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CUSPS AND CORES IN GALAXIES:PROBLEMS AND SOLUTIONS

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October, UFES, Vitoria

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LECTURE 2:

Proposed Solutions 1

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PROPOSED SOLUTIONS (HISTORICAL OVERVIEW)

- A. Systematic effects in observations (e.g., Hayashi+2004)

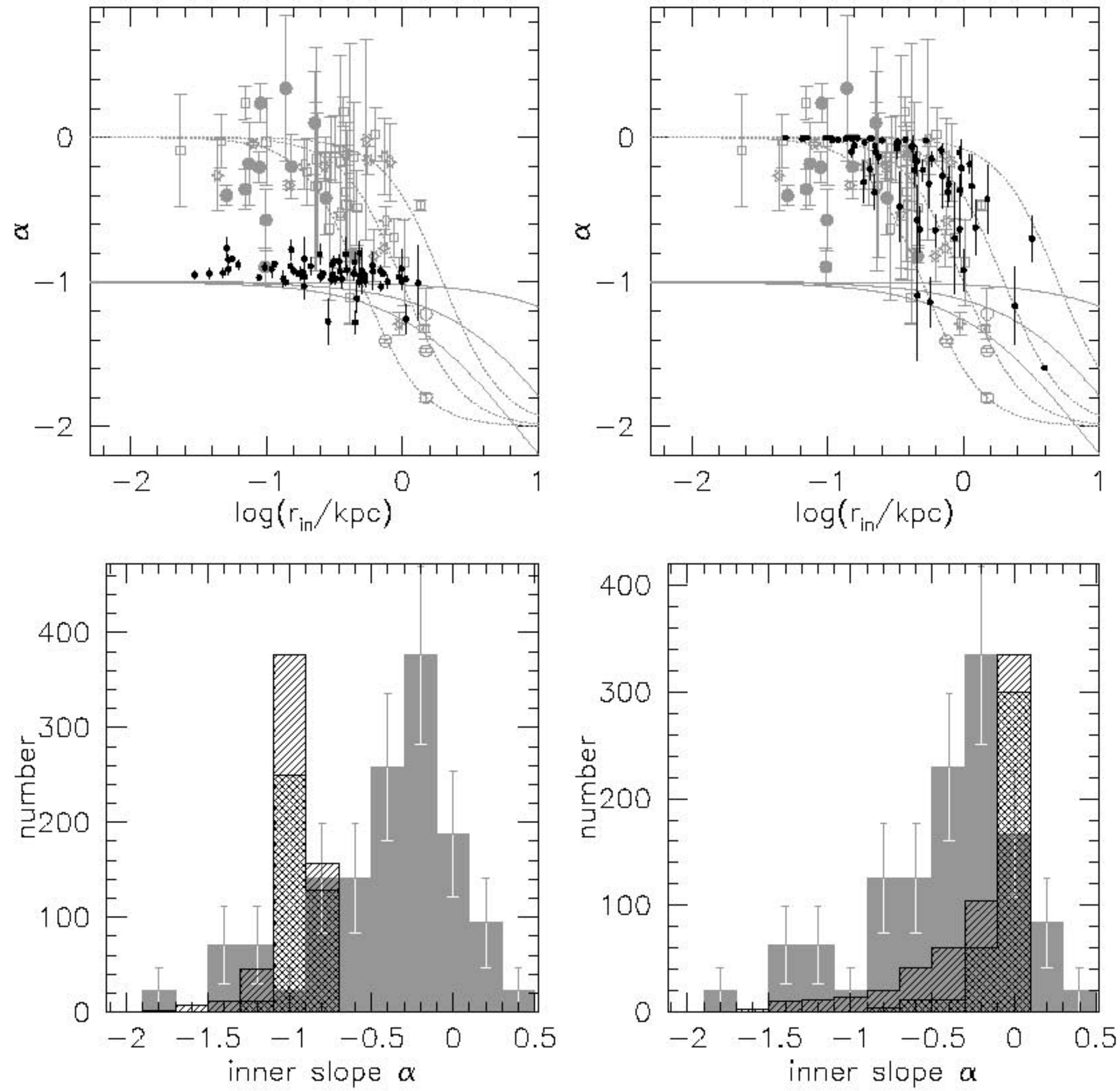
- B. Failure of the CDM model or problems with simulations? **(del Blok et al 2001, 2003; Borriello & Salucci 2001) (resolution; relaxation; overmerging; BARYONS NOT TAKEN INTO ACCOUNT !) Convergence Tests (Dieman et al. 2004)**

Cusp compared to LSB and dwarfs observations

De Blok et al (2003)
Constant density cores

Data cannot be
Reconciled with CDM
→No universal
halo profile

Also Gentile et al (2004)



ISO haloes

$$\rho(R) = \frac{\rho_0}{1+(\frac{R}{R_C})^2}$$

Figure 6. Comparison of the simulated NFW (left) and ISO (right) haloes. The top panels compare the simulated r_{in}–α data points (black) with the observed distribution from dBMBR and dBB02 (grey data points). No systematic effects are assumed. The grey dotted lines converging on $\alpha = 0$ represent the theoretical variations in slope for ISO haloes with $R_C = 0.5$, 1 and 2 kpc. The lines converging on $\alpha = -1$ show the variation in slope for $c/V_{200} = 9.8/50$, 8.6/100 and 6.1/500 (the slope only depends on the ratio c/V_{200}). The bottom panels show the histograms of α values for the two models. The double-hatched histogram indicates well-resolved galaxies with $r_{in} \leqslant 0.5$ kpc.

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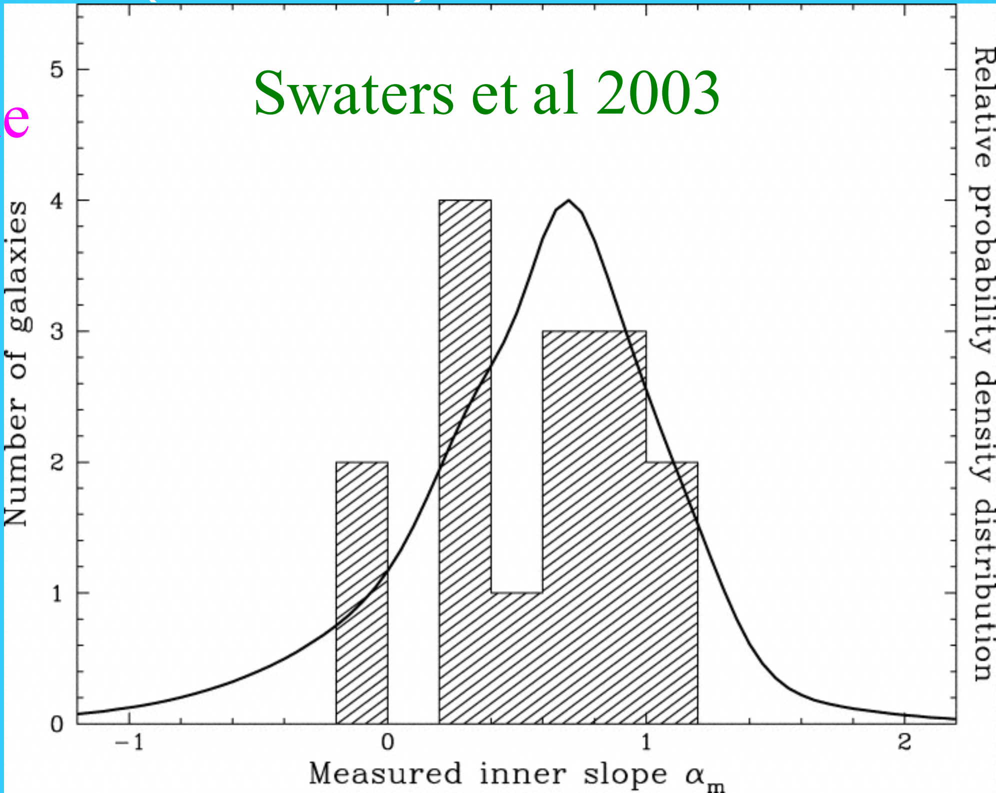
Observed fits

Hot debate about the
Cusps and their confrontation to observations

Van den Bosch & Swaters (2000). Rotation curves from 19 dwarfs. Claim that CDM halos are consistent with data. To do this " they had to throw away half of the Galaxies and adopt unphysicsl (zero) L/M ratios" (Moore 2001)

Swaters et al (2003) claim that some of the rotation curves could be compatible with a steep slope (even if a shallow slope is not ruled out)

But Spekkens et al (2005)
165 LSB, $\alpha = 0.22\text{-}0.28$



Systematic effects in observations

- HI observations, BEAM SMEARING (limited spatial resolution (few tens of arcsec)): finite beam size -> HI emission smeared out -> apparently larger HI disks.

  -Moreover: gradients in the velocity fields determined (e.g.) from Gaussian fits to the line profiles, may become shallower -> RCs obtained from these velocity fields have shallower gradients

  -Effect depending on: combination of the size of the beam, HI distribution, inclination angle of the galaxy, intrinsic velocity gradients.

  -Despite the effects of beam smearing, information on the true rotation curve is still contained in the data

  -Solution:high spatial resolution (< 1kpc, de Blok et al. 2008)

- Hα observations -slit misplacement: missing the dynamic centre of the galaxy

  -Solutions: repeated 1d long-slit spectra observed by independent observers at different telescopes
  -optical 3d spectroscopy (Kuzio de Naray et al. 2006, 2008; Spano et al. 2008)

- Non-circular motions: it is assumed that the gas moves on circular orbits, any deviation will lead to an underestimate of the slope

  -only of the order of a few km/s (Gentile et al. 2005, Trachternach et al. 2008)

- **Kuzio de Naray & Kaufmann 2011:** High resolution observations can distinguish cored and cuspy haloes by deriving their asymptotic inner slopes from rotation curve data

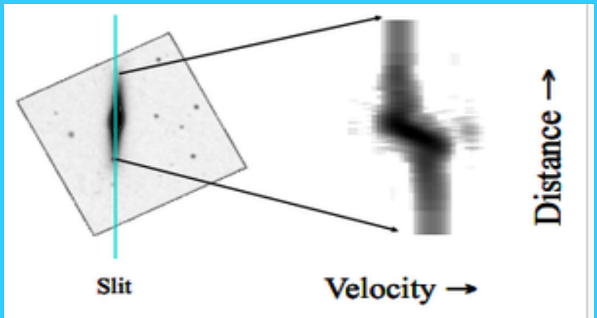
More Recent High Resolution Observations

- Kuzio de Naray et al. (2008): high-resolution optical velocity fields from DensePak integral field (two-dimensional) spectroscopy, along with derived rotation curves, for a sample of low surface brightness galaxies

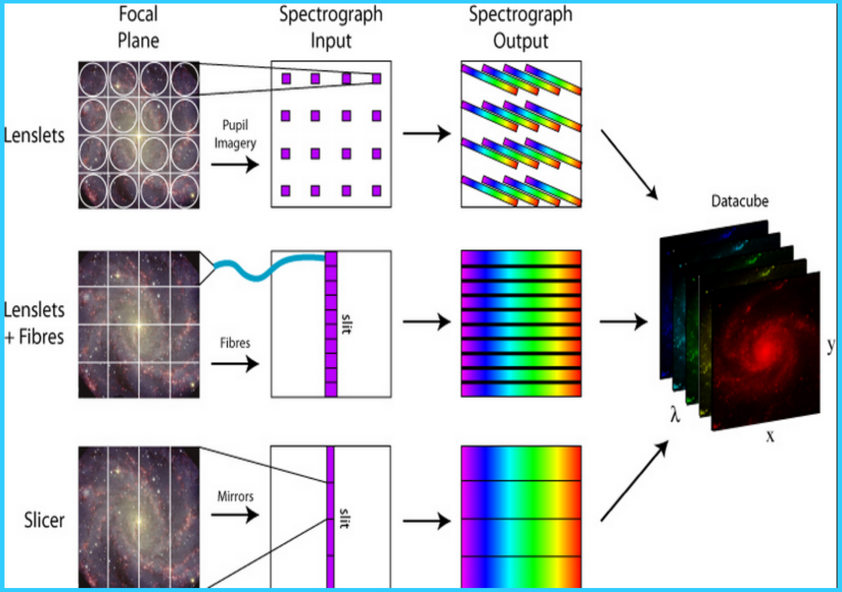
  -> halo central densities better described by the isothermal halo

- Spano et al. 2008: optical velocity fields of 36 galaxies of different morphological types

  -> slowly rising rotation curves, consistent with a core.



Observation through a long slit allows simultaneously taking spectrographs of all parts of the objects which fall onto the slit. When observing spectral lines, different Doppler shifts can be observed along a given spectral line, leading to velocity profiles of the object along the slit.



An **integral field spectrograph** or a spectrograph equipped with an **integral field unit** (**IFU**) is an optical instrument combining spectrographic and imaging capabilities, used to obtain spatially resolved spectra in astronomy and other fields of research

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Theory Vs. observations: what could have gone wrong?

- Error bars large enough so that the cores are favoured but cusps can usually not be ruled out (Hayashi et al. 2004).

- Power et al. , Hayashi et al. and Navarro et al. 2003 suggest ways to reconcile with the observations:

1. With better simulations, the halos become progressively shallow from the virial radius inwards and show no sign of turning into a power-law (actually minimum slope -0.8 (Stadel+ 2009)
2. Simulated haloes Vc are consistent with 70% of LSB rotation curves.
3. Observational disagreement is with the fitting formulae, rather than with simulated haloes.
4. CDM haloes are non-spherical and 3D. Comparing rotation speeds of gaseous disks to spherically-averaged circular velocity of DM haloes, differences should be expected.

Hayashi et al. 2004

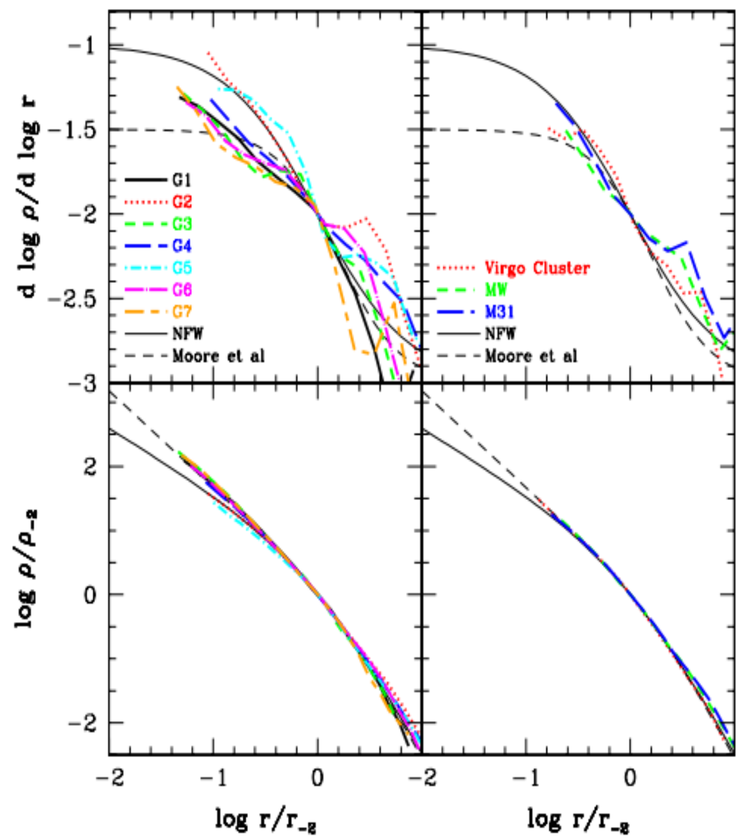
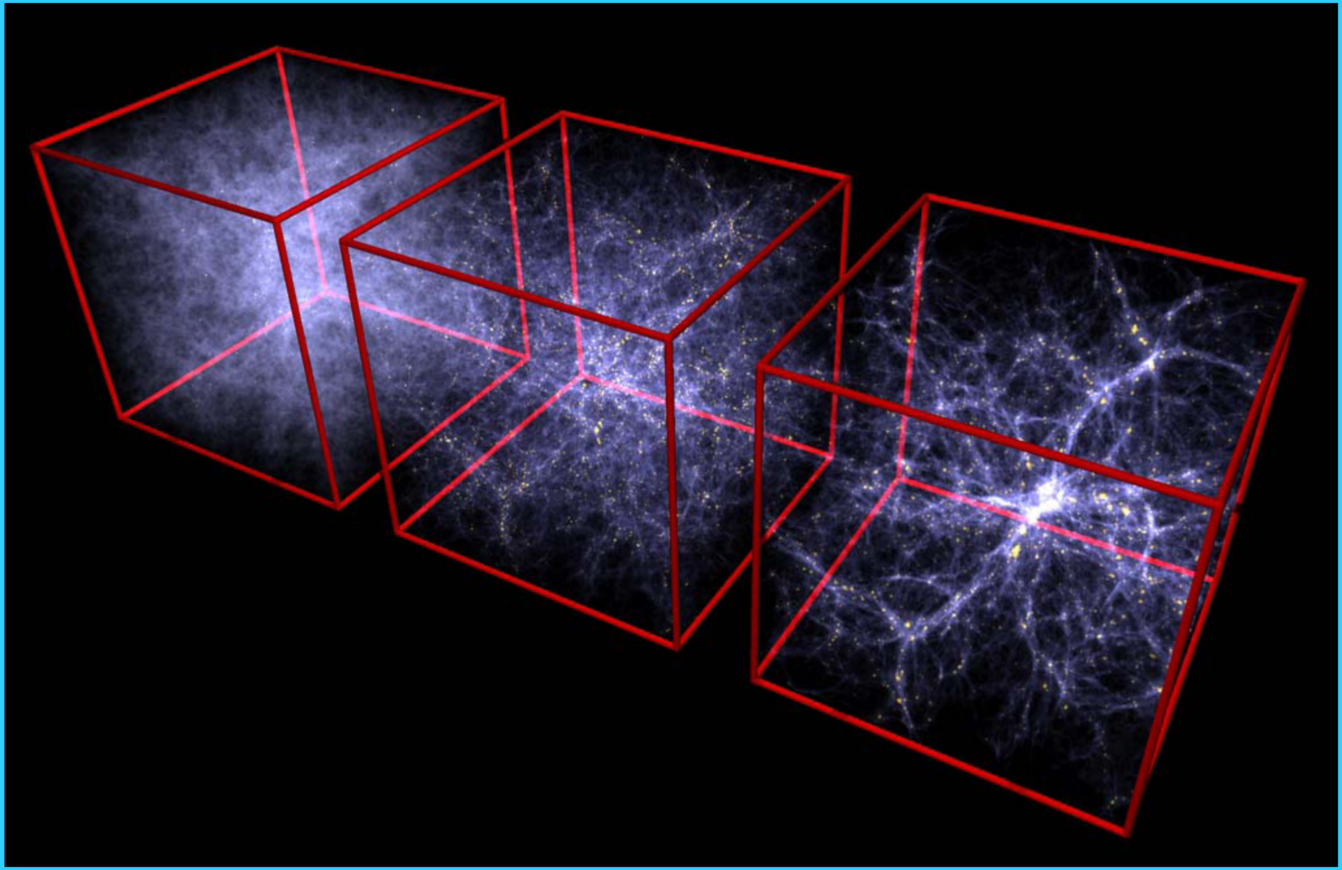


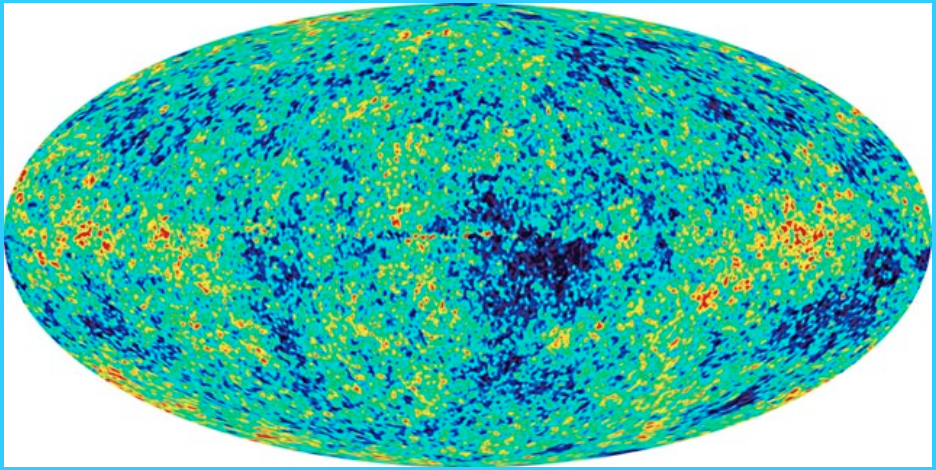
Figure 4. Upper-left panel: logarithmic slope of the density profile of haloes simulated at our highest resolution $N_{\rm sbox} = 256^3$, plotted for $r \geqslant r_{\rm conv}$. Curves are scaled horizontally to the radius r_{-2}, where the slope takes the isothermal value, $d \log \rho / d \log r \ (r_{-2}) = -2$. NFW and M99 profiles are shown as solid and dashed curves, respectively. The logarithmic slope increases monotonically with decreasing radius and there is no obvious convergence to a particular asymptotic value of the central slope. Upper-right panel: same as upper-left panel but for the SCDM Virgo cluster of Ghigna et al. (2000) and SCDM MW and M31 galaxy haloes of M99. The profiles of the two galaxy-sized haloes appear to be consistent with those of our haloes. The logarithmic slope of the cluster halo appears to be slightly steeper than the others, fluctuating about a value of -1.4 at the innermost resolved point. Lower-left panel: halo density profiles scaled horizontally to radius r_{-2} and vertically to the corresponding density at that radius $\rho_{-2} \equiv \rho(r_{-2})$. Lower-right panel: same as lower-left panel but for the Ghigna et al. (2000) and M99 haloes.

