Universidade Federal do Espirito Santo

The Curvature parameter and its implications in cosmological models

Alan Miguel Velásquez Toribio
Núcleo-cosmo ufes
Departamento de Física
Centro de Ciências Exatas

Summary

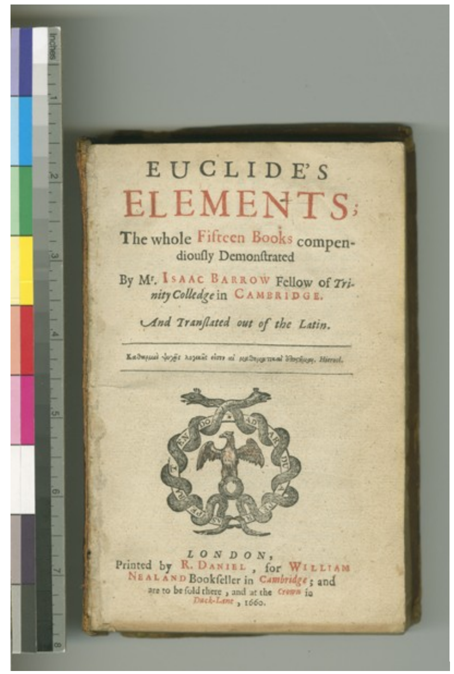
- On the history of curvature
- The curvature tension?
- Observational constraints for non-flat LCDM.
- Observational constraints in different models.
- Some conclusion

On the history of curvature

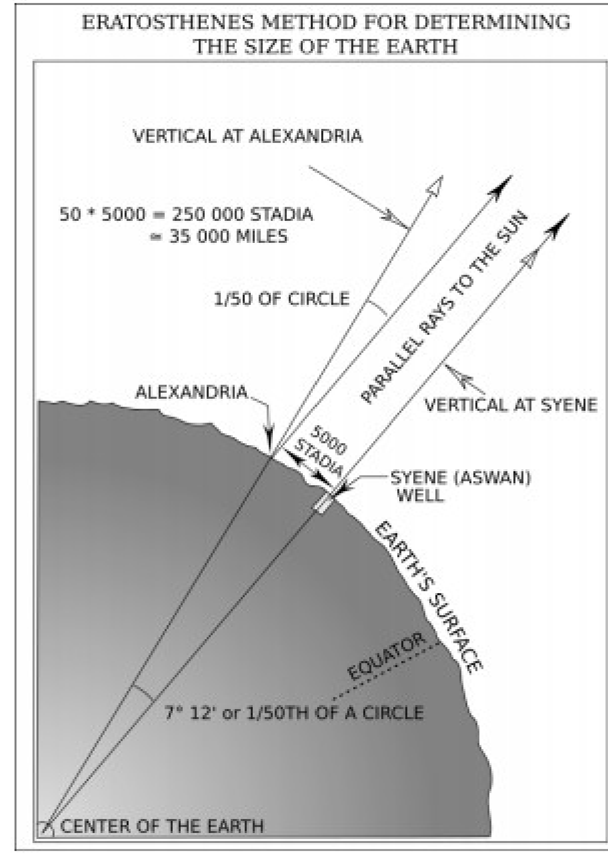
- Euclidean geometry 300 B.C
- the fifth postulate of euclid:

  A straight line falling on parallel straight lines makes the alternate angles equal to one another, the exterior angle equal to the interior and opposite angle, and the interior angles on the same side equal to two right angles.

- Proclus V A.D, Nasiraddin XIII, John Wallis XVII, Saccheri XVIII, etc.
- Eratosthenes of Cyrene II B.C., Greek scientific writer, astronomer, and poet, who made the first measurement of the size of Earth for which any details are known.



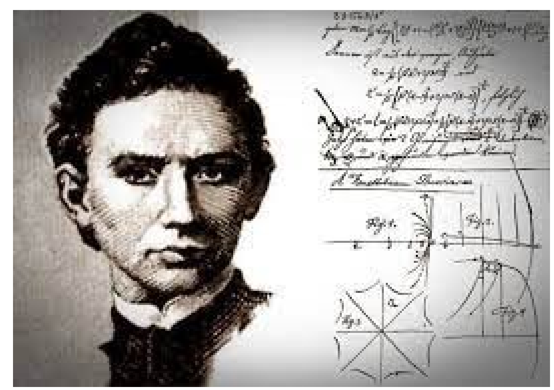
This is the title page of Isaac Barrow’s (1630-1677) Euclide's Elements, 1660 edition

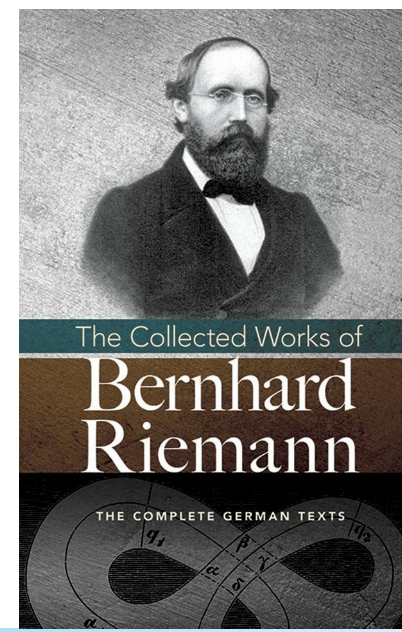


Universe in the Classroom No. 91 • Spring 2016

On the history of curvature

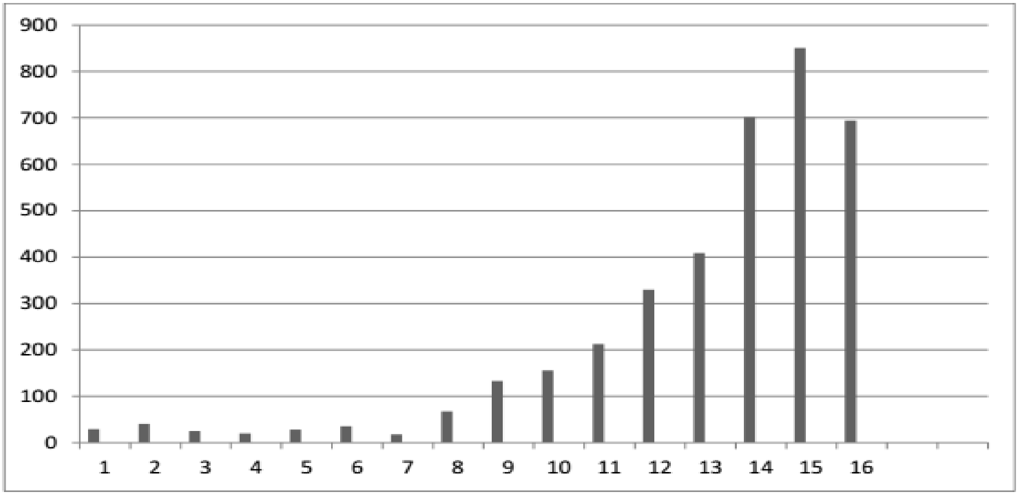
- C. F. Gauss did not want to get involved in philosophical problems.
- Kant in the “Critique of Pure Reason” had established Euclidean geometry as an a priori synthetic concept.
- Bolyai publishes in 1832 a hyperbolic geometry and Gauss comment on his unpublished discoveries
- Lobatchevski 1929 publishes a hyperbolic geometry and a book in German in 1837. Gauss appointed Lobatchevski to the Gottingen Academy.
- In 1854 the young Göttingen mathematician and physicist Bernhard Riemann put the concept of curvature as an intrinsic property of space on a firmer basis.





