Cold Black Holes in Einstein-Maxwell-Scalar Theory

Tales Gomes

Programa de Pós-Graduação em Física
Universidade Federal do Espírito Santo

V José Plínio Baptista School of Cosmology,
September 30, 2021

Universidade Federal do Espírito Santo

Table of Contents

1. Einstein-Maxwell-Scalar Theroy
2. Field's equations
3. Cold Black Holes
4. Conclusion

Universidade Federal do Espírito Santo

Table of Contents

1 Einstein-Maxwell-Scalar Theroy

2 Field's equations

3 Cold Black Holes

4 Conclusion

Universidade Federal do Espírito Santo

Starting from the action of Einstein-Maxwell-Scalar Tensor Theory with a minimally coupled massless scalar field, in Einstein's frame,

$$S = \int d^4x\sqrt{-g}(R - \epsilon\nabla_\alpha\Phi\nabla^\alpha\Phi + F_{\alpha\beta}F^{\alpha\beta}), \tag{1}$$

The constant ϵ can assume the values $+1$, which will give us a positive kinetic energy for the scalar field, and -1, that will represent the "phantom" scalar field with negative kinetic energy.

Using the variational principle, one obtain, from the action (1),

$$R_{\mu\nu} = \epsilon \nabla_\mu \Phi \nabla_\nu \Phi - \left(2F_{\mu\lambda}F_\nu^\lambda - \frac{1}{2} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right), \tag{2}$$

$$\nabla_\alpha F^{\alpha\beta} = 0, \tag{3}$$

$$\nabla_\alpha \nabla^\alpha \Phi = 0. \tag{4}$$

To solve these equations, we consider a static, spherically symmetric metric

$$ds^2 = e^{2\gamma} dt^2 + e^{2\alpha} dr^2 + e^{2\beta} d\Omega^2 \tag{5}$$

Table of Contents

1. Einstein-Maxwell-Scalar Theroy
2. Field's equations
3. Cold Black Holes
4. Conclusion

Universidade Federal do Espírito Santo

Maxwell's equation

The general form of Maxwell's tensor for a point charge in a static spherically symmetric space-time can be written as,

$$F_{\alpha\beta} = E(u)(-\delta_{\mu 0}\delta_{\nu 1} + \delta_{\mu 1}\delta_{\nu 0}), \tag{6}$$

in this sense, the Maxwell's equation (3) gives us

$$\partial_u E(u) - (\gamma' + \alpha' - 2\beta')E(u) = 0 \tag{7}$$

which can be integrated to obtain the solution

$$E(u) = Qe^{\alpha+\gamma-2\beta}; \tag{8}$$

Klein-Gordon equation

On the other hand, since we are working with a static, spherically symmetric space-time,$\Phi \equiv \Phi(u)$, thus

$$\Phi'' - (\alpha' - \gamma' - 2\beta')\Phi' = 0, \tag{9}$$

where the solution is

$$\Phi' = Ce^{\alpha - \gamma - 2\beta} \tag{10}$$

Einstein's filed equations

A suitable choice of coordinate is the harmonic coordinates, where $\alpha(u) = \gamma(u) + 2\beta(u)$, with this choice the equations become

$$\gamma'' = Q^2 e^{2\gamma}, \tag{11}$$

$$-2\beta'' + 2(\beta')^2 + 4\beta'\gamma' = \epsilon C^2, \tag{12}$$

$$\gamma'' + \beta'' = e^{2\gamma+2\beta}; \tag{13}$$

where the scalar and eletric field are writen

$$E(u) = Qe^{2\gamma} \qquad \Phi(u) = Cu + \Phi_0 \tag{14}$$

The solutions for Einstein's equations are

$$e^{-\gamma-\beta} = s(k,u) = \begin{cases} k^{-1}\sinh(ku), & k>0, \\ u, & k=0, \\ k^{-1}\sin(ku), & k<0; \end{cases} \tag{15}$$

$$e^{-\gamma} = h(u) = Qs(\lambda, u+u_0) = \begin{cases} \frac{Q}{\lambda}\sinh[\lambda(u+u_0)], & \lambda>0, \\ u, & \lambda=0,\ ; \\ \frac{Q}{\lambda}\sin[\lambda(u+u_0)], & \lambda<0; \end{cases} \tag{16}$$

where λ and k are constants of integration, thus, we can write the metric as

$$ds^2 = \frac{s^2(\lambda, u_0)dt^2}{s^2(\lambda, u+u_0)} - \frac{s^2(\lambda, u+u_0)}{s^2(\lambda, u_0)s^2(k,u)}\left(\frac{du^2}{s^2(k,u)} + d\Omega^2\right) \quad (17)$$

where all constants are related by

$$m^2 - Q^2 = \lambda^2 \mathrm{sign}\,\lambda = k^2 \mathrm{sign}\,k - \frac{\epsilon C^2}{2}; \quad (18)$$

