${}^{(3)}R = \frac{4}{a^2}\left[\nabla^2\psi + 3k\left(\psi + \frac{1}{2}\right)\right] \qquad \psi \quad \text{is associated with curvature}$$

- Two gauge independent variables

$$\mathscr{R} = \psi + \mathscr{H}\frac{\delta\varphi}{\varphi'} \qquad \text{curvature pert. on uniform-density foliation}$$

$$-\zeta = \psi + \mathscr{H}\frac{\delta\rho}{\rho'} \qquad \text{curvature pert. on comoving foliation (no heat flux)}$$

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- Expand the action $S = \int \sqrt{-g}\, R = S^{(0)} + S^{(1)} + S^{(2)}$ .... Abracadabra!

$$S^{(2)} = \frac{1}{2}\int \mathrm{d}\eta\, \mathrm{d}x^3 \left(a\frac{\varphi'}{\mathscr{H}}\right)^2 \left[\mathscr{R}'^2 - (\partial_i\mathscr{R})^2\right]$$

$$= \frac{M_{\mathrm{Pl}}^2}{2}\int \mathrm{d}\eta\, \mathrm{d}x^3 \left[v'^2 - (\partial_i v)^2 + \frac{z_s''}{z_s}v^2\right] \qquad v \equiv z_s\mathscr{R} \qquad z_s = a\frac{\dot{\varphi}}{H}$$

Testing Primordial Universe (Observations)

General Relativity

- scalar perturbations (fourier mode)

$$\boxed{v''_{\mathbf{k}} + \left(k^2 - \mu_s^2\right) v_{\mathbf{k}} = 0 \qquad \mu_s^2 = \frac{z_s''}{z_s}} \qquad\qquad v \equiv z_s \mathscr{R} \qquad z_s = a\frac{\dot{\varphi}}{H}$$

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- gravitational waves

$$\boxed{v_{\mathbf{k}}^{\lambda\,\prime\prime} + \left(k^2 - \mu_t^2\right) v_{\mathbf{k}}^{\lambda} = 0 \qquad \mu_t^2 = \frac{a''}{a}} \qquad\qquad v_{\mathbf{k}}^{\lambda} = \frac{a}{2} h_{\mathbf{k}}^{\lambda} \qquad z_t = a$$

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$f(R)$ theories

- scalar perturbations

$$Q_s = 3M_{\rm Pl}^2 F'^2 / 2F\left[\mathscr{H} + \left(\tfrac{F'}{2F}\right)\right]^{-2}$$

$$\boxed{v''_{\mathbf{k}} + \left(k^2 - \mu_{fs}^2\right) v_{\mathbf{k}} = 0 \qquad \mu_{fs}^2 = \frac{z''_{fs}}{z_{fs}}} \qquad z_{fs} = a\sqrt{Q_s} \quad v_{\mathbf{k}} = z_{fs}\, \mathscr{R}_{\mathbf{k}}$$

- gravitational waves

$$\boxed{v^{\lambda\,\prime\prime}_{\mathbf{k}} + \left(k^2 - \mu_{ft}^2\right) v^{\lambda}_{\mathbf{k}} = 0 \qquad \mu_{ft}^2 = \frac{z''_{ft}}{z_{ft}}} \qquad z_{ft} = a\sqrt{F} \quad v^{\lambda}_{\mathbf{k}} = \frac{M_{\rm Pl}}{2}\, z_{ft}\, h^{\lambda}_{\mathbf{k}}$$