Simulated Gravitational Instability

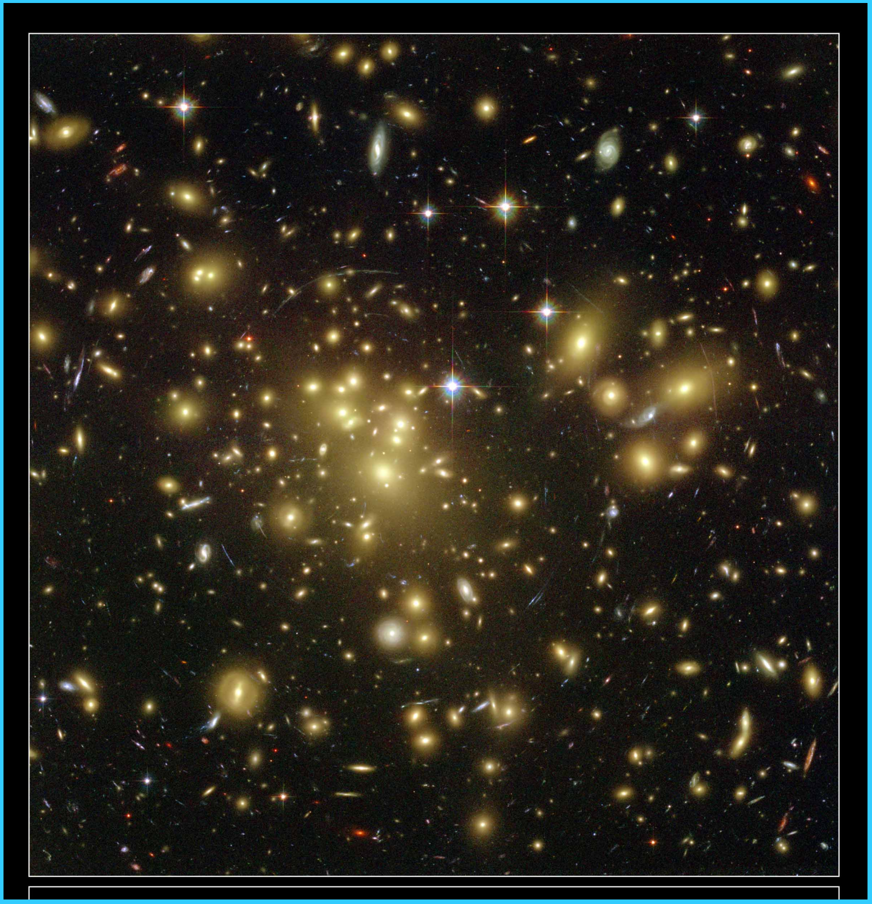


How to study/follow the Universe: why numerical simulations?

Initial conditions from the CMB



$$\frac{\partial T}{T} \approx \frac{\partial \rho}{\rho} \approx 10^{-5}$$



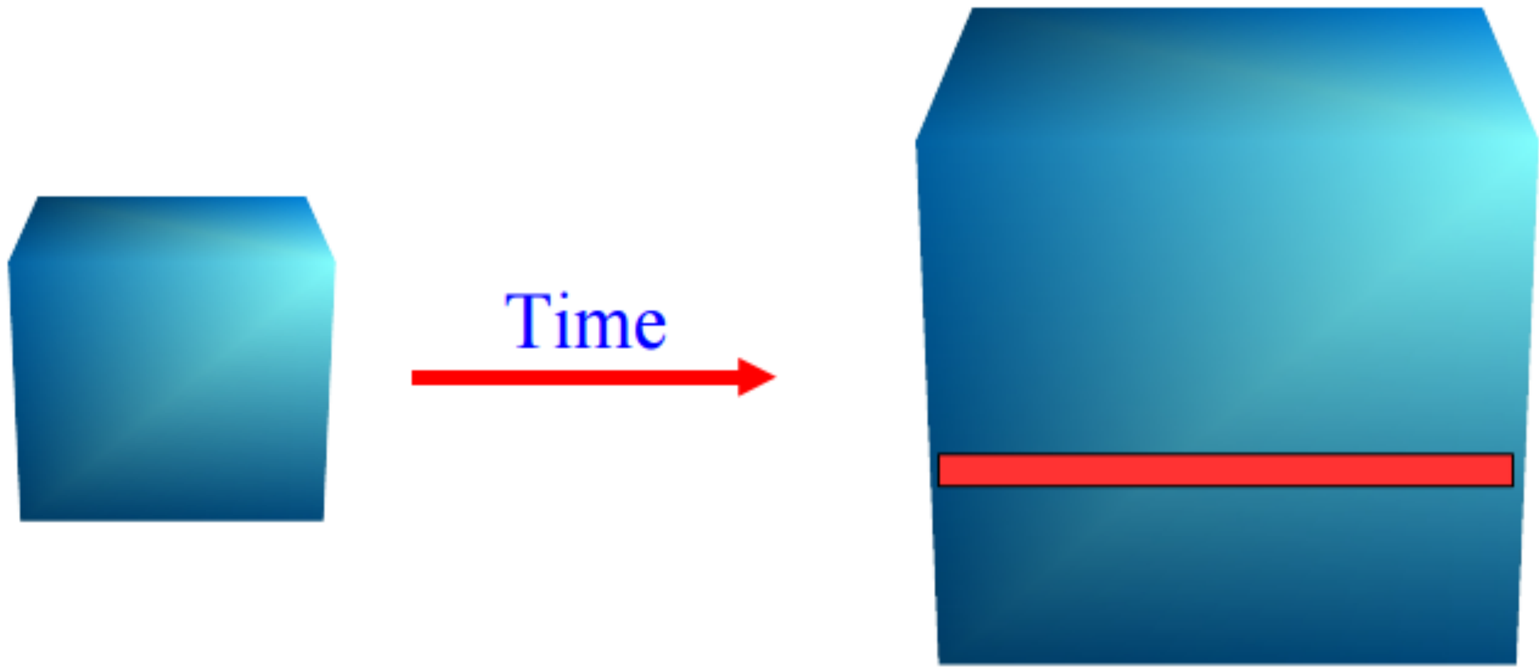
$$\frac{\partial \rho}{\rho}(cluster\ center) \approx 10^{5}$$

10 orders of magnitude

(break down of linear theory)

-> Numerical simulations

Evolving the Universe in a computer



- Follow the matter in an expanding cubic region
- Start 400,000 years after the Big Bang
- Match initial conditions to the observed Microwave Background
- Calculate evolution forward to the present day

Simulating the evolution of the DM distribution

- Represent the distribution of the N_{true} particles of DM in the simulated region by N_{sim} simulation particles.

  Typically $N_{true} / N_{sim} \sim 10^{60}$ or more!

- Place these uniformly within the computational volume

- Perturb their positions and velocities with a random realisation of the perturbation field predicted at early times by a theoretical model, e.g. ΛCDM – Cold Dark Matter + a cosmological constant + baryons + inflationary initial structure

- $\ddot{\mathbf{x}}_i = \nabla_i \Phi = -\Sigma_j G\, m_i\, m_j\, \mathbf{r}_{ij} / r_{ij}^3 \qquad i, j = 1, N$

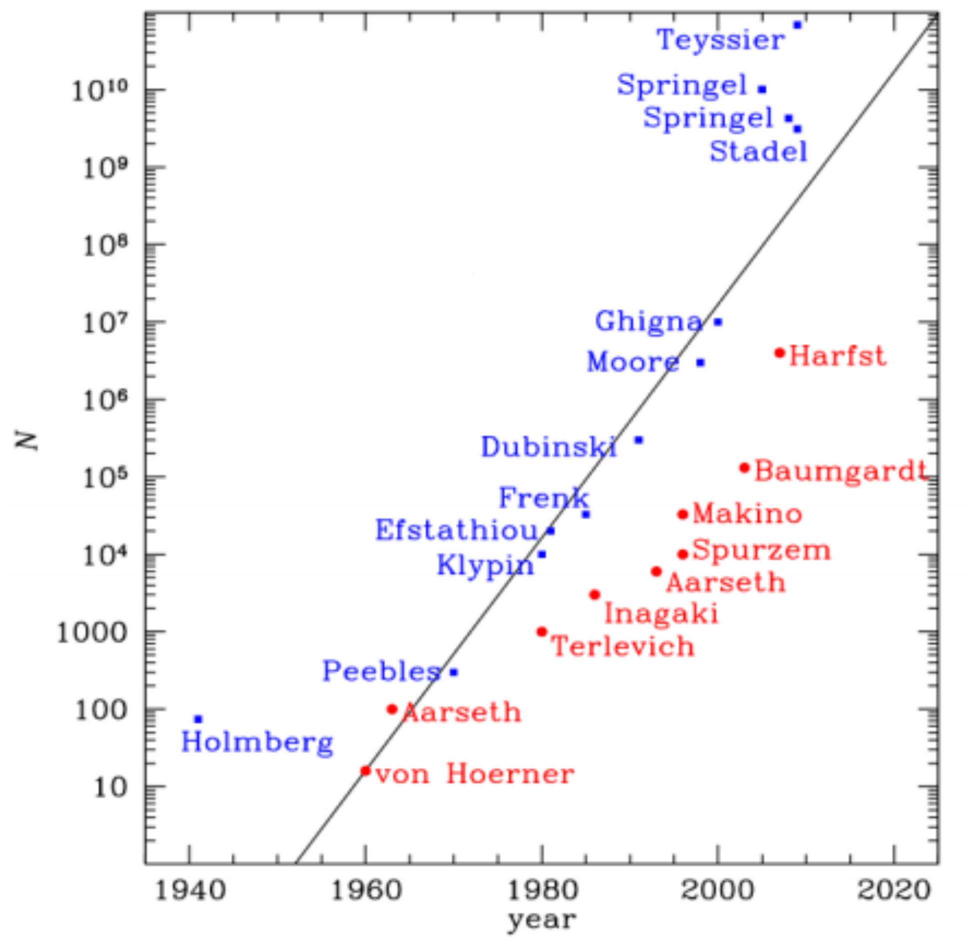
- Solve the equations of motions of N particles directly.
- Choose an integration scheme, a time step, and define the softening parameter

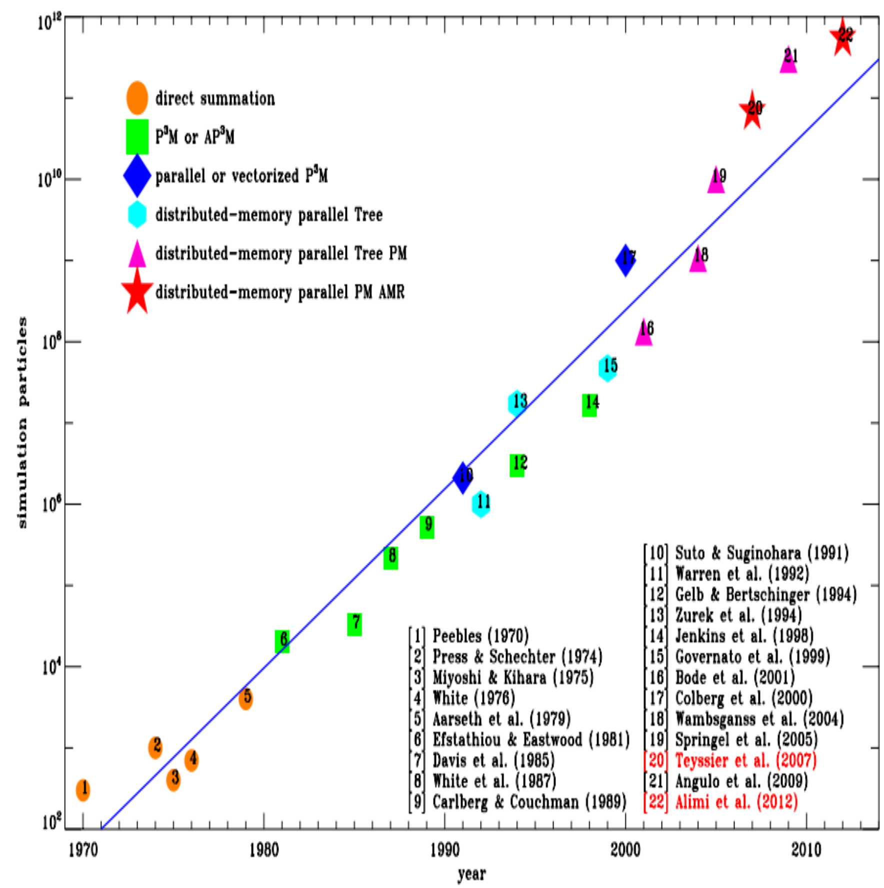
- Solve for the gravitational forces
  Step the particles forward a few million years

  Repeat until the present

Particles for a numerical cosmologist

Modern computer can handle more than 10^{10} particles





$$m_p \approx 10^7 M_{sun}$$

Clusters

Initial Conditions

- Inflation, large scale structure, CMB -> structures started to grow from an initially Gaussian stocastic field of density fluctuations

- The properties of a Gaussian Random Field can be completely specified by its power spectrum, whose shape is theoretically well motivated and depends on cosmological parameters and on the nature of DM

The Power Spectrum evolves according linear theory untill:

$$P(k) = Ak^n T^2(k,z)$$

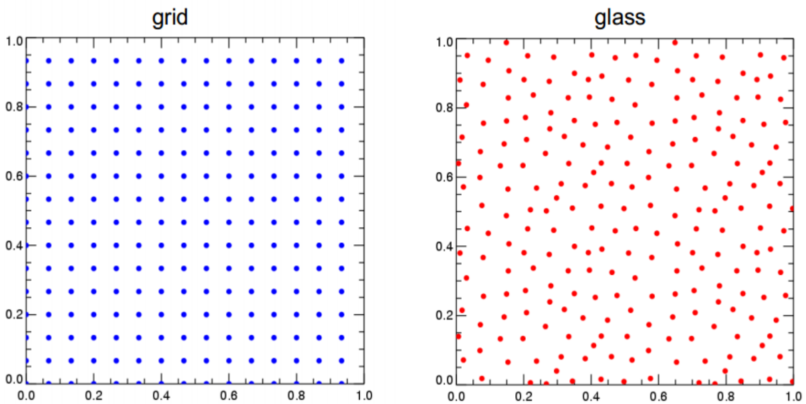
$$\frac{\delta\rho}{\rho} \approx 0.2 \quad z \sim 20-50$$

T(k,z) (transfer function): change of the initial P(K) due to different physical processes

Then we should obtain a realization of this P(k) using particles:

To create a realization of the perturbation spectrum, a model for an unperturbed density field is needed

GLASS OR CARTESION GRID



Using the Zeldovich approximation, the density fluctuations are converted to displacements of the unperturbed particle load

SETTING INITIAL DISPLACEMENTS AND VELOCITIES

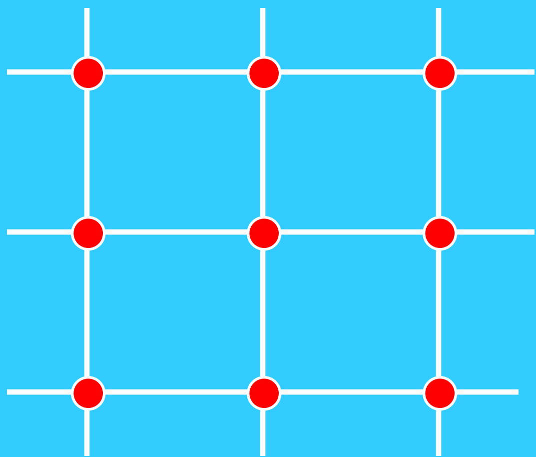
Zeldovich's Lagrangian version of linear theory implies:

The force field simply follows from the initial realization of density fluctuations

Particle displacements: $$\mathbf{d} = \mathbf{x} - \mathbf{x}_0 = -\frac{\nabla\Phi}{4\pi G\bar{\rho}a^2}$$

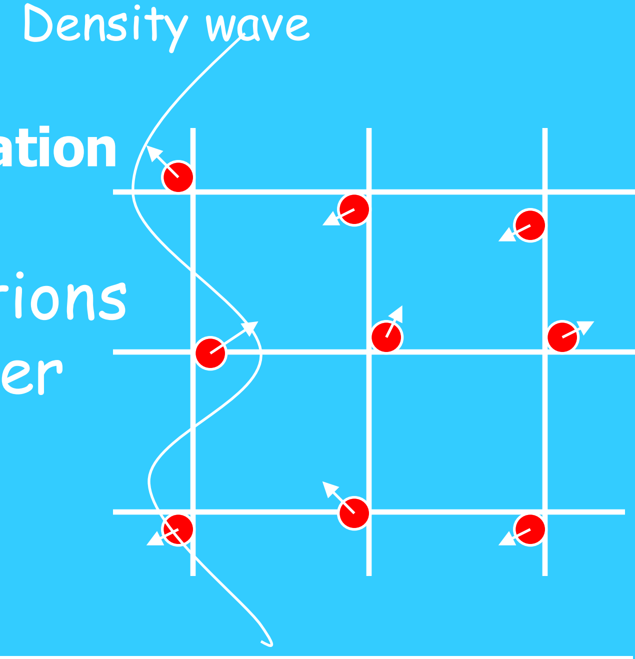
Particle velocities: $$\mathbf{v} = \frac{a\dot{D}}{D}\mathbf{d}$$

Note: Particles move on straight lines in the Zeldovich approximation.



