Greybody factors for higher-dimensional rotating black holes

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Black Holes and their Analogues: 100 years of General Relativity
A workshop in honor of Kirill Bronnikov's 70th birthday

Outline

Introduction

We present the study of greybody factors for rotating black holes (BHs) in higher(odd)-dimensional spacetimes with nonzero cosmological constant.

- Importance:
  - Cosmological constant is one of the best candidates to explain an accelerated expanding universe;
  - AdS/CFT correspondence;
  - Hawking radiation.
- Objectives:
  - Understanding how the cosmological constant influences the greybody factors of rotating BHs;
  - Analyzing super-radiance mainly on asymptotically dS and AdS BHs.
- We consider rotating BHs in odd-dimensional spacetimes;
- We also consider that all BH angular momenta are equal;
- Therefore the geometry becomes cohomogeneity-1, and we only need to concentrate on solving the radial part of the field equation.

Metric

The metric is

$$ds^2 = -f(r)^2 dt^2 + g(r)^2 dr^2 + r^2 \widehat{g}_{ab} dx^a dx^b + h(r)^2 \left[d\psi + A_a dx^a - \Omega(r) dt \right]^2 , \qquad (1)$$

where

$$g(r)^2 = \left(1 + \kappa^2 r^2 - \frac{2M}{r^{2N}} + \frac{2\kappa^2 M a^2}{r^{2N}} + \frac{2Ma^2}{r^{2N+2}} \right)^{-1} ,$$

$$h(r)^2 = r^2 \left(1 + \frac{2Ma^2}{r^{2N+2}} \right) , \quad \Omega(r) = \frac{2Ma}{r^{2N} h(r)^2} , \quad f(r) = \frac{r}{g(r) h(r)} ,$$

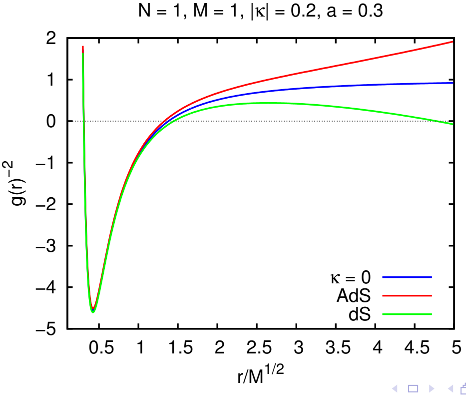
that is solution of Einstein equations with cosmological constant, $R_{\mu\nu} = -(D-1)\kappa^2 g_{\mu\nu}$ ($D = 2N+3$, the number of dimensions).

- $\kappa = 0$ refers to asymptotically flat spacetimes;
- $\kappa^2 < 0$ refers to asymptotically dS spacetimes;
- $\kappa^2 > 0$ refers to asymptotically AdS spacetimes.

Horizons

Roots of $g(r)^{-2}$ define the horizons:

- If $\kappa^2 \geq 0$, the largest root corresponds to r_h;
- if $\kappa^2 < 0$, the largest root corresponds to r_c, the cosmological horizon, and the second largest root corresponds to r_h.



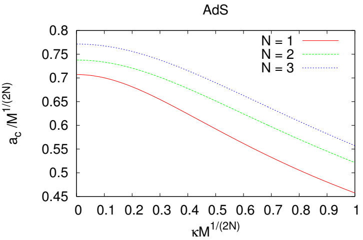
Extremal limit

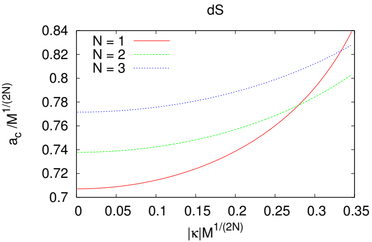
Black holes correspond to the cases in which $a \leq a_c$.

- It is not possible to find a closed form of a_c for all cases. For the asymptotically flat case:

$$a_c \equiv \sqrt{N/(N+1)}\,[2M/(N+1)]^{\frac{1}{2N}}\ .$$

- In other cases, results for each N may be found, but are not elucidating.





