Black Holes: classical and quantum aspects

Black holes and their analogues: 100 years of GR

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Plan of the lectures

1. Black holes in General Relativity
2. The Hawking effect and physical implications
3. Analog gravity and analog Hawking radiation: the experimental search

Black holes in General Relativity

Dark stars

- Mitchell 1783 and Laplace 1796 theorise the existence of dark stars, compact objects with an escape velocity $\geq c$

$$\frac{mv^2}{2} - \frac{GMm}{R} = 0, \ v \geq c \Rightarrow R \leq \frac{2GM}{c^2} \equiv r_G$$

- This prediction was based on Newton's corpuscular nature of light
- Despite the quite different nature of Newton and Einstein gravity (to which it reduces in the weak-field limit), the gravitational radius r_G plays a crucial role also in GR

General Relativity

- In Einstein 1915 theory matter curves the space-time geometry $g_{\mu\nu}$
- The Einstein-Hilbert action

$$S = \frac{c^3}{16\pi G}\int d^4x\sqrt{-g}\; R + S_{Matter}$$

leads to the equations of motion

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

- Newton's theory (and Poisson's equation $\nabla^2\phi = 4\pi G\rho$) is recovered in the weak field limit

$$g_{tt} \sim 1 + \frac{2\phi}{c^2}\;,\;\; \phi = -\frac{GM}{r}$$

The Schwarzschild solution

- Schwarzschild 1916 finds the first exact solution of Einstein's field equations in vacuum ($R_{\mu\nu} = 0$)

$$ds^2 = -(1 - \frac{r_G}{r})c^2dt^2 + \frac{dr^2}{(1 - \frac{r_G}{r})} + r^2 d\Omega^2$$

  to describe the space-time exterior to a (static and spherically symmetric) star ($r > R$)
- By Birkhoff's theorem it is the unique spherically symmetric solution of vacuum GR
- Nobody 'dared', for a longtime, to look at the properties of this solution as r approaches r_G, where $g_{tt} \to 0,\ g_{rr} \to \infty$ (coordinate singularity), since $r_G \sim 3\ km$ for the Sun and $\sim 0.9\ cm$ for the earth...

Mathematical properties of the Schwarzschild solution

- The curvature invariants ($c = G = 1$)

$$R = 0,\ R_{\mu\nu} = 0,\ R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = \frac{48M^2}{r^6}$$

  show that $r_H = 2M$ is not a curvature singularity

- Finkelstein 1958 writes down the first coordinate system regular at the horizon

$$ds^2 = -(1 - \frac{2M}{r})dv^2 + 2dvdr + r^2 d\Omega^2,$$

  where $v = t + r_* = t + \int \frac{dr}{1-\frac{2M}{r}} = r + 2M \ln \frac{r-2M}{2M}$ is a (radial) ingoing null coordinate ($v = const. \to ds^2 = 0$).

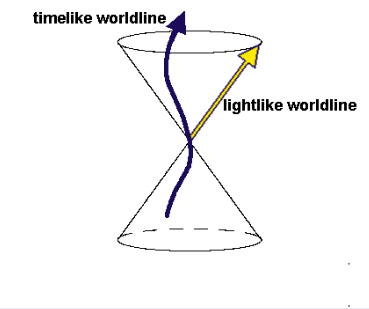
Light-cone structure: Minkowski

- The (radial) ingoing $v = t + r = const.$ (towards decreasing r) and outgoing $u = t - r = const.$ (towards increasing r) light rays

$$ds^2 = -dt^2 + dr^2 + r^2 d\Omega^2 = -dudv + r^2 d\Omega^2$$

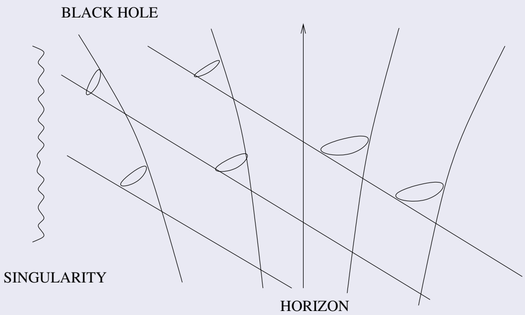
are 45^o lines

- For any point (event) P they define its future and past light cone



Light-cone structure: Schwarzschild

- (Radial) Light rays $ds^2 = 0$ are given by $v = const.$ (ingoing) and $\frac{dr}{dv} = \frac{1}{2}(1 - \frac{2M}{r})$ $(u = t - r_* = const.)$ (outgoing)

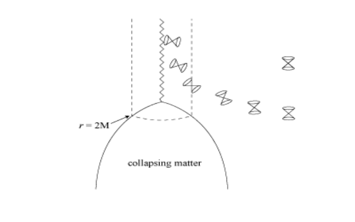


- As we approach r_H light-cones are more and more bent inward
- At $r = r_H$ outgoing null rays are stuck on it (event horizon) and future light cones of any point inside $(r < r_H)$ terminate into the central curvature singularity at $r = 0$ (trapped region)

Physics of gravitational collapse

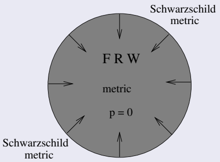
Depending on the total mass of the collapsing star we have three different final outcomes:

- Chandrasekhar 1931: For $M \leq 1.4\ M_{Sun}$ the end-point is a white dwarf (supported by the quantum degeneracy pressure of electrons) with $R \gg r_G$
- Oppenheimer and Volkoff 1939: For $1.4\ M_{Sun} \leq M \leq 2-3\ M_{Sun}$ a neutron star forms (neutrons' degeneracy pressure), $R \gtrsim r_G$
- Harrison, Thorne, Wakano, Wheeler 1965: above the neutron star threshold the star collapses inside its Schwarzschild (gravitational) radius: a black hole forms (Wheeler 1967)

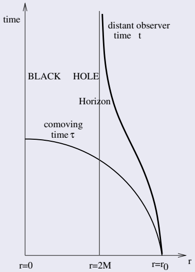


First model of black hole formation

- Oppenheimer and Snyder 1939 considered collapsing dust



- An observer on the surface of the 'star' sees it reaching r_H, and subsequently $r = 0$, in a finite proper time τ
- For an exterior (asymptotic) observer the star is frozen at its Schwarzschild radius ($t \to +\infty$, infinite redshift surface)

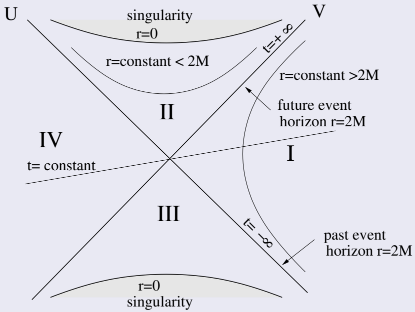


Maximal analytical extension of the Schwarzschild solution

- Use the (null) Kruskal coordinates (Kruskal 1960; Szekers 1960) $-\frac{U}{4M} = e^{-u/4M}$, $\frac{V}{4M} = e^{v/4M}$ ($\frac{1}{4M} \equiv$ horizon's surface gravity) for which

$$ds^2 = -\frac{2M}{r} e^{-\frac{r}{2M}} dU dV + r^2 d\Omega^2$$

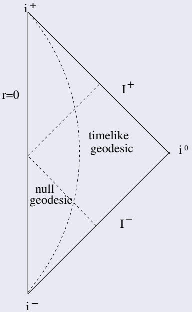
- This divides the full manifold in four regions



- There are two (I and IV) external regions and two interiors, II (black hole) and II (white hole)

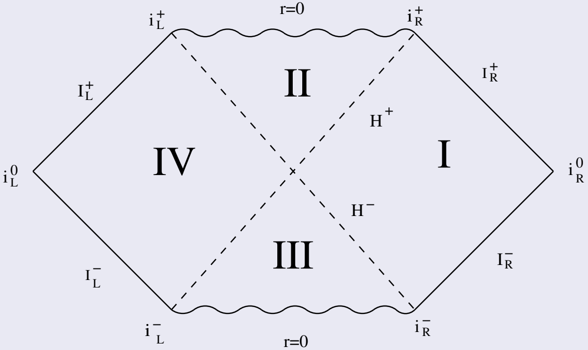
Penrose (causal) diagrams

- Map the original metric to a new one via a Weyl (conformal) transformation $ds^2 \to d\bar{s}^2 = \Omega^2(x^\mu) ds^2$
- Causal relationship between points is preserved
- The transformation $u = \tan \bar{u},\ v = \tan \bar{v}$ maps Minkowski to a triangle



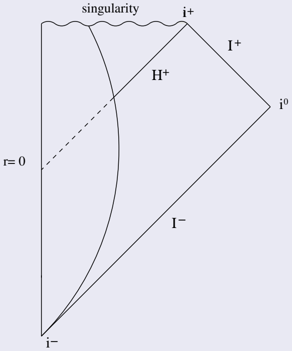
$i^-(i^+)$ is where timeline geodesics start (end), $I^-(I^+)$ are null past (future) infinity, i^0 is space like infinity

From the Kruskal diagram, via $V = 4M \tan \bar{V}, \; U = 4M \tan \bar{U}$ we get



where $H^-(H^+)$ is the past (future) horizon

Finally, for black holes formed by gravitational collapse



Only (a part of) previous regions I and II is physically relevant, the rest being replaced by the (regular) geometry inside the collapsing matter

Einstein-Maxwell theory

In Einstein-Maxwell theory we have a 'simple' generalisation of the Schwarzschild solution

$$ds^2 = -(1 - \frac{2M}{r} + \frac{Q^2}{r^2})dt^2 + \frac{dr^2}{(1 - \frac{2M}{r} + \frac{Q^2}{r^2})} + r^2 d\Omega^2$$

due to Reissner 1916 - Nordström 1918, and a generalisation of Birkhoff's theorem holds.