Zel’dovich Approximation

Velocities and Positions are linked together

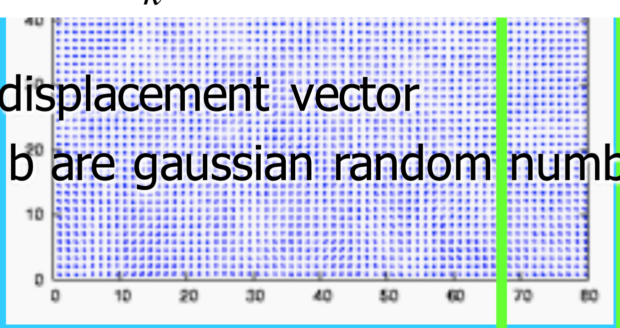


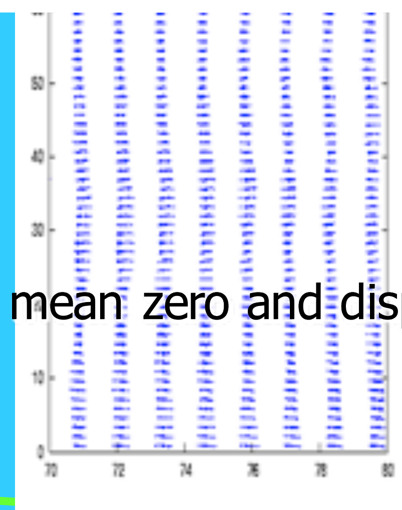
$$r(q,t) = a(t)\left[q + b(t)S(q)\right]$$

$$S(q) = \nabla \phi_0(q)$$

$$\phi_0(q) = \sum_k a_k \cos(kq) + b_k \sin(kq)$$

$$a_k, b_k = \sqrt{P(|k|)}\,\frac{Gauss(0,1)}{|k|^2}$$





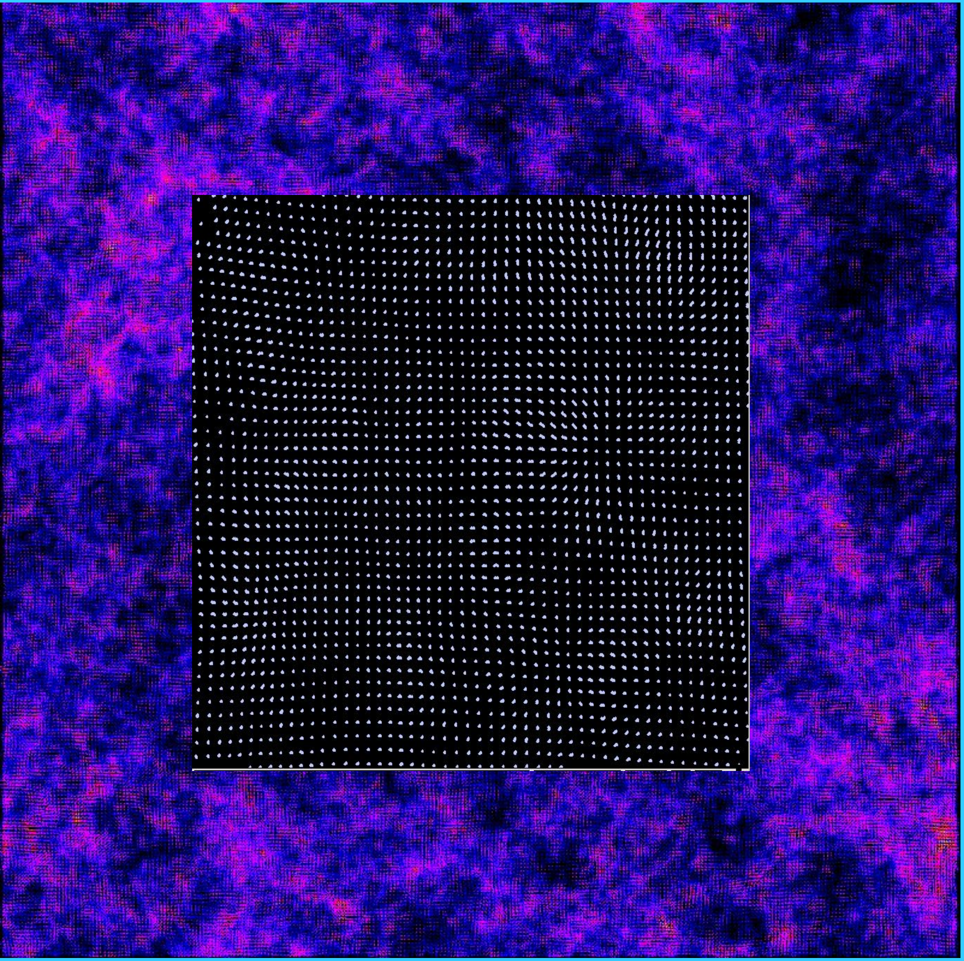
- S(q): displacement vector
- a and b are gaussian random numbers with the mean zero and dispersion $\sigma^2 = P(k)/k^4$

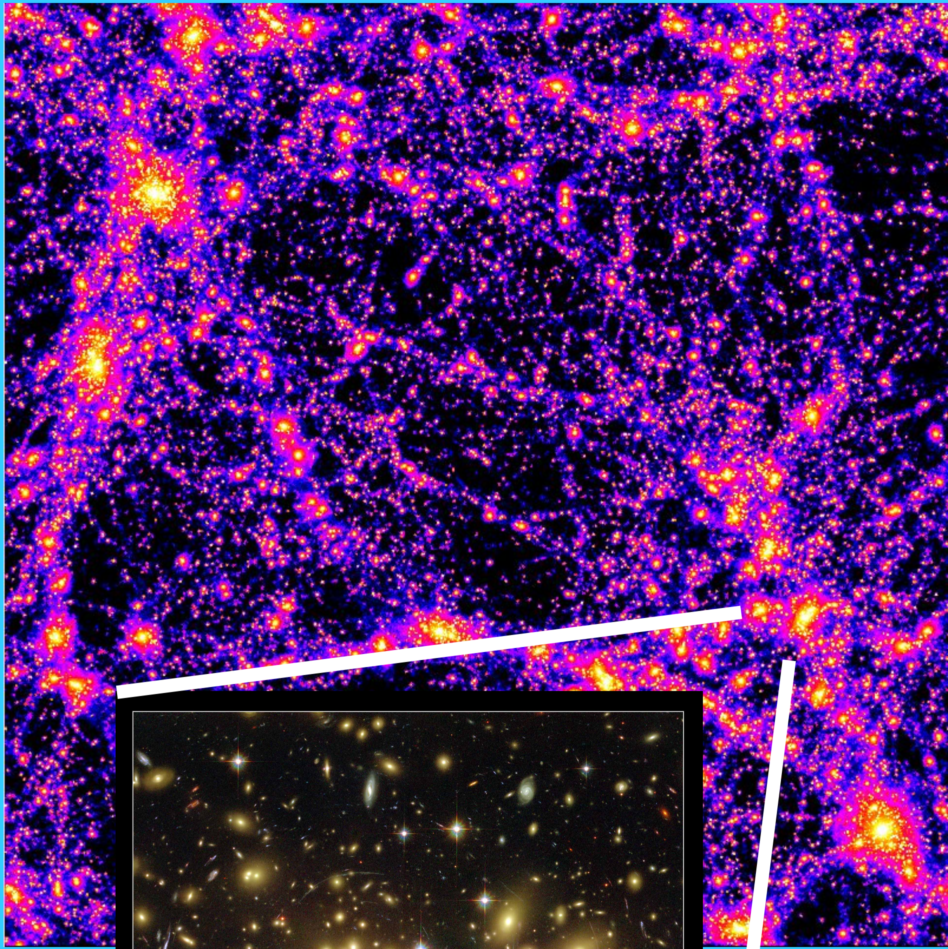
50 Mpc - 300^3 part

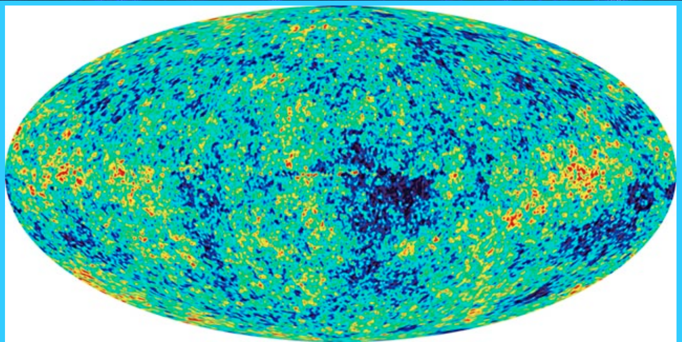
z=25

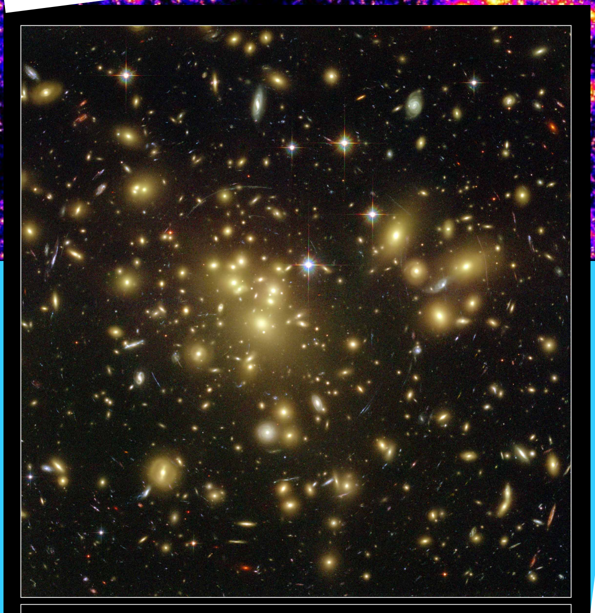
Maccio'+06,07

z=0



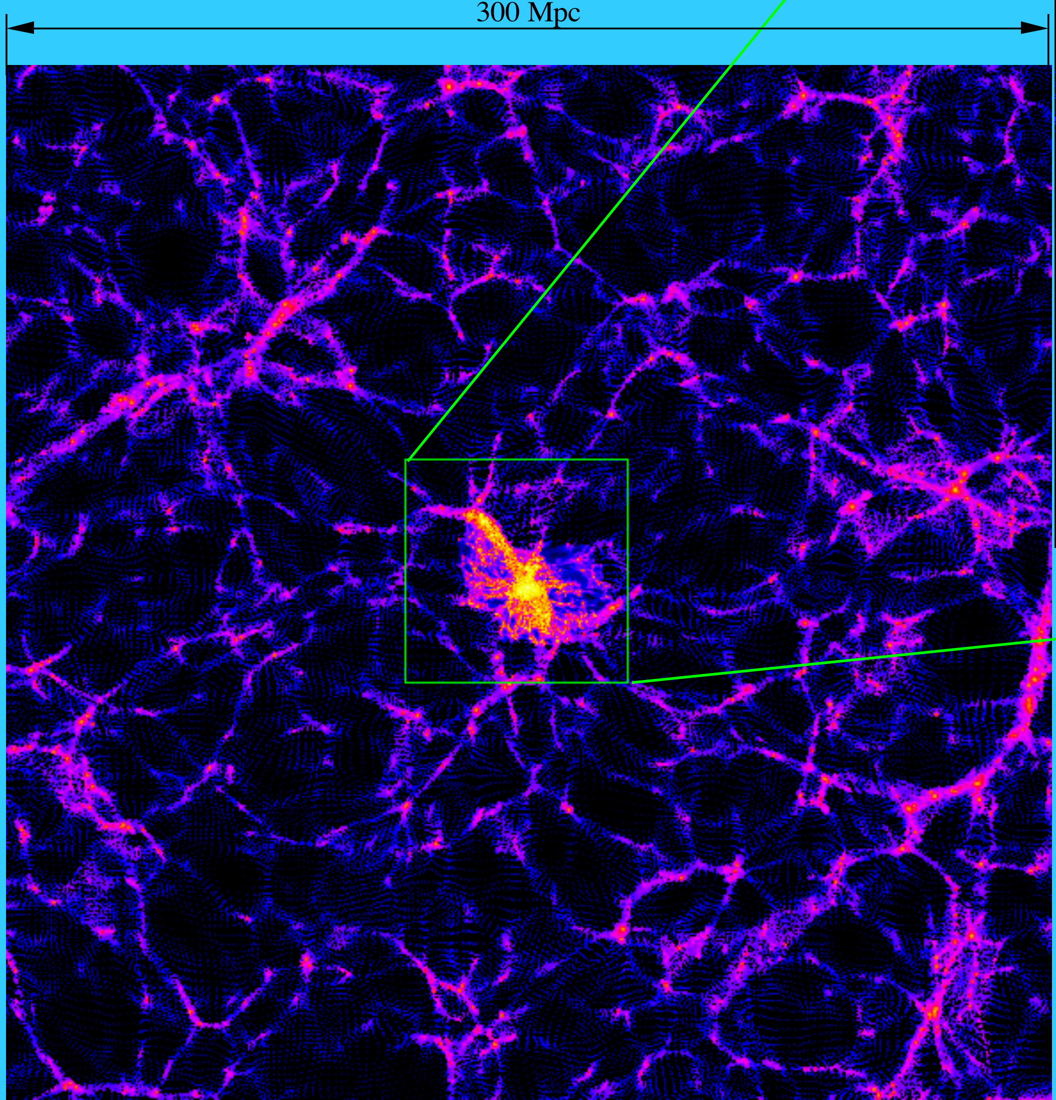


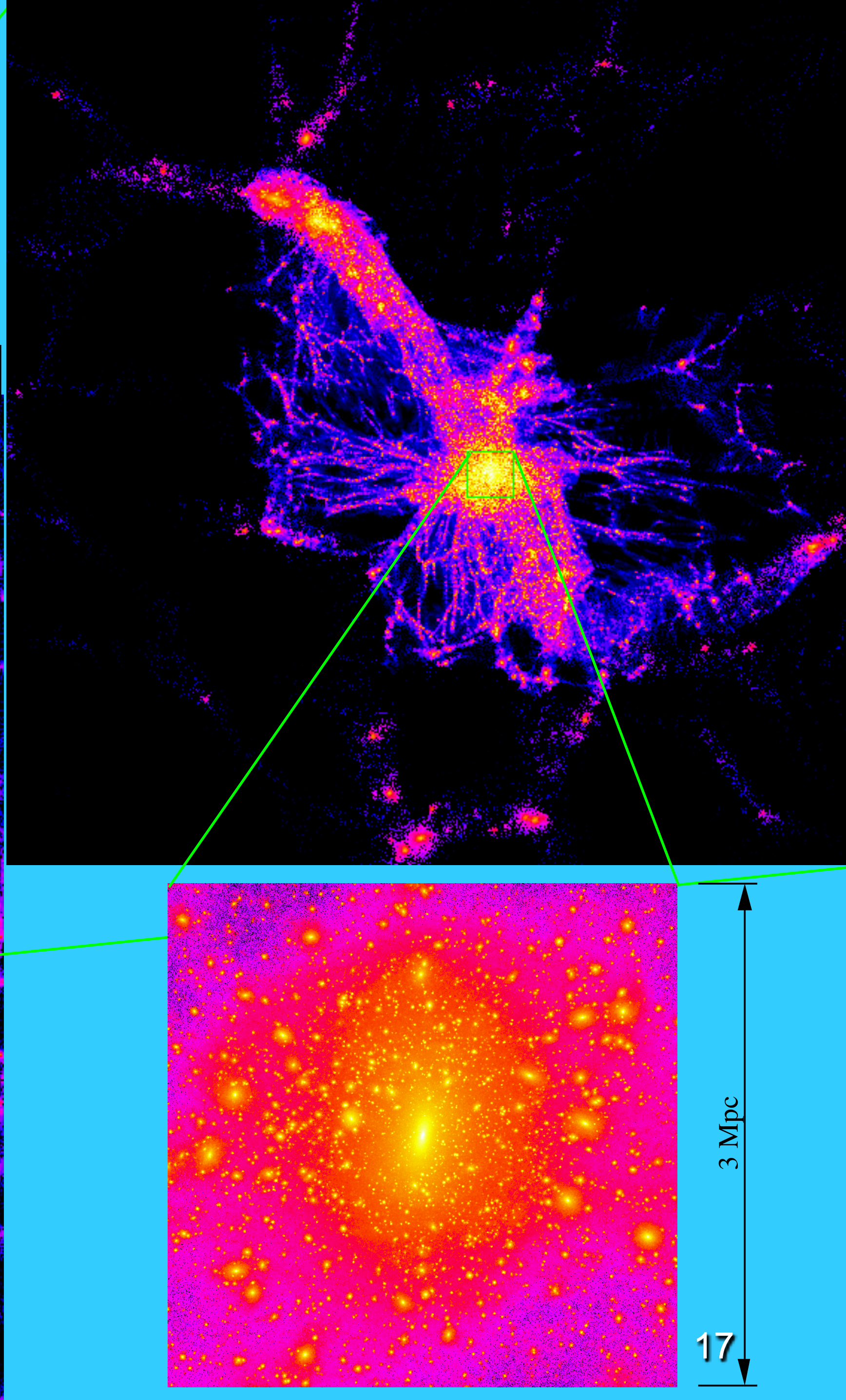




Refinement:

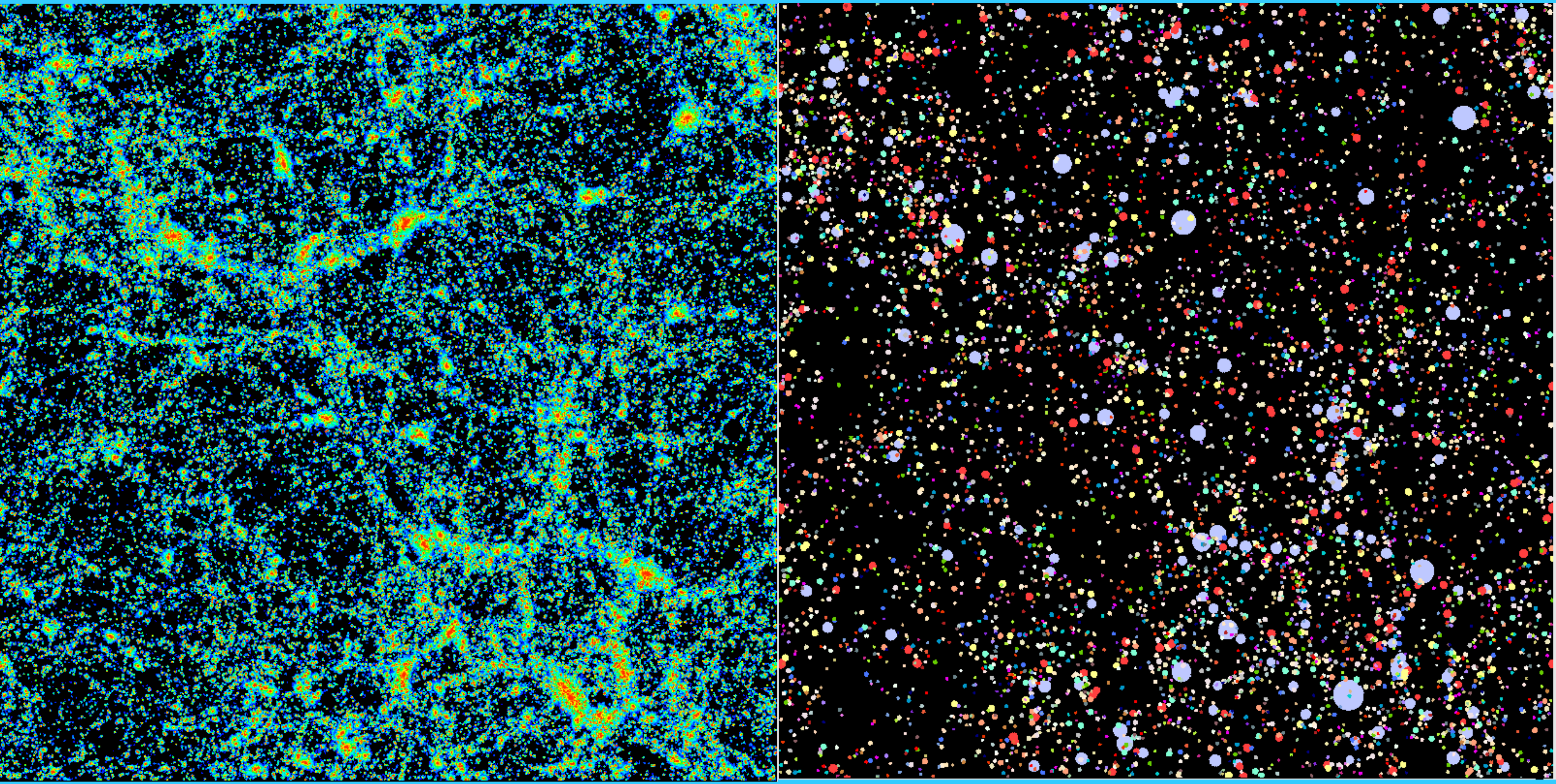
Re-simulating one halo with better mass resolution



