

CMB theory

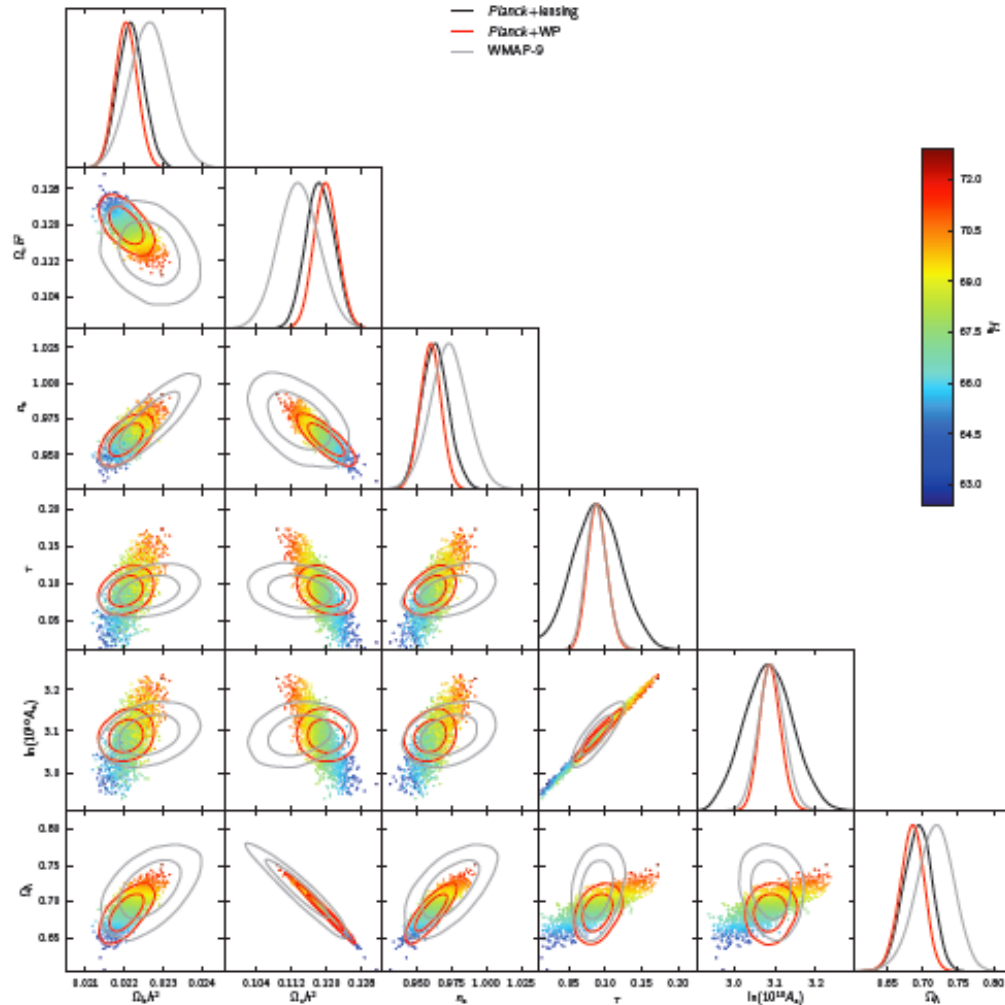
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Part 4: outlook

- Parameter constraints from angular power spectrum
- Polarisation
 - Density perturbations
 - Gravitational waves
- New frontiers in CMB theory?
 - Non-linearity
 - Weak lensing
 - Non-Gaussianity

Planck 6 parameter Λ CDM cosmology



$$\Omega_m h^2, \Omega_b h^2, n_s, \tau_{re}, P_\zeta, \Omega_\Lambda$$

Parameter constraints from peak structure

- Angular scale of first acoustic peak

$$\theta_* = \frac{s_*}{D_A} = (1.04148 \pm 0.00066) \times 10^{-2} \quad [\text{Planck 2013}]$$

- Sound horizon at recombination

$$s_* = \int_0^* c_s d\eta \propto (\Omega_m h^2)$$

- Horizon distance to recombination

$$D_* = \eta_0 - \eta_* = \int_0^* \frac{dz}{H(z)} \propto h^{-1}$$

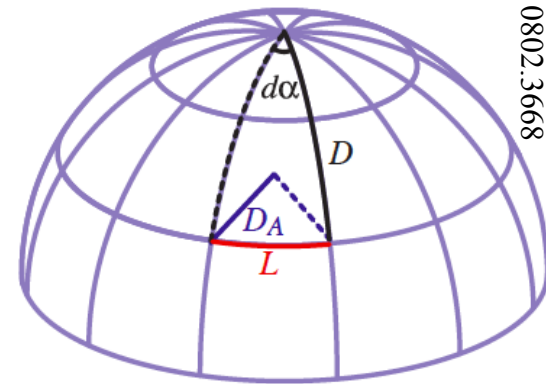
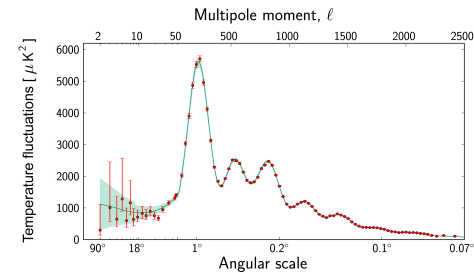
- Curved space

$$D_A = R \sin\left(\frac{D_*}{R}\right) \approx D_* \left(1 + \frac{\Omega_\kappa H_0^2 D_*^2}{6}\right)$$