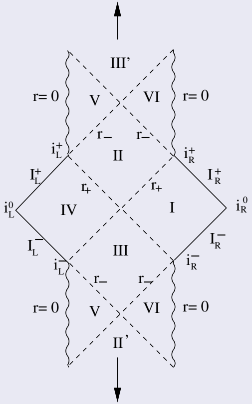
To have a black hole we need $M \geq |Q|$:

- In the non-extremal ($M > |Q|$) case we have two horizons ($g_{tt} = 0$)

  $$r_\pm = M \pm \sqrt{M^2 - Q^2},$$

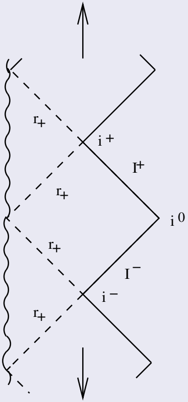
  r_+ is the outer (event) horizon and r_- the inner horizon
- In the extremal ($M = |Q|$) case we have only one (degenerate) horizon at $r_+ = M$
- For $M < |Q|$ no horizon and just a naked singularity

Inside the event horizon the full structure of this solution is much more involved



r_- is also a Cauchy horizon. It is a surface of infinite bluseshift, and is unstable to small perturbations in the external region (mass-inflation singularity Poisson and Israel 1990)

- In the extremal case the trapped region $r_- < r < r_+$ has disappeared



- for $M < |Q|$ we only have a triangle as in Minkowski, but with $r = 0$ singular (by Penrose' cosmic censorship conjecture it should not be realised in physically realistic configurations)

Outside spherical symmetry

- Price 1972: In the collapse of non spherical matter all multipole moments of the asymmetric body are radiated away in the form of gravitational and e.m. radiation. Only mass, charge and angular momentum are protected by conservation laws
- No-hair theorem (Israel 1968, Carter 1971, Hawking 1972, Robinson 1975): stationary black holes are only characterized by M, Q and J (Kerr 1963, Newman 1965 solutions)

Kerr solution

The metric of a rotating black hole ($a \equiv \frac{J}{M}$ angular momentum for unit mass) is

$$
\begin{aligned}
ds^2 &= -\frac{(\Delta - a^2 \sin^2\theta)}{\rho^2} dt^2 - 2ar\frac{2Mr\sin^2\theta}{\rho^2} dt d\phi \\
&+ \frac{(r^2+a^2)^2 - \Delta a^2 \sin^2\theta}{\rho^2} \sin^2\theta^2 d\phi^2 + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 \ ,
\end{aligned}
$$

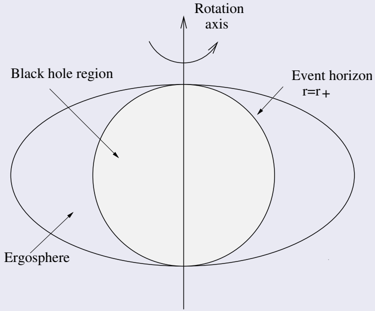
where

$$\rho^2 \equiv r^2 + a^2 \cos^2\theta, \ \Delta \equiv r^2 - 2Mr + a^2.$$

- It is stationary ($g_{\mu\nu,t} = 0$), but not static (i.e. no $t \to -t$ symmetry), and axisymmetric.
- The invariances $t \to t + const.$, $\phi \to \phi + const.$ are expressed by the two Killing vectors

$$\xi^\mu = (1,0,0,0), \ \psi^\mu = (0,0,0,1).$$

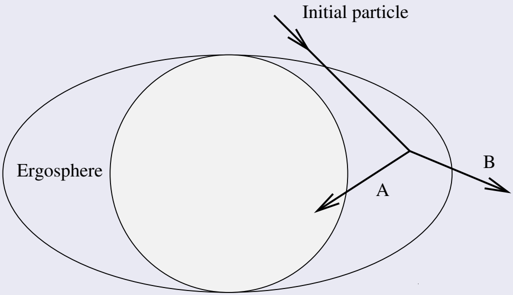
- The horizon is $\Delta = 0$ and not $g_{tt} = 0$. We have two ($r_\pm = M \pm \sqrt{M^2 - a^2}$), one (degenerate) or zero horizons as in Reissner-Nordström
- $g_{tt} = 0$ defines the ergosphere $r_{erg} = M + \sqrt{M^2 - a^2 \cos^2\theta}$



- In the ergoregion ($r_+ < r < r_{erg}$) timelike curves corotate with the black hole $\frac{d\phi}{dt} \geq \frac{a}{a^2 + r_+^2} \equiv \Omega_H$.
- $\xi^2 = \xi^\mu \xi_\mu = 0$ at r_{erg} while the norm of $\chi^\mu = \xi^\mu + \Omega_H \psi^\mu$ vanishes at r_+

Penrose process

- Penrose 1969 pointed out the possibility to extract energy from a rotating black hole (impossible in Schwarzschild).
- A particle following a geodesic (with momentum p^μ and constant of motion $E = -p_\mu \xi^\mu$) enters the ergoregion where it splits into two, A and B, of which only B comes back out, while A falls down the hole



- From momentum conservation $E = E_A + E_B$.
  While $E, E_B > 0$ (energies, measured at infinity) no such constraint exists for E_A: if $E_A < 0$ then
  $$E_B > E$$
- In the ergoregion $-p_\mu \xi^\mu$ has no definite sign (ξ^μ is spacelike) but $\chi^\mu = \xi^\mu + \Omega_H \psi^\mu$ is timelike and $-p_\mu \chi^\mu > 0$. This implies
  $$E_A - \Omega_H L_A > 0$$
  In this process the hole looses energy ($\delta M = E_A < 0$) and angular momentum ($\delta J = L_A < \frac{E_A}{\Omega_H} < 0$)
- The wave analog of this process, for which an incident wave with amplitude A_{in} is scattered by the geometry into a final one with $A_{out} > A_{in}$ ($\omega < m\Omega_H$) is called superradiance (Zeldovich 1972, Starobinski 1973)

Irreducible mass

Christodoulou 1970, Christodoulou and Ruffini 1971

The inequality

$$\Omega_H \delta J - \delta M < 0$$

can be rewritten, by introducing the irreducible mass

$$M_{irr}^2 = \frac{1}{2}[M^2 + \sqrt{M^4 - J^2}],$$

as

$$\delta M_{irr}^2 = \frac{r_+^2 + a^2}{r_+ - r_-}(\delta M - \Omega_H \delta J) > 0.$$

Two 'curiosities' (to be continued)

- $A_H = \int_{r=r_+} d\theta d\phi \sqrt{g_{\theta\theta} g_{\phi\phi}} = 16\pi M_{irr}^2 \Rightarrow \delta A_H > 0$
- $\delta M = \frac{\kappa_+}{8\pi}\delta A_H + \Omega_H \delta J$, where $\kappa_+ = \frac{r_+ - r_-}{2(r_+^2 + a^2)}$ is the horizon's surface gravity

Stability analysis

- Regge and Wheeler 1957 studied the classical linear perturbations $h_{\mu\nu}$ around the Schwarzschild solution satisfying $\delta R_{\mu\nu} = 0$.
- (Infinitesimal) diffeomorphism invariance

$$x'^{\alpha} = x^{\alpha} + \xi^{\alpha} \Rightarrow h_{\mu\nu} \to h_{\mu\nu} + \xi_{\mu;\nu} + \xi_{\nu;\mu}$$

  allows to simplify the equations
- The transverse traceless gauge $\nabla^{\nu} h_{\mu\nu} = 0 = h^{\mu}_{\mu}$ leads to the massless Lichnerowicz equation

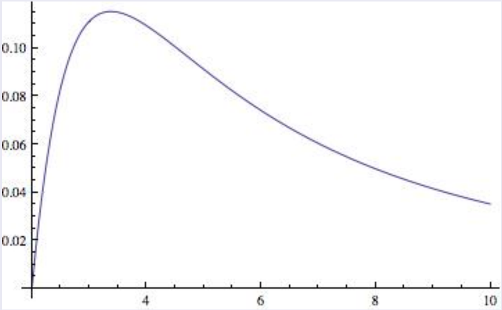
$$\Delta_L h_{\mu\nu} \equiv \Box h_{\mu\nu} + 2R^{\sigma}{}_{\mu}{}^{\lambda}{}_{\nu} h_{\lambda\sigma} = 0$$

  but it does not eliminate all the gauge freedom

Perturbations were expanded in spherical harmonics and Schrödinger-like equations (see also Zerilli 1970) were derived for the radial part of $h_{\mu\nu} = e^{-iwt} H_{\mu\nu}$

$$\partial^2_{r_*} \Psi + (w^2 - V_{eff}(r,l))\Psi = 0,$$

where $V_{eff} \to 0$ both at the horizon ($r_* \to -\infty$) and at infinity ($r_* \to +\infty$).



and no sign of instability (i.e. spatially normalizable modes with $Im\ w \equiv \Omega > 0$) was found.