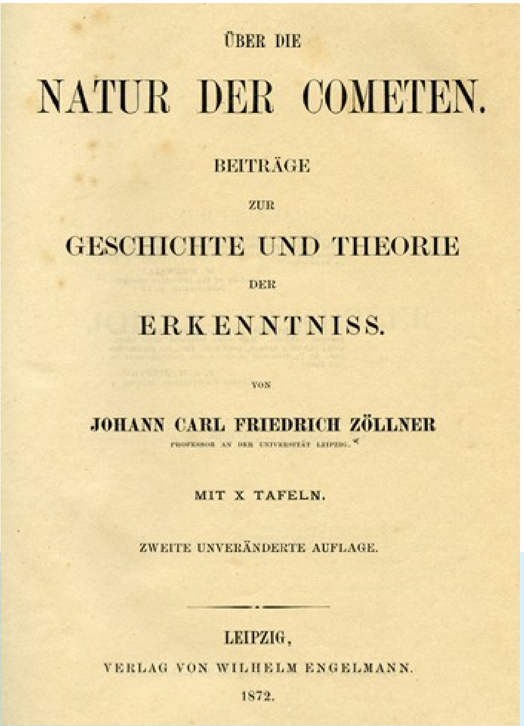
On the history of curvature

- Lobatchevski proposes to investigate a hyperbolic geometry using astronomical methods: astronomical parallaxes:
- He concluded that in a triangle whose vertices are the sun, the earth, and a fixed star (Sirius), the angle sum cannot differ from two right angles by more than 0.00000372". (Daniels, N. (1975). Isis, 66(1), 75–85. doi:10.1086/351377)
- William Kingdon Clifford translated the lecture of Riemann into English in 1873.
- Popular lectures by Hermann von Helmholtz on Riemann em 1867.



The diagram shows the number of titles per 5-year interval in Sommerville's 1911 bibliography on non-Euclidean geometry, starting with 1830-1834 and ending with 1905-1909. Of the 4,016 titles listed in the bibliography, by far most were written in German (1,149 or 28.6%), French (884 or 22.0%), Italian (848 or 21.1%), and English (723 or 18.0%).

Kragh, Helge, arXiv:1205.4909



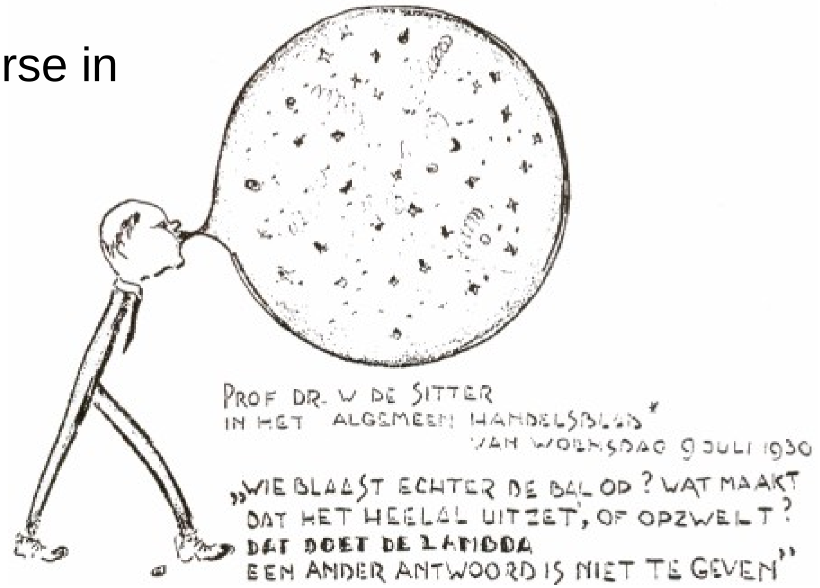
On the history of curvature

- General Relativity: Einstein 1915

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Einstein Universe 1917, represented a finite three-dimensional spatial hyperspheric surface of constant radius r embedded in 4D spacetime.

De Sitter’s model 1917: A model of an expanding universe in which there is no matter or radiation but the expansion is driven by a cosmological constant.



Friedmann 1922: the general theory of relativity (GR) admits nonstatic solutions: It can, Friedmann found, describe a cosmos that expands, contracts, collapses, and might even have been born in a singularity (Ari Belenkiy).

Georges Lemaître 1927 include cosmological constant

Observational evidence for the curvature

Observational constraints...

A. Einstein and W. de Sitter

"...and we must conclude that at the present time it is possible to represent the facts without assuming a curvature of three dimensional space. The curvature is, however, essentially determinable, and an increase in the precision of the data derived from observations will enable us in the future to fix its sign and to determine its value".

PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES

Volume 18 March 15, 1932 Number 3

ON THE RELATION BETWEEN THE EXPANSION AND THE MEAN DENSITY OF THE UNIVERSE

By A. Einstein and W. de Sitter

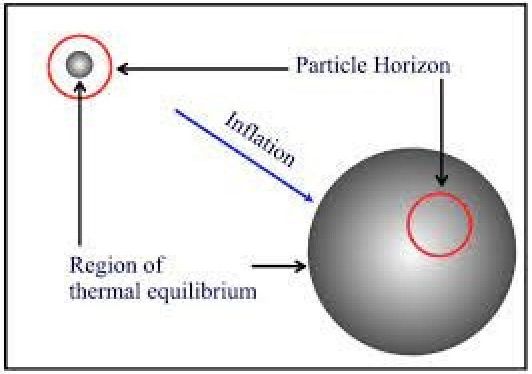
Communicated by the Mount Wilson Observatory, January 25, 1932

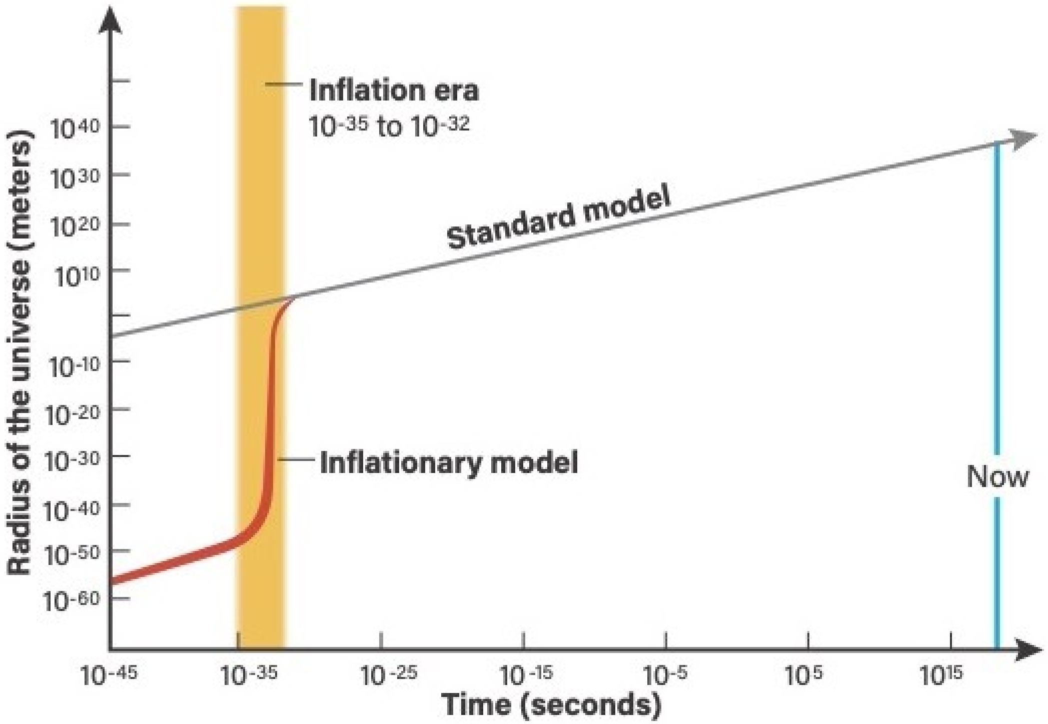
In a recent note in the *Göttinger Nachrichten*, Dr. O. Heckmann has pointed out that the non-static solutions of the field equations of the general theory of relativity with constant density do not necessarily imply a positive curvature of three-dimensional space, but that this curvature may also be negative or zero.