The scalar field

Massless minimally coupled field

The equation for the massless scalar field is

$$\nabla_\mu \nabla^\mu \Phi = 0.$$

In the case of the cohomogeneity-1 spacetime, we may perform the separation ansatz $\Phi = e^{-i\omega t - im\psi} R(r) \mathbb{Y}(x^a)$. The angular part, $\mathbb{Y}(x^a)$, satisfies a eigenvalue equation

$$-\mathcal{D}^2 \mathbb{Y} = -\widehat{g}^{ab}(D_a - imA_a)(D_b - imA_b)\mathbb{Y} = \left[\ell(\ell + 2N) - m^2\right]\mathbb{Y}.$$

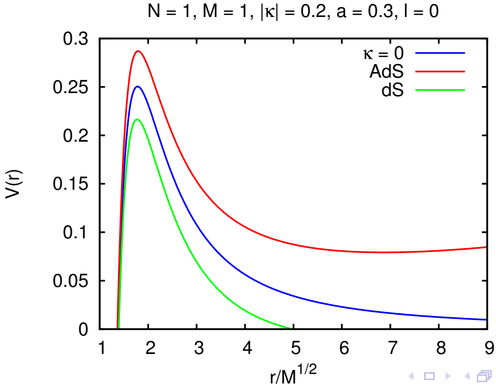
- Therefore, we do not need to worry about searching for the engeinvalues of the angular part neither considering only small rotations;
- Let us then concentrate in the radial part of the solution, $R(r)$.

Radial equation

Effective potential behavior

$$-\frac{f}{g}\frac{d}{dr}\left(\frac{f}{g}\frac{d\Psi}{dr}\right)+\left[V-(\omega-m\Omega)^2\right]\Psi=0, \qquad (2)$$

where $V(r)=\frac{1}{\sqrt{h}\,r^N}\frac{f}{g}\frac{d}{dr}\left[\frac{f}{g}\frac{d}{dr}\left(\sqrt{h}\,r^N\right)\right]+\frac{f^2}{r^2}\left[\ell(\ell+2N)-m^2\left(1-\frac{r^2}{h^2}\right)\right].$



Radial equation

Asymptotic solutions

Asymptotic solutions are necessary to define the greybody factors.

- At the event horizon, the solution is (independently of the value of κ):

$$\Psi = B^{+} e^{-i\widetilde{\omega}x},$$

  with $\widetilde{\omega} = \omega - m\Omega(r_h)$, and x is the the tortoise coordinate.

- An intermediate region solution is possible for low frequencies:

$$\Phi_{\omega}(r) = \frac{\Psi(r)}{\sqrt{h(r)}\, r^N} = A_{II} + B_{II} \int_{\infty}^{r} \frac{g(r')^2}{r'^{2N+1}} dr'.$$

- By matching this with the near-horizon solution we can write the integration constants in terms of B^{+};
- By matching it with the asymptotic solutions we can obtain low-frequency greybody factors.

Matched asymptotic expansions

- Far from the event horizon the solution does depend on the value of κ. For each case, the solution behaves differently.
- Analytical results for the greybody factors also depend on extra approximations:
  - Low-frequency approximations;
  - Small BHs in the case of asymptotically (A)dS spacetimes;
  - One needs also restrict the computations to $\ell = 0$ (no super-radiant regime probed).
- A full analysis requires matching with numerical solutions;
- We consider both analytical and numerical results.

Asymptotically flat case

Low-frequency result

Asymptotic solution for $\kappa = 0$ is

$$\Phi_\omega(r) = (\omega r)^{-N}\left[C_1 H^{(1)}_{N+\ell}(\omega r) + C_2 H^{(2)}_{N+\ell}(\omega r)\right].$$

The greybody factor is defined as

$$\gamma(\omega) = \frac{J_{hor}}{J_{in}} = 1 - \frac{|C_1|^2}{|C_2|^2},$$

where J_{in} and J_{hor} are the the incident and the absorbed fluxes. (We are considering “in” modes.)