Wands' Duality

- There is a hidden symmetry in the dynamic equation

$$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0$$

- invariant under transformation

$$\tilde{z}(\eta) = c_0\, z(\eta) \int \frac{\mathrm{d}\eta'}{z^2(\eta')}$$

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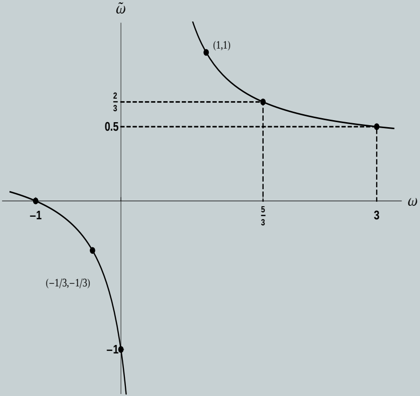
Example: perfect fluid ($p = \omega\,\rho$)

- slow-roll $\quad H \approx \dot{\varphi} \quad (z \approx a)$

$$\omega \to \tilde{\omega} = \frac{1+\omega}{-1+3\omega}$$

dust		de Sitter
$a \propto \eta^2$	$\Leftrightarrow$	$\tilde{a} \propto \dfrac{1}{\eta}$



Starobinsky Inflation

- The action in the Jordan frame

$$S = \frac{M_{\rm Pl}^2}{2}\int \mathrm{d}^4x\,\sqrt{-g}\,f(R) = \frac{M_{\rm Pl}^2}{2}\int \mathrm{d}^4x\,\sqrt{-g}\left(R + \frac{R^2}{6M^2}\right)$$

$$F \equiv \frac{\partial f}{\partial R} = 1 + \frac{R}{3M^2}$$

- in leading order of slow-roll $a(t) = a_0(t_e - t)^{1/2} exp\left[-\frac{M^2}{12}(t_e - t)^2\right]$

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- in leading order of slow-roll $a(t) = a_0(t_e - t)^{1/2} exp\left[-\frac{M^2}{12}(t_e - t)^2\right]$
- the next order correction $a(t) = a_0(t_e - t)^{-1/6} exp\left[-\frac{M^2}{12}(t_e - t)^2\right]$
- There is three relevant slow-roll parameters

$$\epsilon_1 \equiv -\frac{\dot{H}}{H^2} \quad , \quad \epsilon_3 \equiv \frac{\dot{F}}{2HF} \quad , \quad \epsilon_4 \equiv \frac{\ddot{F}}{H\dot{F}}$$

Starobinsky Inflation

- let us assume the form $a(t) = a_0(t_e - t)^p exp\left[-\frac{M^2}{12}(t_e - t)^2\right]$

$$\epsilon_1 = \frac{6}{M^2\,(t_s - t)^2}\,\frac{\left(1 + \frac{6p}{M^2(t_s-t)^2}\right)}{\left(1 - \frac{6p}{M^2(t_s-t)^2}\right)^2} = \frac{6}{M^2\,(t_s - t)^2} + \mathcal{O}(M^{-4})$$

$$\epsilon_3 = -\frac{M^2}{6H^2}\,\frac{\left[1 + \frac{p}{H(t_s-t)}\left(1 + \frac{3(1-2p)}{M^2(t_s-t)^2}\right)\right]}{\left[1 + \frac{M^2}{6H^2}\left(1 - \frac{3p}{M^2(t_s-t)^2}\right)\right]^1} = -\frac{6}{M^2\,(t_s - t)^2} + \mathcal{O}(M^{-4})$$

$$\epsilon_4 = -\frac{M^2}{6H^2}\,\frac{\left[1 - \frac{54p(1-2p)}{M^4(t_s-t)^4}\right]}{\left[1 + \frac{p}{H(t_s-t)}\left(1 + \frac{3(1-2p)}{M^2(t_s-t)^2}\right)\right]^1} = -\frac{6}{M^2\,(t_s - t)^2} + \mathcal{O}(M^{-4})$$

- in order $\mathcal{O}(M^{-2})$, the slow-roll are independent of p

$$\epsilon_1 = \frac{6}{M^2\,(t_s - t)^2} \quad , \qquad \epsilon_3 = -\epsilon_1 \quad , \qquad \epsilon_4 = \epsilon_3$$