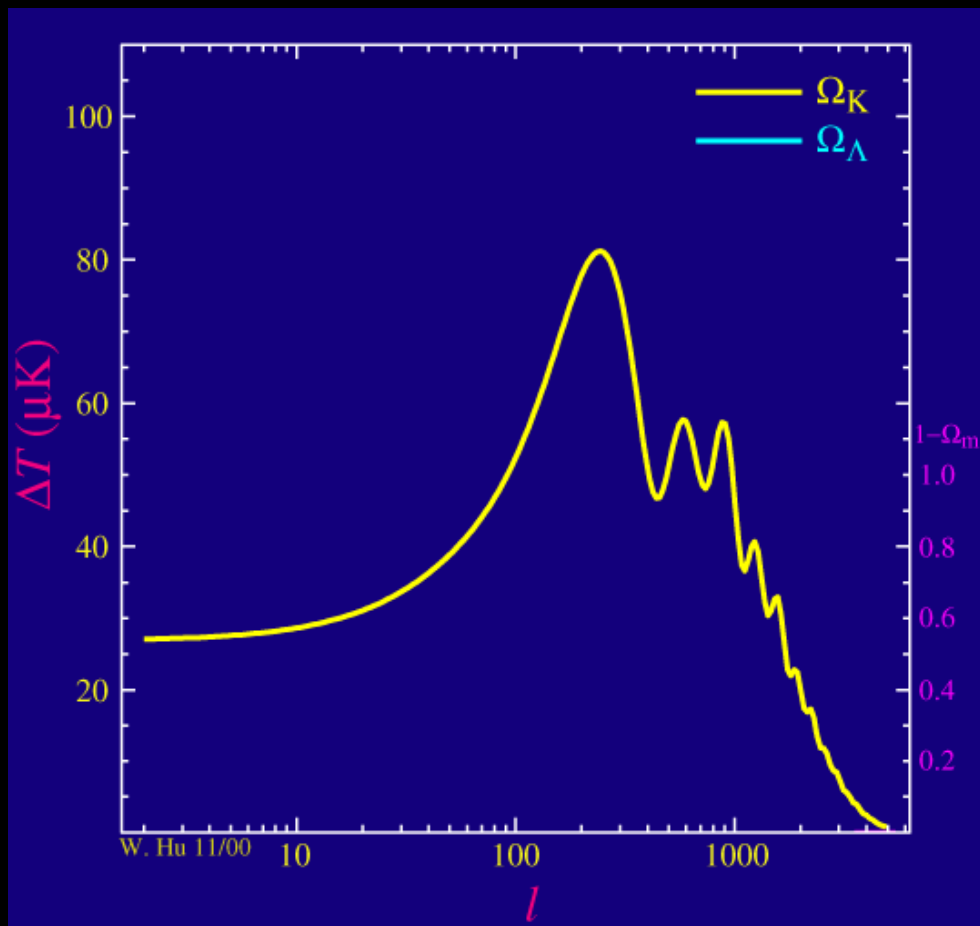
$$\Rightarrow \Omega_\kappa = -0.042^{+0.043}_{-0.048} \quad [\text{Planck 2013}]$$

- Flat space

$$\theta_* = \frac{s_*}{D_*} \propto \Omega_m h^3 = 0.0959 \pm 0.0006 \quad [\text{Planck 2013}]$$



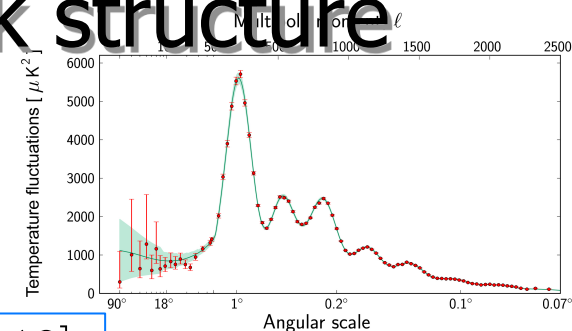
Precision cosmology from angular power spectrum



background.uchicago.edu/~whu

- Acoustic scale
 - strongly dependent on curvature
 - weakly dependent on Λ in flat space
- Integrated Sachs-Wolfe on large scales due to dark energy
 - correlated with large-scale structure along line-of sight (Crittenden&Turok'96)

Parameter constraints from peak structure



- Second peak relative height
 - Baryon loading suppresses even peaks (rarefaction)

$$\Omega_b h^2 = 0.02207 \pm 0.00033 \quad [\text{Planck 2013}]$$

- Third and higher peaks
 - Decay of potential in radiation era enhances higher peaks, suppressed by matter density

$$\Omega_m h^2 = 0.1423 \pm 0.0029 \quad [\text{Planck 2013}]$$

- Damping tail
 - Fixed in basic Lambda-CDM cosmology
 - Requires spectral tilt

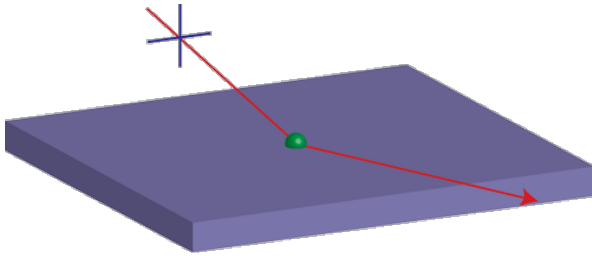
$$n = 0.9603 \pm 0.0073 \quad [\text{Planck 2013}]$$

- Reionisation
 - $\tau = 0.097 \pm 0.038 \quad [\text{Planck 2013}]$
 - Suppresses all small angle anisotropies below $l=20$
 - Approximately degenerate with spectral tilt – *can be broken by polarisation*

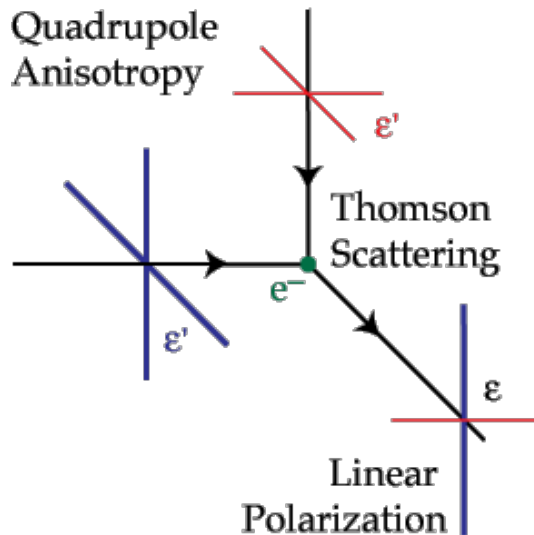
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Polarisation