By matching the above asymptotic solution with the low-frequency one, we get:

$$\gamma(\omega) = \frac{4\pi\,\omega^{2N+1}\,r_h^{2N+1}}{2^{2N+1}[\Gamma(N+1)]^2}\,\frac{h(r_h)}{r_h}.$$

Asymptotically flat case

Low-frequency result

About the low-frequency result

- It recovers previously obtained results for $a = 0$ [Harmark et al. (2010)];
- We can see that absorption decreases as the rotation parameter, a, increases;
- It can be used to compute the low-frequency absorption cross section:

$$\sigma(\omega) = \frac{(2\pi)^{2N+1}}{\omega^{2N+1}\Omega_{2N+1}}\gamma(\omega) \quad \xrightarrow{\omega \to 0} \quad \Omega_{2N+1} r_h^{2N} h(r_h) = \mathcal{A}_h .$$

- The absorption cross section coincides with the BH area in the limit $\omega \to 0$ [Higuchi (2001)].

Asymptotically dS case

Small BHs

For dS rotating BHs it is possible to obtain analytical results for $\omega \ll |\kappa|$. Near the cosmological horizon, we have:

$$\Psi \sim e^{-i\omega' x} + A\, e^{i\omega' x},$$

where $\omega' \equiv \omega - m\Omega(r_c)$. The greybody factor is defined as

$$\gamma(\omega) = 1 - |A|^2.$$

In the zero-frequency limit, for small BHs, we obtain:

$$\gamma \to 4\mathcal{A}_h/\mathcal{A}_c.$$

Asymptotically dS case

Small BHs

About small dS BH results:

- The analytical result agrees with previously obtained results for the static case [Kanti et al. (2005)];
- Greybody factors for the minimally coupled scalar field are nonzero in the zero-frequency limit;
- This does not happen in asymptotically flat spacetimes and neither in asymptotically dS spacetimes if the scalar field coupling with the curvature is non-minimal, $\xi \neq 0$ [Crispino et al. (2013)];

Asymptotically AdS case

Low-frequency result

For AdS rotating BHs, the solution at infinity is

$$\Phi_\omega(u) = u^{N+1}\left[\widehat{C}_1 H^{(1)}_{N+1}(u) + \widehat{C}_2 H^{(2)}_{N+1}(u)\right],$$

with $u \equiv \omega/(\kappa^2 r)$. The greybody factors are defined by:

$$\gamma(\omega) = 1 - \frac{|\widehat{C}_2|^2}{|\widehat{C}_1|^2}.$$

At low frequencies, we obtain:

$$\gamma(\omega) \simeq \frac{\pi^{N+2}}{2^{2(N-1)}\left[\Gamma(N+1)\right]^3 \mathcal{A}_h}\left(\frac{\omega}{\kappa^2}\right)^{2N+1}.$$

Asymptotically AdS case

Small BHs

Some considerations about analytical results in AdS case:

- Because of the potential asymptotic divergence, one of the terms in the previous radial solution falls off with $\mathcal{O}(r^{-2(N+1)})$;
- This makes the numerical analysis very challenging and imprecise in some cases;
- A alternative solution can be found for $r \gg r_h$:

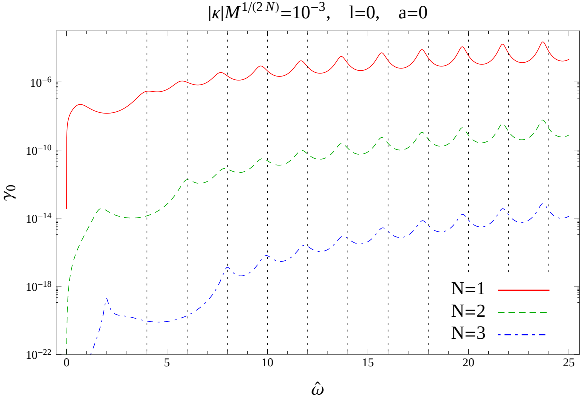
$$r^{\frac{2N+1}{2}}\Phi_\omega = C_1 z^{\frac{2N+1+2\ell}{4}}(1-z)^{-\frac{2N+1}{4}}\,{}_2F_1\left(\frac{\hat{\omega}+\ell}{2}, -\frac{\hat{\omega}-\ell}{2}, N+1+\ell, z\right)$$
$$+ C_2 z^{-\frac{2N-1+2\ell}{4}}(1-z)^{\frac{2N+3}{4}}\,{}_2F_1\left(1-\frac{\hat{\omega}+\ell}{2}, 1+\frac{\hat{\omega}-\ell}{2}, N+2, 1-z\right),$$

  where $\hat{\omega} \equiv \omega/k$ and $z = \frac{\kappa^2 r^2}{1+\kappa^2 r^2}$;
- With this solution we can obtain numerical results as well as analytical results for small BHs.