There is no direct observational evidence for the curvature, the only directly observed data being the mean density and the expansion, which latter proves that the actual universe corresponds to the non-statical

Curvature and inflation

Inflationary model:





Magazine Astronomy, 2021

A flat universe is a key piece of the standard cosmological model

Curvature tension?

- Di Valentino E., Melchiorri A., Silk J., Nature Astronomy, 484, (2019)
- Park C.-G., Ratra B., ApJ, 882, 158, (2019)
- Handley W., arXiv:1908.09139, Phys. Rev. D 103, 041301 (2021)

But observational constraints of COBE and BOOMERANG have confirmed that the universe is consistent with a flat cosmology to with $\longrightarrow$

$$\Omega_K = \pm 10\%$$

In all cases the articles show evidence for a curvature parameter of a closed universe with more than 3.4σ

Explanation: statistical fluctuation, systematics, new physics or a combination.

Curvature tension?

- Two different methodologies:
  Plik or CamSpec
- BAO, SNIa, Planck

MNRAS **000**, 000–000 (0000) Preprint 18 February 2020 Compiled using MNRAS LaTeX style file v3.0

The evidence for a spatially flat Universe

George Efstathiou and Steven Gratton
Kavli Institute for Cosmology Cambridge and Institute of Astronomy, Madingley Road, Cambridge, CB3 OHA.

18 February 2020

ABSTRACT
We revisit the observational constraints on spatial curvature following recent claims that the *Planck* data favour a closed Universe. We use a new and statistically powerful *Planck* likelihood to show that the *Planck* temperature and polarization spectra are consistent with a spatially flat Universe, though because of a geometrical degeneracy cosmic microwave background anisotropy spectra on their own do not lead to tight constraints on the curvature density parameter Ω_K. When combined with other astrophysical data, particularly geometrical measurements of baryon acoustic oscillations, the Universe is constrained to be spatially flat to extremely high precision, with $\Omega_K = 0.0004 \pm 0.0018$ in agreement with the 2018 results of the *Planck* team. In the context of inflationary cosmology, the observations offer strong support for models of inflation with a large number of e-foldings and disfavour models of incomplete inflation.

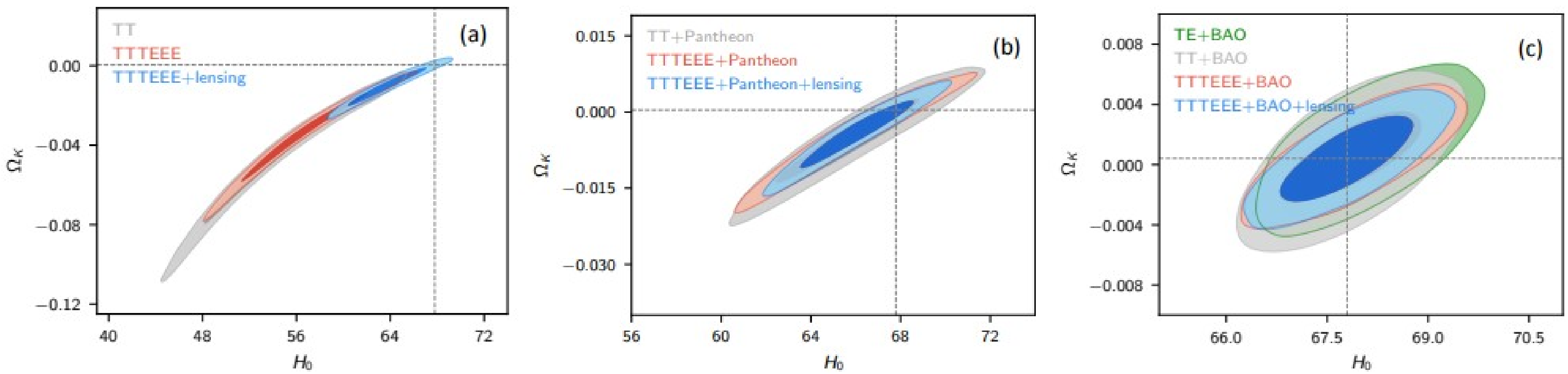


Figure 1. 68% and 95% contours on Ω_K and H_0 *assuming a flat prior in* Ω_K *over the range* $-0.3 < \Omega_K < 0.3$ for: (a) *Planck* data alone; (b) *Planck* data combined with the Pantheon supernova sample; (c) *Planck* data combined with BAO.

cosmological models with curvature

Non-flat LCDM

Model non-flat LCDM

$$H = H_0\sqrt{\Omega_{m0}(1+z)^3 + \Omega_{k0}(1+z)^2 + \Omega_{\Lambda 0}},$$

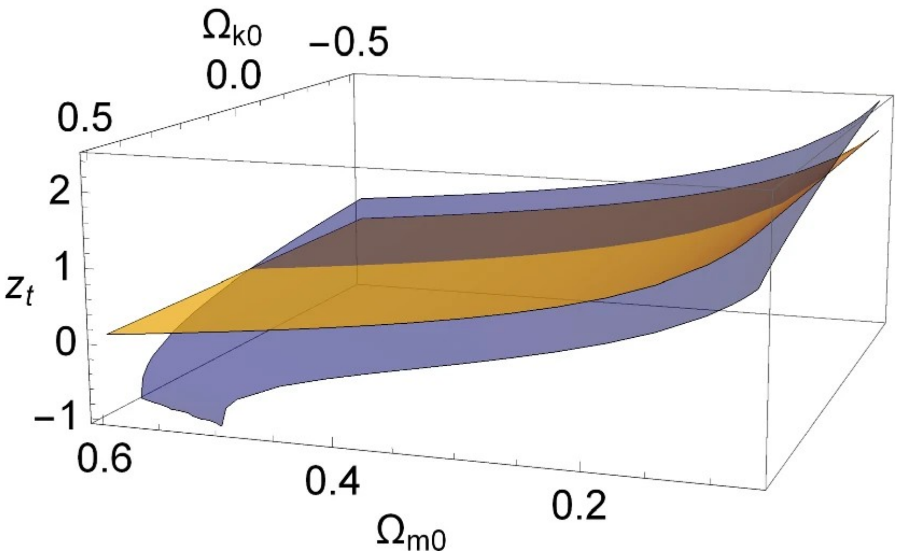
$$\Omega_{m0} = \frac{8\pi G\rho_{m0}}{3H_0^2},\ \Omega_{\Lambda 0} = \frac{\Lambda}{3H_0^2} \text{ and } \Omega_{k0} = \frac{-k}{a^2H^2}.$$