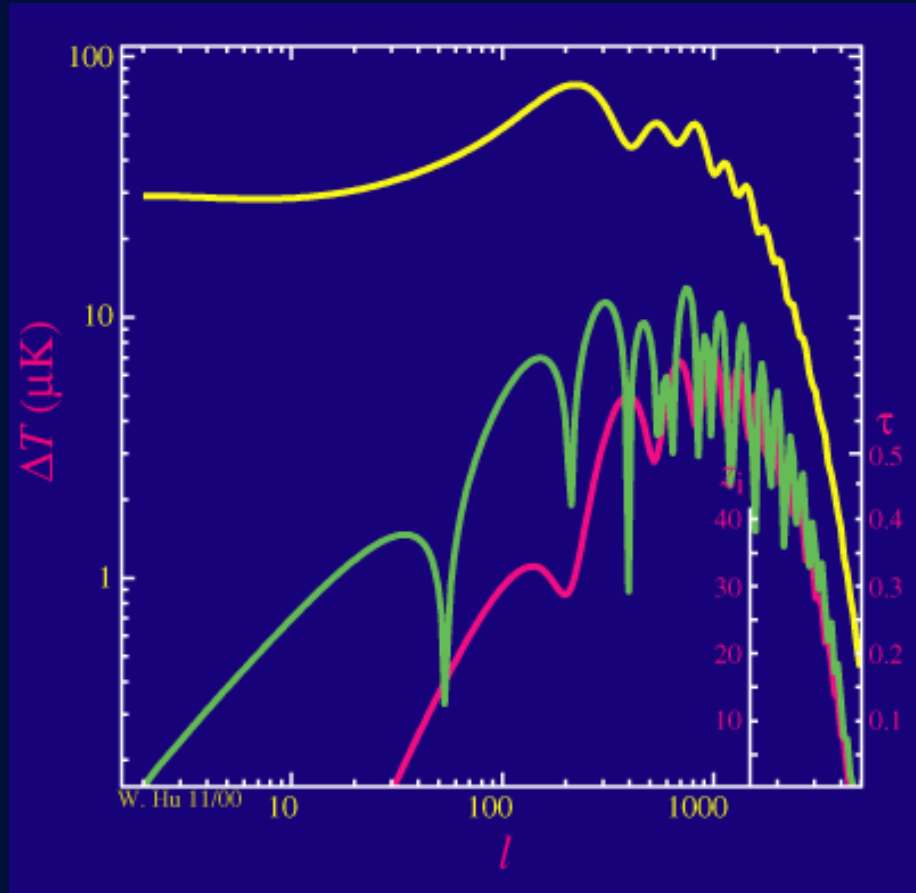


- requires
 - scattering
 - inhomogeneity
- Weak polarisation at last-scattering
 - small quadrupole moment due photon diffusion
- Strong polarisation at re-ionisation
 - large angular scales



$$P_{ij} \propto \langle \vec{E} \vec{E}^\dagger \rangle \propto \begin{pmatrix} \Theta + Q & U \\ -U & \Theta - Q \end{pmatrix}$$

Effect of reionisation



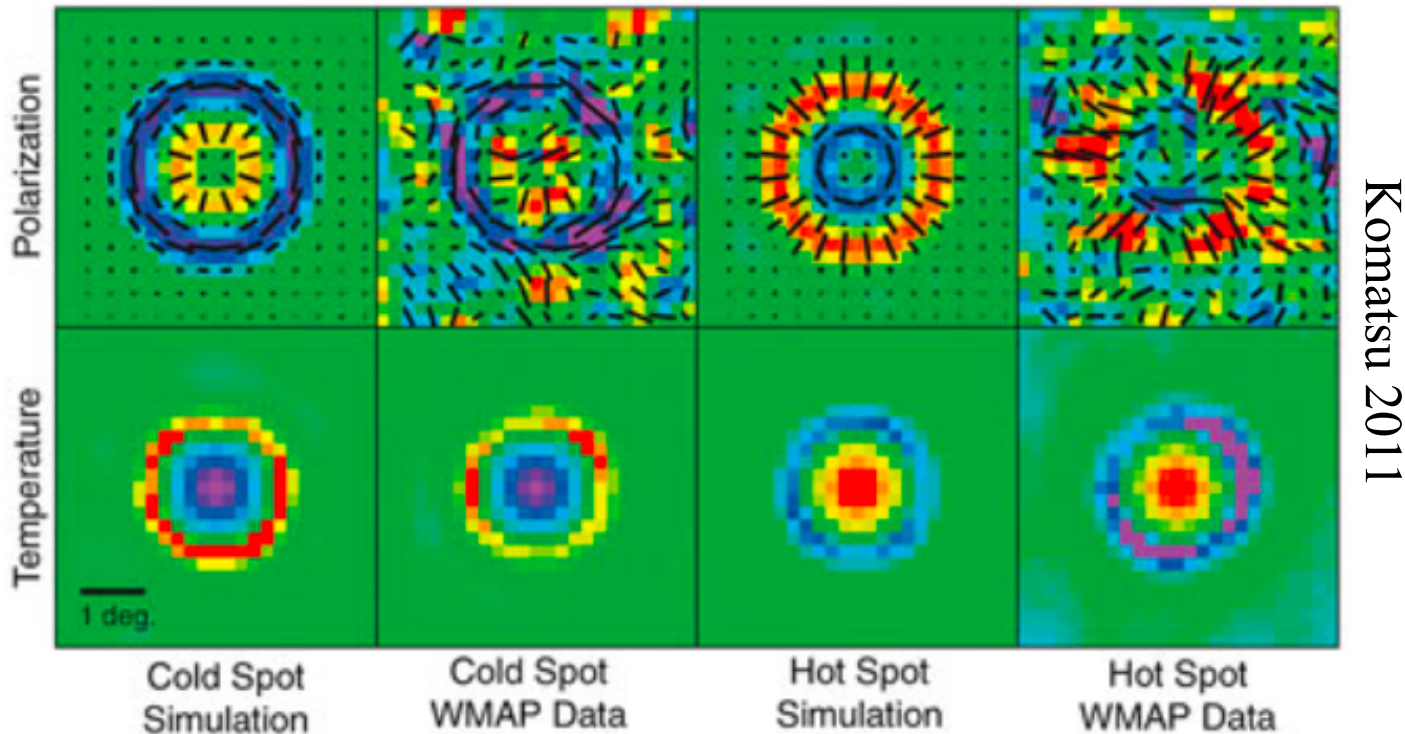
$$C_\ell^{TT}$$

$$C_\ell^{TE}$$

$$C_\ell^{EE}$$

E mode polarisation from scalar perturbations

$$Q \pm iU = \sum_{\ell, m} (E_{\ell m} \pm iB_{\ell m}) \pm 2 Y_{\ell m}(\hat{n})$$