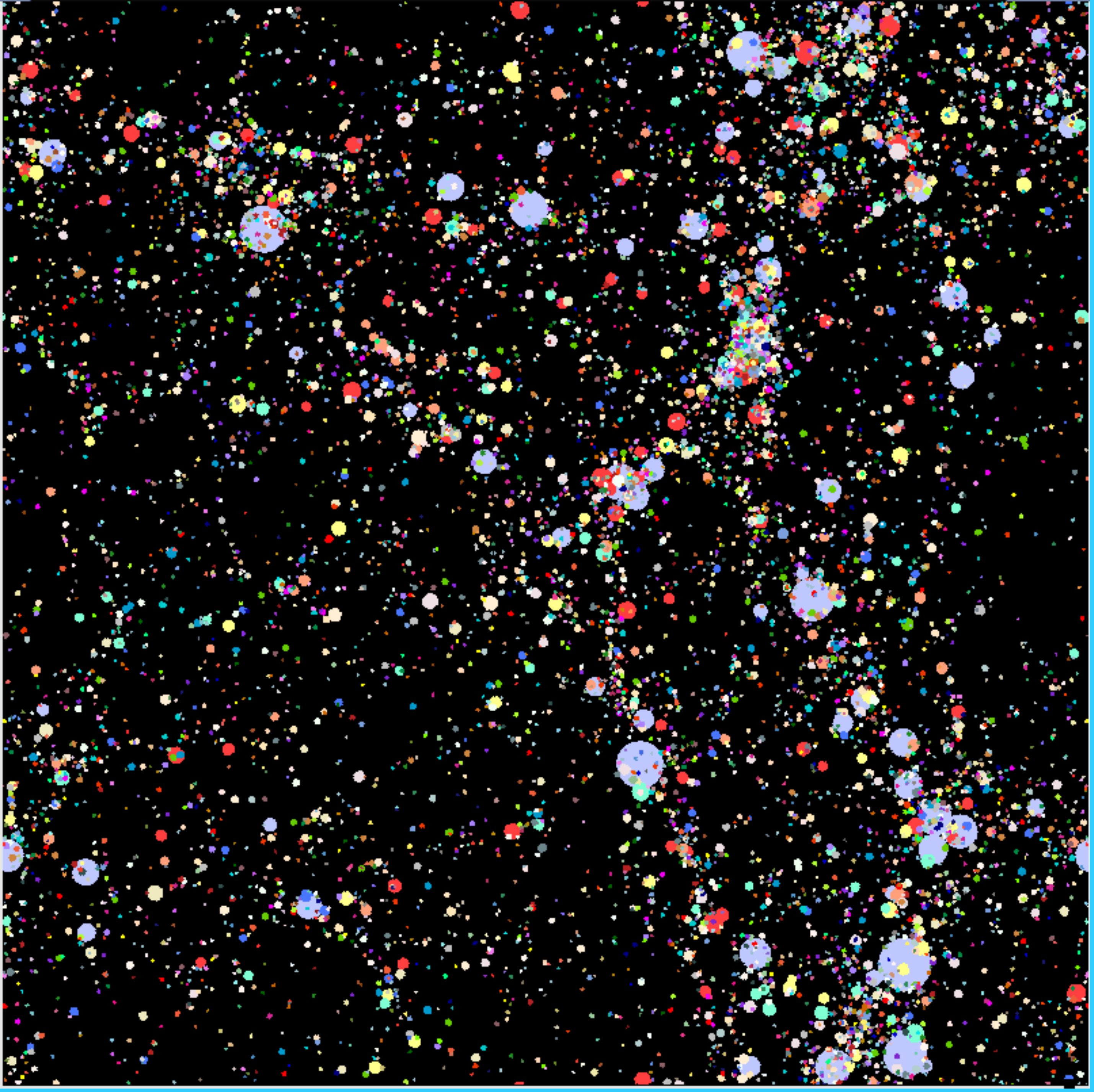


Finding Halos:

Spherical Over-density algorithm (Lacey & Cole 1994):
Virial density contrast fixed by linear theory: Dvir = 220*background



180 Mpc



For each halo:

M_{vir}

R_{vir}

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NUMERICAL ISSUES

Overmerging, two body relaxation, choises of softening length; multi-mass simulations

- In cosmological simulations of the dark matter each particle represents a coarse grained sampling of phase space which sets a mass and spatial resolution.
- Unfortunately these super-massive particles will undergo two body encounters that lead to energy transfer as the system tends towards equipartition (two-body relaxation). In the real Universe the dark matter particles are essentially collisionless and pass unperturbed past each other.
- The artificial smoothing of the density distribution in these regions (disruption of dark matter halos within dense environments) is referred to as `overmerging' (for a review of the problem see Moore 2000).
- A modification of the $1/r^2$ law through a softening length diminishes two body scattering and relaxation and allows larger time steps.
- Simulation results are least reliable in the densest parts of the halo, where the dynamical timescale is shortest and artificial heating has the greatest effect. In early simulations of the formation of galaxy clusters, overmerging erased substructure completely (e.g. White 1976). When simulations reached sufficient resolution to resolve roughly as many subsystems as there are galaxies in a cluster, the problem was considered `solved' (e.g. Ghigna et al. 2000), although the scale invariance of halo properties quickly lead to an excess dwarf satellite problem in galaxy haloes.
- The assumption that the overmerging problem is now solved has not been fully tested (Taylor et al. 2003). The effects of particle discreteness in N-body simulations of Λ CDM are still an intensively debated issue (Romeo et al. 2008).
- The processes of relaxation is difficult to quantify, but in the large N limit the discreteness effects inherent to the N-body technique vanish, so one tries to use as large a number of particles as computationally possible. Increasing N helps, but slowly ($N^{-0.25}$)

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$$\ddot{\mathbf{x}}_i = -\nabla_i \, \Phi(\mathbf{x}_i)$$

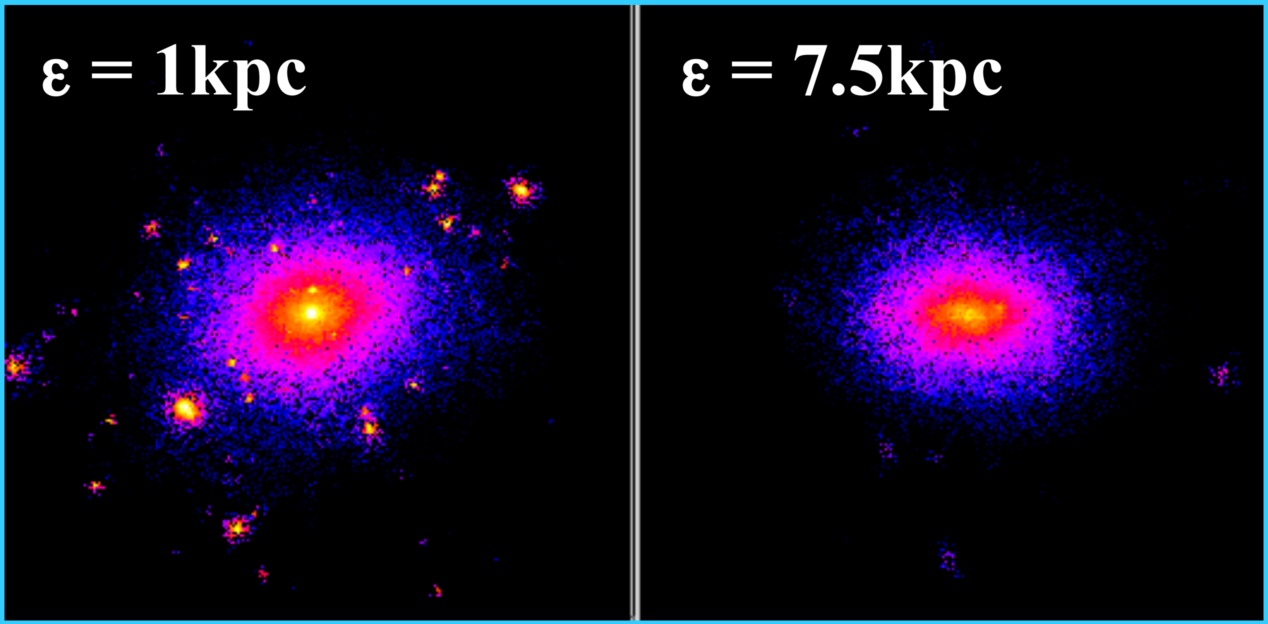
$$\Phi(\mathbf{x}) = -G \sum_{j=1}^{N} \frac{m_j}{[(\mathbf{x}-\mathbf{x}_j)^2 + \epsilon^2]}$$

The need for **gravitational softening**:

- Prevent large-angle particle scatterings and the formation of bound particle pairs.
- Ensure that the two-body relexation time is sufficiently large.

} Needed for faithful collisionless behaviour

- low mass/force resolutions
  ⇒ shallower potential than real
  ⇒ artificial disruption/overmerging
  (especially serious for small systems)



central 500kpc region of a simulated halo in SCDM

Moore (2001)

Relaxation Time

- Def: Time when rms of energy changes due to encounters becomes equal to mean energy.
- Fokker-Planck estimate:

$$T_{relax} = 0.34 \frac{\sigma^3}{G^2 m \rho \ln(b_{max}/b_{min})} \simeq \frac{N}{8 \ln(N)} T_{cross}$$

- Globular clusters: T $\simeq$ 100 Myr to 10 Gyr
- Stars in a galaxy: T $\simeq 10^{15}$ yr
- CDM in a galaxy halo: T $\simeq 10^{65}$ yr

Two body relaxation

- DM in N-body simulation: T = one Hubble time in a halo with N = 5'000, and
- T(r) = one Hubble time, for r = 0.01 rvir in a system with N = 1 million. (Power et al. 2003)
- "Resolved radius" ~ $N^{-0.5}$

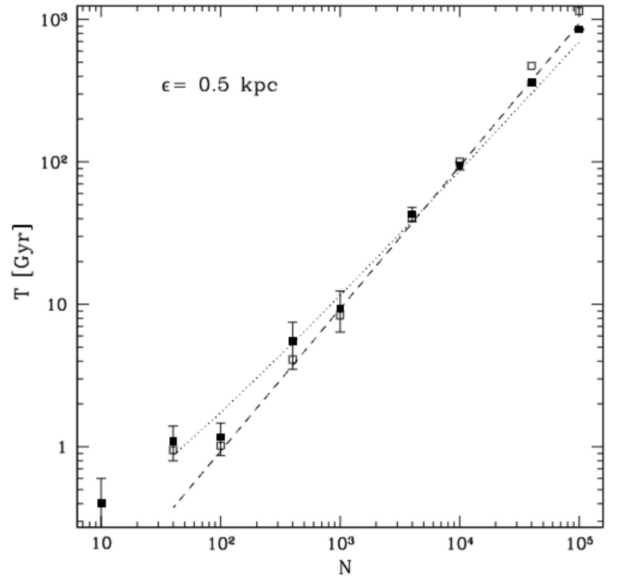


Figure 2. Average relaxation times of isotropic Hernquist models versus particle number N, with a constant softening $\epsilon = 0.5$ kpc. The filled squares are the measured relaxation times, with error bars form the linear fit of $\overline{\Delta E^2(\Delta t)}$. The open squares are the local estimates of the relaxation time (4).

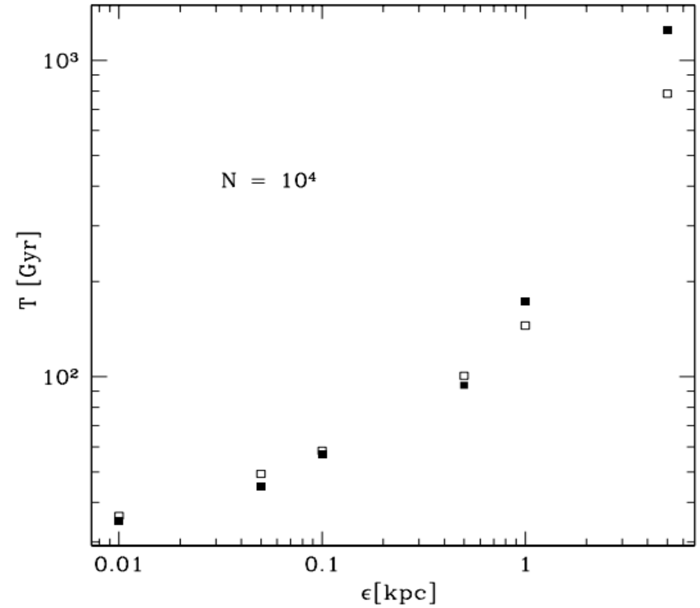


Figure 3. Average relaxation times of an $N = 10^4$ Hernquist models versus softening length ϵ. The filled squares are the measured relaxation times, the open squares are the local estimate (4).

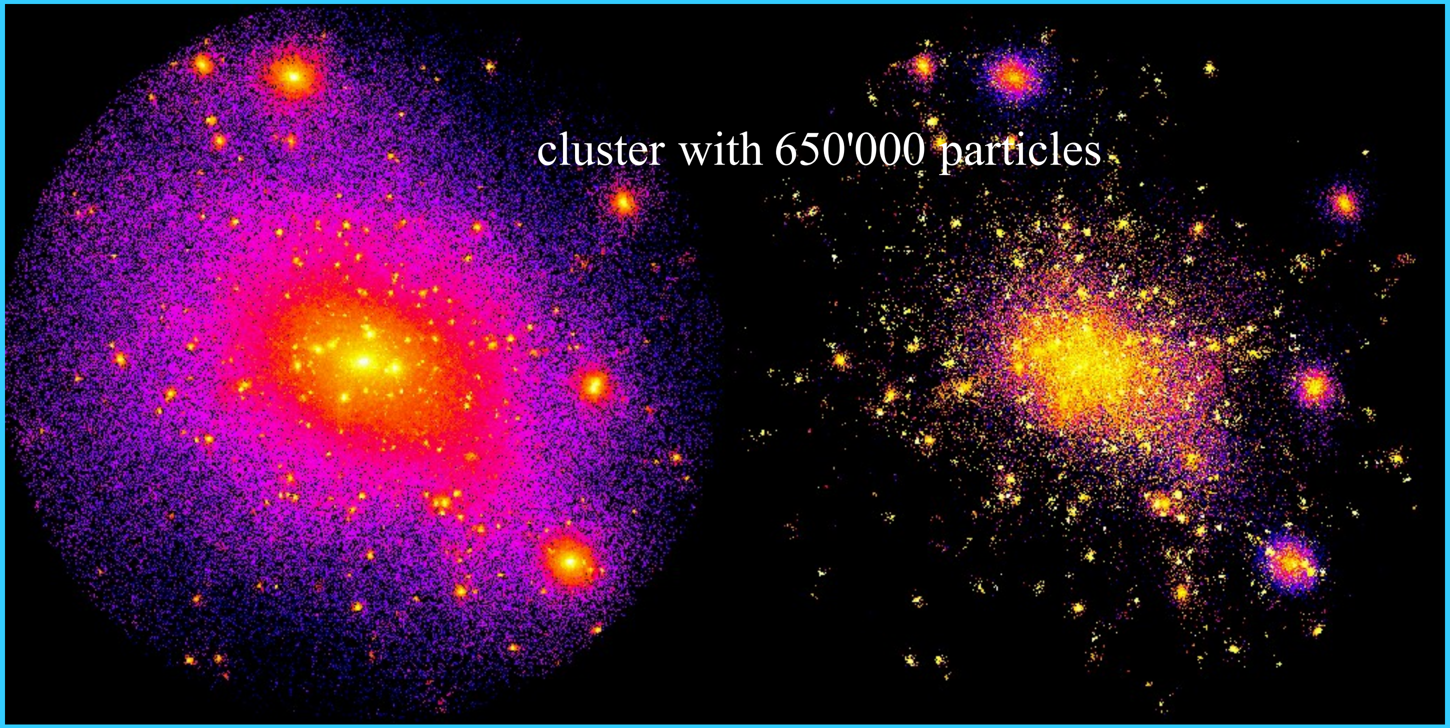
Measured relaxation time proportional to N, instead of N/log(N)

- BUT:

N is always small in the first CDM objects, also at high resolution! (Moore 2001, Binney & Knebe, 2002)

Relaxation in cosmological runs

cluster with 650'000 particles

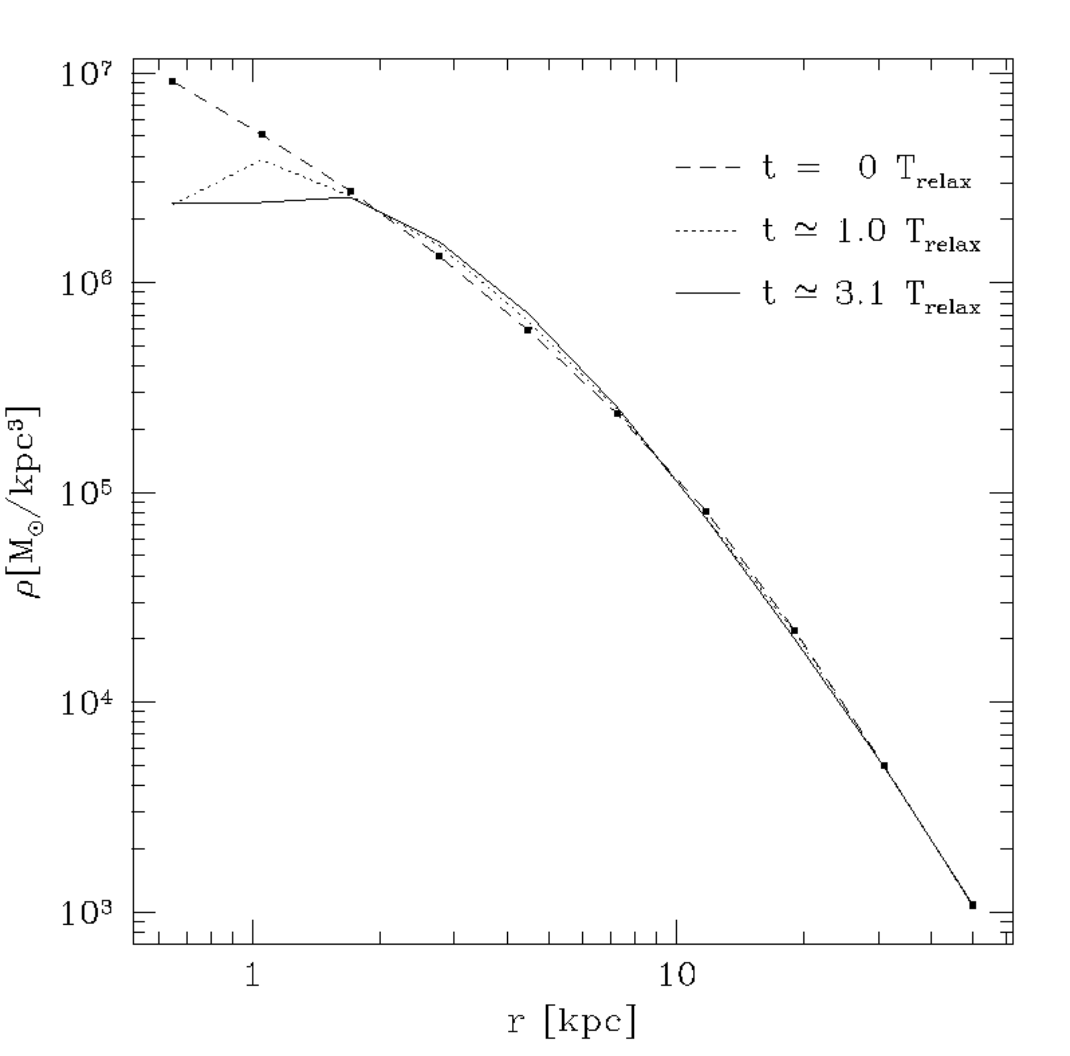


log density

log "relaxation"

Effect on final object?