The Schwarzschild black string instability

Gregory and Laflamme 1993 considered the classical linear perturbations around the Schwarzschild black string

$$ds^2 = -(1 - \frac{2M}{r})dt^2 + \frac{dr^2}{(1 - \frac{2M}{r})} + r^2 d\Omega^2 + dz^2$$

and found an instability to perturbations with large wavelength in the fifth dimension

In the transverse trace-free gauge the perturbation equations from the 5D Einstein equations take the form

$$\Box h_{ab} + 2R^{c\ \ d}_{\ a\ \ b} h_{cd} = 0$$

and the relevant unstable modes take the form

$$h^{ab}_{unst} = e^{ikz} e^{\Omega t} \begin{pmatrix} H^{zz}(r) & H^{zt}(r) & H^{zr}(r) & 0 & 0 \\ H^{zt}(r) & H^{tt}(r) & H^{tr}(r) & 0 & 0 \\ H^{zr}(r) & H^{tr}(r) & H^{rr}(r) & 0 & 0 \\ 0 & 0 & 0 & K(r) & 0 \\ 0 & 0 & 0 & 0 & \frac{K(r)}{\sin^2\theta} \end{pmatrix}$$

- From a 4D perspective, we have a scalar h^{zz}, a vector $h^{\mu z}$ and a tensor $h^{\mu\nu(4)}$.
- h^{zz} and $h^{\mu z}$, regular at the horizon and exponentially vanishing asymptotically, show no presence of unstable modes ($h^{zz}_{unst} = h^{\mu z}_{unst} = 0$)
- For the remaining components we are left with 4D massive Lichnerowicz eqs.

$$\Box h^{(4)}_{\mu\nu} + 2R^{\sigma}{}_{\mu}{}^{\lambda}{}_{\nu} h^{(4)}_{\lambda\sigma} = m^2 h^{(4)}_{\mu\nu}, \quad m^2 = k^2$$

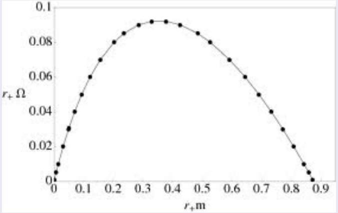
- A master equation can be obtained for, say, H^{tr} and crucial is the choice of the correct boundary conditions at the horizon (say, regularity in ingoing EF coordinates)

$$H^{tr} \sim (r - 2M)^{-1+2M\Omega}, \ \Omega > 0$$

while at infinity

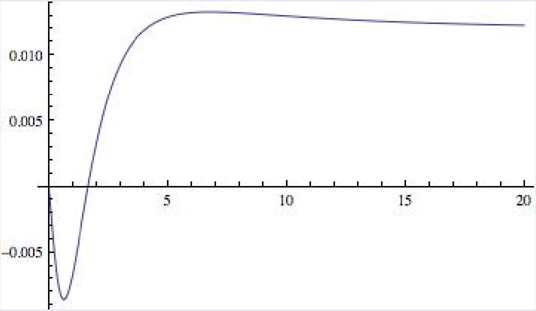
$$H^{tr} \sim e^{-r\sqrt{\Omega^2+m^2}}$$

- Numerical analysis shows the presence of unstable modes in the range $0 < m < m_{crit} \sim \frac{0.43}{M}$ along the curve



- Analytical long-wavelength approximations give $\Omega \sim \frac{m}{\sqrt{2}} + O(m^2)$ Emparan, Harmark, Niarchos and Obers 2010
- The static mode $(\Omega = 0,\ m = m_{crit})$ was already noticed by Gross, Perry and Jaffe 1982

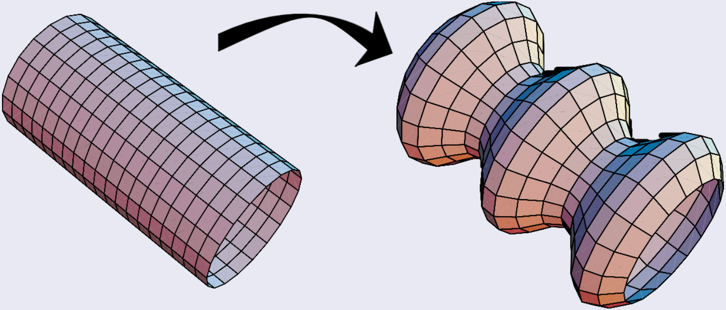
A (lengthy) calculation allows to put the perturbation eqs. into a Schrödinger-like form with an effective potential



that develops a well close to the horizon where the bound state/instability lives

Endpoint of the GL instability

The GL instability leads to the breaking of the string into an array of spherical holes



(In)stability of Schwarzschild black holes in massive gravity

- Nonlinear completions of the quadratic massive gravity theory by Fierz and Pauli 1939

$$S_{PF,m} \sim -m^2 \int d^4x (h_{\mu\nu} h^{\mu\nu} - h^2),$$

  in which diffeomorphism invariance is broken and the graviton acquires 5 propagating degrees of freedom, have been achieved de Rham, Gabadadze and Tolley 2011

- They are compatible with solar system experiments
- They require the introduction of a second metric $f_{\mu\nu}$ which could be either fixed of dynamical: when $f = \omega^2 g$, $\omega^2 = const.$, the e.o.m. reduce to those of GR.

- Linear perturbation analysis shown that the class of (bi-)Schwarzschild black holes has an unstable GL mode Babichev and Fabbri 2013 (the same is true in fourth-order gravity ..) provided m is in the instability range $0 < m < O(\frac{1}{r_G})$
- In physically relevant situations $m \ll \frac{1}{r_G}$ (to satisfy solar system experiments), i.e. we are in the initial linear regime of the GL instability curve $\Omega = \Omega(m)$
- If the graviton mass is responsible for the cosmic acceleration $m \sim H$, implying an instability timescale $\tau^{GL} \sim \frac{1}{m}$ of the order of the Hubble time
- Brito, Cardoso and Pani 2013 confirmed numerically these results (the bi-Kerr solutions are also unstable due to superradiant modes). They also found likely candidate end-point configurations - black holes with a massive graviton halo - but only in the regime $m \sim \frac{\mathcal{O}(1)}{r_G}$

The laws of black hole mechanics

Hawking 1972 showed that under general conditions (in particular the weak energy condition) the area of the event horizon never decreases

$$\delta A_H \geq 0$$

Moreover Bardeen, Carter and Hawking 1973 showed that the parameters of two nearby (stationary) black holes satisfy

$$c^2 \delta M = \frac{c^2}{8\pi G} \kappa \delta A_H + \Omega_H \delta J \ (+\Phi_H \delta Q)$$

and that the surface gravity κ is constant on the horizon.

Analogy with thermodynamics

Provided $M \leftrightarrow E,\ \kappa \leftrightarrow T,\ A_H \leftrightarrow S$ we recover the laws of thermodynamics:

- T is constant at equilibrium (0th-law)
- $\delta E = T\delta S - P\delta V$ (1st law)
- S (closed system) never decreases (2nd law)

There is also an analogy with the third law (Nernst version), in the sense that the closer the state of the black hole is to (extremal) $\kappa = 0$ ($\leftrightarrow T = 0$) the harder it is to get even closer

The analogy with thermodynamics led Bekenstein 1972 to conjecture that A_H measures the entropy of the black hole, but how can we assign a temperature to it, since classically a black hole is a perfect absorber and has $T = 0$??