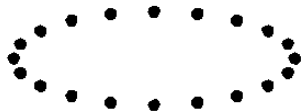
- ***Planck polarisation data coming November 2014***

E+B modes from tensor metric perturbations

- spatial metric perturbation

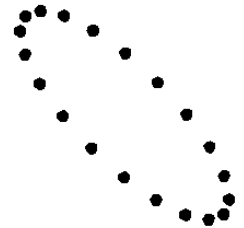
$$h_{ij}(\mathbf{x}, \eta) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{x}} [h_{\mathbf{k}}(\eta) \mathbf{e}_{ij}(\mathbf{k}) + \bar{h}_{\mathbf{k}}(\eta) \bar{\mathbf{e}}_{ij}(\mathbf{k})] ,$$

- two polarisation states

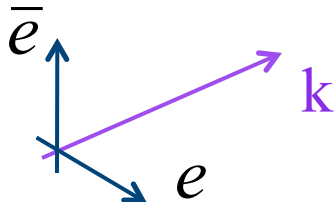


$$\mathbf{e}_{ij}(\mathbf{k}) \equiv \frac{1}{\sqrt{2}} [\mathbf{e}_i(\mathbf{k}) \mathbf{e}_j(\mathbf{k}) - \bar{\mathbf{e}}_i(\mathbf{k}) \bar{\mathbf{e}}_j(\mathbf{k})]$$

$$\bar{\mathbf{e}}_{ij}(\mathbf{k}) \equiv \frac{1}{\sqrt{2}} [\mathbf{e}_i(\mathbf{k}) \bar{\mathbf{e}}_j(\mathbf{k}) + \bar{\mathbf{e}}_i(\mathbf{k}) \mathbf{e}_j(\mathbf{k})]$$



transverse and trace-free



$$k_i e_{ij} = k_i \bar{e}_{ij} = 0 \quad , \quad e_{ii} = \bar{e}_{ii} = 0$$

gravitational waves from inflation

- amplitude, $h(t)$, obeys **free wave equation** for massless field in an unperturbed FRW cosmology, $\delta\varphi = M_{Pl} h / (32\pi)^{1/2}$

$$\ddot{h}_k + 3H\dot{h}_k + \frac{k^2}{a^2} h_k = 0$$

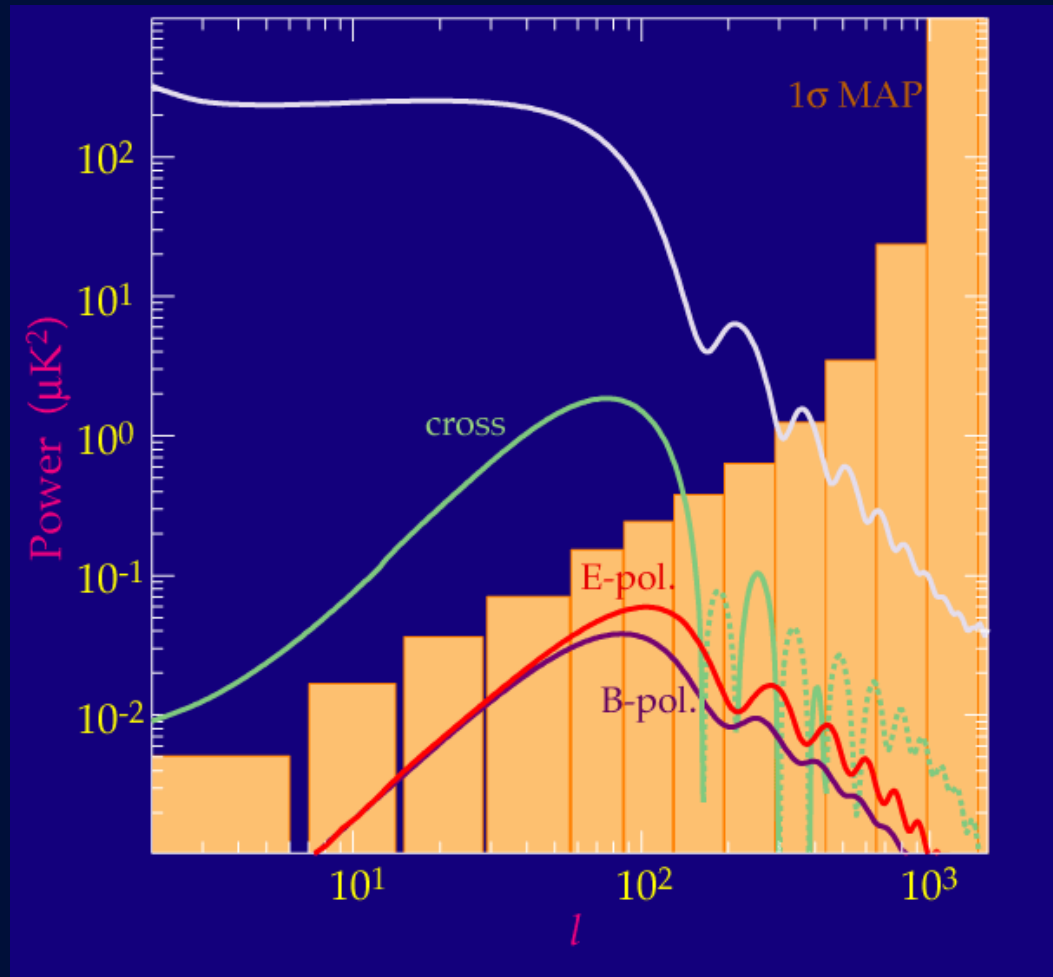
- vacuum fluctuations during inflation
→ primordial perturbations on super-Hubble scales $k < aH$

$$\Rightarrow P_T(k) = 2 \frac{4\pi k^3}{(2\pi)^3} \langle h^2 \rangle \approx 2 \left(\frac{32\pi}{M_{Pl}^2} \right) \left(\frac{H}{2\pi} \right)^2_{k=aH}$$