$$\Omega_{m0} + \Omega_{\Lambda 0} + \Omega_{k0} = 1.$$

$$q(z) = -\frac{\ddot{a}}{aH^2} = \frac{d}{dt}\left(\frac{1}{H}\right) - 1.$$

$$z_t(\Omega_{m0}, \Omega_{k0}) = \left(\frac{2\Omega_{\Lambda 0}}{\Omega_{m0}}\right)^{1/3} - 1$$

$$= \left(\frac{2\,(1-\Omega_{m0}-\Omega_{k0})}{\Omega_{m0}}\right)^{1/3} - 1,$$



Velasquez-Toribio and Magnago, EPJ C, 562 (2020)

$$H = H_0\sqrt{\frac{(1-\Omega_{k0})(1+z)^3}{\frac{1}{2}(1+z_t)+1} + \Omega_{k0}(1+z)^2 + \frac{(1-\Omega_{k0})(1+z_t)^3}{2(\frac{1}{2}(1+z_t)+1)}}.$$

$$H = H_0\sqrt{\Omega_{m0}(1+z)^3 + (1-\Omega_{m0}(1+\frac{(1+z_t)^3}{2}))(1+z)^2 + \frac{\Omega_{m0}}{2}(1+z_t)^3}.$$

Non-flat LCDM

- Model non-flat LCDM

$$r(z) = \frac{c}{H_0}\int_0^z \frac{dz'}{E(z')}.$$

$$r_t = \begin{cases} \frac{c}{H_0}\frac{1}{\sqrt{\Omega_{k0}}}\sinh\left[\sqrt{\Omega_{k0}}\,\frac{H_0}{c}r(z)\right] & \text{for } \Omega_{k0} > 0 \\ r(z) & \text{for } \Omega_{k0} = 0 \\ \frac{c}{H_0}\frac{1}{\sqrt{|\Omega_{k0}|}}\sin\left[\sqrt{|\Omega_{k0}|}\,\frac{H_0}{c}r(z)\right] & \text{for } \Omega_{k0} < 0 \end{cases}$$

$$d_L = (1+z)r_t$$

Supernovae Ia called “Pantheon” sample

1048

$$\chi^2_{SNIa} = \Delta\boldsymbol{\mu}^T.\mathbf{C}^{-1}.\Delta\boldsymbol{\mu}, \qquad \Delta\boldsymbol{\mu} = \boldsymbol{\mu}_{obs} - \boldsymbol{\mu}_{model}$$

$$- M,$$

$$= M + 5Log_{10}(D_L) + 5Log_{10}(\frac{c/H_0}{1Mpc}) + 25$$

$$= \bar{M} + 25 + 5Log(D_L).$$

$$H(z) = -\frac{1}{1+z}\frac{dz}{dt}.$$

Cosmic chronometers

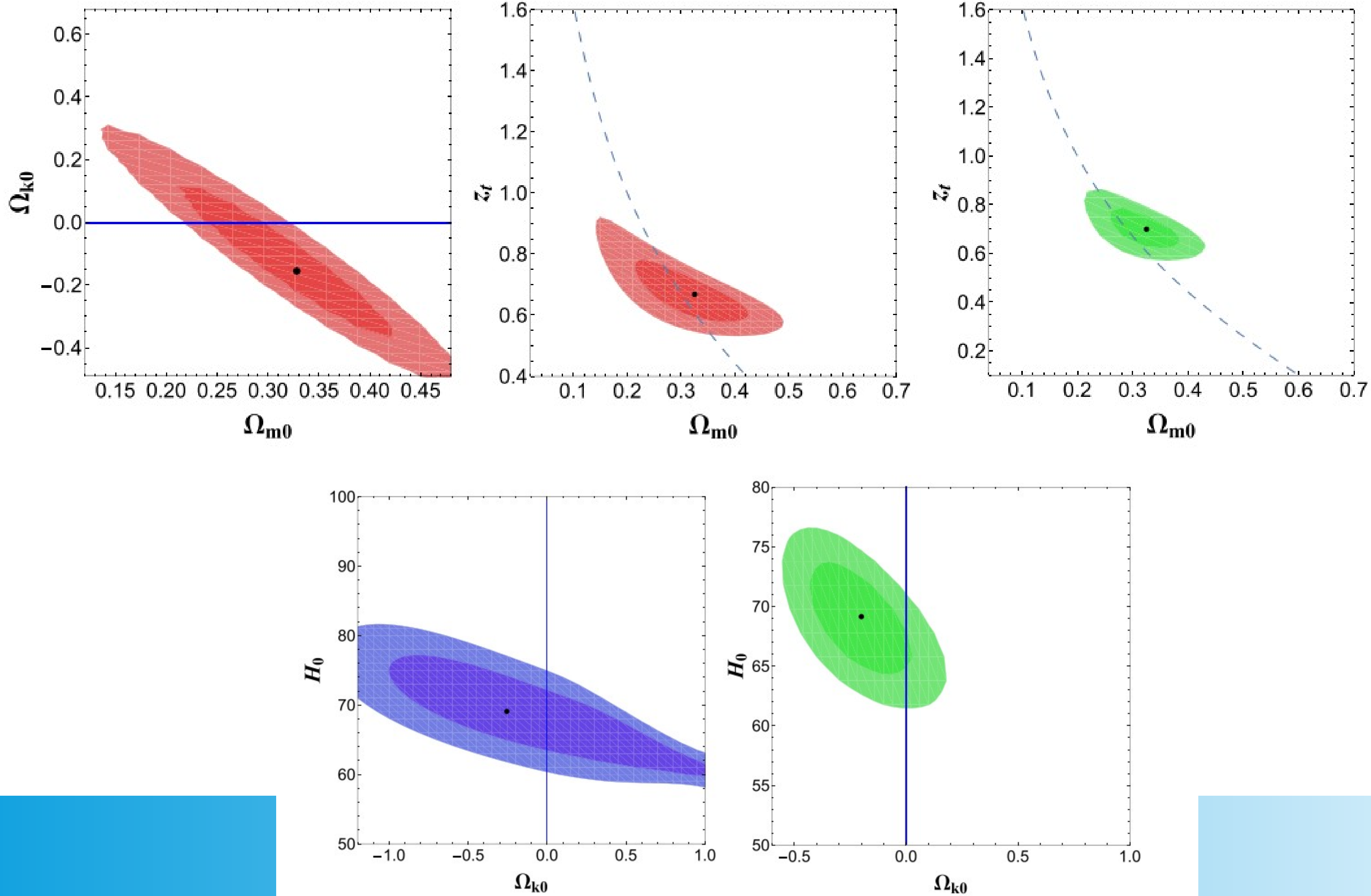
$$\chi^2_H(\Omega_{m0}, \Omega_\Lambda, H_0) = \sum_{i=1}^{34} \frac{(H_{obs,i} - H_{model}(z_i, \Omega_{m0}, \Omega_\Lambda, H_0))^2}{\sigma^2_{obs,i}},$$

Total chi square:

$$\chi^2_{total}(\Omega_{m0}, \Omega_{\Lambda 0}, H_0) = \chi^2_{SNIa,marg} + \chi^2_H.$$

