Kerr-de Sitter(Revisited) Black Hole Shadows

Presented by: Eunice Monyenye Omwoyo

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Introduction

- Black holes are regions of very strong gravity that is sufficient to warp space, bend light and give rise to space-time singularities.
- Since black holes results from the supernovae explosions that leaves behind a positive angular momentum of the remaining matter it is most probable that all black holes in nature are rotating and are therefore described by the Kerr solution
- In the presence of a cosmological constant, a generalization of the Kerr metric is given by the Kerr-de Sitter metric.
- Moreover, Stuchlik and Ovalle have recently found a revisited solution, the Kerr-de Sitter Revisited metric.
- As a consequence of the cosmological constant, the space-time becomes asymptotically de sitter and there arises another horizon(the Cosmological horizon).

- For astrophysical processes, another radius associated to cosmic repulsion is relevant, the static radius.
- On the static radius, the gravitational attraction due to the central compact object and cosmic repulsion counterbalance each other.
- When a black hole is in front of a lucent background, its unstable photon region will be projected on an observer's sky to form the black hole's shadow.
- In non asymptotically flat space-times, calculation of the angular radius of the shadow requires the use of finite distance observers.
- For such observers, a choice of an orthonormal tetrad is necessary.
- On the other hand, by the use of embedding diagrams, it has been shown that in the vicinity of the static radius, de sitter space-time is the analogue of an asymptotically flat space-time.
- In this work,we will analyze the shadows considering observers at least in the vicinity of the static radius.

In Boyer-Lindquist coordinates, the Kerr-de Sitter Metric is given by

$$ds^2 = \left(\frac{a^2\Delta_\theta \sin^2(\theta)}{L^2\Sigma} - \frac{\Delta_r}{L^2\Sigma}\right)\mathrm{d}t^2 - \frac{2a\sin^2(\theta)\left(\Delta_\theta\left(a^2+r^2\right)-\Delta_r\right)}{L^2\Sigma}\mathrm{d}t\mathrm{d}\phi$$
$$+\frac{\sin^2(\theta)\left(\Delta_\theta\left(a^2+r^2\right)^2 - a^2\Delta_r\sin^2(\theta)\right)}{L^2\Sigma}\mathrm{d}\phi^2 + \frac{\Sigma}{\Delta_\theta}\mathrm{d}\theta^2 + \frac{\Sigma}{\Delta_r}\mathrm{d}r^2 \quad (1)$$

where

$$\Delta_\theta = 1 + \frac{\Lambda a^2\cos^2\theta}{3} \quad (2)$$

$$\Delta_r = (1 - \frac{\Lambda r^2}{3})(r^2+a^2) - 2Mr \quad (3)$$

$$L = 1 + \frac{\lambda a^2}{3} \quad (4)$$

$$\Sigma = r^2 + a^2\cos^2\theta \quad (5)$$

The equations for Null geodesics in this space time are given by

$$\frac{\Sigma}{E} p^r = \pm\sqrt{R(r)} \tag{6}$$

$$\frac{\Sigma}{E} p^\theta = \pm\sqrt{\Theta(\theta)} \tag{7}$$

$$\frac{\Sigma}{E} p^\phi = \frac{aL^2}{\Delta_r}(a(a-\lambda)+r^2) - \frac{L^2}{\Delta_\theta \sin^2\theta}(a\sin^2\theta - \lambda) \tag{8}$$

$$\frac{\Sigma}{E} p^t = \frac{L^2}{\Delta_r}((r^2+a^2)^2 - a\lambda(a^2+r^2)) - \frac{aL^2}{\Delta_\theta}(a\sin^2\theta - \lambda) \tag{9}$$

Where

$$R(r) = L^2(r^2+a^2-a\lambda)^2 - \Delta_r(\eta + L^2(\lambda - a)^2) \tag{10}$$

$$\Theta(\theta) = a^2\Delta_\theta L^2 + a^2L^2\cos^2(\theta) - a^2L^2 - 2a\Delta_\theta\lambda L^2 + 2a\lambda L^2 + \Delta_\theta\eta + \Delta_\theta\lambda^2L^2 - \lambda^2L^2\cot^2(\theta) - \lambda^2L^2 \quad (11)$$