Table of Contents

1. Einstein-Maxwell-Scalar Theroy
2. Field's equations
3. Cold Black Holes
4. Conclusion

Universidade Federal do Espírito Santo

All BH are found in the Phantom sector, where $\epsilon = -1$. The first BH solution is for the case $\lambda > k = 0$. The metric in this case is written as,

$$ds^2 = \frac{\sinh^2(\lambda u_0)dt^2}{\sinh^2[\lambda(u+u_0)]} - \frac{\sinh^2[\lambda(u+u_0)]}{\sinh^2(\lambda u_0)u^2}\left(\frac{du^2}{u^2} + d\Omega^2\right) \quad (19)$$

Where we have the relation

$$m^2 - Q^2 = \lambda^2 = \frac{C^2}{2} \quad (20)$$

The appropriate coordinate transformation for this solution is

$$x = \frac{1}{u} \tag{21}$$

In consequence the metric become

$$ds^2 = h(x)dt^2 - \frac{dx^2}{h(x)} - \frac{x^2}{h(x)}d\Omega^2 \tag{22}$$

where

$$h(x) = \frac{4\lambda^2 e^{-2\frac{\lambda}{x}}}{[(m+\lambda) - (m-\lambda)e^{-2\frac{\lambda}{x}}]^2}, \tag{23}$$

and the geometric mass is

$$m = \lambda \coth\left(\lambda u_0\right) \tag{24}$$

Universidade Federal do Espírito Santo

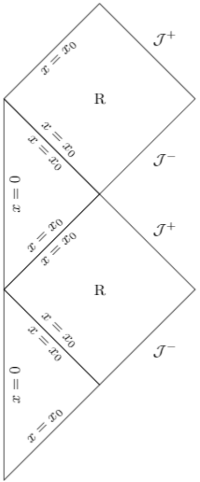

Figure: Carter-Penrose Diagram for the case $\lambda > k = 0$, with one event horizon.

The other solution, for $\lambda > k > 0$, has the metric in the form

$$ds^2 = \frac{\sinh^2(\lambda u_0)dt^2}{\sinh^2[\lambda(u+u_0)]} - \frac{k^2 \sinh^2[\lambda(u+u_0)]}{\sinh^2(\lambda u_0)\sinh^2(ku)} \left(\frac{k^2 du^2}{\sinh^2(ku)} + d\Omega^2 \right) \tag{25}$$

with constants relation

$$m^2 - Q^2 = \lambda^2 = k^2 + \frac{C^2}{2} \tag{26}$$

the transformation for the another solution is

$$e^{-2ku} = 1 - \frac{2k}{\rho} = P(\rho) \tag{27}$$

so the metric writes

$$ds^2 = f(\rho)dt^2 - \frac{d\rho^2}{f(\rho)} - \frac{P(\rho)}{f(\rho)}\rho^2 d\Omega^2; \tag{28}$$

where

$$f(\rho) = \frac{4\lambda^2 P(\rho)^a}{[(m+\lambda) - (m-\lambda)P(\rho)^a]^2} \tag{29}$$

and $a = \lambda/k$.

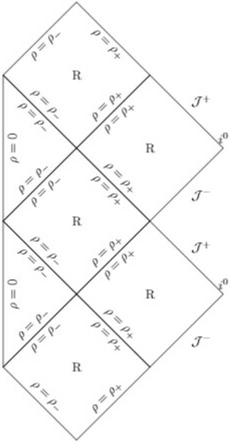

Figure: Carter-Penrose Diagram for an even integer a in the case where $\lambda > k > 0$.

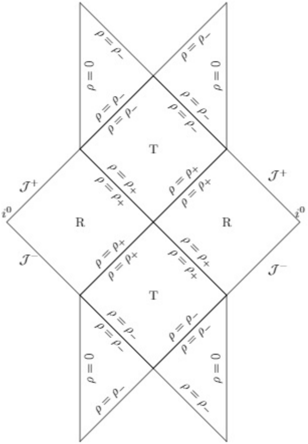

Figure: Carter-Penrose Diagram for $\lambda > k > 0$ and a an odd integer

Table of Contents

1 Einstein-Maxwell-Scalar Theroy

2 Field's equations

3 Cold Black Holes

4 Conclusion

Universidade Federal do Espírito Santo

Conclusions

- We study the Cold Black Holes in Einstein-Maxwell-Scalar theory;
- This BHs have interesting characteristics, such as infinity surface area and zero Hawking temperature;
- We will study the geodesics and stability of these solutions,
- We look forwards to obtain a rotating solution from the static ones we studied;
- Investigate the methods of rotate solutions, such as JNA.
- Study the thermodynamics of this BHs.

Obrigado!