N=4'000 Hernquist halo,
after 1 and 3 mean relaxation times:



merging... → final profile also shallower

Relaxation in cosmological simulations

- Relaxation is present in CDM simulations
- Increasing N helps, but slowly ($N^{-0.25}$)
- Convergence by using more particles

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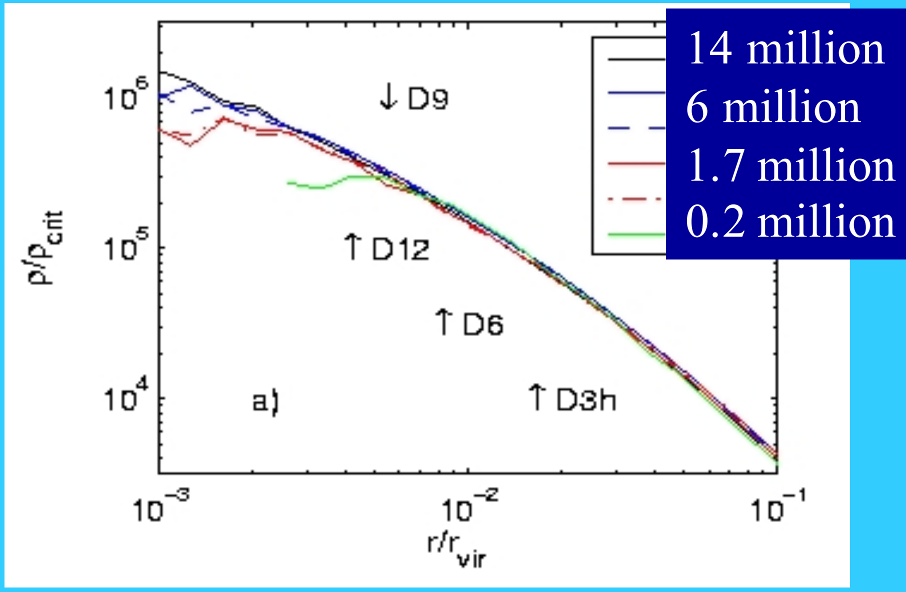
Convergence tests in CDM clusters

- Numerical flattening due to two body relaxation:
  slow convergence, $r \propto N^{-1/3}$
  1 million to resolve 1% of Rvirial,
  1000 to resolve 10% !

  (Moore et al. 1998; Diemand et al. 2004)

- To have such a high number of particles per halo the use of a special technique is required. This technique ("multi-mass technique", "refinement") consists of simulating with high mass resolution only the particles that will end up in the halo of interest at *z* = 0, while having less mass resolution for the particles that end up far away from the halo of interest.

- This has the drawback that each simulation can resolve only one halo at a time, therefore selection effect biases and a poor statistical sample could affect the reliability of the final result even if the simulation is very accurate and has high resolution



SUMMARY

- In the last 2 decades several solution has been proposed to the Cusp/Core problem
- Systematic errors in observations: nowadays no longer important
- Limitations in dissipationless simulations: nowadays it seems that simulations are correctly predicting DM density profiles BUT they do not include baryonic physics (Hydrodynamical simulations does)
- Modifying the power spectrum or the DM particles can solve the problem, but it can introduce other problems (see next lecture)
- Modifying Gravity: can also solve the problem introducing sometime other problems. Moreover there are not compelling evidences showing knots in GR (see next lecture)

INITIAL CONDITIONS

- Inflation, large scale structure, CMB -> structures started to grow from an initially Gaussian stocastic field of density fluctuations

- The properties of a Gaussian Random Field can be completely specified by its power spectrum, whose shape is theoretically well motivated and depends on cosmological parameters and on the nature of DM

$$P(k) = \langle |\delta_k|^2 \rangle = Ak^n T(k)^2$$

- The overdensity, δ (Gaussian random process) at location x and time t is

$$\delta(x,t) = \frac{\rho(x,t) - \bar{\rho}(t)}{\bar{\rho}(t)} = \sum_k \hat{\delta}(k,t)\, e^{ikx}$$

(expansion in Fourier components)

- First Step: Generating a Gaussian random field: Easiest to generate realisations in Fourier (or k-) space

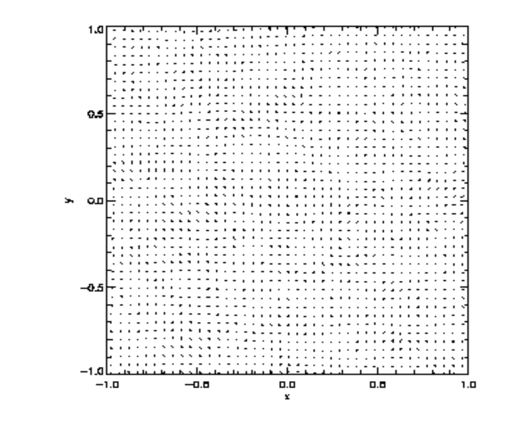
- Perturbations are distributed according to a Gaussian distribution, so phases are drown from a uniform distribution, and the amplitude of a particular mode (i.e. a given k) is drawn from a distribution of the form,

$$f(\delta_k) = \frac{1}{2\pi P(k)} e^{-|\delta_k|^2 / 2P(k)}$$

- In other words, in order to generate the initial conditions, it is necessary to generate a set of complex numbers with a phase φ distributed in an aleatory fashion and with and amplitude Normally distributed and with a variance determined from the spectrum (e.g., Bardeen et al. 1986). This can be obtained choosing two aleatory numbers φ in] 0, 1] and A in] 0, 1] for each point in the k-space

$$\hat{\delta}_{\mathbf{k}} = \sqrt{-2P(|\mathbf{k}|)\ln(A)}\,\mathrm{e}^{i2\pi\phi}.$$

- **Second step**: Generate a particle distribution -- either a grid or a glass



- **Third step**: Use the Zel'dovich approximation to initialise particle displacements and velocities

$$
\begin{aligned}
r(q,t) &= a(t)\left[q + b(t)S(q)\right] \\
S(q) &= \nabla \phi_0(q) \\
\phi_0(q) &= \sum_k a_k \cos(kq) + b_k \sin(kq)
\end{aligned}
$$

- The displacement vector S is related to the velocity potential Φo and the power spectrum of fluctuations P(k)

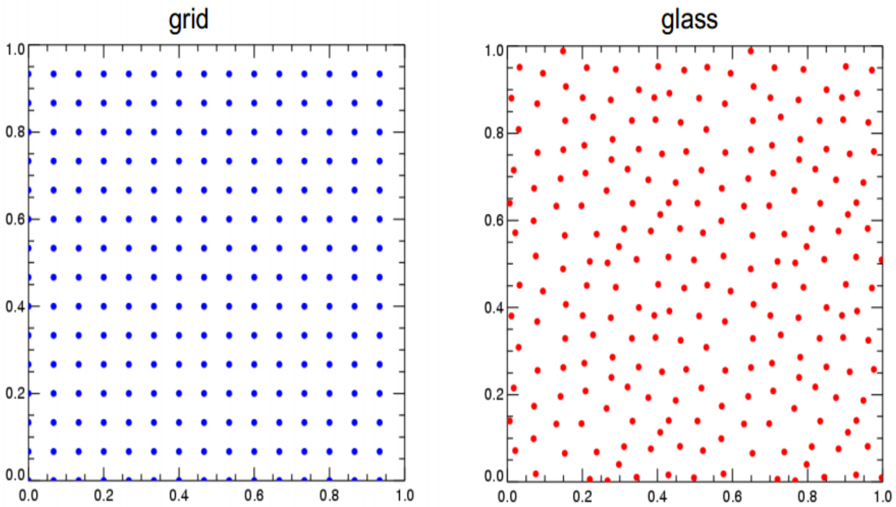
- a and b are gaussian random numbers with the mean zero and dispersion $\sigma^2 = P(k)/k^4$

$$a_k, b_k = \sqrt{P(|k|)}\,\frac{Gauss(0,1)}{|k|^2}$$

- In order to set the initial conditions, we choose the size of the computational box L and the number of particles N^3. The phase space is divided into small equal cubes of size 2π/L. Each cube is centered on a harmonic $k_{Nyquist} = 2\pi/L \times \{i, j, k\}$, where {i, j, k} are integer numbers with limits from zero to N/2. We make a realization of the spectrum of perturbations ak and bk, and find displacement and momenta of particles with q = L/N × {i, j, k} using Ze'ldovich approximation. Here i, j, k = 1, N.

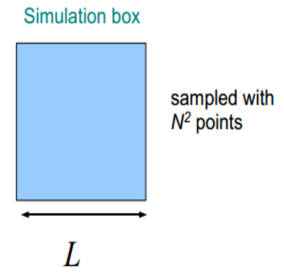
To create a realization of the perturbation spectrum, a model for an unperturbed density field is needed

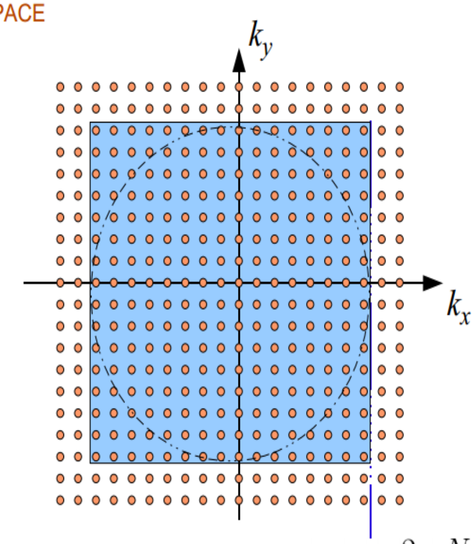
GLASS OR CARTESION GRID



One usually assigns random amplitudes and phases for individual modes in Fourier space

GENERATING THE FLUCTUATIONS IN K-SPACE





$$k_{\text{Nyquist}} = \frac{2\pi}{L}\,\frac{N}{2}$$

$$\delta_{\mathbf{k}} = B_{\mathbf{k}} \exp^{i\,\phi_{\mathbf{k}}}$$

For each mode, draw a random phase, and an amplitude from a Rayleigh distribution.

$$\left\langle \delta_{\mathbf{k}}^2 \right\rangle = P(k)$$

Using the Zeldovich approximation, the density fluctuations are converted to displacements of the unperturbed particle load

SETTING INITIAL DISPLACEMENTS AND VELOCITIES

Zeldovich's Lagrangian version of linear theory implies:

The force field simply follows from the initial realization of density fluctuations

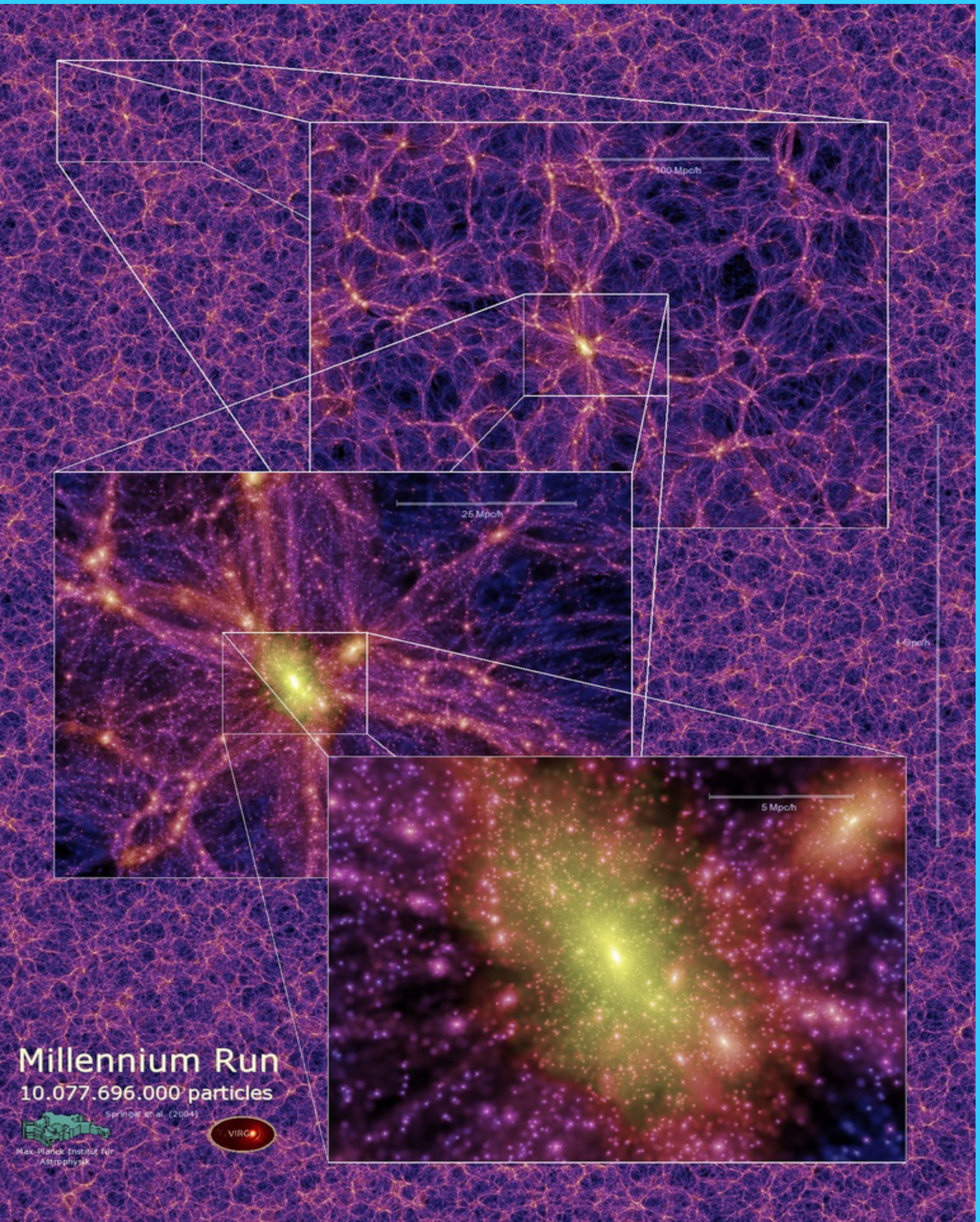
Particle displacements:

$$\mathbf{d} = \mathbf{x} - \mathbf{x}_0 = -\frac{\nabla\Phi}{4\pi G\bar{\rho}a^2}$$

Particle velocities:

$$\mathbf{v} = \frac{a\dot{D}}{D}\,\mathbf{d}$$

Note: Particles move on straight lines in the Zeldovich approximation.



N-body simulations

Study "nonlinear" structure formation in an expanding Universe after the cosmic decoupling epoch (z~1100) governed by the *gravity*

- Large Scale Structure: cosmic structure on scales of ~10-50 Mpc/h in mildly-nonlinear regime (overdensity~1), representing forming superclusters, low-density voids, filaments of galaxies.

- Clusters of galaxies: largest self-gravitating systems (aka, DM halos) with overdensity>>1, composed of 100-1000 galaxies.

Identified as dense nodes of "Cosmic Web", being building blocks of LSS

Millennium simulation in a 500Mpc/h box

Springel et al. 2005)

N-body simulations

Cold Dark Matter: non relativistic, collisionless fluid of particles

$$\frac{\partial f}{\partial t} + \frac{\bar{p}}{ma^2}\nabla f - m\nabla\Phi\frac{\partial f}{\partial \bar{p}} = 0$$

Boltzmann collisionless equations
(Vlasov Equation)
in an expanding Universe

$$f = f(\bar{x}, \bar{p}, t)$$

Phase Space density

$$\rho(x,t) = \int f(x,p,t)d^3 p$$

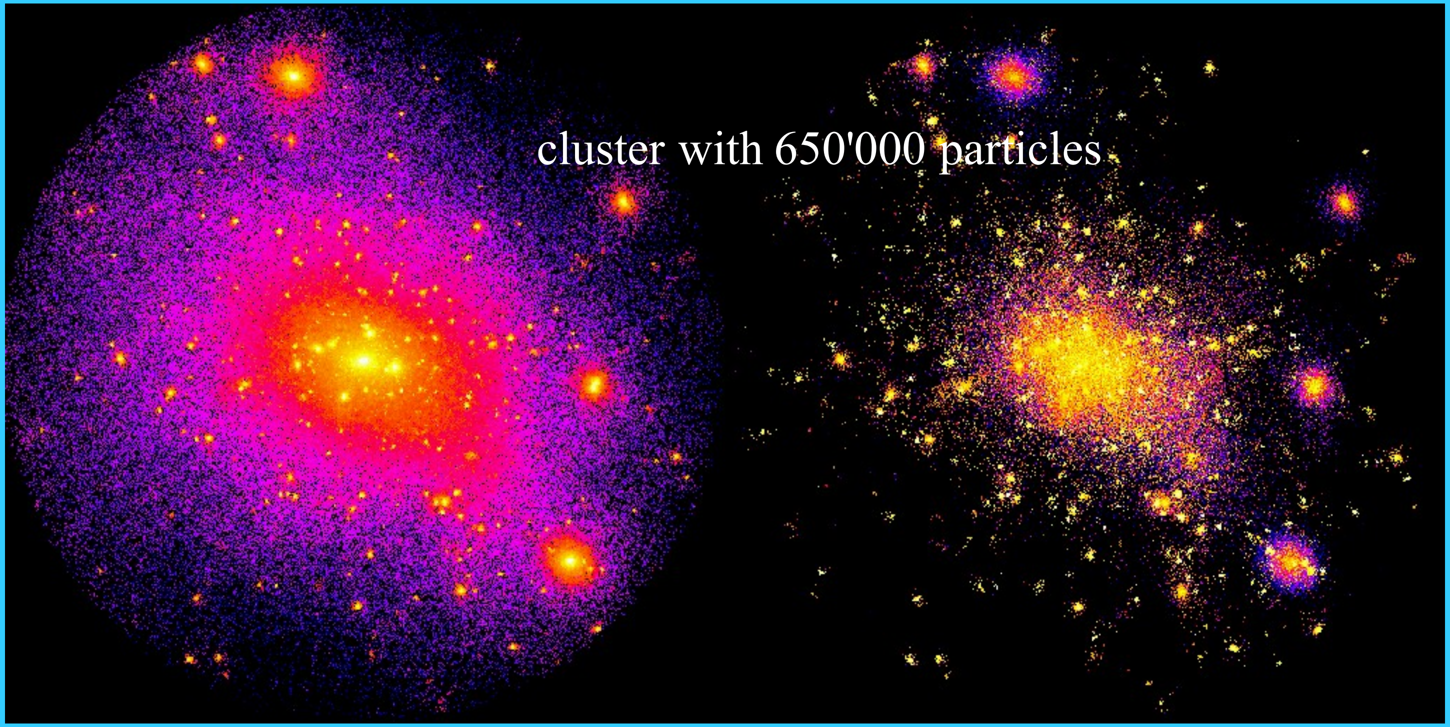
$$\nabla^2\Phi(x,t) = 4\pi G a^2[\rho(x,t) - \bar{\rho}(t)]$$