For photon orbits at a constant radius ;

$$R(r) = 0, R'(r) = 0 \quad (12)$$

Simultaneously solving eq.12 yields,

$$\eta = -\frac{L^2r^3\left(6a^2\left(\Lambda r^2(3M+r) - 6M\right) + a^4\Lambda^2r^3 + 9r(r-3M)^2\right)}{a^2\left(r\left(a^2\Lambda + 2\Lambda r^2 - 3\right) + 3M\right)^2} \quad (13)$$

$$\lambda = \frac{r\left(a^2\left(6-\Lambda r^2\right) + 3r(r-3M)\right)}{a\left(r\left(a^2\Lambda + 2\Lambda r^2 - 3\right) + 3M\right)} + a \quad (14)$$

- η vanishes on the equatorial plane. Thus solving for r in $\eta = 0$ yields the radii of equatorial circular photon orbits,

$$r_{ph+,KDS} = -\frac{6M\left(a^2\Lambda - 3\right)}{\left(a^2\Lambda + 3\right)^2} + 6\sqrt{\frac{M^2\left(a^2\Lambda\left(a^2\Lambda - 42\right) + 9\right)}{\left(a^2\Lambda + 3\right)^4}} \times \cos\left(\frac{\phi}{3} + \frac{4\pi}{3}\right) \tag{15}$$

$$r_{ph-,KDS} = -\frac{6M\left(a^2\Lambda - 3\right)}{\left(a^2\Lambda + 3\right)^2} + 6\sqrt{\frac{M^2\left(a^2\Lambda\left(a^2\Lambda - 42\right) + 9\right)}{\left(a^2\Lambda + 3\right)^4}} \times \cos\left(\frac{\phi}{3}\right) \tag{16}$$

Thus the photon region : $r \in [r_{ph+,KDS}, r_{ph-,KDS}]$

Kerr-de Sitter Revisited

The Kerr-de Sitter revisited solution is defined by the metric,

$$ds^2 = -\left(\frac{\Delta_\Lambda - a^2 \sin 2}{\rho^2}\right) dt^2 + \frac{\rho^2}{\Delta_\Lambda} dr^2 + \rho^2 d\theta^2 + \frac{\Sigma_\Lambda \sin^2\theta}{\rho^2} d\phi^2 - \frac{2a \sin^2\theta}{\rho^2}(r^2 + a^2 - \Delta_\Lambda) dt d\phi \quad (17)$$

where

$$\Delta_\Lambda = r^2 - 2Mr + a^2 - \frac{\Lambda r^4}{3} \quad (18)$$

$$\Sigma_\Lambda = (r^2 + a^2)^2 - \Delta_\Lambda a^2 \sin^2\theta \quad (19)$$

$$\rho^2 = r^2 + a^2 \cos^2\theta \quad (20)$$

The null geodesics in this space-time are given by

$$\frac{\rho^2}{E} p^r_\Lambda = \pm\sqrt{R_\Lambda(r)} \tag{21}$$

$$\frac{\rho^2}{E} p^\theta_\Lambda = \pm\sqrt{\Theta_\Lambda(\theta)} \tag{22}$$

$$\frac{\rho^2}{E} p^\phi_\Lambda = \frac{(ar^2 + a^3 - a\Delta_\Lambda - a^2\lambda_\Lambda)}{\Delta_\Lambda} + \frac{\lambda_\Lambda}{\sin^2\theta} \tag{23}$$

$$\frac{\rho^2}{E} p^t_\Lambda = \frac{(r^2 + a^2)(r^2 + a^2 - a\lambda_\Lambda)}{\Delta_\Lambda} + a\lambda_\Lambda - a^2\sin^2\theta \tag{24}$$

where

$$R_\Lambda(r) = (r^2 + a^2 - a^2\lambda_\Lambda)^2 - \Delta_\Lambda(\eta_\Lambda + (\lambda_\Lambda - a)^2) \tag{25}$$

$$\Theta_\Lambda(\theta) = \eta_\Lambda + a^2\cos^2\theta - \lambda_\Lambda^2\cot^2\theta \tag{26}$$

Solving simultaneously for $R_\Lambda(r) = R'_\Lambda(r) = 0$, yields,

$$\eta_\Lambda = -\frac{3r^3\left(4a^2\left(\Lambda r^3 - 3M\right) + 3r(r-3M)^2\right)}{a^2\left(3M + 2\Lambda r^3 - 3r\right)^2} \tag{27}$$

$$\lambda_\Lambda = \frac{3a^2M + 2a^2\Lambda r^3 + 3a^2 r - 9Mr^2 + 3r^3}{a\left(3M + 2\Lambda r^3 - 3r\right)} \tag{28}$$

which are the constants governing motion of photon orbits at a constant radius in Kerr-de Sitter Revisited space-time.
Solving for r in $\eta = 0$ results in the radii of equatorial circular photon orbits ;