By looking at it in more detail

$$Mc^2 \leftrightarrow E,\ \frac{\alpha c^2 \kappa}{G} \leftrightarrow T,\ \frac{A_H}{8\pi\alpha} \leftrightarrow S$$

we see that in order for $\frac{A}{\alpha}$ to have the dimension of an entropy then $[\alpha] = \frac{L^2}{k_B}$ and with only c and G this is not possible.. we need l_P !

$$[\alpha] = \frac{l_p^2}{k_B} \Rightarrow S \sim \frac{k_B c^3 A}{G\hbar},\ T \sim \frac{\hbar\kappa}{k_B c}$$

S, $O(\frac{1}{\hbar})$, needs quantum gravity, $T = O(\hbar)$ QFT in curved space (Hawking 1974)

Quantum Field Theory in Minkowski space

Canonical quantisation of a (free) scalar field $\hat{f}$ (action $S = -\frac{1}{2}\int d^4x \partial_\mu f \partial^\mu f$)

$$\hat{f} = \sum_i [\hat{a}_i u_i + \hat{a}_i^\dagger u_i^*]$$

where u_i are (ortho-normal, wrt $(f_1, f_2) = -i \int d^3\vec{x} f_1 \overleftrightarrow{\partial_t} f_2$) modes of the field (satisfying the classical e.o.m. $\partial^\mu \partial_\mu f = 0$) of positive frequency with respect to global inertial Minkowski time t

$$\frac{\partial}{\partial t} u_i = -i\omega_i u_i, \omega > 0$$

and $\hat{a}_i / \hat{a}_i^\dagger$ are annihilation/creation operators.
The vacuum state is defined by

$$\hat{a}_i |0\rangle = 0$$

and is unique (i.e. independent of the particular inertial time t considered)

Transition to curved space (Parker 1969)

- Because of general covariance we loose, in general, a unique criterium to define positive frequency modes, vacuum states and particles..
- When the space-time is stationary we have a privileged time t (and Killing vector ξ^μ) allowing to construct these quantities.
- We have a natural definition of scalar product

$$(f_1, f_2) = -i \int_\Sigma d\Sigma^\mu (f_1 \partial_\mu f_2^* - f_2^* \partial_\mu f_1)$$

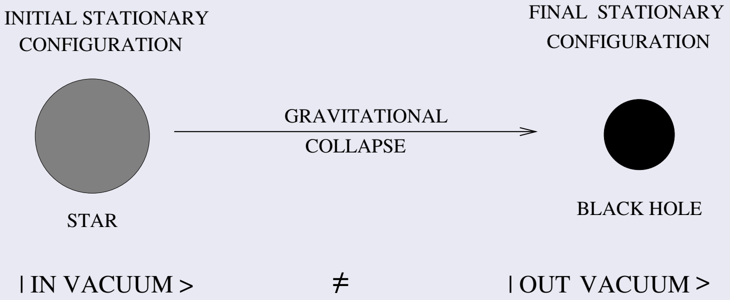
and of positive frequency

$$\xi^\mu \nabla_\mu u_i = -i\omega_i u_i, \ \omega > 0$$

and the standard particle interpretation (of states in the Fock space) in Minkowski space can be extended

Black holes

For a black hole formed by gravitational collapse, despite the global nonstationarity we have well defined initial and final stationary configurations



The fact that the initial and final vacuum states are not the same is the essence of the Hawking effect

Birrell and Davies 1982, Frolov and Novikov 1998, Fabbri and Navarro-Salas 2005

A crucial tool: Bogoliubov transformations

Our field $\hat{f}$ has two natural in/out decompositions

$$\hat{f} = \sum_i [\hat{a}^{in/out} u_i^{in/out} + \hat{a}^{in/out\ \dagger} u_i^{in/out*}]$$

each with its associated vacuum state

$$\hat{a}^{in}|0\rangle_{in} = 0,\ \ \hat{a}^{out}|0\rangle_{out} = 0.$$

Being both sets of modes complete, the general relation between them is

$$u_j^{out} = \sum_i (\alpha_{ji} u_i^{in} + \beta_{ji} u_i^{in*}),\ \ \hat{a}^i_{out} = \sum_j (\alpha^*_{ij} \hat{a}_j^{in} + \beta^*_{ij} \hat{a}_j^{in\dagger})$$

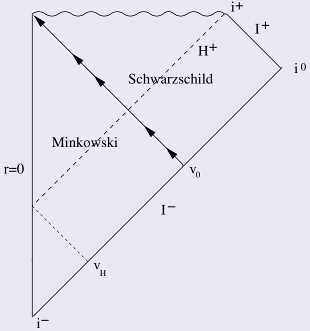
If $\beta \neq 0$ we have that

$$_{in}\langle 0| N_i^{out} |0\rangle_{in} \equiv {}_{in}\langle 0| \hat{a}_i^{out\dagger} \hat{a}_i^{out} |0\rangle_{in} = \sum_j |\beta_{ij}|^2 \neq 0,$$

i.e. $|0\rangle_{in} \neq |0\rangle_{out}$.

Choice of the background

- We will simplify our scenario at a maximum by considering the collapsing star as an ingoing null shock wave



- The 'in' region is Minkowski and the 'out' one Schwarzschild.
- The motivation comes from the fact that the results do not depend on the type of collapse, but just on the black hole parameters (M in this case)

Matter field: a convenient approximation

Our scalar field satisfies the Klein-Gordon equation

$$\Box f = g_{\mu\nu}\nabla^{\mu}\nabla^{\nu} f = 0$$

and for a spherically symmetric background we can expand f in spherical harmonics

$$f(x^{\mu}) = \sum_{l,m} \frac{f_l(t,r)}{r} Y_{lm}(\theta,\phi)$$

($l = 0$ is the s-wave component, $l \neq 0$ have non vanishing angular momentum). We have infinite equations, one for each l

The wave equation in the 'in' region (Minkowski) is

$$\left(-\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial r^2}-\frac{l(l+1)}{r^2}\right)f_l(t,r)=0$$

and in the 'out' region (Schwarzschild)

$$\left(-\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial r_*^2}-V_{eff}\right)f_l(t,r)=0$$

The effective potential

$$V_{eff}=(1-\frac{2M}{r})(\frac{l(l+1)}{r^2}+\frac{2M}{r^3})$$

is a positive barrier whose height grows with l. Particles created by the black hole will be mainly in s-wave ($\sim 90\ \%$), so we make the 'crude' approximations

$$l=0,\ V_{eff}^{l=0}=0$$

which 'only' overestimate the final result by a factor $\sim 5-6$

'In' and 'out' Modes

In this way the solutions for f_0 ($\equiv$ 2d conformal scalar field) are just plane waves. In the 'in' region, where

$$ds^2 = -du_{in}dv_{in} + r_{in}^2 d\Omega^2$$

we have

$$u_\omega^{in} = \frac{1}{4\pi\sqrt{\omega}} \frac{e^{-i\omega v_{in}}}{r}$$

and in the 'out' Schwarzschild region

$$ds^2 = -(1 - \frac{2M}{r_{out}})dv_{out}du_{out} + r_{out}^2 d\Omega^2$$

it is

$$u_\omega^{out} = \frac{1}{4\pi\sqrt{\omega}} \frac{e^{-i\omega u_{out}}}{r}$$

Important: in the 'out' region u only covers the external region, u_ω^{out} is not a complete set. We'll come back to this point later.

Particle creation by black holes

We need to calculate

$$ {}_{in}\langle 0|N^{out}_{\omega}|0\rangle_{in} = \int_0^{\infty} d\omega' |\beta_{\omega\omega'}|^2, $$

i.e.

$$ \beta_{\omega\omega'} = -(u^{out}_{\omega}, u^{in*}_{\omega'}) = i \int_{I^-} dvr^2 d\Omega (u^{out}_{\omega}\partial_v u^{in}_{\omega'} - u^{in}_{\omega'}\partial_v u^{out}_{\omega}). $$

Since the modes u^{out}_{ω} are defined at I^+ we need to propagate them 'back' to I^- where the scalar product is (conveniently) evaluated:

1. From I^+ to the shock-wave (trivial, free propagation)
2. across the shock-wave (need to know the 'matching' between 'in' and 'out' coordinates)
3. reflection at $r = 0$ and propagation to I^-

$$ e^{-i\omega u_{out}}|_{I^+} \to e^{-i\omega u_{out}(u_{in})} - e^{-i\omega u_{out}(v_{in})}\theta(v_H - v_{in})|_{I^-} $$

Matching of the 'in' and 'out' metrics

Continuity of the 'in' and 'out' metrics at the shock-wave requires

$$r_{in} = r_{out} \equiv r, \; v_{in} = v_{out} \equiv v$$

implying

$$u_{out} = u_{in} - 4M \ln \left| \frac{v_H - u_{in}}{4M} \right|,$$

where $v_H = v_0 - 4M$ is the location of the null ray that will form the event horizon $u = +\infty$.