Matter density

We want to solve the equations of motions of N particles directly. We have to choose an integration scheme, a time step, and define the softening parameter

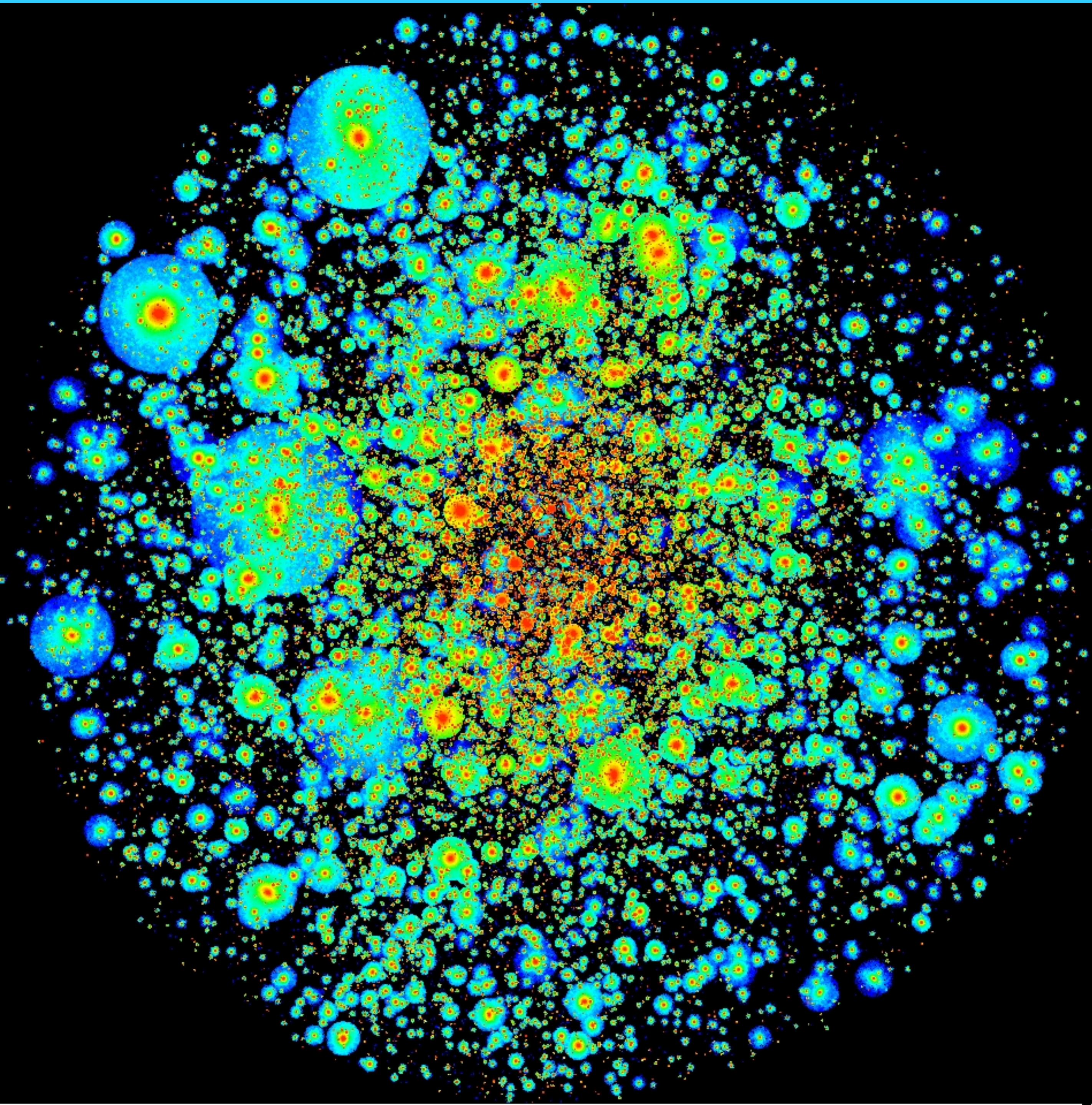
Relaxation in cosmological runs

cluster with 650'000 particles



log density

log "relaxation"



36.000
DM satellites
(within 300 kpc)

25 Millions part

(Diemand+08
Maccio'+10)

Initial Conditions

The Power Spectrum evolves according linear theory untill:

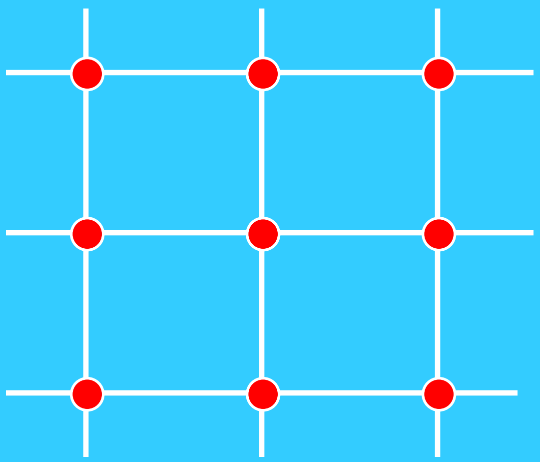
$$P(k) = Ak^n T^2(k,z)$$

$$\frac{\delta\rho}{\rho} \approx 0.2 \quad z \sim 20-50$$

T(k,z) provided by linear theory

Then we should obtain a realization of this P(k) using particles:

Zel'dovich Approximation

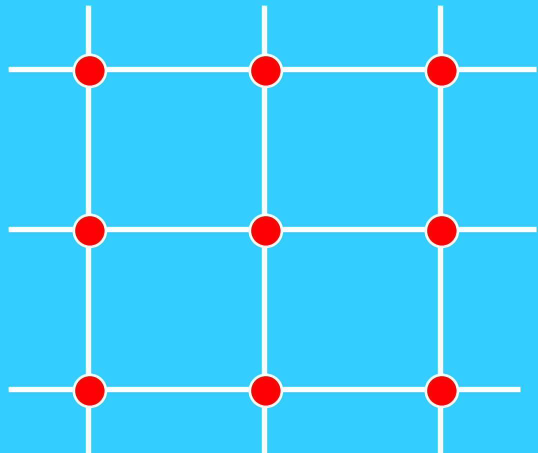


$$r(q,t) = a(t)\left[q + b(t)S(q)\right]$$

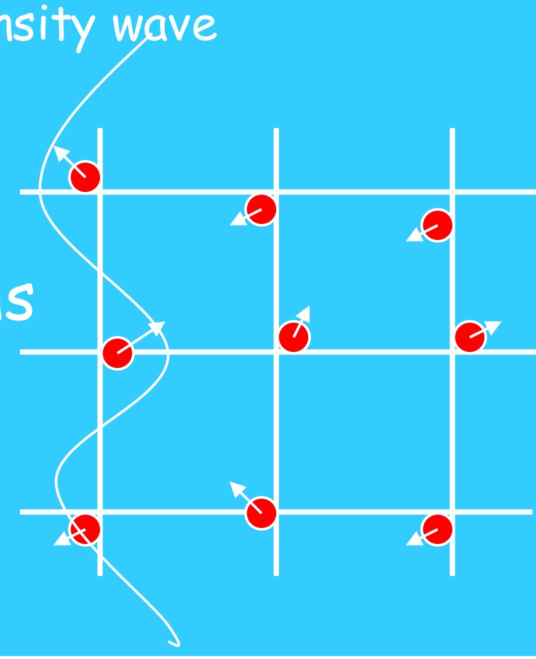
$$S(q) = \nabla\phi_0(q)$$

$$\phi_0(q) = \sum_k a_k \cos(kq) + b_k \sin(kq)$$

$$a_k, b_k = \sqrt{P(|k|)}\,\frac{Gauss(0,1)}{|k|^2}$$



Zeldovich



Velocities and Positions are linked together

$$r(q,t) = a(t)\left[q + b(t)S(q)\right]$$

$$S(q) = \nabla\phi_0(q)$$

$$\phi_0(q) = \sum_k a_k\cos(kq) + b_k\sin(kq)$$

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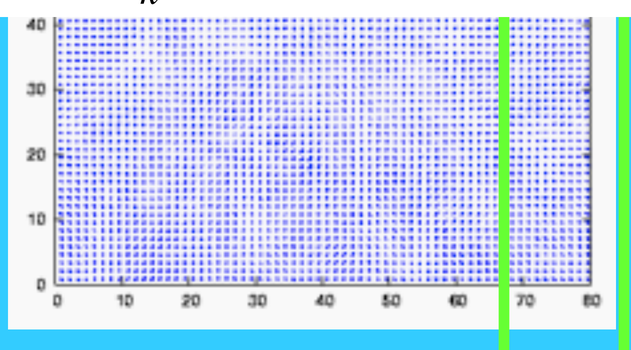
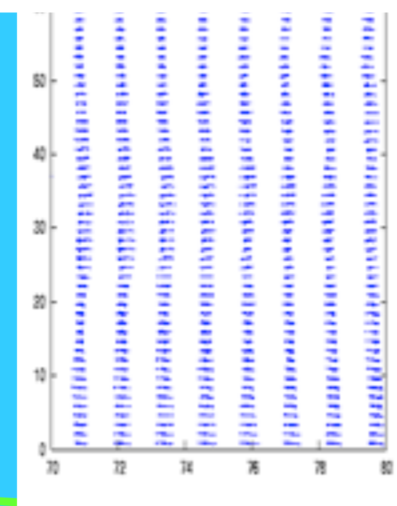


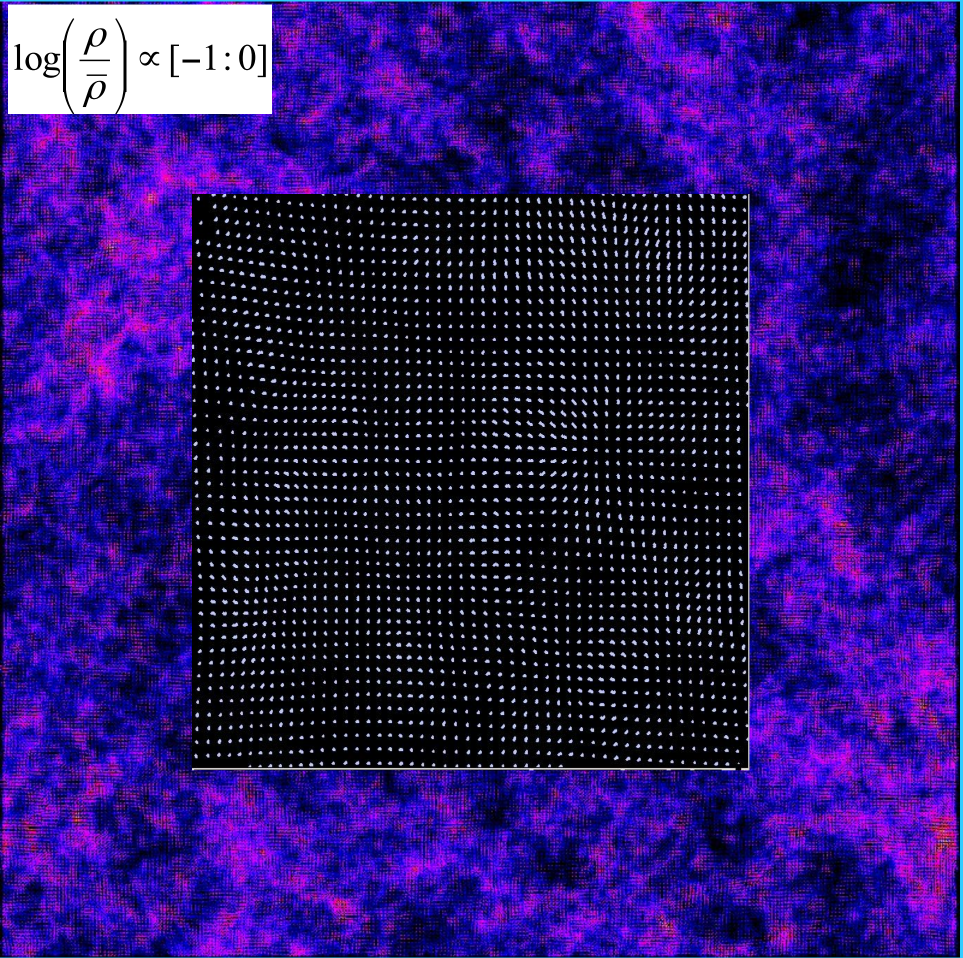


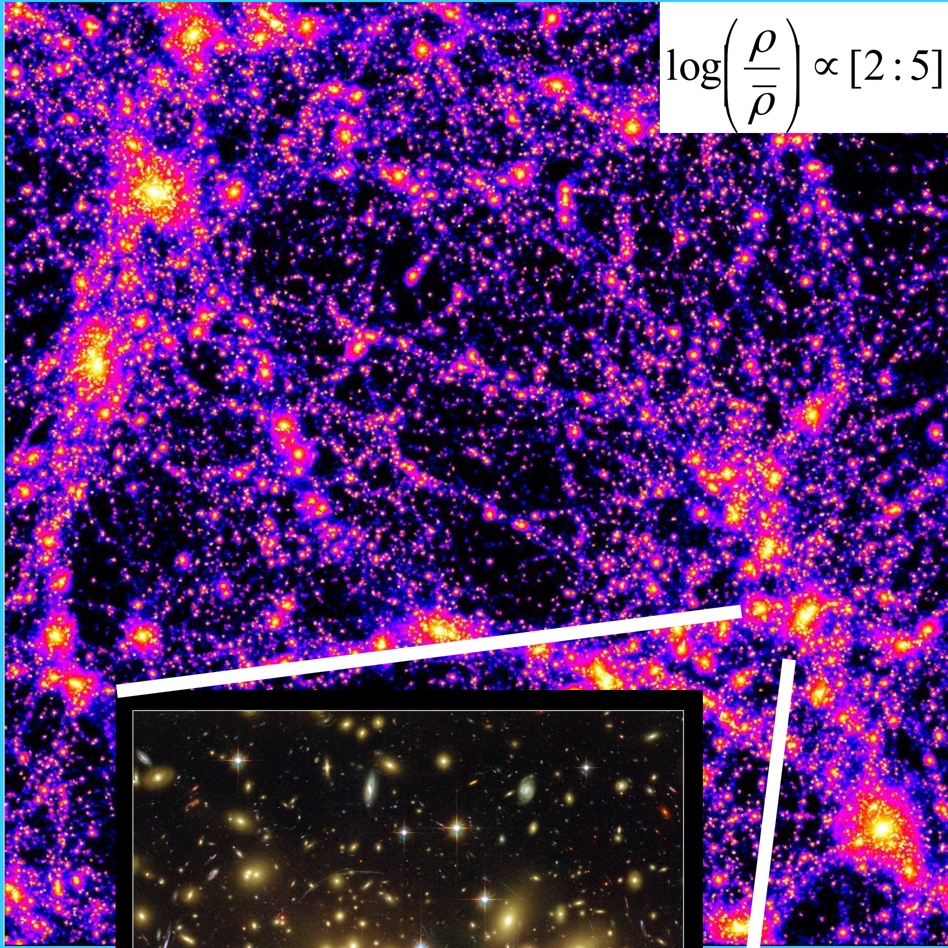
50 Mpc – 300^3 part

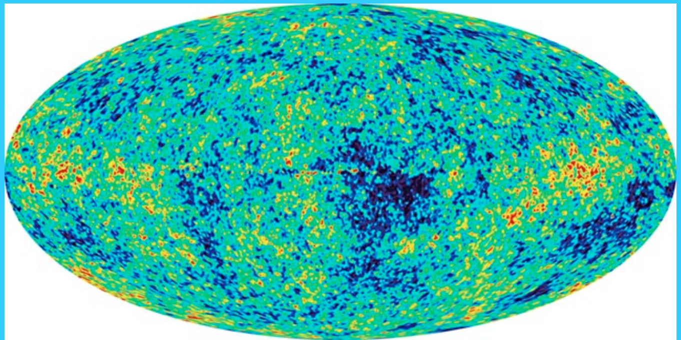
z=25

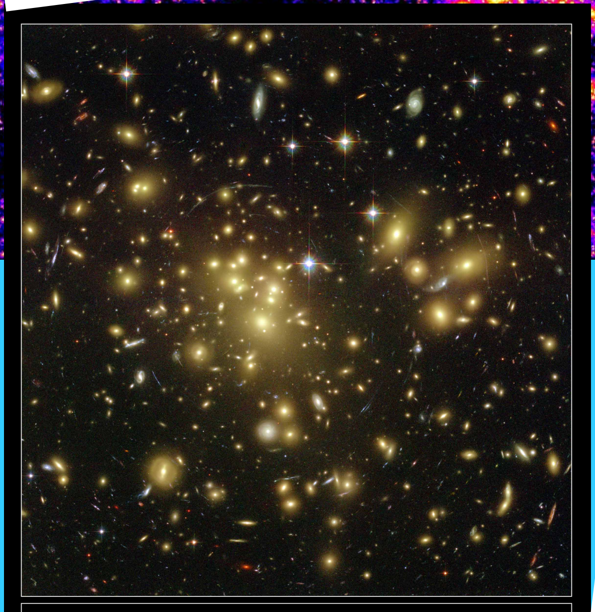
Maccio'+06,07

z=0









INITIAL CONDITIONS

- Inflation: structures started to grow from an initially Gaussian stocastic field of density fluctuations
- The properties of a Gaussian Random Field can be completely specified by its power spectrum, whose shape is theoretically well motivated and depends on cosmological parameters and on the nature of DM

$$P(k) = \langle |\delta_k|^2 \rangle = Ak^n T(k)^2$$

- The overdensity, δ (Gaussian random process) at location x and time t is

$$\delta(x,t) = \frac{\rho(x,t) - \overline{\rho}(t)}{\overline{\rho}(t)} = \sum_k \hat{\delta}(k,t)\, e^{ikx}$$

(expansion in Fourier components)