Asymptotically AdS case

Small BHs

The expression for greybody factors of small AdS BHs is very complex and not elucidating. However, they present very different behavior from the asymptotic flat and dS cases.



Asymptotically AdS case

Small BHs

Some important considerations can be done about the previous result:

- Pure AdS spacetimes have normal modes with frequencies

$$\hat{\omega} = 2(N+1) + \ell + 2n, \qquad n = 0, 1, 2, \ldots$$

- The greybody factors of small AdS BHs present peaks that are consistent with the normal frequencies of pure AdS spacetimes [Cardoso et al. (2003)];

Numerical results

Applied methods

Numerical results for the greybody factors are obtained by:

- Applying a fourth-fifth order Runge-Kutta method to solve the radial equation;
- The boundary condition is set at $r = (1 + \varepsilon)r_h$, where, typically, $\varepsilon = 10^{-3}$;
- The equation is evolved up to
  - $r_{\max} \gg r_h$ in the case of asymptotically flat spacetimes;
  - $r_{\max} \lesssim r_c$ in the case of asymptotically dS spacetimes;
  - $r_{\max}$ in a region where $\kappa r \gg 1/(\kappa M^{1/(2N)})$.
- The numerical solution is matched with the corresponding asymptotic solution to obtain the coefficients that define the greybody factors.

Numerical results

Precision

In order to measure the precision of our numerical computations, we

- Watch violations in the flux conservation. We keep the flux difference between the event horizon and asymptotic region smaller than 0.01 %;
- We perform a convergence test by verifying that our results do not depend on the choice of r_{max};
- We also compare the numerical results with the analytical results when possible.

Asymptotically flat case

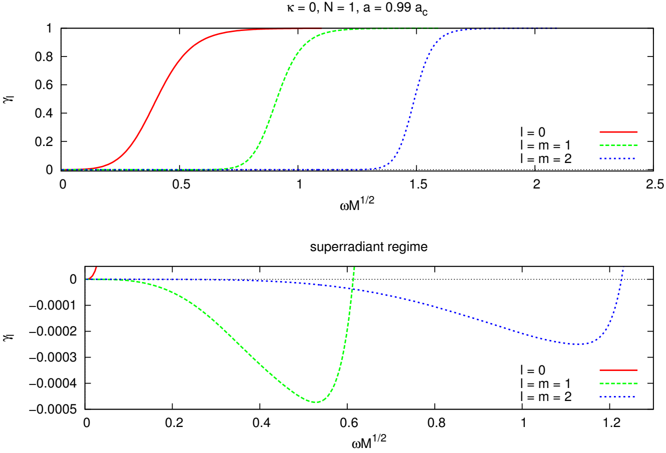


Figure : Greybody factors for asymptotically flat BHs. Maximum amplification is obtained for $l = m = 1$ (0.046 %).

Asymptotically flat case

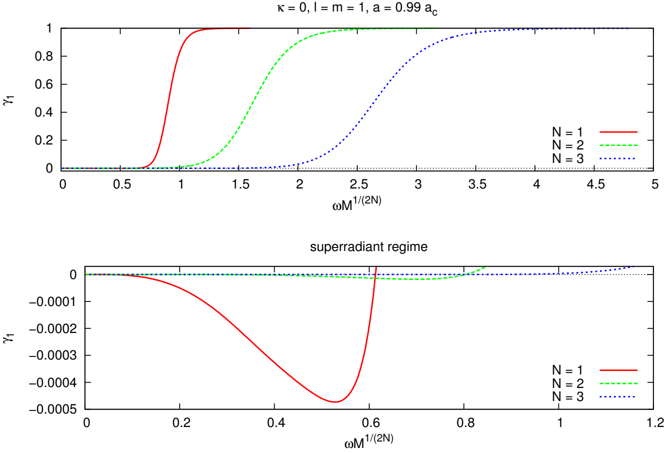


Figure : Main super-radiated mode, $l = m = 1$ for $N = 1, 2, 3$. Superradiance is highly suppressed for higher dimensions.