We identify two regimes:

- 'early times' $u_{out} \to -\infty, u_{out} \sim u_{in}$ and

$$e^{-i\omega u_{out}}|_{I^+} \to -e^{-i\omega v}|_{I^-} \Rightarrow \beta = 0$$

- 'late times' $u_{out} \to +\infty, \; u_{in} \to v_H$ and

$$u_{out} \sim v_H - 4M \ln \frac{|v_H - u_{in}|}{4M} \; , v_H - u_{in} \sim U$$

We have $u^{in}_{\omega'} = \frac{1}{4\pi\sqrt{\omega'}}\frac{e^{-i\omega' v}}{r}$ and

$$u^{out}_{\omega}|_{I^-} = -\frac{1}{4\pi\sqrt{\omega}}\frac{e^{-i\omega'(v_H - 4M\ln|\frac{v_H - v}{4M}|)}}{r}\theta(v_H - v).$$

Note that $\partial_v u^{out}_{\omega} = -i\frac{\omega}{v_H - v}u^{out}_{\omega}$ (infinite blueshift transplanckian problem)

The calculation

$$\beta_{\omega\omega'} = i\int_{-\infty}^{v_H} dv r^2 d\Omega (u^{out}_{\omega}\partial_v u^{in}_{\omega'} - u^{in}_{\omega'}\partial_v u^{out}_{\omega})$$

leads to

$$\int_0^{+\infty} d\omega' \beta_{\omega_1\omega'}\beta^*_{\omega_2\omega'} = \frac{\delta(\omega_1 - \omega_2)}{e^{8\pi M\omega_1} - 1}$$

which shows that particles are spontaneously produced thermally at the Hawking temperature ($\frac{\hbar\omega}{k_B T_H} = 8\pi M\omega$)

$$T_H = \frac{\hbar}{8\pi k_B M} = \frac{\hbar\kappa}{2\pi}$$

An apparent contradiction

We started with our matter field in the initial vacuum $|0\rangle_{in}$ (pure state), but for the 'out' observer this is just (uncorrelated) thermal radiation, described by a (thermal) density matrix ρ_{th}, i.e.

$$\langle \hat{N}^{out}_{\omega} \rangle = Tr\left(\rho_{th} \hat{N}^{out}_{\omega}\right),$$

where $\rho_{th} = \Pi_{\omega} \sum_{N=0}^{\infty} P(N^{out}_{\omega}) |N^{out}_{\omega}\rangle\langle N^{out}_{\omega}|$. How can it be??

We have 'forgotten' the region inside the black hole... as already remarked, in the 'out' region u^{out}_{ω} is not a complete set (I^{+} is not a Cauchy hypersurface), we need to add the modes u^{int}_{ω} inside the black hole

$$\hat{f} = \int_{0}^{+\infty} d\omega [\hat{a}^{out}_{\omega} u^{out}_{\omega} + \hat{a}^{int}_{\omega} u^{int}_{\omega} + h.c.]$$

The form of u_ω^{int} is not univocally defined (the interior of the black hole does not have a natural time nor an asymptotically flat region). A convenient choice is (Wald 1975, Parker 1975)

$$u_\omega^{int} \sim e^{i\omega(v_H - 4M \ln \frac{v-v_H}{4M})}\theta(v - v_H)$$

allowing to simplify calculations and clarify the nature of the Hawking effect.

Writing Bogoliubov transformations between 'in' and 'out' + 'int' decompositions one has four Bogo coeffs. $\alpha, \beta, \gamma, \delta$ and, from them,

$$|0\rangle_{in} \sim \Pi_\omega \sum_N e^{-4\pi N M\omega} |N_\omega^{out}\rangle \times |N_\omega^{int}\rangle.$$

We recover $|0\rangle_{in}$ as a pure state - correlations are between the 'ext' and the 'int' regions - and we see that uncorrelated thermal radiation arises only after tracing over the internal states

$$\rho_{th}^{in} = Tr_{int}|0\rangle_{in\,in}\langle 0|.$$

A look at the approximations made

The luminosity (energy emitted per unit time) that we get is

$$L = \frac{1}{2\pi}\int_0^\infty d\omega \frac{\hbar\omega}{e^{8\pi M\omega}-1} = \frac{\hbar}{768\pi M^2}.$$

By taking into account $V_0(r)$ we get (Balbinot et al 2001)

$$L_{l=0} = \frac{1}{2\pi}\int_0^\infty d\omega \frac{\hbar\omega\Gamma_\omega^{l=0}}{e^{8\pi M\omega}-1} \approx \frac{1.62\hbar}{7680\pi M^2},$$

where $\Gamma_\omega^{l=0} \equiv |t_\omega^{l=0}|^2$ is the $(l=0)$ gray-body factor, and including the contribution of all l (De Witt 1975, Elster 1983)

$$L_{tot} = \sum_l L_l \approx \frac{1.79\hbar}{7680\pi M^2}.$$

Including angular momentum and charge

$$_{in}\langle 0|N^{lm}_\omega|0\rangle_{in} = (1-\frac{m\Omega_H}{\omega})\frac{|t^{lm}|^2}{e^{\frac{2\pi}{\kappa_+}(\omega - m\Omega_H)}-1}$$

which for $\kappa_+ \to 0,\ \omega < m\Omega_H$ becomes

$$\frac{(m\Omega_H-\omega)}{\omega}|t^{lm}_\omega|^2,$$

i.e. the black hole spontaneously looses angular momentum (superradiance). Similar results are valid for a charged (q) field in a Reissner-Nordstrom black hole ($m \to q,\ \Omega_H \to \phi_H$): by a Schwinger-like effect, the black hole (spontaneously) looses its charge.

Spin and mass

Hawking analysis applies to any field, the only difference between them being their gray-body factor: it is maximum for $l = 0$ and $s = 0$, and decreases as l and/or s are increased.
Moreover, most of the emission is in massless particles, massive ones are allowed only when $k_B T_H > m$.

Beyond the fixed background approximation

- The total energy emitted by the black hole in Hawking's analysis

$$E = \int_{-\infty}^{+\infty} du L$$

  is infinite.
- We get this (absurd) result because we have neglected to take into account the effects of the Hawking flux on the black hole which will modify the background geometry (backreaction problem).
- By energy conservation, the emission process must be accompanied by a decrease of the mass of the black hole.

Quasistatic approximation

Since for a macroscopic BH T_H is very small ($T_H \sim 10^{-7} \frac{M_{Sun}}{M}$) one can plausibly describe the evaporation as a series of Scharzschild BHs with decreasing mass satisfying

$$\frac{dM(t)}{dt} = -\frac{\alpha}{M(t)^2} \Rightarrow M = (M_0^3 - 3\alpha t)^{1/3}$$

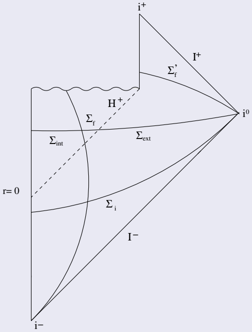
This approximation is no more valid when $M \sim M_{Planck}$, and according to the quasi static evolution it takes a time

$$\Delta t \sim 10^5 t_P (\frac{M}{m_P})^3$$

to get there, during which the black hole emits (to a good approximation) thermally. Δt is huge for a solar mass bh ($\gg$ age of the Universe).

What happens when M reaches M_P??

Hawking 1976 extrapolated his results to $M = 0$: the bh disappears, the evolution $|0\rangle_{in} \to \rho_{th}$ is non unitary and information about the initial state is lost



Alternatives to restore predictability

- Page 1980 The bh disappears but the radiation is sufficiently correlated to determine a pure state
- Hawking radiation stops at the Planck scale, leaving a (Planck mass) remnant (Giddings 1994, Banks 1995)
- correlations are maintained through a 'baby' universe (Hawking 1988)
- black hole complementarity ('t Hooft 1990, Susskind et al. 1993, ..), Maldacena's AdS/CFT, firewalls (Almeheiri, Marolf, Polchinski, Sully 2013)

Backreaction

- To improve the quasi-static picture of black hole evaporation we consider the semiclassical Einstein equations

$$G_{\mu\nu} = 8\pi\langle T_{\mu\nu}\rangle,$$

  which requires quantization of our matter fields in a generic geometry.
- Schwarzschild is solution when the rhs is zero, but not when it is nonzero.
- Already in Schwarzschild to get an expression of $\langle T_{\mu\nu}\rangle$ is very difficult, only approximate expressions valid near the horizon and at infinity are known.
- One usually circumvents this problem by considering lower-dimensional models

2d conformal scalar field - classical considerations

We already considered this field in deriving Hawking radiation..and we know the results are physically meaningful.

The classical action

$$S = -\frac{1}{2}\int d^2x\sqrt{-g}(\nabla f)^2 \ ,$$

leads to a stress-energy tensor

$$T_{\mu\nu} = \partial_\mu f \partial_\nu f - \frac{g_{\mu\nu}}{2}(\nabla f)^2$$

which is traceless (conformal invariance) and conserved .

2d conformal scalar field - quantum considerations

- It turns out that at the quantum level (1-loop) we cannot maintain both a traceless and a conserved $\langle T_{\mu\nu} \rangle$.
- Maintaining conservation, as required by the semiclassical Einstein eqs., gives a (local) trace proportional to the scalar curvature (Capper and Duff 1974)

$$\langle T \rangle = \frac{\hbar}{24\pi} R$$

  called the trace anomaly
- trace anomaly + conservation eqs. completely characterize $\langle T_{\mu\nu} \rangle$ up to a radiation term which, in turn, characterises the quantum state in which the expectation value is taken

2d stress tensor

- By writing the metric in double-null coordinates (e.o.m. $\partial_+\partial_- f = 0$)

$$ds^2 = -e^{2\rho}dx^+dx^-$$

we have

$$\langle T_{\pm\pm}\rangle = -\frac{1}{2\pi}[(\partial_\pm\rho)^2 - \partial_\pm^2\rho + t_\pm(x^\pm)]$$

while

$$\langle T_{+-}\rangle = -\frac{\hbar}{12\pi}\partial_+\partial_-\rho$$

is fixed by the trace anomaly.