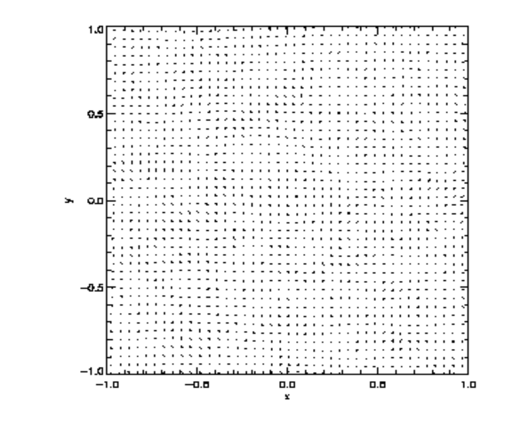
- First Step: Generating a Gaussian random field: Easiest to generate realisations in Fourier (or k-) space
- Perturbations are distributed according to a Gaussian distribution, so phases are drown from a uniform distribution, and the amplitude of a particular mode (i.e. a given k) is drawn from a distribution of the form,

$$f(\delta_k) = \frac{1}{2\pi P(k)} e^{-|\delta_k|^2 / 2P(k)}$$

- In other words, in order to generate the initial conditions, it is necessary to generate a set of complex numbers with a phase φ distributed in an aleatory fashion and with and amplitude Normally distributed and with a variance determined from the spectrum (e.g., Bardeen et al. 1986). This can be obtained choosing two aleatory numbers φ in] 0, 1] and A in] 0, 1] for each point in the k-space

$$\hat{\delta}_{\mathbf{k}} = \sqrt{-2P(|\mathbf{k}|)\ln(A)}e^{i2\pi\phi}.$$

- **Second step**: Generate a particle distribution -- either a grid or a glass



- **Third step**: Use the Zel'dovich approximation to initialise particle displacements and velocities

$$
\begin{aligned}
r(q,t) &= a(t)\left[q + b(t)S(q)\right] \\
S(q) &= \nabla \phi_0(q) \\
\phi_0(q) &= \sum_k a_k \cos(kq) + b_k \sin(kq)
\end{aligned}
$$

- The displacement vector S is related to the velocity potential Φ_0 and the power spectrum of fluctuations P(k)

- a and b are gaussian random numbers with the mean zero and dispersion $\sigma^2 = P(k)/k^4$

$$
a_k, b_k = \sqrt{P(|k|)}\,\frac{Gauss(0,1)}{|k|^2}
$$

INITIAL CONDITIONS

- Observations of galaxies on large scale and of the CMB lead to think that structures started to grow from an initially Gaussian stocastic field of density fluctuations

- Because of the Gaussianity, the field is completely described by the power spectrum, whose shape is theoretically well motivated and depends on cosmological parameters and on the nature of DM

$$P(k) = Ak^n T^2(k,z)$$

- In order to generate the initial conditions, it is necessary to generate a set of complex numbers with a phase φ distributed in an aleatory fashion and with and amplitude Normally distributed and with a variance determined from the spectrum (e.g., Bardeen et al. 1986). This can be obtained choosing two aleatory numbers φ in] 0, 1] and A in] 0, 1] for each point in the k-space

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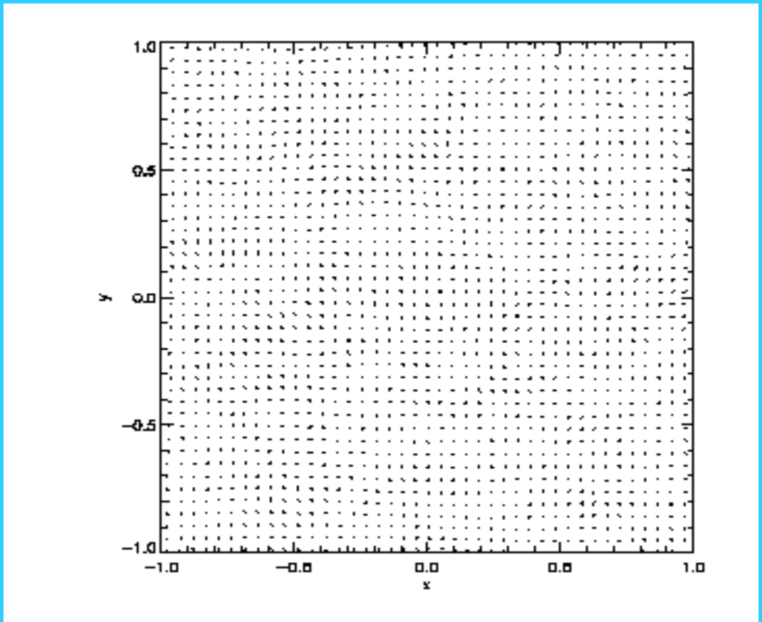


In order to obtain the perturbation field, generated from the quoted distribution, it is necessary to generate the potential Φ (q) (originated by the initially linear fluctuations) in a grid q in the real space through a Fourier transform, like

$$\Phi(\mathbf{q}) = \sum_k \frac{\hat{\delta}_\mathbf{k}}{\mathbf{k}^2} \mathrm{e}^{i\mathbf{kq}}.$$

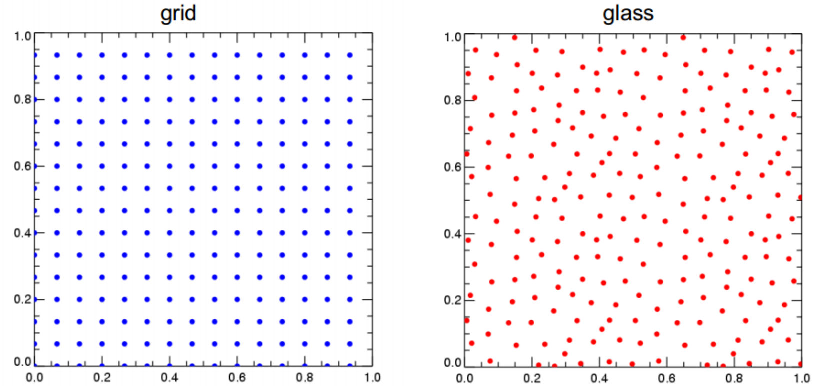
The posterior application of the Zel'dovich (Zel'dovich 1970) approximation allows to find the inicial positions and velocities

$$\mathbf{v} = \dot{D}^+(z)\nabla\Phi(\mathbf{q})$$

$$\mathbf{x} = \mathbf{q} - D^+(z)\Phi(\mathbf{q})$$

To create a realization of the perturbation spectrum, a model for an unperturbed density field is needed

GLASS OR CARTESION GRID



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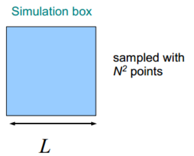

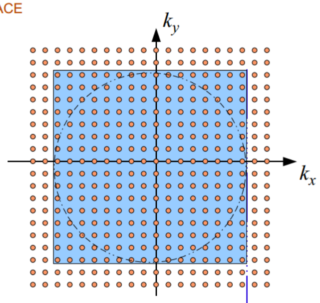

$$\delta_{\mathbf{k}} = B_{\mathbf{k}} \exp^{\mathrm{i}\,\phi_{\mathbf{k}}}$$

For each mode, draw a random phase, and an amplitude from a Rayleigh distribution.

$$\left\langle \delta_{\mathbf{k}}^2 \right\rangle = P(k)$$

$$k_{\rm Nyquist} = \frac{2\pi}{L}\,\frac{N}{2}$$

Using the Zeldovich approximation, the density fluctuations are converted to displacements of the unperturbed particle load

SETTING INITIAL DISPLACEMENTS AND VELOCITIES

Zeldovich's Lagrangian version of linear theory implies:

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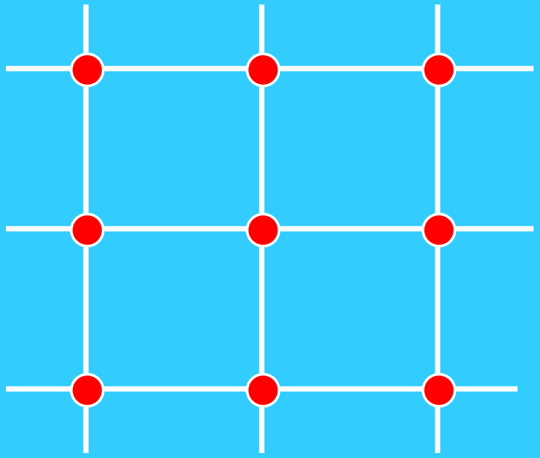
T(k,z) provided by linear theory

Then we should obtain a realization of this P(k) using particles:

Zel'dovich Approximation



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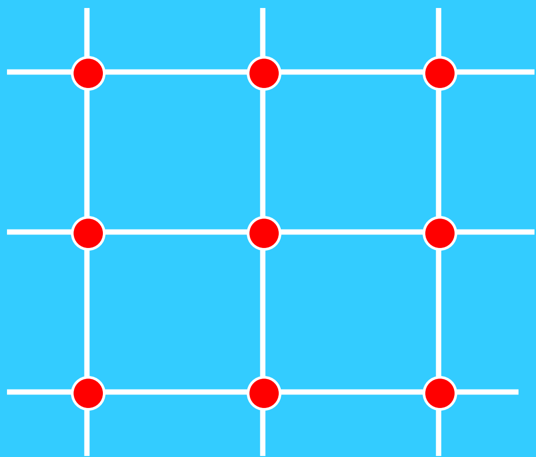
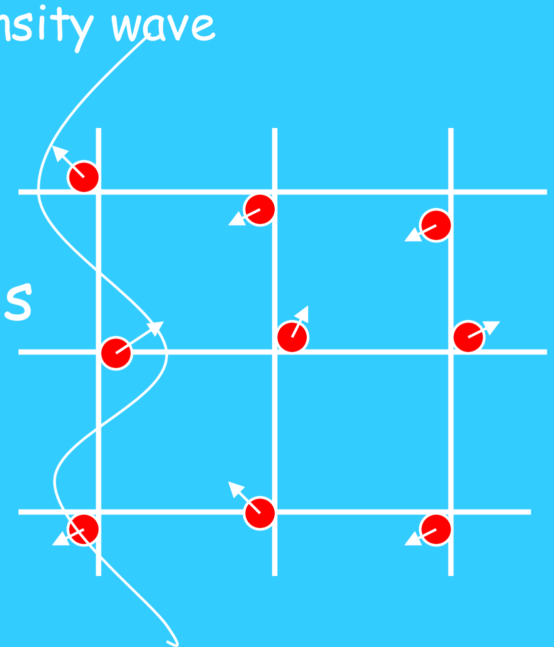
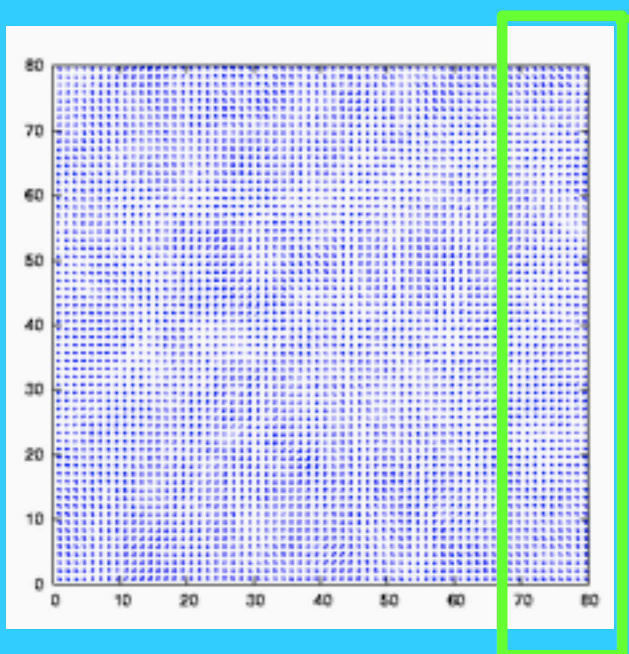
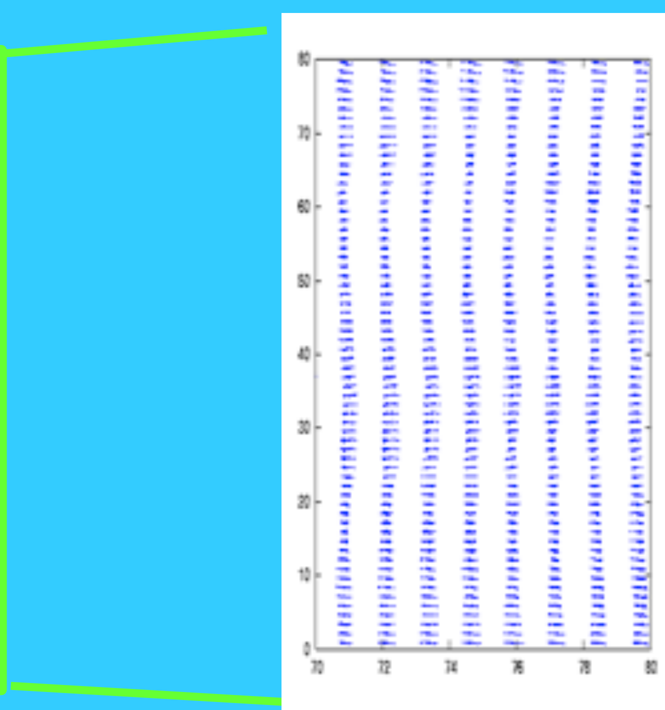
Zeldovich

Velocities and Positions
are linked together









FREEZE OUT: MORE QUANTITATIVE

- **The Boltzmann equation:**

L [f]=C [f] ⟹ $$\frac{dn_X}{dt} + 3Hn_X = -<\sigma_{X\bar{X}}|v|>(n_X^2 - n_{X,\mathrm{eq}}^2)$$

Dilution from expansion

$\chi\chi \rightarrow f\bar{f}$

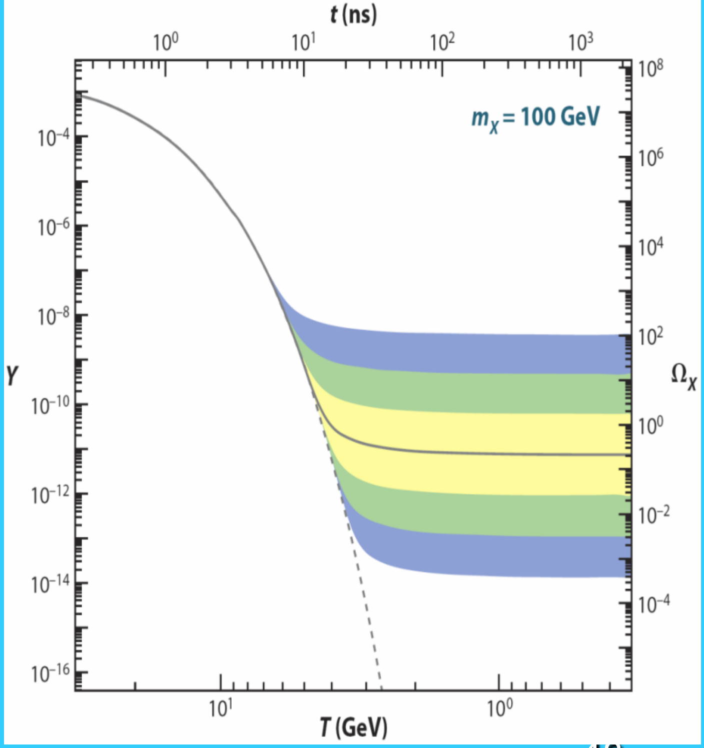
$f\bar{f} \rightarrow \chi\chi$

$<\sigma_{X\bar{X}}|v|>$ Thermally averaged anihilation cross section

- $T >> m_x$ ($\Gamma > H$) $n \approx n_{eq}$ until interaction rate drops below expansion rate **$T << mx$ ($\Gamma < H$)** :

$$n_{\mathrm{eq}}\langle\sigma v\rangle \sim H$$

$(mT)^{3/2}e^{-m/T}$ m^{-2} T^2/M_{Pl}



•$T << mx$ ($\Gamma < H$) equilibrium density small: $3Hn_x$ and $<\sigma_{X\bar{X}}|v|> n_X^2$ deplete number density

• For sufficiently small n_x annihilation insignificant with respect dilution due to expansion -> Freez-out

- Might expect freeze out at T ~ m, but the universe expands *slowly*! First guess: $m/T \sim \ln(M_{Pl}/m_W) \sim 40$

Resulting (relic) density today:

$$\Omega_X h^2 \approx \frac{1.04\times 10^9\,\mathrm{GeV}^{-1}}{M_{\rm Pl}} \frac{x_{\rm FO}}{g_\star^{1/2}(a+3b/x_{\rm FO})}$$

~ x_{Fo} / $<\sigma v>$

$$x_{\rm FO} \equiv \frac{m_X}{T_{\rm FO}} \approx \ln\left[c(c+2)\sqrt{\frac{45}{8}}\frac{g_X}{2\pi^3}\frac{m_X M_{\rm Pl}(a+6b/x_{\rm FO})}{g_\star^{1/2} x_{\rm FO}^{1/2}}\right]$$

$c \sim 0.5$ $\quad < \sigma_{X\bar{X}}|v| >= a + b < v^2 > + \mathcal{O}(v^4)$

Non-relativistic expansion for heavy states

$g_\star$ number external degrees of freedom =65, 1 GeV
=120, 1 TeV
in SM

If X has a GeV-TeV scale mass and a roughly weak-scale annihilation cross section, freeze-out occurs at $x_{\rm FO} \approx 20-30$, resulting in a relic abundance of

$$\Omega_X h^2 \approx 0.1 \left(\frac{x_{\rm FO}}{20}\right)\left(\frac{g_\star}{80}\right)^{-1/2}\left(\frac{a+3b/x_{\rm FO}}{3\times 10^{-26}\mathrm{cm}^3/\mathrm{s}}\right)^{-1}$$

In case of presence of other species

$$\frac{dn}{dt} = -3Hn - \langle\sigma_{\rm eff} v\rangle\left(n^2 - n_{\rm eq}^2\right) \qquad \langle\sigma_{\rm eff} v\rangle = \sum_{ij}\langle\sigma_{ij} v_{ij}\rangle \frac{n_i^{\rm eq}}{n^{\rm eq}}\frac{n_j^{\rm eq}}{n^{\rm eq}} \qquad \langle\sigma_{\rm eff} v\rangle = \frac{\int_0^\infty dp_{\rm eff}\, p_{\rm eff}^2 W_{\rm eff} K_1\left(\frac{\sqrt{s}}{T}\right)}{m_1^4 T\left[\sum_i \frac{g_i}{g_1}\frac{m_i^2}{m_1^2}K_2\left(\frac{m_i}{T}\right)\right]^2}$$

$$W_{\rm eff} = \sum_{ij}\frac{p_{ij}}{p_{11}}\frac{g_i g_j}{g_1^2}W_{ij} = \sum_{ij}\sqrt{\frac{[s-(m_i-m_j)^2][s-(m_i+m_j)^2]}{s(s-4m_1^2)}}\frac{g_i g_j}{g_1^2}W_{ij},$$

where $W_{ij} = 4E_iE_j\sigma_{ij}v_{ij}$ and p_{ij} is the momentum of the particle X_i (or X_j) in the center-of-mass frame of the pair X_iX_j, and $s = m_i^2 + m_j^2 + 2E_iE_j - 2|\mathbf{p}_i||\mathbf{p}_j|\cos\theta$, with the usual meaning of the symbols.

i,j=1 -> WIMP

Numerical coincidence? Or an indication that dark matter originates from EW physics?