$$r_{ph+,RKD} = \frac{6M}{4a^2\Lambda + 3} + 6\sqrt{\frac{M^2\left(1 - 4a^2\Lambda\right)}{\left(4a^2\Lambda + 3\right)^2}} \cos\left(\frac{\phi}{3} + \frac{4\pi}{3}\right) \tag{29}$$

$$r_{ph-,RKD} = \frac{6M}{4a^2\Lambda + 3} + 6\sqrt{\frac{M^2\left(1 - 4a^2\Lambda\right)}{\left(4a^2\Lambda + 3\right)^2}} \cos\left(\frac{\phi}{3}\right) \tag{30}$$

Thus the photon region : $r \in \left[r_{ph+,RKD}, r_{ph-,RKD}\right]$

Shadows

Using the null geodesic equations, we obtain the celestial coordinates for observers in Kerr-de Sitter space-time as

$$\alpha_{KDS} = \frac{\sqrt{3}L^2 \sin(\theta)\left(a - \lambda \csc^2(\theta)\right)}{\Delta_\theta \sqrt{L^2\left(a^2\Lambda - 2a\lambda\Lambda + \lambda^2\Lambda + 3\right) + \eta\Lambda}} \tag{31}$$

$$\beta_{KDS} = \frac{\sqrt{3}(\pm\sqrt{\Theta(\theta)})}{\sqrt{L^2\left(a^2\Lambda - 2a\lambda\Lambda + \lambda^2\Lambda + 3\right) + \eta\Lambda}} \tag{32}$$

while in Kerr-de Sitter Revisited we obtain,

$$\alpha_{RKD} = -\frac{\sqrt{3}\csc(\theta)(a\cos(2\theta) - a + 2\lambda_\Lambda)}{2\sqrt{a^2\Lambda - 2a\lambda_\Lambda\Lambda + \eta_\Lambda\Lambda + \lambda_\Lambda^2\Lambda + 3}} \tag{33}$$

$$\beta_{RKD} = \frac{\pm\sqrt{3}\sqrt{a^2\cos^2(\theta) + \eta_\Lambda - \lambda_\Lambda^2\cot^2(\theta)}}{\sqrt{a^2\Lambda - 2a\lambda_\Lambda\Lambda + \eta_\Lambda\Lambda + \lambda_\Lambda^2\Lambda + 3}} \tag{34}$$

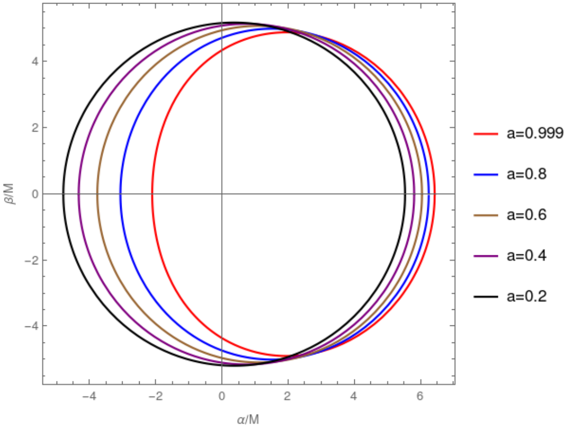

Figure – Kerr-De Sitter Black Hole shadows for different values of black hole spin, $\Lambda = 0.06$ and $\theta = \pi/2$.

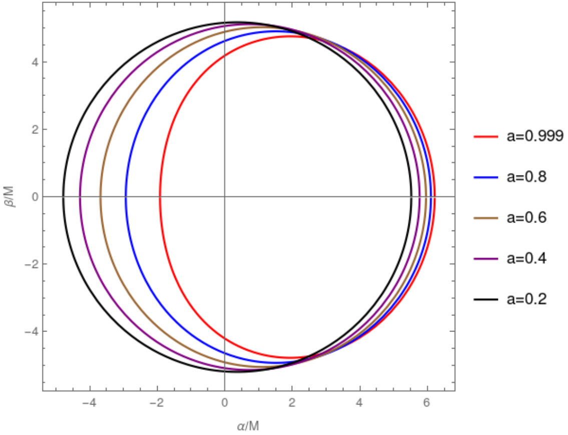

Figure – Kerr-De Sitter Revisited Black Hole shadow for different values of a,$\Lambda = 0.06$ and $\theta = \pi/2$