- The functions $t_\pm$ give the information on the quantum state. If we choose the vacuum state associated to the modes $e^{-i\omega y^\pm(x^\pm)}$, $t_\pm$ are the Schwarzian derivatives between the two sets of coords.

$$t_\pm = \frac{1}{2}\{y^\pm, x^\pm\} = \frac{1}{2}\left(\frac{\dddot{y}^\pm}{\dot{y}^\pm} - \frac{3}{2}(\frac{\ddot{y}^\pm}{\dot{y}^\pm})^2\right).$$

2d stress tensor in Schwarzschild

By considering $x^+ = v,\ x^- = u,\ e^{2\rho} = (1 - \frac{2M}{r})$ we have

$$\begin{aligned}
\langle T_{uu} \rangle &= \frac{\hbar}{24\pi}[-\frac{M}{r^3} + \frac{3}{2}\frac{M^2}{r^4}] - \frac{t_u}{12\pi}, \\
\langle T_{vv} \rangle &= \frac{\hbar}{24\pi}[-\frac{M}{r^3} + \frac{3}{2}\frac{M^2}{r^4}] - \frac{t_v}{12\pi}\ , \\
\langle T_{uv} \rangle &= -\frac{\hbar}{24\pi}(1 - \frac{2M}{r})\frac{M}{r^3}
\end{aligned}$$

Regularity at the future horizon (i.e. $\langle T_{\mu\nu} \rangle$ finite in a regular frame there) is expressed through the conditions (Christensen and Fulling 1977)

$$|\langle T_{vv} \rangle| < \infty,\ \frac{|\langle T_{uv} \rangle|}{(r-2M)} < \infty,\ \frac{|\langle T_{uu} \rangle|}{(r-2M)^2} < \infty$$

and similar conditions (with u and v interchanged) at the past horizon.

Three possible choices

- Boulware state $t_u = t_v = 0$. Vacuum of the modes $e^{-i\omega u}$, $e^{-i\omega v}$: it reproduces Minkowski ground state at infinity (where $\langle T_{\mu\nu}\rangle \to 0$), but is badly divergent at the horizon (vacuum polarization around a static star)
- Hartle-Hawking state $t_u = t_v = -\frac{1}{64M^2}$. Vacuum of the Kruskal modes $e^{-i\omega U}$, $e^{-i\omega V}$: regular on $H^{\pm}$ and far away

$$\langle T_{uu}\rangle = \langle T_{vv}\rangle \to \frac{\hbar}{768\pi M^2} = \frac{\pi T_H^2}{12\hbar}$$

  (thermal equilibrium of a bh in a box with its own radiation)
- Unruh state $t_u = -\frac{1}{64M^2}$, $t_v = 0$. Vacuum of $e^{-i\omega U}$, late-time behaviour of $e^{-i\omega u_{in}}$, and $e^{-i\omega v}$. Outgoing flux of radiation at I^+ (Hawking radiation) and ingoing negative energy influx across H^+

$$\langle T_{uu}\rangle|_{I^+} \to \frac{\hbar}{768\pi M^2}, \ \langle T_{vv}\rangle|_{H^+} = -\frac{\hbar}{768\pi M^2}$$

  (gravitational collapse and black hole evaporation)

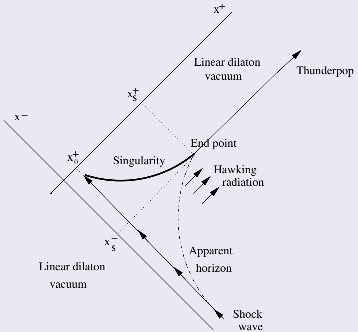
Evaporating black holes: semiclassical evolution

- The picture of black hole evaporation becomes more clear.. but the horizon structure more involved.
- Due to the evaporation we loose staticity (stationarity): we have a local horizon apparent horizon ($\equiv$ outer boundary of the trapped surfaces) and a global one event horizon ($\equiv$ boundary of the past-light cone of future null infinity I^+) which are now different
- The negative energy influx makes the apparent horizon to shrink (because we violate there the weak energy condition)
- To identify the event horizon of an evaporating black hole we need to know the evolution at all times...

Models for evaporating black holes

By considering 2d conformal matter one can construct models which solve analytically the backreaction problem:

- CGHS bhs (Callan, Giddings, Harvey and Strominger 1992, Russo, Susskind and Thorlacius 1992)



  Energy is conserved, information is lost

- Near-extremal Reissner-Nordström bhs: it takes an infinite time to get back go extremity ($\kappa = 0,\ T_H = 0$) (Fabbri, Navarro and Navarro-Salas 2000)

Beyond 2d conformal fields

Outside 2d conformal fields the situation becomes quickly very involved:

- s-wave component of a 4D field ($V_0(r) \neq 0$): $\langle T \rangle$ is known, but not the exact form of $\langle T_{\mu\nu} \rangle$
- 4d conformal fields: the trace anomaly is more complicated

$$\langle T \rangle = -\frac{1}{4\pi^2}(aC^2 + bE + c\Box R),$$

  $a,\ b,\ c$ depends on the matter species

- By integrating

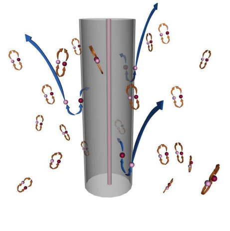
$$\frac{2}{\sqrt{-g}} g_{\mu\nu} \frac{\delta S_{an}^{eff}}{\delta g_{\mu\nu}} = \langle T \rangle$$

  one can get an effective action which reproduces (reasonably well) known approximate results for $\langle T_{\mu\nu} \rangle$ in Schwarzschild (Balbinot, Fabbri, Shapiro 1999) and can be used in backreaction calculations..

Analog gravity and analog Hawking radiation: the experimental search

Black hole evaporation

- We have seen that BHs are not 'black', but emit thermal radiation with a characteristic temperature $T_H = \frac{\hbar\kappa}{2\pi k_B}$ (κ is the horizon's surface gravity)
- Basic mechanism: conversion of quantum vacuum fluctuations into on shell particles due to horizon formation

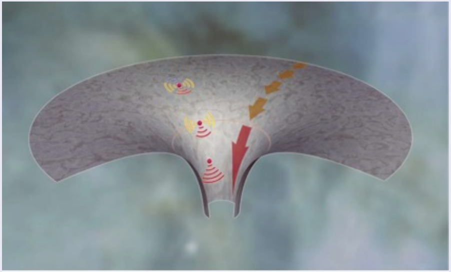


Experimental black hole evaporation?

- Limits on the experimental search: for BHs formed by gravitational collapse $T_H \sim 10^{-7} \frac{M_{Sun}}{M}\ K \ll T_{CMB} \sim 3\ K$...
- Alternative possibilities are based on the possible existence of minibhs, either produced in the early Universe by density fluctuations (Carr 1975) or in particle accelerators due to the existence of large extra dimensions (Arkani-Hamed, Dimopoulous, Dvali 1998, Randall and Sundrum 1999)
- Moreover if we look back at Hawking's derivation we see that nowhere Einstein field eqs. have been used: it is a kinematical effect depending only on the details of wave propagation in the vicinity of a horizon..

Analog Hawking radiation (Unruh '81):

- A stationary fluid undergoing transition from subsonic to supersonic motion is for sound what a black hole is for light (acoustic black hole)



- A process identical to that found by Hawking for BHs works for acoustic bhs: they emit a thermal flux of phonons from their acoustic horizon with

$$T_H = \frac{1}{4\pi k_B c_s} \frac{d(c_s^2 - \vec{v}_0^2)}{dn}\bigg|_{hor}$$

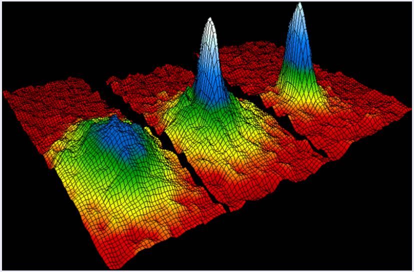
Domain of application and advantages wrt gravity

- Domain of application: fluids and other Condensed Matter systems in the long wavelength (hydrodynamic) approximation
  Barceló, Liberati and Visser 2005
- They allow to test experimentally the existence of Hawking radiation
- The existence of HR can be studied from first principles, overcoming the transplanckian problem in gravity Jacobson '91

Different avenues are explored

- Water tanks experiments (stimulated HR from white hole flows)
  Weinfurtner et al 2011
  (the results have recently been reinterpreted by Michel and Parentani 2014)
- Quantum optics experiments with laser pulse filaments Faccio et al 2010
  (the results are controversial Schutzhold and Unruh 2011)
- Experimental realization of an acoustic black hole in a BEC