Starobinsky Inflation

- Assuming Bunch-Davies initial state

$$v_{\mathbf{k}}(\eta) = \frac{\sqrt{\pi|\eta|}}{2} e^{i(1+2\gamma)\pi/4} H_{\gamma}^{(1)}(k|\eta|)$$

scalar perturbations $$\mathrm{P}_{\mathscr{R}} \approx \frac{1}{Q_s}\left(\frac{H}{2\pi}\right)^2 \left(\frac{|k\eta_c|}{2}\right)^{n_s-1} , \qquad n_s - 1 \approx -4\epsilon_1 + 2\epsilon_3 - 2\epsilon_4 ,$$

tensor perturbations $$\mathrm{P}_{\mathscr{T}} \approx \frac{2}{\pi^2 F}\left(\frac{H}{M_{\mathrm{Pl}}}\right)^2 \left(\frac{|k\eta_c|}{2}\right)^{n_T} , \qquad n_T \approx -2\epsilon_1 - 2\epsilon_3$$

- in terms of the number of e-folds

$$n_s - 1 \approx -3,51 \times 10^{-2}\left(\frac{N}{57}\right)^{-1} , \qquad r \approx 3,69 \times 10^{-3}\left(\frac{N}{57}\right)^{-2}$$

The Planck 2018 release:

- $n_s = 0.9649 \pm 0.0042$ at 68% c.l. ($50 < N < 65$)
- upper limit $r_{0.002} < 0.10$ at 95% c.l.
- including BICEP2/Keck Array BK14 data $r_{0.002} < 0.064$

Mimicking Starobinsky Inflation

- Starobinsky-like scale factor

$$a(t) = a_0\,(t_s - t)^{-1} \exp\left[-\frac{M^2}{12}\,(t_s - t)^2\right]$$

$$\eta = \int \frac{\mathrm{d}t}{a(t)} = \frac{-6}{a_0 M^2} \exp\left[\frac{M^2}{12}\,(t_s - t)^2\right]$$

$$a(\eta) = -\frac{1}{\eta}\,\frac{\sqrt{3}}{M\,\sqrt{\ln(\bar{\eta})}} \quad , \qquad \bar{\eta} \equiv -\frac{a_0 M^2}{6}\eta$$

- Since $M >> 1$: $\ln(\bar{\eta}) \approx \ln\left(a_0 M^2/6\right) \gg 1$ and approximately constant

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- Since $M >> 1$: $\quad \ln(\bar{\eta}) \approx \ln\left(a_0 M^2/6\right) \gg 1 \quad$ and approximately constant
- in leading order

$$\epsilon_1 = -\epsilon_3 = -\epsilon_4 = \frac{1}{2\ln(\bar{\eta})} \qquad\qquad z_s(\eta) = -\frac{\sqrt{6}}{\eta}\,\frac{1}{\ln(\bar{\eta})}$$

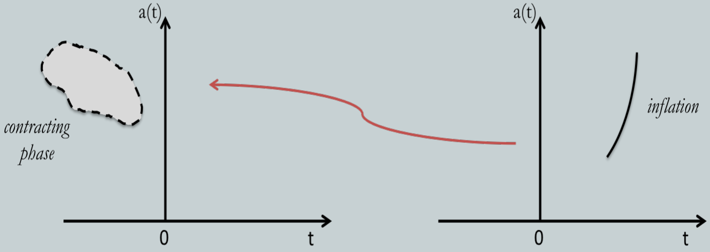
- using wands' duality

$$z_s^B(\eta) = c_0.z_s(\eta)\int_{\eta*}^{\eta} \frac{\mathrm{d}\eta'}{z_s(\eta')^2} = C_1\eta^2 \ln(\bar{\eta})\left[1 + \mathscr{O}\left(\frac{1}{\ln(\bar{\eta})}\right)\right]$$

Mimicking Starobinsky Inflation

What's the embedding dynamics?