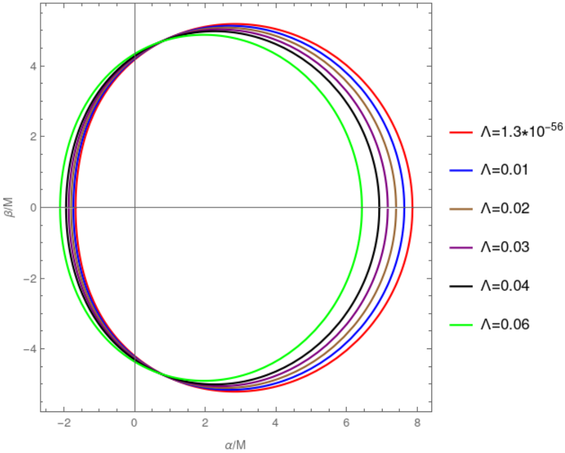

Figure – Kerr-De Sitter Black Hole shadow for different values of Λ, $a = 0.999$ and $\theta = \pi/2$

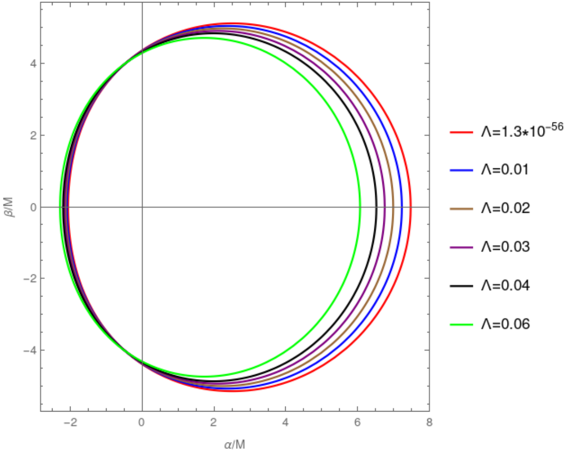

Figure – Kerr-De Sitter Revisited Black Hole shadow for different values of Λ, $a = 0.999$ and $\theta = \pi/2$

Characteristic Points and Diameters

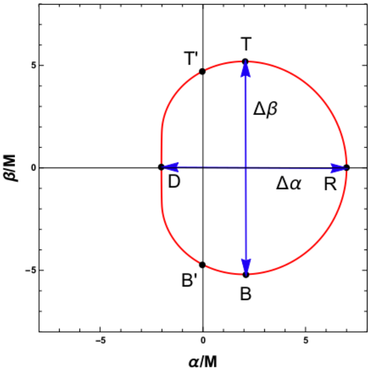

Figure – Illustration of the Characteristic points, D, R, T, B, the horizontal diameter, $\Delta\alpha$ and vertical diameter, $\Delta\beta$.

Table – Points evaluated for $\theta = 16$, $a = 0.5$ and $\Lambda = 1.11 * 10^{-52}$

points	Kerr	KDS,	KDS-R,	KDS"	KDS-R"
$\Delta\beta(\mu as)$	38.9617	38.9617	38.9617	38.9617	38.9617
$\Delta\alpha(\mu as)$	38.9123	38.9123	38.9123	38.9123	38.9123
$R_T(\mu as)$	19.4318	19.4318	19.4318	19.4318	19.4318
$R_D(\mu as)$	19.5096	19.5099	19.5099	19.5099	19.5099
$R_R(\mu as)$	19.502	19.502	19.502	19.502	19.502

Table – Points evaluated for $\theta = 16$, $a = 0.5$ and $\Lambda = 0.01$

points	KDS,	KDS-R	KDS"	KDS-R"
$\Delta\beta(\mu as)$	38.922	38.8599	40.7641	40.6857
$\Delta\alpha(\mu as)$	38.8689	38.8034	40.7336	40.6472
$R_T(\mu as)$	19.4076	19.3738	20.3463	20.3047
$R_D(\mu as)$	19.4929	19.4624	20.4124	20.3665
$R_R(\mu as)$	19.4852	19.4548	20.4044	20.3585

Conclusion

- We have considered Kerr-de Sitter and Kerr-de Sitter Revisited black holes.
- We have obtained the celestial coordinates for their shadows by considering observers in the vicinity of the static radius.
- Using differential geometry techniques we have numerically obtained the vertical diameters, horizontal diameters and radii of curvature at characteristic points along the curve of the shadows.
- We observe that the shadows are indistinguishable for the astrophysically relevant value of the cosmological constant, $\Lambda = 1.11 \times 10^{-52}$. Moreover, they are indistinguishable to the shadow cast by a Kerr-black hole.
- The difference only appears for a larger value of Λ.