Asymptotically dS case

Results for small dS BHs ($\kappa M^{1/(2N)} \ll 1$) are quite similar to asymptotically flat BHs. The results get less similar as we consider large dS BHs ($r_h \lesssim r_c$).

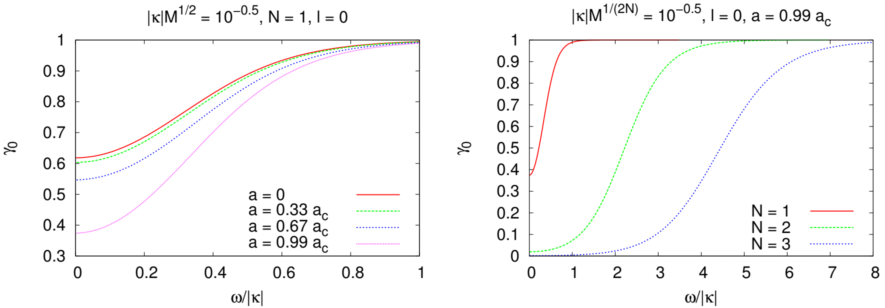


Figure : Large dS BHs greybody factors.

Asymptotically dS case

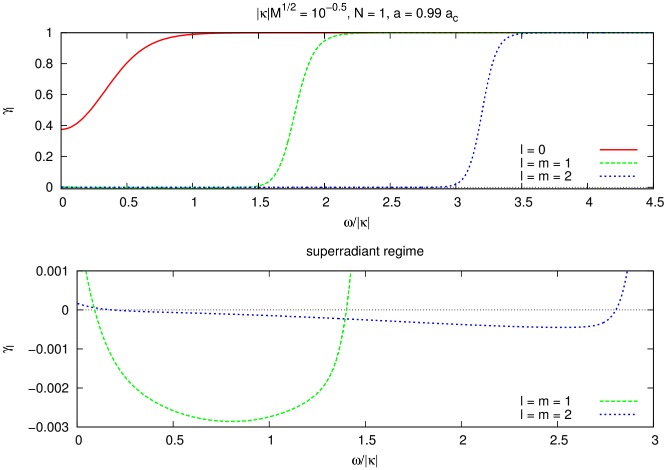


Figure : Greybody factors of s-wave and super-radiant modes for large dS rotating BHs. The super-radiant interval is now $\omega' < \omega < \widetilde{\omega}$.

Asymptotically dS case

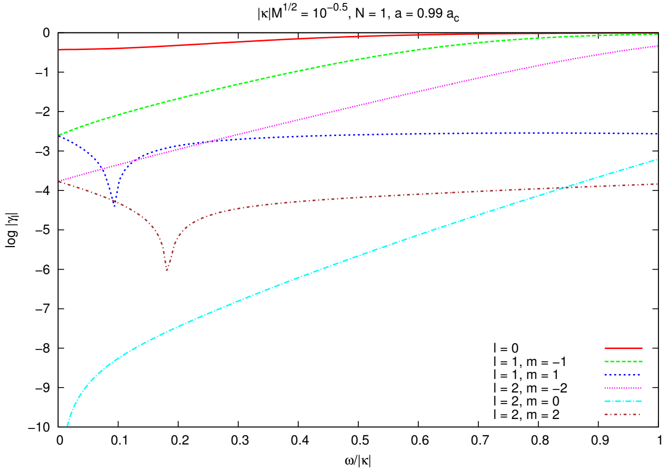


Figure : For static BHs $\gamma \neq 0$ in the $\omega \to 0$ limit for $\xi = 0$ and $\ell = 0$. For rotating BHs, this also applies for modes which have $m \neq 0$.