- GR with perfect fluid / scalar field / ...
- F(R) without matter content / with conventional matter / ...



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- F(R) without matter content / with conventional matter / ...

- Collapsing with scalar field: $z_s^B = a_B\,\dot{\varphi}/H$
- Quasi-de Sitter should be mapped into a quasi-matter dominated universe

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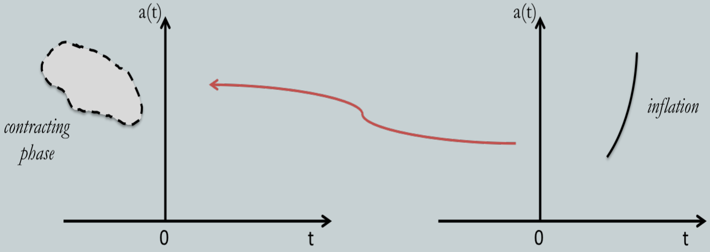

Mimicking Starobinsky Inflation

CBPF 70 ANOS — CENTRO BRASILEIRO DE PESQUISAS FÍSICAS — MINISTÉRIO DA CIÊNCIA, TECNOLOGIA, INOVAÇÕES E COMUNICAÇÕES

PPGCosmo

What's the embedding dynamics?

- GR with perfect fluid / scalar field / ...
- F(R) without matter content / with conventional matter / ...

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$$\dot{\varphi}^2 \simeq 2V \qquad \Rightarrow \qquad \dot{\varphi} \simeq \sqrt{3}H \qquad \Rightarrow \qquad z_s^B(\eta) \approx a_s^B(\eta)$$

- consider the background dynamics

$$a_B(\eta) = a_{B0}\,\eta^2 \ln(\bar{\eta}) \left[1 - \frac{2}{3\ln(\bar{\eta})} + \mathscr{O}\left(\frac{1}{\ln^2(\bar{\eta})}\right)\right]$$

- using FLRW eq.'s

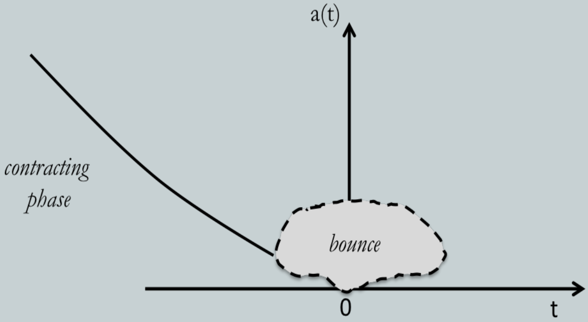
$$\varphi'^2 = 2\left(\mathscr{H}^2 - \mathscr{H}'\right) \quad , \qquad a^2 V = \left(2\mathscr{H}^2 + \mathscr{H}'\right)$$

$$\left.\begin{aligned} V(\eta) &= \frac{6}{a_{B0}^2}\frac{1}{\eta^6\log^2(\bar{\eta})}\left[1 + \frac{15}{6\ln(\bar{\eta})} + \mathscr{O}\left(\epsilon^2\right)\right] \\ \varphi &= -\sqrt{12}\ln\left[\bar{\eta}\ln^{5/12}(\bar{\eta})\right] + \mathscr{O}\left(\epsilon^2\right) \end{aligned}\right\}$$

$$\boxed{V(\varphi) = V_0\sqrt{1 - \varphi/\varphi_*}\,e^{\sqrt{3}\varphi}}$$

Quantum Bounce

- Quantum mechanisms: Wheeler-deWitt, Superstring motivated (Ekpyrotic), LQC



Quantum Bounce

- Wheeler-deWitt: Quantum Cosmology with Fluids

The action reads $$S = \frac{M_{\rm Pl}^2}{2} \int \mathrm{d}^4x \sqrt{-g}\, R - \int \mathrm{d}^4x \sqrt{-g}\, p$$