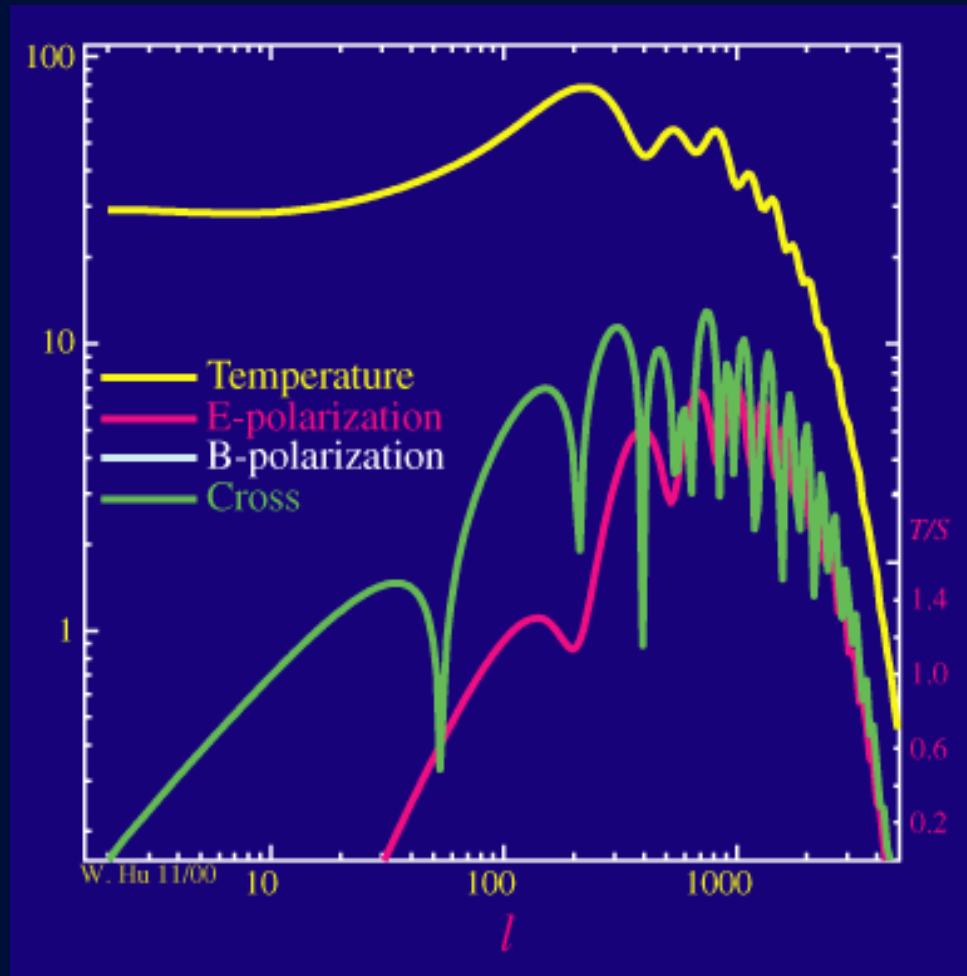
$$\text{tensor-scalar ratio: } r \equiv \frac{P_T}{P_\xi} \approx 16\varepsilon \quad \text{for single-field inflation}$$

$$\text{tilt: } n_T \equiv \frac{d \ln P_T}{d \ln k} \approx -2\varepsilon < 0 \quad \text{where } \varepsilon \equiv -\dot{H}/H^2$$