Bose-Einstein condensates



- Ultracold bosonic systems in which (almost) all constituents occupy the same quantum state Bose and Einstein 1924
- They have been discovered experimentally in 1995 (atomic gases of rubidium and sodium) Cornell, Wieman and Ketterle

Perfect condensates do not exist, not even at zero temperature (quantum depletion): $\hat{\Psi} = \Psi_0(1+\hat{\phi})$

- Gross-Pitaevski equation for the condensate

$$i\hbar\partial_t\Psi_0 = \left(-\frac{\hbar^2\vec{\nabla}^2}{2m} + V_{ext} + g|\Psi_0|^2\right)\Psi_0 \ ,$$

- Bogoliubov-de Gennes equation for the (linear) fluctuations ($c_s = \sqrt{\frac{gn_0}{m}}, \ n_0 = |\Psi_0|^2$)

$$i\hbar\partial_t\hat{\phi} = -\left(\frac{\hbar^2\vec{\nabla}^2}{2m} + \frac{\hbar^2}{m}\frac{\vec{\nabla}\Psi_0}{\Psi_0}\vec{\nabla}\right)\hat{\phi} + mc_s^2(\hat{\phi}+\hat{\phi}^\dagger) \ ,$$

These equations are valid at all scales

Hydrodynamic limit

- It is more convenient to consider the density-phase representation $\hat{\Psi} = \sqrt{\hat{n}} e^{i\hat{\theta}}$ and $\hat{n} = n_0 + \hat{n}_1,\ \hat{\theta} = \theta_0 + \hat{\theta}_1$
- Considering backgrounds that vary on scales bigger than the healing length $\xi = \frac{\hbar}{mc_s}$ (the analogous of the Planck length in gravity) one obtains the continuity and Euler equations for $n_0,\ \theta_0$
- In the same approximation, for the fluctuacions $\hat{n}_1 = n_0(\hat{\phi} + \hat{\phi}^\dagger)$ and $\hat{\theta}_1 = \frac{\hat{\phi} - \hat{\phi}^\dagger}{2i}$ we obtain an algebraic equation for $\hat{n}_1$ ($v_0 = \frac{\hbar \partial_x \theta_0}{m}$)

$$\hat{n}_1 = -\frac{n_0}{mc_s^2}\left[v_0 \partial_x \hat{\theta}_1 + \partial_t \hat{\theta}_1\right] \ ,$$

while the equation for $\hat{\theta}_1$ decouples..

Gravitational analogy

The equation for θ_1 is mathematically equivalent to the Klein-Gordon equation for a massless and minimally coupled scalar field

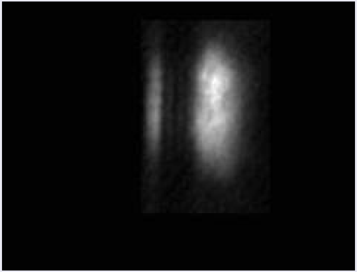
$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\hat{\theta}_1) = 0$$

in the acoustic metric

$$ds^2 = \frac{n_0}{mc_s}\left(-(c_s^2 - v_0^2)dt^2 + 2v_0 dt dx + dx^2 + dy^2 + dz^2\right)$$

The horizon's surface gravity of the acoustic metric $\kappa = \frac{1}{2c_s}\frac{d(c_s^2 - v_0^2)}{dx}|_{hor}$ gives the analog Hawking temperature T_H^{an}

An acoustic black hole in a BEC has already been realised by means of an external step-like potential accelerating the atoms and creating a region of supersonic flow Steinhauer et al 2010



Problems

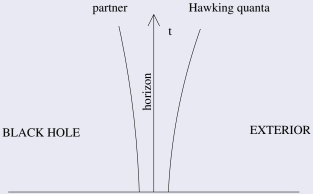
- The Hawking signal is small and there are competing effects (nonzero background temperature, quantum noise, ..)
- In BECs $T_H \sim 10\ nK < T_C \sim 100\ nK$: much better wrt gravity, but not enough to attempt a direct detection of the Hawking flux

Laser effect

- In configurations with a black hole and a white hole horizon, Corley and Jacobson 1999 showed that due to dispersion the partner travels back and forth between the horizons stimulating the emission of more and more Hawking quanta (exponential amplification)
- Coutant and Parentani 2010 reinterpreted it as a dynamical instability of the supersonic region; application to BECs was studied by Finazzi and Parentani 2010
- This phenomena was observed by Steinhauer 2014

The Hawking effect in acoustic black holes in BECs

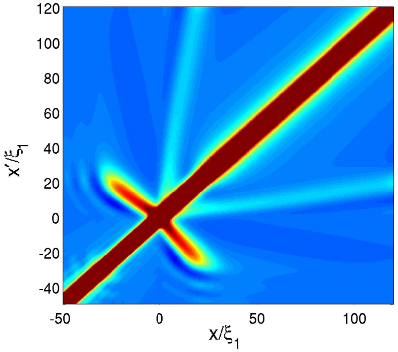
The emission of phonon pairs (Hawking quanta/ partner) on both sides of the acoustic horizon



leads to a characteristic stationary signal in the (equal time) correlation function of the density fluctuations Balbinot et al 2008

$$G^{(2)}_{BH}(t;x,x') \equiv \langle \hat{n}_1(t,x)\hat{n}_1(t,x')\rangle \sim \kappa^2 \cosh^{-2}[\frac{\kappa}{2}(\frac{x}{v+c_l} - \frac{x'}{v+c_r})]$$

This signature is robust and can be exploited to isolate HR from competing processes and experimental noise Carusotto et al 2008



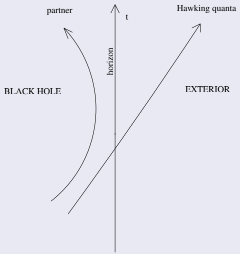
A numerical analysis using QFT in curved space techniques has confirmed the good qualitative and quantitative agreement with the 'ab-initio' CM calculation Anderson, Balbinot, Fabbri, Parentani 2013

Resolution of the transplanckian problem in BECs

- The relativistic dispersion relation for the fluctuations gets modified at large momenta $k \gg k_c = \frac{1}{\xi}$

$$w - vk = \pm c_s k \sqrt{1 + \frac{\xi^2 k^2}{4}}$$

- This makes the Hawking quanta-partner pairs, evolved backwards in time, to emerge both from inside (and not at) the horizon

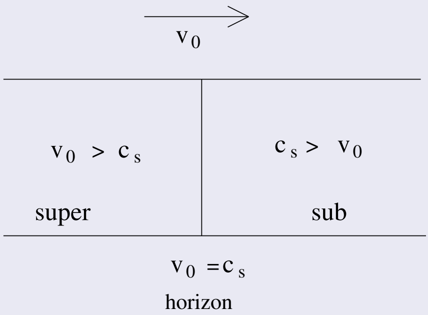


- Redshift is finite and the emitted flux is approximately thermal until $w_{max} \sim \frac{1}{\xi}$

- Density correlations measurements appear to be the most promising way to detect experimentally the analog Hawking effect
- Inserting numbers for realistic experiments one anticipates normalized correlations of order 10^{-3}, not far from the sensitivity of actual experiments
- Cornell (Valencia, 2009) proposed a way to amplify the signal by reducing the atoms' interactions shortly before measurements are made

White holes

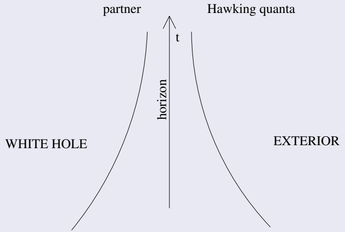
- They are the time reversal of black holes, and their acoustic version is realised with a supersonic fluid that decelerates and becomes subsonic (for acoustic black holes it is the opposite)



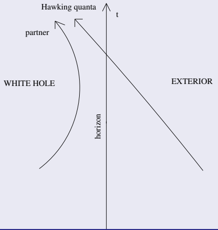
- In gravity, because they need an initial singularity, they have received little attention

Hawking effect in white holes

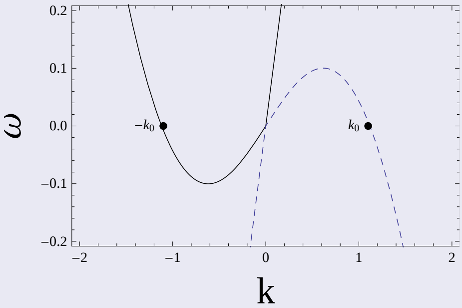
In gravity the pairs of the Hawking effect accumulate at both sides of the horizon with a large (transplanckian) frequency



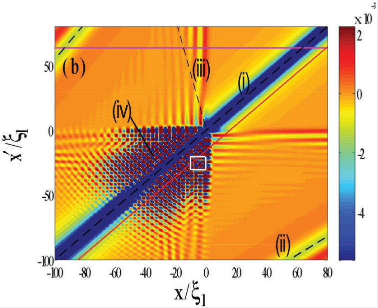
making the horizon unstable, while in BECs they enter the horizon