- Assuming matter content described by dust ($p = 0$) $\oplus$ radiation ($p = \frac{1}{3}\rho$)

$$H = N\left(-\frac{p_a^2}{12a} - 3\kappa a + \frac{P_r}{a} + P_m\right)$$

- Classical motion

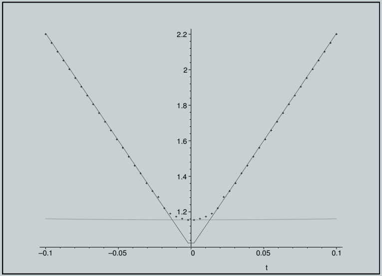
$$\left(\frac{\dot{a}}{a}\right)^2 = N^2\left[-\frac{\kappa}{a^2} + \frac{1}{3}\left(\frac{p_r}{a^4} + \frac{p_m}{a^3}\right)\right]$$

- Quantum WdW ($N = a$)

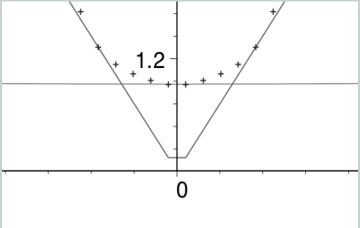
$$i\frac{\partial}{\partial\eta}\Psi(a,\eta) = \left(-\frac{1}{12}\frac{\partial^2}{\partial a^2} + 3\kappa a^2\right)\Psi(a,\eta)\,.$$

- Interpretation of QM: Many-worlds, Consistent histories, De Broglie-Bohm, ~~Copenhagen~~

Quantum Bounce



credit: N.Pinto-Neto, PRD 79, 083514 (2009)



- Bohmian trajectories
- Polar form $\Psi = |\Psi| \exp(i S/\hbar)$
- guidance relations

$$\dot{a} = -\frac{a^{(3\omega-1)}}{2}\frac{\partial S}{\partial a}$$

$$a(T) = a_0\left[1 + \left(\frac{T}{T_0}\right)^2\right]^{\frac{1}{3(1-\omega)}}$$

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Quantum Bounce

- Loop Quantum Cosmology: semi-classical states have a master equation
- LQC has an analytical solution for scalar field
- dynamics

$$H^2 = \frac{M_{\rm Pl}^{-2}}{3}\rho\left(1-\frac{\rho}{\rho_c}\right) \quad , \quad \dot{H} = -\frac{M_{\rm Pl}^{-2}}{6}(\rho+p)\left(1-\frac{2\rho}{\rho_c}\right) \quad , \quad \dot{\rho} + 3H(\rho+p) = 0$$

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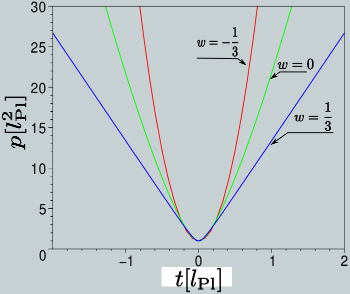
- scalar field such that $p = \omega\rho \quad \Rightarrow \quad \rho = \rho_c\left(\frac{a_B}{a}\right)^{3(1+\omega)}$

$$a(t) = a_B\left(1+\alpha^2(t-t_B)^2\right)^{1/3(1+\omega)}$$

$$\varphi(t)-\varphi_B = \frac{\sqrt{\rho_c(1+\omega)}}{\alpha}\text{arcsinh}\left(\alpha(t-t_B)\right)$$

$$V = \frac{\rho_c(1-\omega)}{2}\text{sech}^2\left[\frac{\alpha(\varphi-\varphi_B)}{\sqrt{\rho_c(1+\omega)}}\right]$$

$$\alpha = \frac{\sqrt{3\rho_c}}{2}(1+\omega)M_{\rm Pl}$$



credit: J. Mielczarek, PLB 675 p. 273,(2009)