Angular power spectrum for GWs only



effect of density+gravitational waves



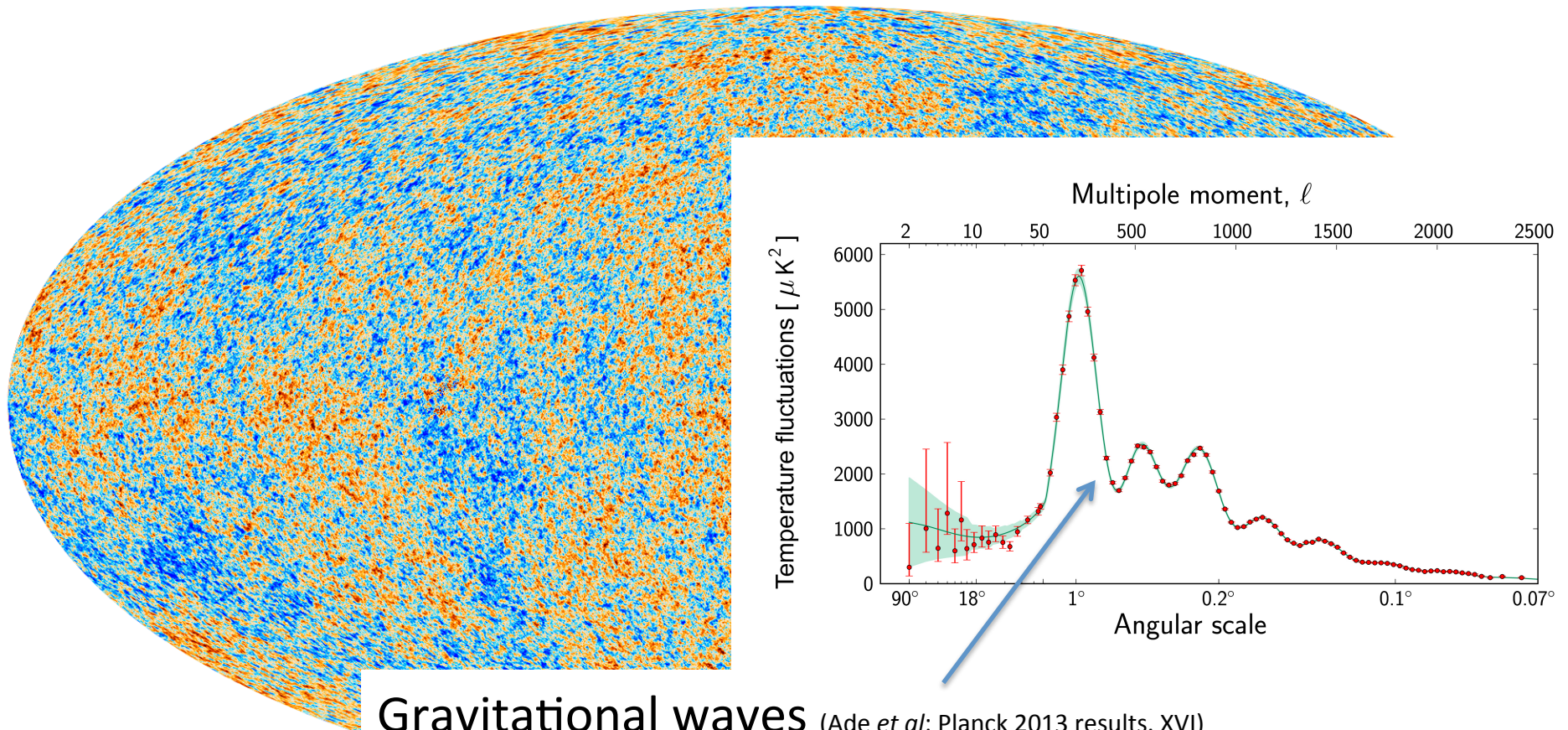
$$C_\ell^{TT}$$

$$C_\ell^{TE}$$

$$C_\ell^{EE}$$

$$C_\ell^{BB}$$

Planck - new standard model of primordial cosmology



Gravitational waves (Ade *et al.*: Planck 2013 results. XVI)

$r < 0.11$ (Planck+WP+ACT+SPT)

$V < 2 \times 10^{16}$ GeV

Next frontier in CMB theory?

- **Nonlinearity in CMB**

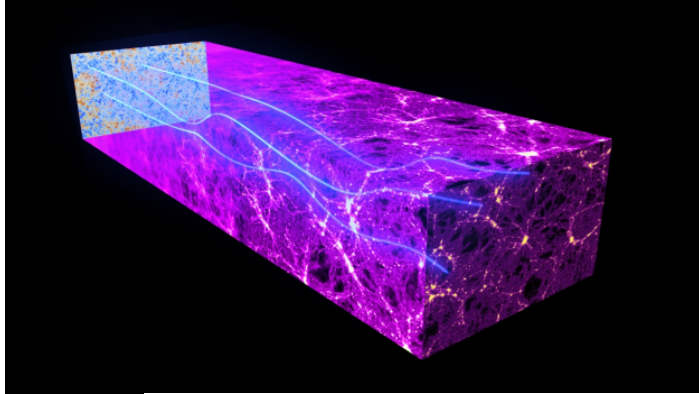
- Intrinsic non-linearity at last scattering

- intrinsic bispectrum
 - second order B-modes from scalar perturbations
 - anisotropic spectrum