Asymptotically AdS case

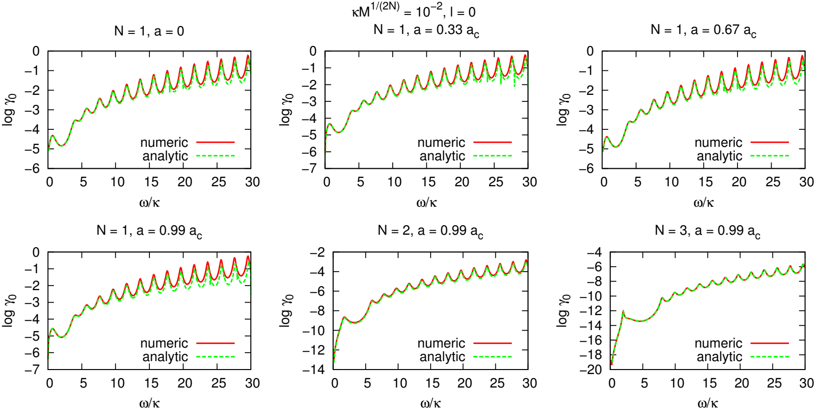


Figure : Comparison between numeric and analytic results for small AdS rotating BHs.

Asymptotically AdS case

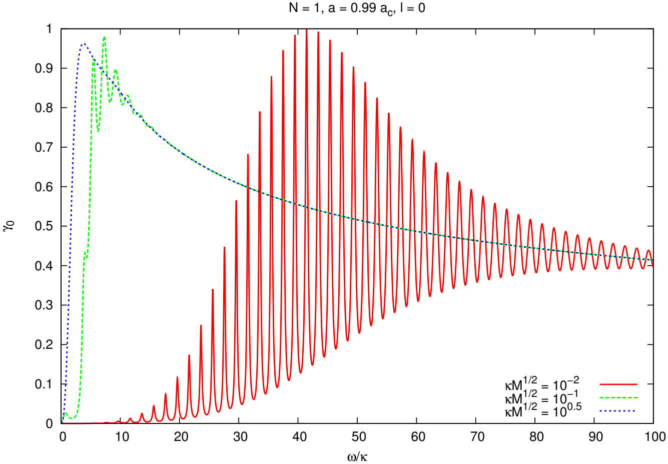


Figure : Peaks of oscillations are only prominent for small BHs. As we consider larger BHs, such peaks wane.

Asymptotically AdS case

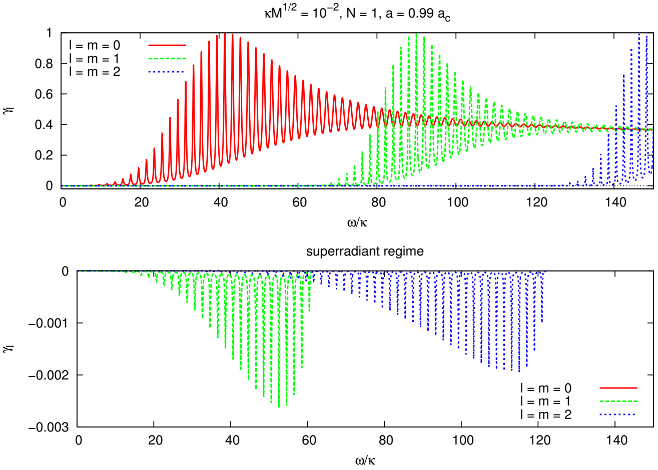


Figure : Super-radiant modes on AdS rotating BHs. Because of the peaks, super-radiance is enhanced on AdS case – 0.26 % for small BHs.

Conclusions I

Some remarks:

- In all cases, super-radiance is suppressed for higher dimensions;
- In dS:
  - Nonzero absorption is obtained in the *s*-wave case and also if the mode has angular momentum component along the BH rotation axis;
  - The interval for super-radiance is now $\omega - m\Omega(r_c) < \omega < \omega - \Omega(r_h)$. This is a consequence of the cosmological horizon having angular velocity.
- In AdS:
  - Peaks consistent with the normal frequencies of pure AdS spacetimes were observed in the greybody factors of small-to-medium black holes;
  - Super-radiance can be highly improved because of greybody factor peaks;

Conclusions II

- For medium BHs, preliminary results indicate that maximum amplification can be of the order of 100 %! However, the numerical analysis is imprecise to be sure. (Instability?)
- For high frequencies, the greybody factors do not tend to unity; instead, they tend to zero because of the divergent characteristic of the effective potential.

Further reading I

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Acknowledgments

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