Non-flat LCDM

- Model non-flat LCDM



Non-flat LCDM

- Model non-flat LCDM

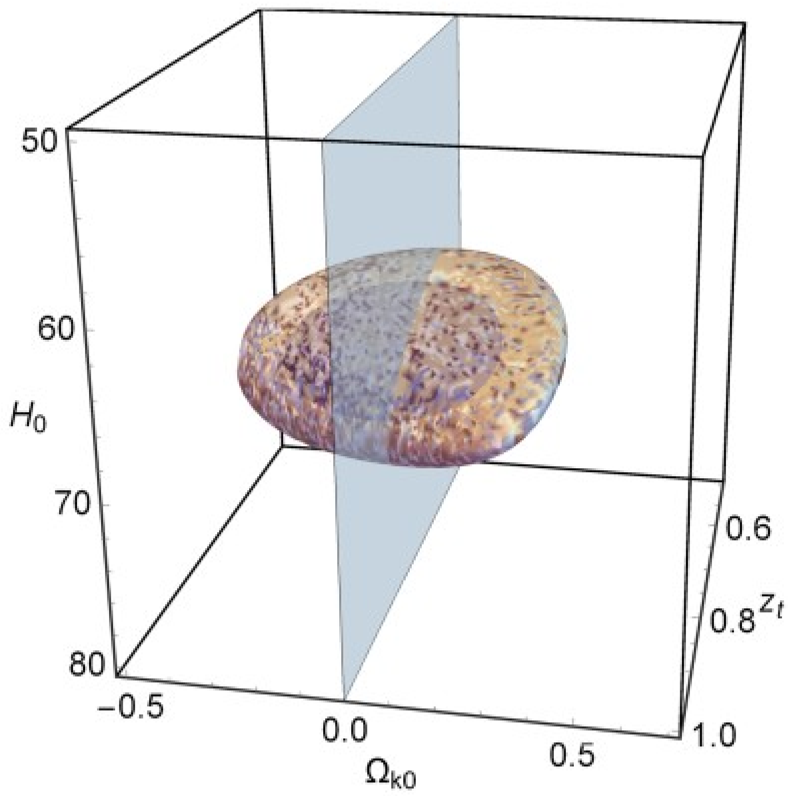


Table 1 Best-fitting parameters 1σ confidence intervals

Parameters	Best-fitting	Marginalization range
Ω_{m0}	0.325 ± 0.0750	$0.07 < \Omega_{m0} < 0.600$
H_0	70.06 ± 1.99	$50 < H_0 < 80$
z_t	0.69 ± 0.25	$0.400 < z_t < 1.00$
Ω_{k0}	-0.195 ± 0.210	$-0.50 < \Omega_{k0} < 0.50$

Non-flat LCDM

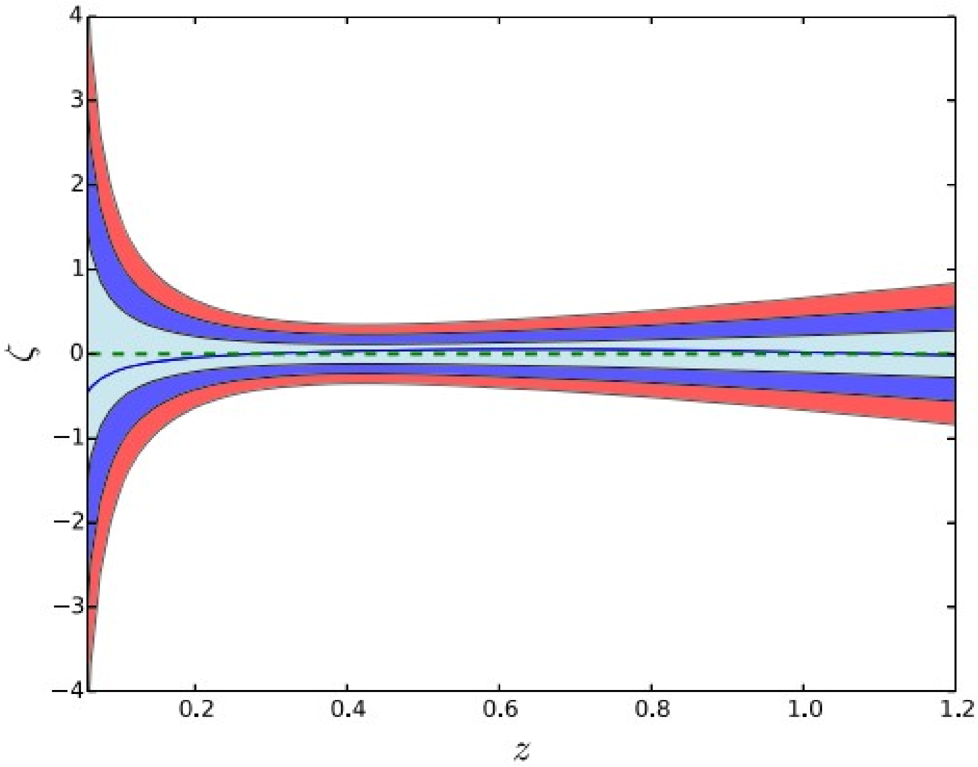
- Model non-flat LCDM

$$z_{t,non-flat} = \left(\frac{\left(\frac{H}{H_0}\right)^2 - \Omega_{m0}(1+z)^3 - (1-\Omega_{m0})(1+z)^2}{\Omega_{m0}\left(1-(1+z)^2\right)} \right)^{1/3} - 1,$$

$$\zeta = z_{t,flat} - z_{t,nonflat},$$

$\zeta = 0$ ► The flat model is preferred

$$\sigma_\zeta = \sqrt{\frac{\partial \zeta}{\partial H}\delta H + \frac{\partial \zeta}{\partial H_0}\delta H_0 + \frac{\partial \zeta}{\partial \Omega_{m0}}\delta \Omega_{m0}}.$$



We use the method of Gaussian processes

Running Cosmological constant and warm dark matter with curvature

- Model:

The mathematical description of the model is based on GR, with a non-zero spatial curvature, energy exchange between cosmological constant density and baryonic matter, and adiabatically expanding ideal gas of WDM, described by RRG. In this way, we arrive at the following system of equations:

$$H^2(z) = \frac{\kappa^2}{3}\left[\rho_\Lambda(z) + \rho_b(z) + \rho_{dm}(z)\right] + H_0^2\Omega_k^0(1+z)^2$$

$$\rho_b' - \frac{3(1+w)}{1+z}\,\rho_b = -\rho_\Lambda',$$

$$\rho_{dm}' = \frac{(4-s)}{1+z}\,\rho_{dm}.$$

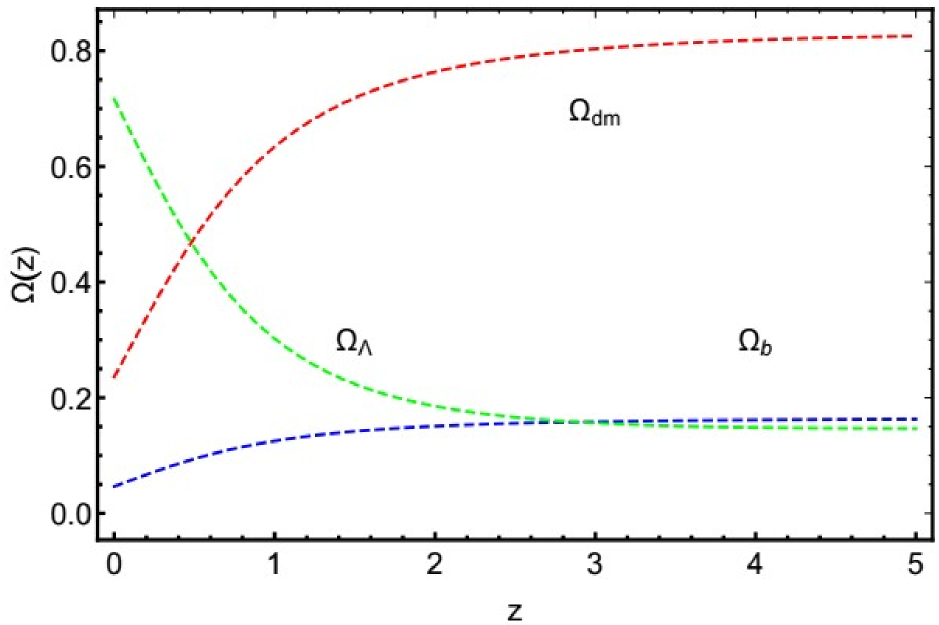
$$\frac{d\rho_\Lambda}{dz} = \frac{3\nu}{8\pi G}\frac{dH^2}{dz}$$