Quantum Bounce

- Our solution:

$$V(\varphi) = V_0 \sqrt{1 - \varphi/\varphi_*}\, e^{\sqrt{3}\varphi} \quad \Rightarrow \qquad \omega = \frac{\varphi'^2 - 2a^2 V}{\varphi'^2 + 2a^2 V} = -\frac{1}{6 \ln(\bar{\eta})} \equiv -\frac{1}{6}\epsilon_c$$

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- Scalar perturbations

$$v'' + \left[\left(1 - \frac{2\rho}{\rho_c}\right)k^2 - \frac{z''}{z}\right]v = 0 \qquad z = \frac{\sqrt{\rho + p}}{H}a$$

$$v(\eta) = B_1 z + B_2 z \int^{\eta} \frac{d\bar{\eta}}{z^2} - k^2 \int^{\eta} \frac{d\bar{\eta}}{z^2} \int^{\bar{\eta}} d\bar{\bar{\eta}}\, z\, v + \frac{2k^2}{\rho_c} z \int^{\eta} \frac{d\bar{\eta}}{z^2} \int^{\bar{\eta}} d\bar{\bar{\eta}}\, z\, v$$

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- Matching Conditions

before crossing $\quad v^{in}(\eta) = \sqrt{\dfrac{-\pi\eta}{4}}\, \mathrm{H}_{\gamma}^{(1)}(-k\eta) \qquad \gamma = \dfrac{3}{2} + \epsilon_c$

through the bounce $\quad v(\eta) \approx B_1 z + B_2 z \displaystyle\int^{\eta} \frac{\mathrm{d}\bar{\eta}}{z^2}$

- power spectrum

$$\mathrm{P}_{\mathscr{R}} = \frac{k^3}{2\pi^2}\left|\frac{v}{z}\right|^2 \approx 1.736 \times 10^{-3} \left(\frac{\rho_c}{M_{\mathrm{Pl}}^4}\right) k^{-2\epsilon_c} \qquad \boxed{n_s - 1 = -2\epsilon_c}$$

Quantum Bounce

- Tensor perturbations

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$$\mathrm{P}_{\mathscr{T}} = 2 \times \frac{k^3}{2\pi^2}\left(2M_{\mathrm{Pl}}\left|\frac{v}{z_T}\right|\right)^2 \approx 4.645\, \epsilon_c^2 \times 10^{-3}\left(\frac{\rho_c}{M_{\mathrm{Pl}}^4}\right) k^{-\epsilon_c} \qquad \boxed{n_t = -2\epsilon_c}$$

- tensor-to-scalar ratio

$$\boxed{r = \frac{\mathrm{P}_{\mathscr{T}}}{\mathrm{P}_{\mathscr{R}}} = \frac{8}{3}\epsilon_c^2}$$

Overview

- Starobinsky Inflation

$$n_s - 1 = -\frac{2}{N} \quad , \quad r = \frac{12}{N^2} \quad \Rightarrow \quad r = 3\ (n_s - 1)^2$$

Consistency relation (single field slow-roll): $\quad n_t = -r/8 \quad \Rightarrow \quad n_t = -\frac{3}{8}\ (n_s - 1)^2$

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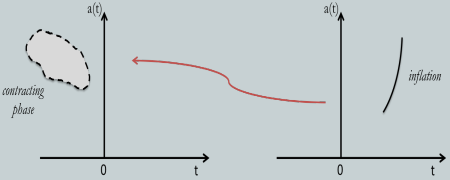

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The mode-mixing

inflation: decreasing $\oplus$ constant mode.

bounce: increasing $\oplus$ constant mode.

$$v = D_1\, z + D_2\, z \int^{\bar\eta} \frac{\mathrm{d}\eta}{z^2}$$

In our case we get a $\frac{\epsilon_c^2}{9}$

Conclusion

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Thanks for your attention!

ArXiv:1902.05031 [gr-qc]