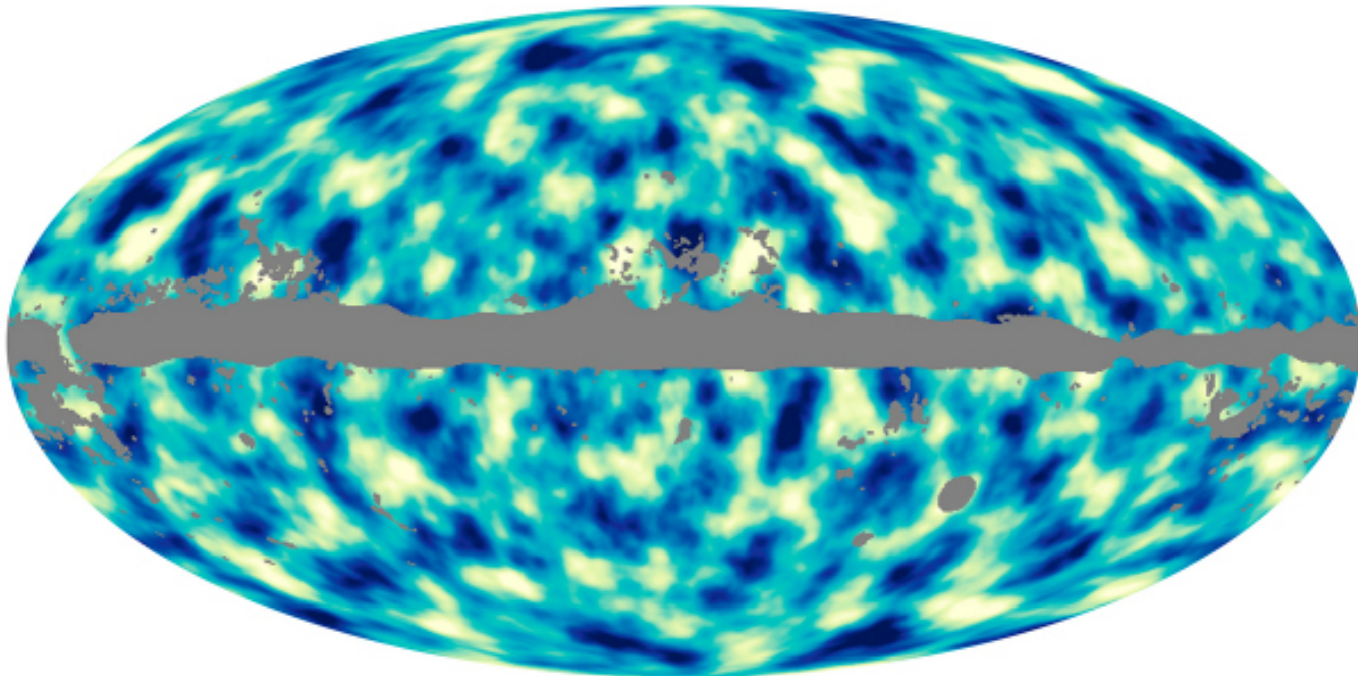
- Weak lensing along line of sight

- redistributes power in small-scale power spectrum

Weak lensing of CMB photons



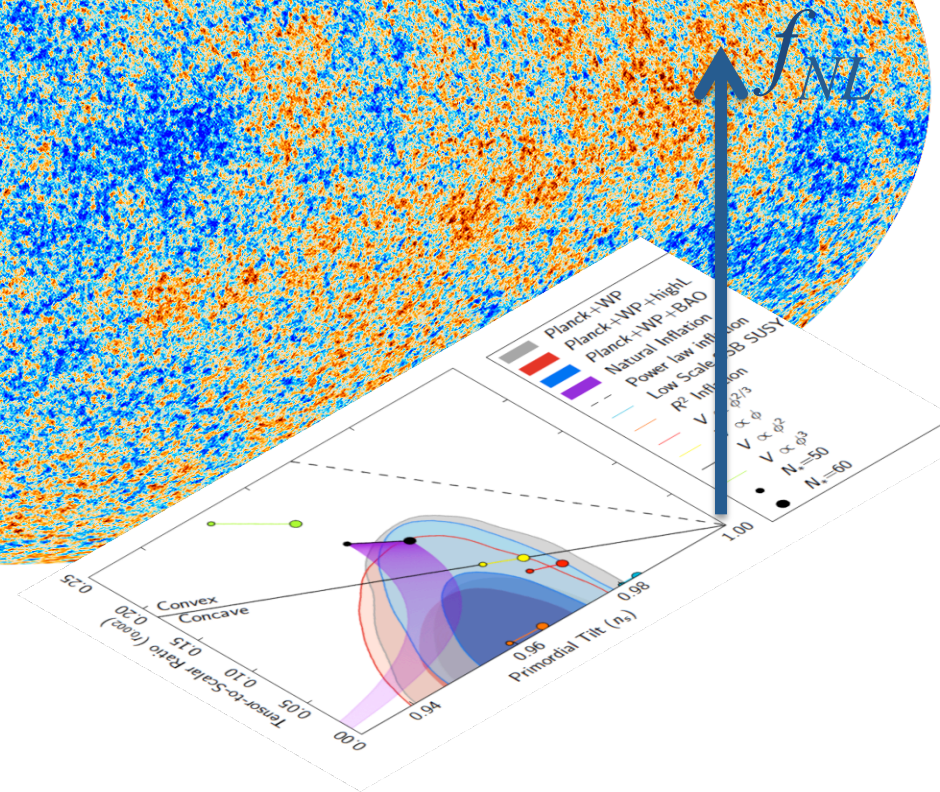
- Many small angle deflections by non-linear structure
- Redistribute power on small angular scales
- Detected by Planck
- Non-Gaussian signal



Planck - new standard model of primordial cosmology

“non-Gaussianity parameter f_{NL} measured by Planck is consistent with zero”

$$f_{NL} = \frac{B_{\xi}(k_1, k_2, k_3)}{P(k_1)P(k_2) + P(k_2)P(k_3) + P(k_3)P(k_1)}$$



Lensing-ISW signal

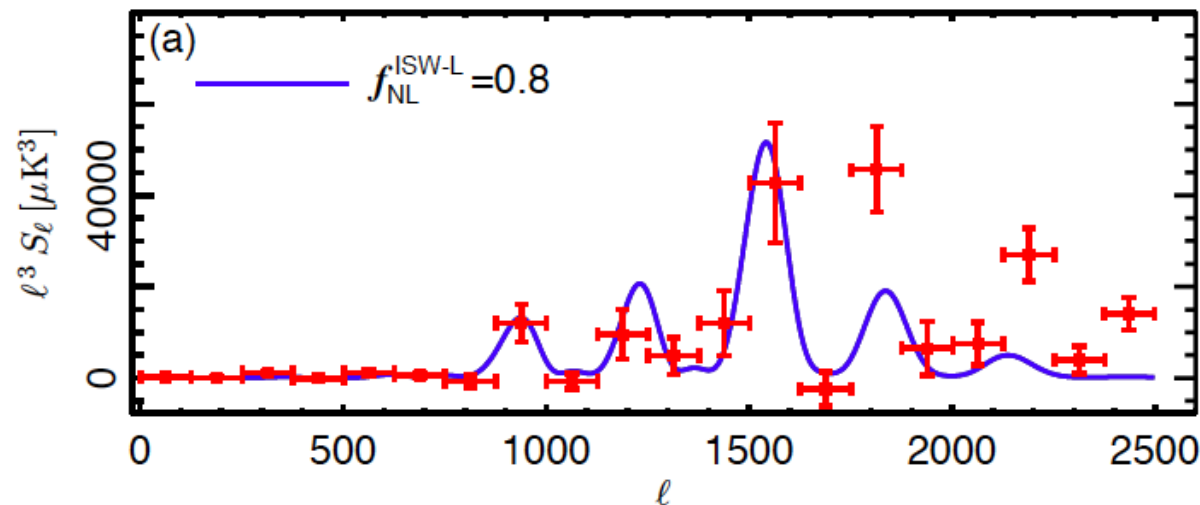
Goldberg & Spergel 1999
Seljak and Zaldarriaga 1999
Lewis, Challinor & Hanson 2011
Lewis 2012

Significant bispectrum comes from combining two late time gravitational effects: gravitational lensing and the integrated Sachs-Wolfe effect.

In the ISW effect, large scale over-densities and under-densities appear hotter or colder as photons can gain or lose energy passing through their time dependent potentials.