$$\rho_\Lambda = \rho_\Lambda^0 + \frac{3\nu}{8\pi G}\left(H^2 - H_0^2\right),$$

Running Cosmological constant and warm dark matter with curvature

- We can show that

$$
\begin{aligned}
\left(\frac{H(z)}{H_0}\right)^2 &= 1+\left(\Omega_b^0+\frac{2\nu\Omega_k^0}{2-\zeta}\right)\left[\frac{(1+z)^\zeta-1}{1-\nu}\right]+\Omega_k^0(z^2+2z)\left[1-\frac{\nu\zeta}{(1-\nu)(2-\zeta)}\right] \\
&+ \frac{\Omega_{dm}^0}{1-\nu}\left\{\left[\nu+\frac{\nu\zeta}{3-\zeta}\,\frac{{}_2F_1(\alpha,\beta;\gamma;-b^2)}{\sqrt{1+b^2}}\right](1+z)^\zeta-1\right\} \\
&+ \frac{\Omega_{dm}^0(1+z)^3}{\sqrt{1+b^2}}\left[\sqrt{1+b^2(1+z)^2}-\frac{\nu\zeta}{(1-\nu)(3-\zeta)}\;{}_2F_1(\alpha,\beta;\gamma;Z)\right]. \qquad (
\end{aligned}
$$



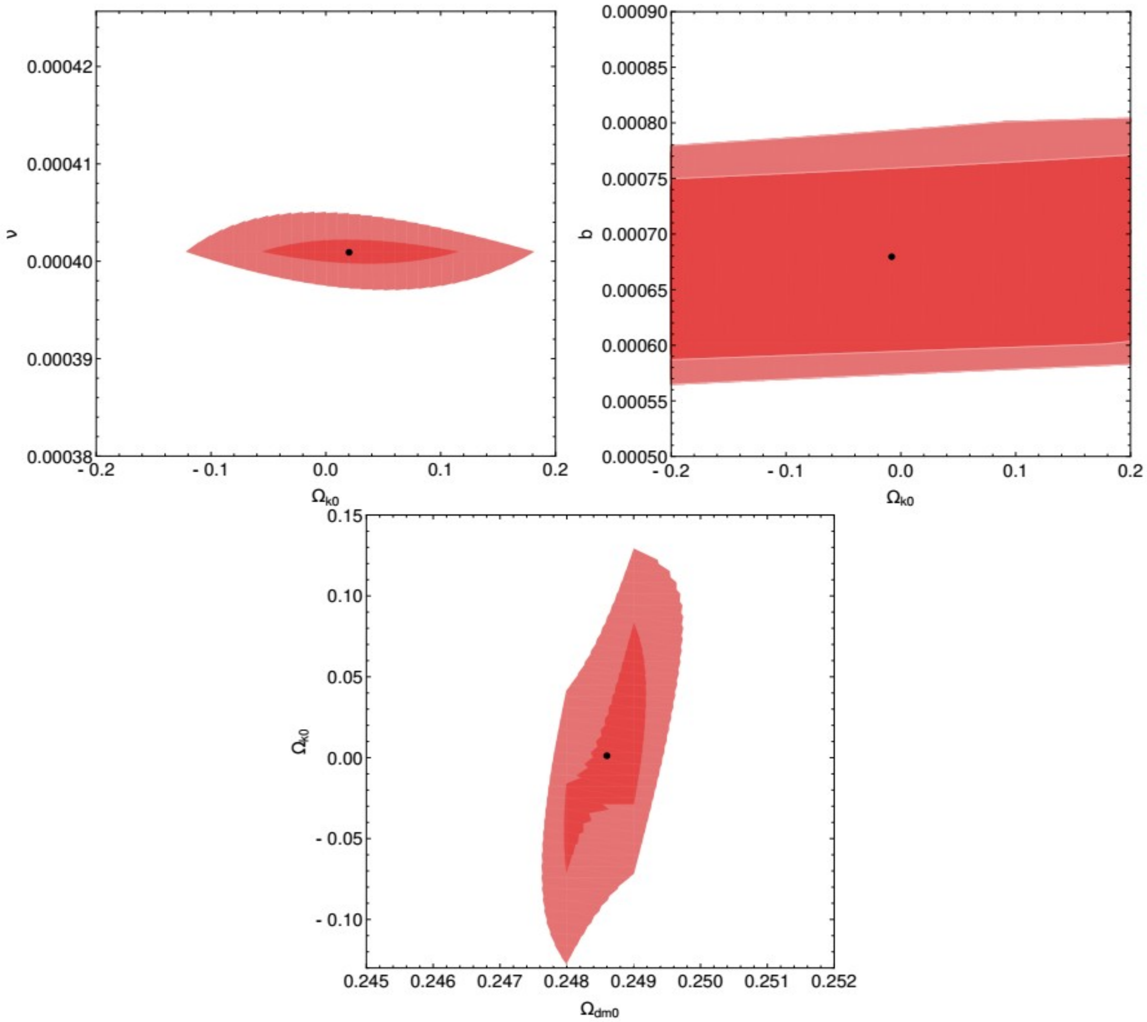
J. A. Agudelo Ruiz, JCF, AMVT and I.L.Shapiro, Gravitation and Cosmology, 26, Issue 4 (2020) 316-325

Running Cosmological constant and warm dark matter with curvature

- Observational constraints

Supernovae Ia + CC

Parameters	Best-fitting
Ω_{dm}^0	0.249 ± 0.107
Ω_k^0	-0.05 ± 0.120
b	0.000655 ± 0.000300
ν	0.000407 ± 0.000070



Influence of curvature on linear perturbations in some cosmological models

Numerical solution versus Growth factor parametrization

(Velasquez-Toribio and Fabris, EPJ C,80,1200, 2020)

Flat and non-flat *LCDM*

$$\ddot{\delta}_m(t) + 2H\dot{\delta}_m(t) - 4\pi G\rho_m\delta_m(t) = 0, \longrightarrow$$

$$\delta_m(a) = a_2F_1 \times \left(-\frac{1}{3w}, \frac{1}{2} - \frac{1}{2w}, 1 - \frac{5}{6w}, a^{-3w}\left(1 - \frac{1}{\Omega_m}\right)\right),$$

Flat model

$$f(z)\sigma_8(z) = -\sigma_8(1+z)\frac{\delta'_m(z)}{\delta_m(0)}$$

where $$f(a) \equiv \frac{d\ln\delta_m}{d\ln a} = -(1+z)\frac{d\ln\delta_m}{dz}$$

$$f(z) = \frac{d\ln\delta_m}{d\ln a} \approx \Omega_m^{\gamma}(z).$$

(P.J.E. Peebles, Astrophys. J. 205, 318 (1976))