- For a particle with a GeV-TeV mass, to obtain a thermal abundance equal to the observed dark matter density, we need an annihilation cross section of $<\sigma v>$ ~ pb

WIMP MIRACLE

- Generic weak interaction yields:

$<\sigma v>$ ~ α^2 (100 GeV)$^{-2}$ ~ pb

- 1998: Universe acceleration

$\Longrightarrow$ Thousands of work in Modified Gravity

(f(R), Gauss-Bonnet, Lovelock, nonminimal scalar coupling, nonminimal derivative coupling, Galileons, Hordenski etc)

[Copeland, Sami, Tsujikawa Int.J.Mod.Phys.D15], [Nojiri, Odintsov Int.J.Geom.Meth.Mod.Phys. 4]

- Almost all in the curvature-based formulation of gravity

*

C. New Physics ?

WDM (**disperson velocity (now) 100 m/s; reduce the small scale power, fewer low mass halos, all halos have less steep inner profile)**

Self-interacting DM (SIDM) (QCD interaction but no EM. Scattering strips the halos from small clumps of dark matter orbiting larger structures, making them vulnerable to tidal stripping and reducing their number. Q-BALLS)

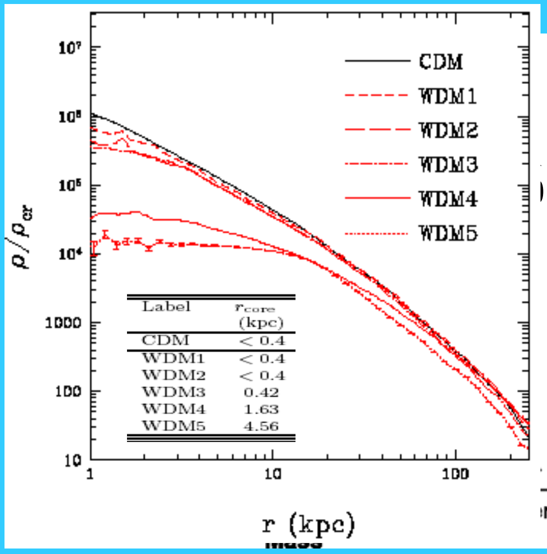
Repulsive Dark matter (RDM)

Fuzzy DM (FDM) **(ultra-light scalar particles,** similar to axion dark matter models)

Decaying DM (DDM)

Self-Annihilating DM (SADM)

Modified gravity (F(R); F(T); …MOND)



D. **Possible Solutions in ΛCDM** ("heating" of dark matter, BARYONIC SOLUTIONS)

Rotating bar

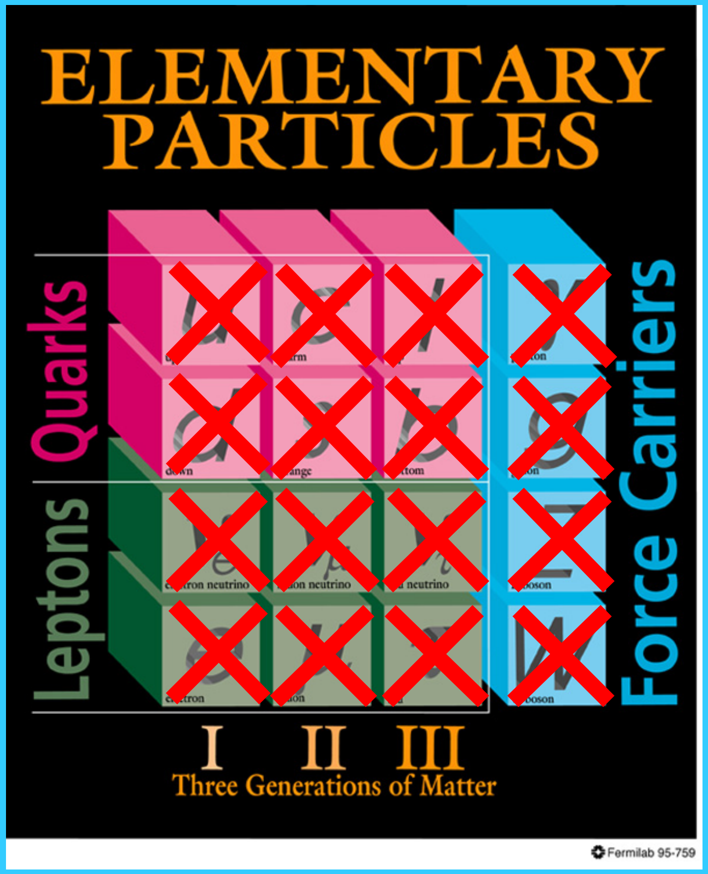
Passive evolution of cold lumps (El Zant et al., 2001)

AGN, Spernovas (Governato et al.. 2010)

"Maximal stellar feedback"/"blowout"

A 1 pc core requires a 0.1 keV thermal candidate. Candidates satisfying large scale structure constrains (m_ν larger than 1-2 keV) the expected size of the core is of the order of 10 (20) pc

First place to look for candidates: SM



Desired DM properties

- Gravitationally interacting
- Not short-lived
- Not hot
- Not baryonic

Unambiguous evidence for new particles

Newtonian Gravity

$$\vec{a} = -\vec{\nabla}\Phi$$

$$\nabla^2\Phi = 4\pi G\rho_B$$

Geodesic

$$a^\mu + \Gamma^\mu_{\alpha\beta}v^\alpha v^\beta = 0$$

Einstein-Hilbert

$$\frac{1}{16\pi G}\int d^4x\sqrt{-g}R(g)$$

$$g_{\alpha\beta}$$

Spacetime metric

How to get a MG theory

Simplest approach: modify the Einstein-Hilbert action

$$\frac{1}{16\pi G}\int d^4x\sqrt{-g}\mathcal{F}(R, R_{\alpha\beta}, R_{\alpha\beta\mu\nu}, \dots)$$

OR

Keep Einstein Hilbert action and modify the metrics $g_{\mu\nu}$

Example

Use two different metrics (bimetric theories)

"Physical" in Geodesic equations

"Geometric" metric in Einstein equations

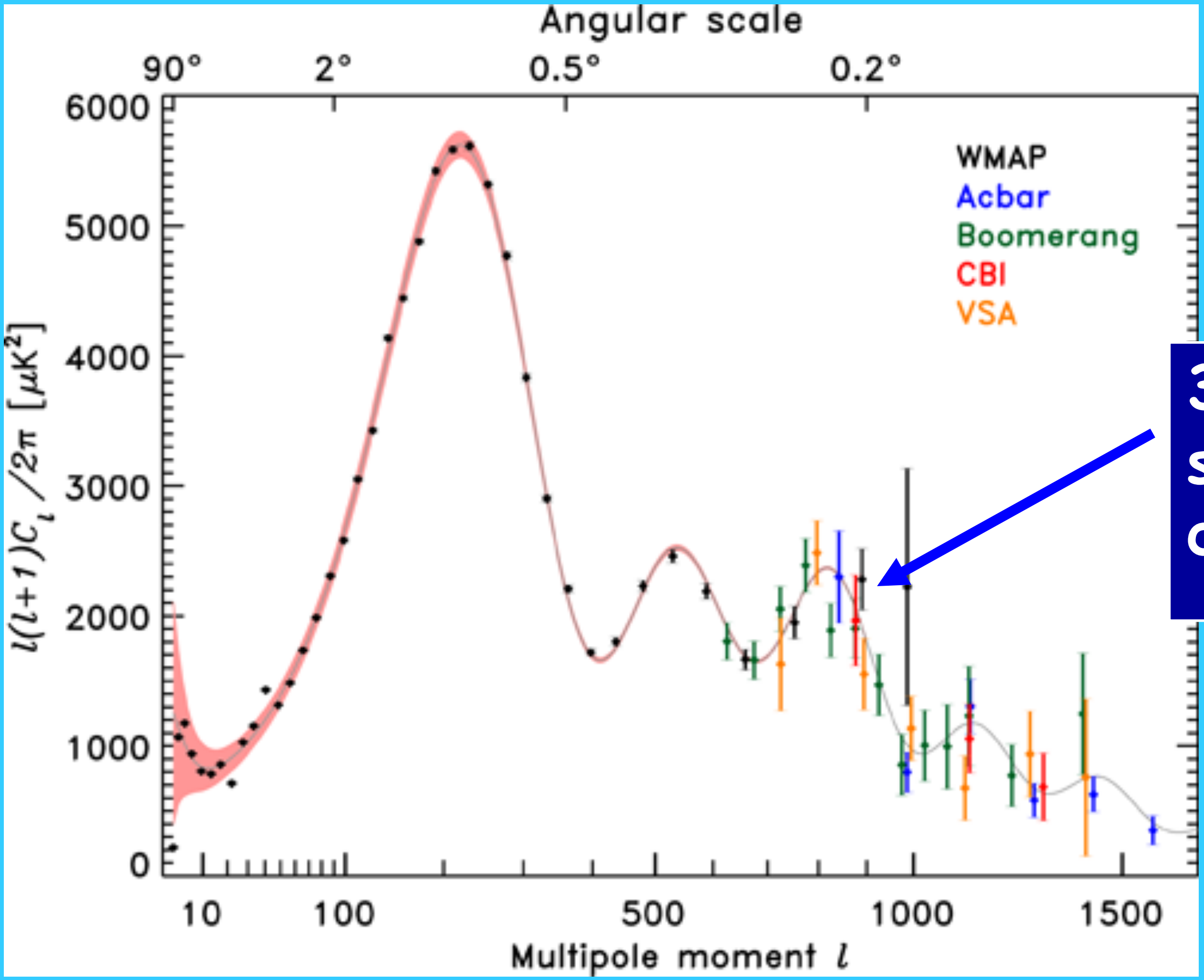
$$g_{\mu\nu} = e^{-2\phi}\tilde{g}_{\mu\nu}$$

Dirac, Jordan, Brans, Dicke

How can we test these theories?

And how generic are these tests?

The Cosmic Microwave Background



3rd Peak is sensitive do dark matter

Remember: extra degrees of freedom drive gravity

$$\nabla^2\Phi = 4\pi G\delta\rho_B + \mathcal{S}_1(\alpha,\Phi,\Psi)$$

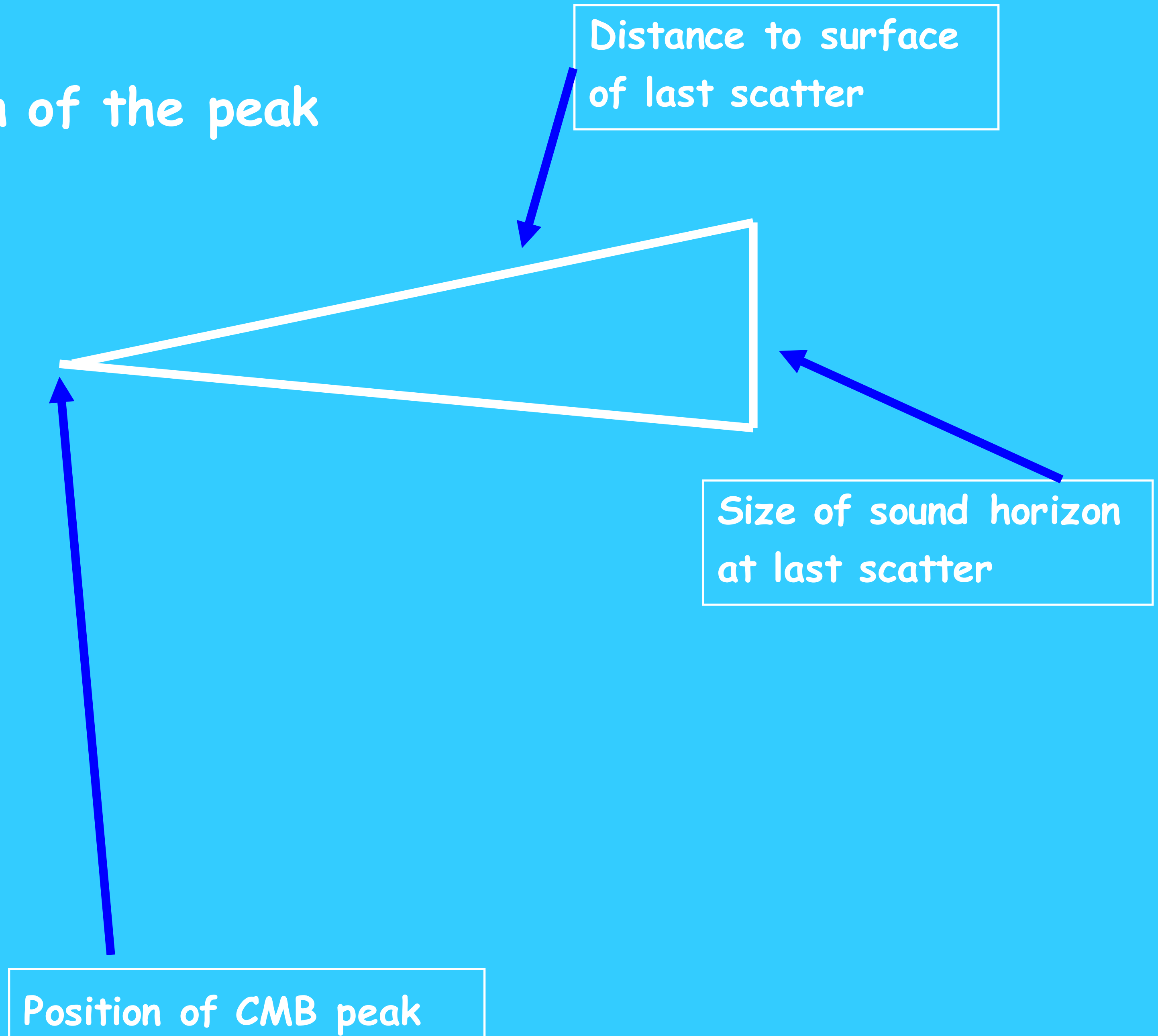
Pushes up peaks

but …

Boosts large scales as well

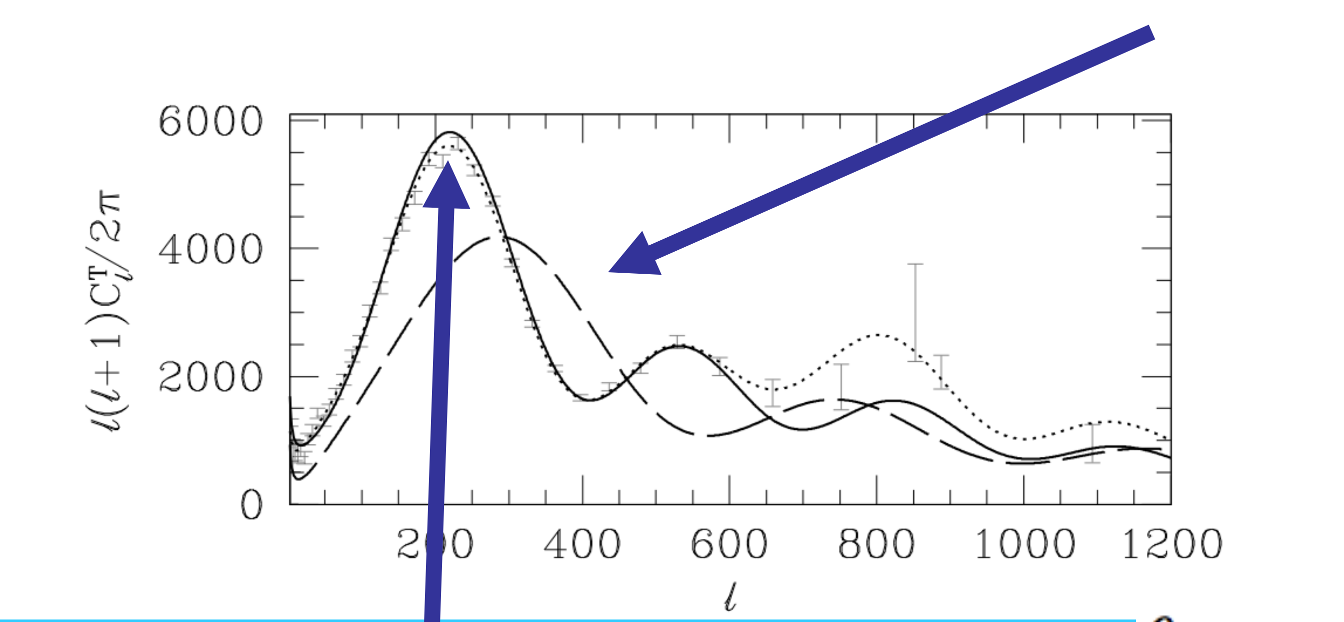
$$\nabla^2(\Phi - \Psi) = \mathcal{S}_2(\alpha,\Phi,\Psi)$$

The position of the peak



Position of CMB peak is very well constrained:

$\Omega_\Lambda = 0.95 \qquad \Omega_B = 0.05$



$$\theta = \frac{D_{ls}}{D_0[\Omega_B, \Omega_\Lambda, \cdots]}$$

Need $m_\nu = 2.2\ eV$
to fit peaks …

… not generically true (Ferreira et al)

Is there a smoking gun for modified gravity ?

Analogy with PPN approach:

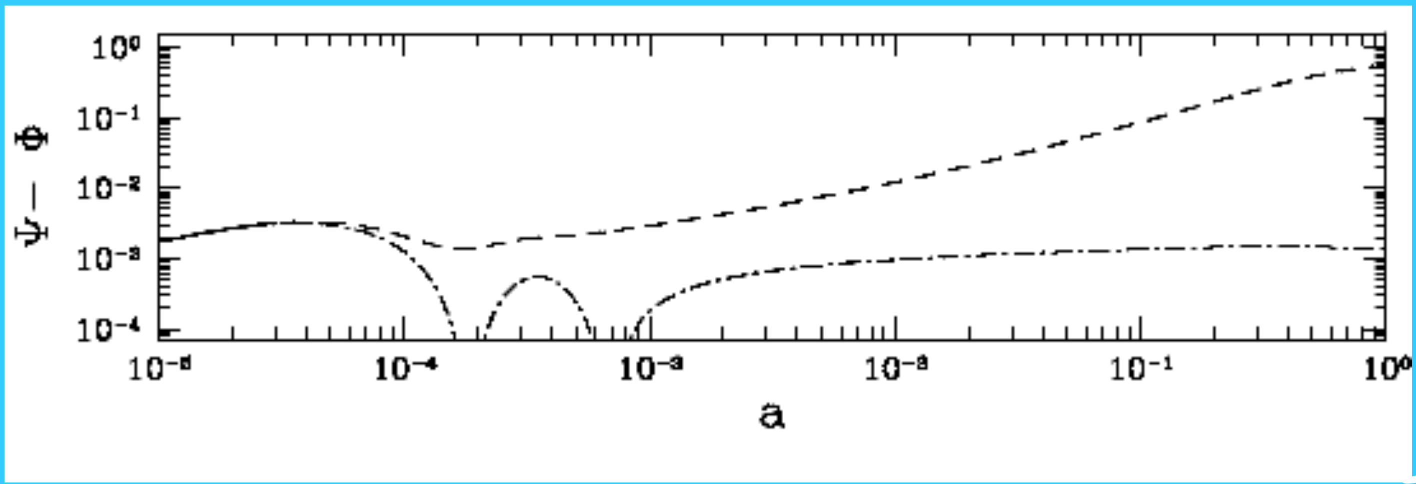
$$ds^2 = -(1 - \frac{2GM}{r})dt^2 + (1 - \frac{2\gamma GM}{r})dr^2 + r^2 d\Omega^2$$

$$\Phi \neq \Psi$$

γ : deviations from Einstein

$$|1-\gamma| \sim \begin{cases} 10^{-4} & \text{Solar System} \\ 10^{-2} & \text{Galaxies} \\ 10^{-1} & \text{Clusters} \end{cases}$$

$\Phi - \Psi \neq 0$ on cosmological scales



TeVeS

Liguori et al

And changes the growth of structure

Conclusions:

- Does Einstein/Newton gravity break down in regions of very weak field?
- Is there a consistent theory for this deviation from standard gravity?
- How can we test it, i.e. how can we distinguish it from dark matter models?

- There are now a range of relativistic theories:
  - All have some problems
  - All have extra degrees of freedom (just like dark matter)

- Tests should be chosen wisely to disentangle extra degrees of freedom, modified gravity and non-"Birkoffianess"

- More generally, searches for deviations in the PPN-like parameter, on cosmological scales.

- A real opportunity with future surveys: satellites (Euclid, JDEM), groundbased (SKA, LSST, ...).