To understand the correlation signal inside the horizon we have to take into account the presence of a nontrivial outgoing zero mode in the dispersion relation of the supersonic region



The density correlator inside the horizon displays a checkerboard pattern



well approximated, analytically, by

$$G^{(2)}_{WH}(x,x') \sim [A\cos k_0(x+x') + B\sin k_0(x+x') + C\cos k_0(x-x')]\, I_\epsilon,$$

where the overall amplitude $I_\epsilon = \int \frac{dw}{w}$ is infrared divergent (regularised by introducing a IR cutoff $\epsilon = \frac{1}{t}$, where t is time elapsed from horizon formation)
Mayoral et al 2011

Undulations

- A closer inspection shows that the low-frequency dominant contribution to the 2pt function factorizes Coutant et al 2012

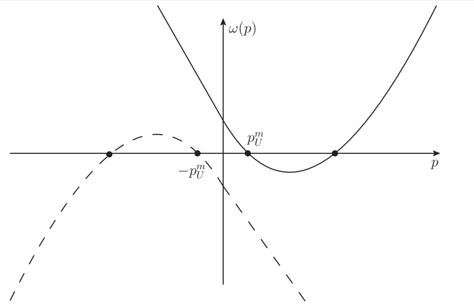
$$G_{WH} \sim \int dw |\beta_w^H|^2 \psi(x)\psi(x')$$

  describing the emission of a classical zero-frequency wave propagating away from the horizon
- Its macroscopic growing amplitude $\propto \ln(t)$ ($\sim t$ if the initial state is thermal) is fixed by $|\beta_w^H|^2 \sim \frac{\kappa}{2\pi w}$, the low-frequency spectrum of spontaneously produced massless Hawking phonons: nonlinear effects are expected to saturate this growth Busch, Michel and Parentani 2014

Undulations have been observed in water tanks experiments, but their possible connection to the Hawking effect was not pointed out

Transverse momentum

Excitations of quasi-1D systems can also have transverse momentum $p_\perp = 2\pi n/L_\perp$ - a mass term in the 1D (hydrodynamic) theory - that changes the dispersion relation in the black hole supersonic region to

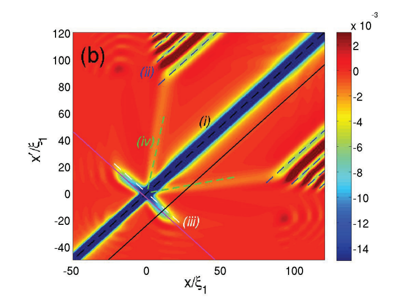


and a (small) undulation appears whose amplitude now saturates (no IR div) to $|\beta_\omega^{\rm Tot}|^2 \sim \frac{\kappa}{2\pi p_\perp}$, while the radiation in the exterior starts at $w = p_\perp$

Coutant, Fabbri, Parentani, Balbinot, Anderson 2012

Momentum correlations in acoustic black holes

In our numerical simulations in acoustic black holes we observed, at early times, the transient feature (ii) (due to the time-dependent formation of the bh)

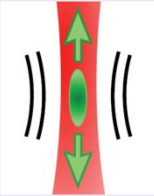


that we interpreted to be due to dynamical Casimir effect

Carusotto, Balbinot, Fabbri, Recati 2010

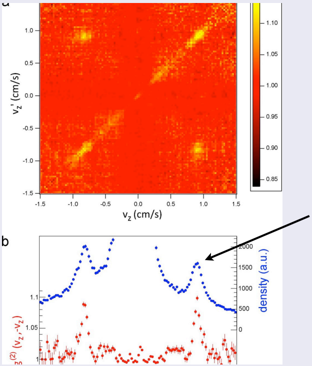
Inspired by our work, Westbrook et al 2012 considered an experiment in which:

- They varied rapidly in time the confining potential of a homogeneous 1D condensate
- They subsequently (adiabatically) opened the trap (during which excitations are converted into particles expelled from both ends of the condensate)



- They measured the velocity of the particles from their time of arrival at the detector (time of flight measurements)

- They measured the correlations between particles' (vertical) velocities v and v' and found a peak along $v = -v'$, indicating the creation of correlated excitations with opposite momentum $k = -k'$ (typical in homogeneous configurations)

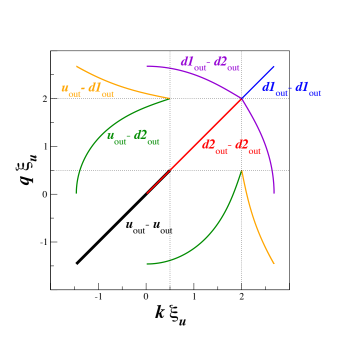


- The same phenomena is responsible for particle creation in the early Universe

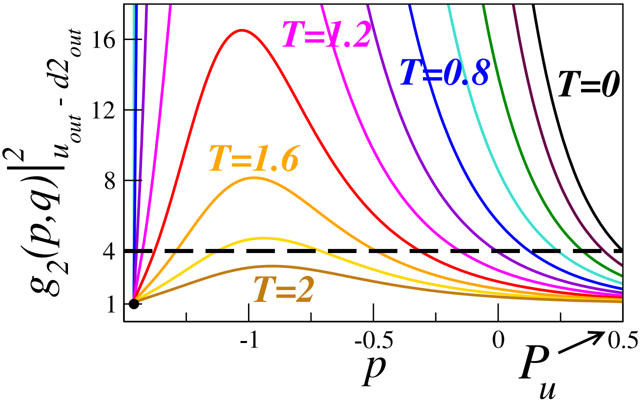
Westbrook and his group are now moving to acoustic black holes. We are joining forces to adapt our original proposal to study the features of the Hawking effect in the momentum correlation function

$$G_2(k,q) = \langle \hat{n}(k)\hat{n}(q)\rangle - \langle \hat{n}(k)\rangle \langle \hat{n}(q)\rangle$$

to be later compared with the experimental data.



The interest of this observable is that it gives a clear signature of the quantum nature of the Hawking signal via the violation of the Cauchy-Schwarz inequality even at $T > 10\ T_H$



Boiron, Fabbri, Larré, Pavloff, Westbrook and Zin 2015

Graybody factor and infrared divergences

Black holes do not emit as perfect black bodies, indeed particles' emission

$$N_\omega = |\beta_w^H|^2 = \frac{\Gamma(\omega)}{e^{\frac{\hbar\omega}{k_B T_H}} - 1}$$

is modulated by the gray-body factor $\Gamma = |T|^2$, T being the transmission factor for modes propagating between the horizon and infinity

- In gravity, for BHs in asymptotically flat space (Schwarzschild) we have, for small ω, $\Gamma \sim A_H \omega^2$, where A_H is the area of the horizon: this regulates the IR divergence of the Planckian distribution
- Surprisingly, for acoustic black holes $\Gamma \to const.$ at low frequency: the analog Hawking emission is dominated by an infinite number $(\frac{1}{w})$ of soft phonons. A similar behaviour is found for BHs immersed in an expanding universe (Schwarzschild - de Sitter)
  Anderson, Balbinot, Fabbri and Parentani 2014
- Despite this fact, however, the density correlation function is IR finite
  Anderson, Fabbri and Balbinot 2015

Backreaction

- In gravity we study the evolution of black holes due to Hawking radiation by solving the semiclassical Einstein equations

$$G_{\mu\nu} = 8\pi\langle T_{\mu\nu}\rangle$$

  which are valid until the BH reaches the Planck scale
- In BECs we need to solve the modified Gross-Pitaevski equation

$$i\hbar\partial_t\Psi_0 = \left(-\frac{\hbar^2\vec{\nabla}^2}{2m} + V_{ext} + g|\Psi_0|^2 + 2\langle\hat{\phi}^\dagger\hat{\phi}\rangle\right)\Psi_0 + \langle\hat{\phi}\hat{\phi}\rangle\Psi_0^* \ ,$$

  where $\langle\hat{\phi}^\dagger\hat{\phi}\rangle$ and $\langle\hat{\phi}\hat{\phi}\rangle$ are, respectively, the depletion and the anomalous density
- The search of analytical solutions in the near-horizon region will tell us how an evaporating acoustic black hole evolves and will guide the numerical resolution of the full problem

Information loss paradox

- We do not know whether or not black holes violate the rules of Quantum Mechanics, as predicted by Hawking '76
- Almeheiri, Marolf, Polchinski, Sully 2013 conjectured that the Hawking quanta - partner correlation across the horizon will be destroyed at some (Page) time if correlations between early time and late time Hawking radiation are such that unitarity is preserved
- It is interesting to address these issues in the context of BEC black holes, where concrete calculations can be performed

Conclusions

1. The study of Hawking radiation in BECs allows us to better understand its origin (negative energy states and energy conservation, the role played by the horizon)
2. The existence of Hawking radiation in BECs makes us confident that it exists also in gravity
3. Its experimental detection has allowed to establish a bridge between different fields in Physics