Dynamical dark energy: CPL-parametrization

$$w(z) = w_0 + w_a\frac{z}{1+z}, \qquad f = \Omega_m(z)^{\gamma(z)} \qquad \gamma(z) = \gamma_0 + \gamma_a\frac{z}{1+z}.$$

Influence of curvature on linear perturbations in some cosmological models

- Runing cosmological constant

$$\ddot{\delta}_m + (2H + Q)\dot{\delta}_m - (4\pi G\rho_m - 2HQ - \dot{Q})\delta_m = 0 \qquad Q = \frac{\dot{\rho}_\Lambda}{\rho_m}.$$

$$\frac{d^2\delta_m}{dz^2} + \left[\frac{d\ln H}{dz} - \frac{1}{(1+z)}\left(1 + \frac{Q}{H}\right)\right]\frac{d\delta_m}{dz}$$
$$= \left(\frac{3}{2}\Omega_m - \frac{2Q}{H} + \frac{(1+z)}{H}\frac{dQ}{dz}\right)\frac{\delta_m}{(1+z)^2}.$$

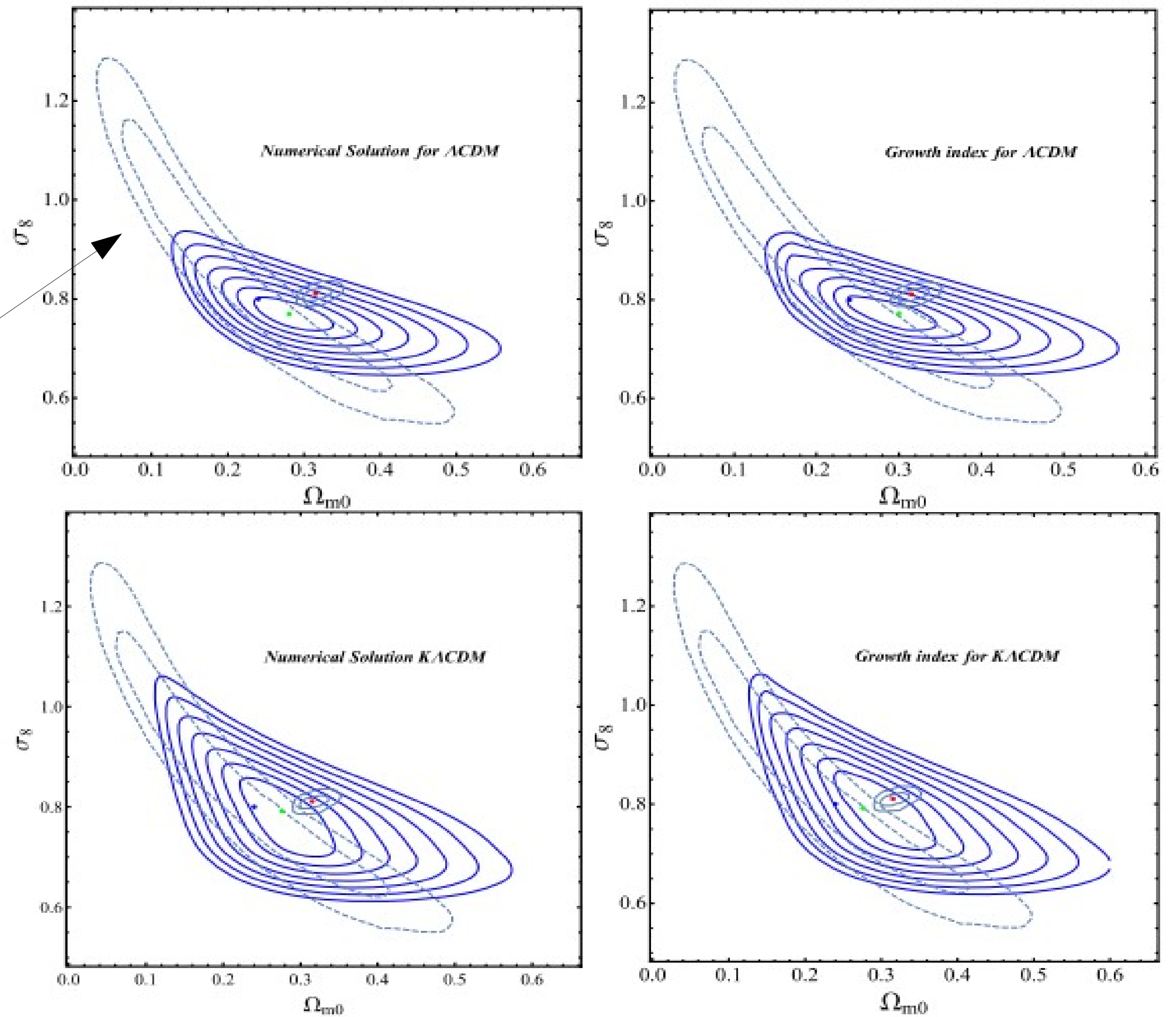
$$f = \frac{d\ln\delta_m}{d\ln a} \approx \tilde{\Omega}_m^{\gamma(a)} = \frac{\Omega_m(a)^{\gamma(a)}}{1-\nu}, \qquad \Omega_m(a) = \frac{\Omega_{m0}a^{-3(1-\nu)}}{H^2(a)/H_0^2}$$

Influence of curvature on linear perturbations in some cosmological models

- Observational constraints

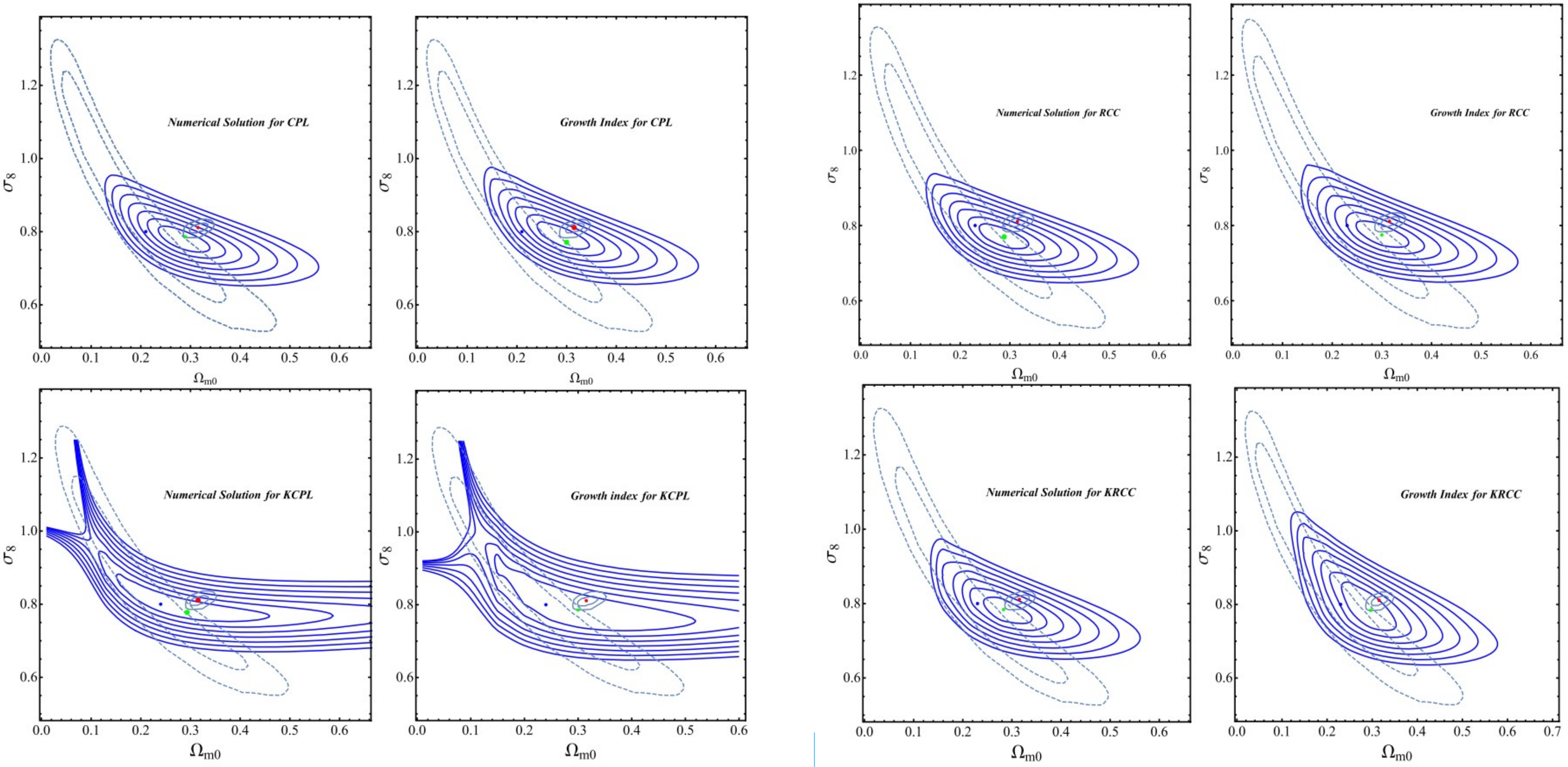
$f\sigma_8$ data

the data of the project Kids



Influence of curvature on linear perturbations in some cosmological models

- Observational constraints

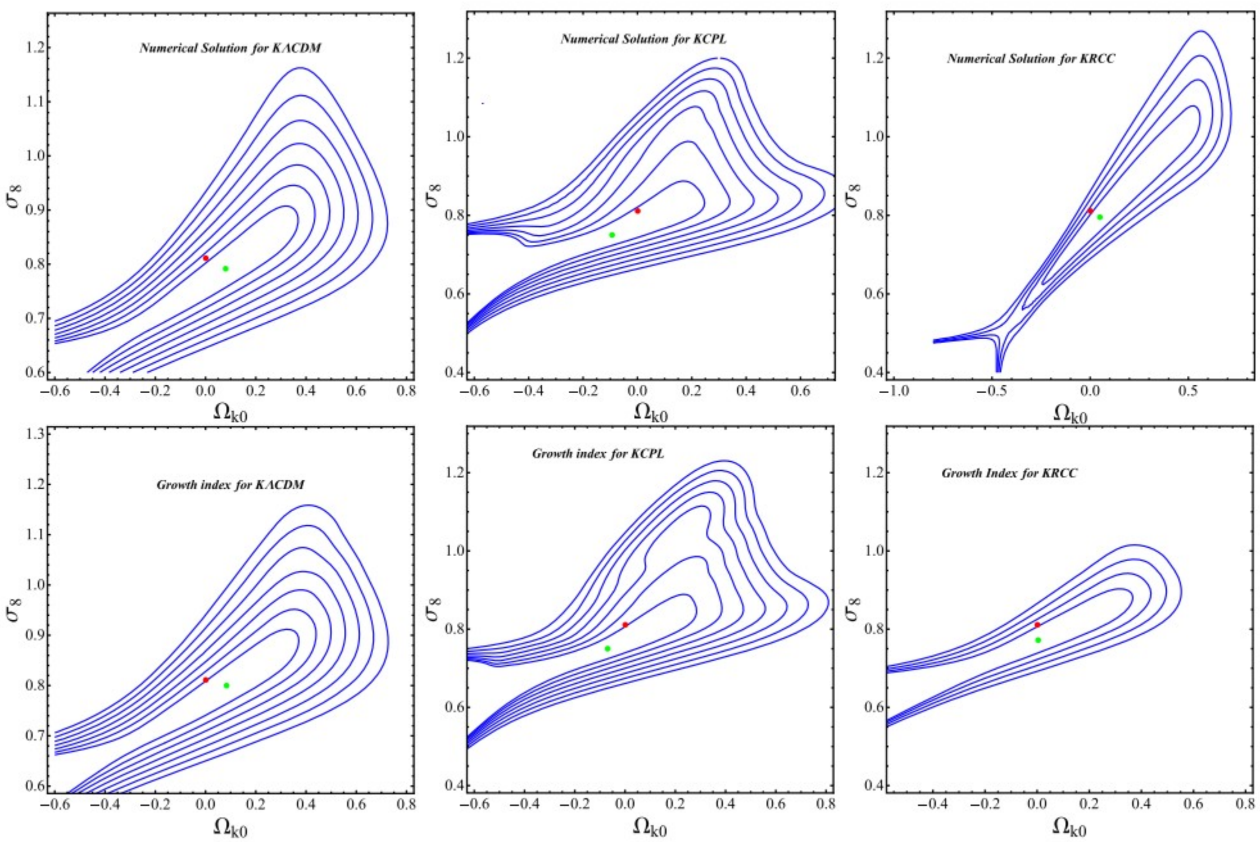


Influence of curvature on linear perturbations in some cosmological models

- Degeneracy

Table 1 Best-fitting parameters 1σ confidence intervals

Parameter	Numerical Solution	Growth Index
$K\Lambda CDM$		
Ω_{m0}	0.277 ± 0.118	0.277 ± 0.165
Ω_{k0}	0.075 ± 0.204	0.083 ± 0.165
σ_8	0.791 ± 2.00	0.799 ± 2.00
γ_0		0.599 ± 0.080
$KCPL$		
Ω_{m0}	0.303 ± 0.70	0.299 ± 0.83
Ω_{k0}	-0.043 ± 0.123	-0.069 ± 0.170
σ_8	0.749 ± 0.050	0.774 ± 0.050
w_0	-0.950 ± 0.250	-0.900 ± 0.400
w_a	0.0965 ± 0.365	-0.107 ± 0.367
γ_0		0.561 ± 1.07
γ_a		0.068 ± 0.100
$KRCC$		
Ω_{m0}	0.283 ± 0.070	0.299 ± 0.81
Ω_{k0}	0.065 ± 0.195	0.050 ± 0.132
σ_8	0.795 ± 0.204	0.770 ± 181
ν	0.00001 ± 0.00005	0.005 ± 0.012
γ_0		0.58 ± 0.212
γ_a		-0.01 ± 0.070



Conclusion and perspectives

- We determined observational constraints on non-flat LCDM and non-flat RRG+WDM models.
- We study implications on first order perturbations for some dark energy models. The curvature increases the CL in all models.
- The non-flat RCC model is a strong candidate versus flat and non-flat LCDM.
- Use model-independent methods to determine observational constraints on curvature: we can directly use the Taylor series by selecting data with $z<1$. In this way we avoid conceptual and mathematical consistency problems.
- Study observationally alternative models of gravitation plus curvature.