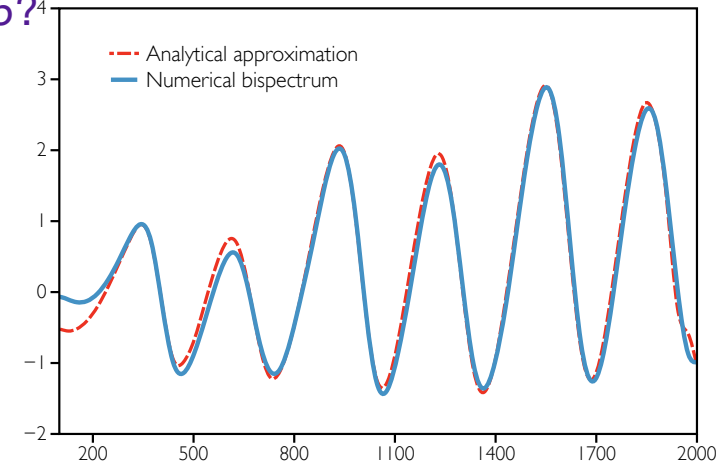
These potentials correlate to where gravitational lensing (seen on smaller scales) is largest or smallest, creating mode coupling.

Significant bias, f_{NL} of order 7, seen in the Planck analysis.

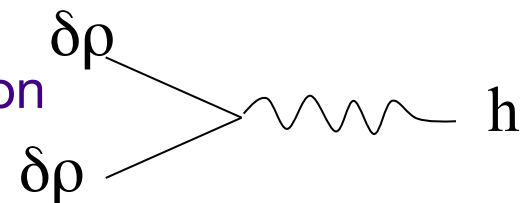


Intrinsic bispectrum of CMB

- Existing non-Gaussianity templates based on non-linear primordial perturbations + linear Boltzmann codes (CMBfast, CAMB, etc)
- Second-order general relativistic Boltzmann codes now available
 - **Pitrou (2010)**: CMBquick in Mathematica: $f_{NL} \sim 5?$
 - Huang & Vernizzi (Paris)
 - **Pettinari, Fidler et al (Portsmouth)** →
 - Su, Lim & Shellard (Cambridge & London)

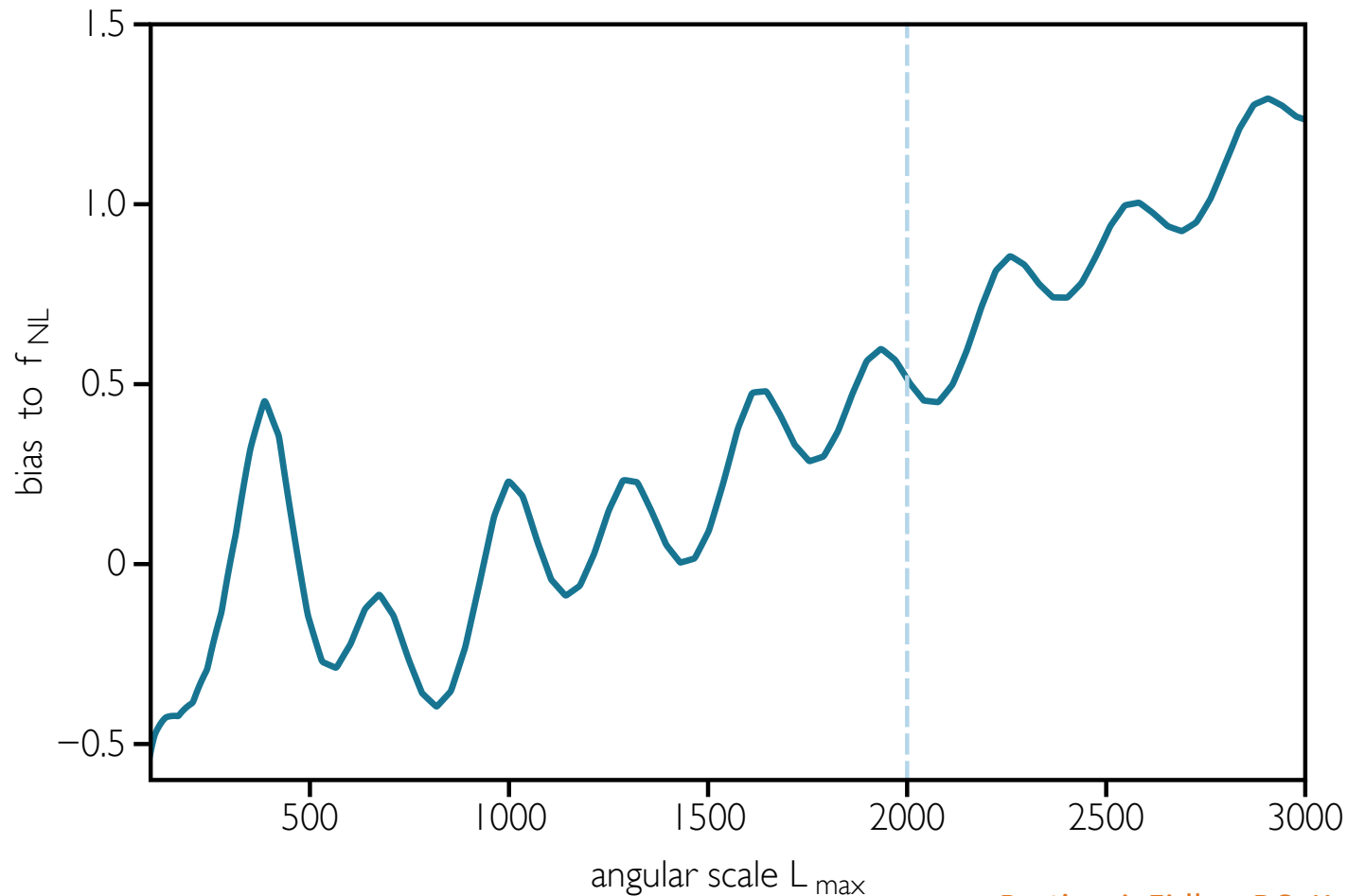


- need templates for secondary non-Gaussianity (e.g. lensing-ISW)
- induced tensor and vector modes from density perturbations
- testing interactions at recombination
 - e.g., gravitational wave production



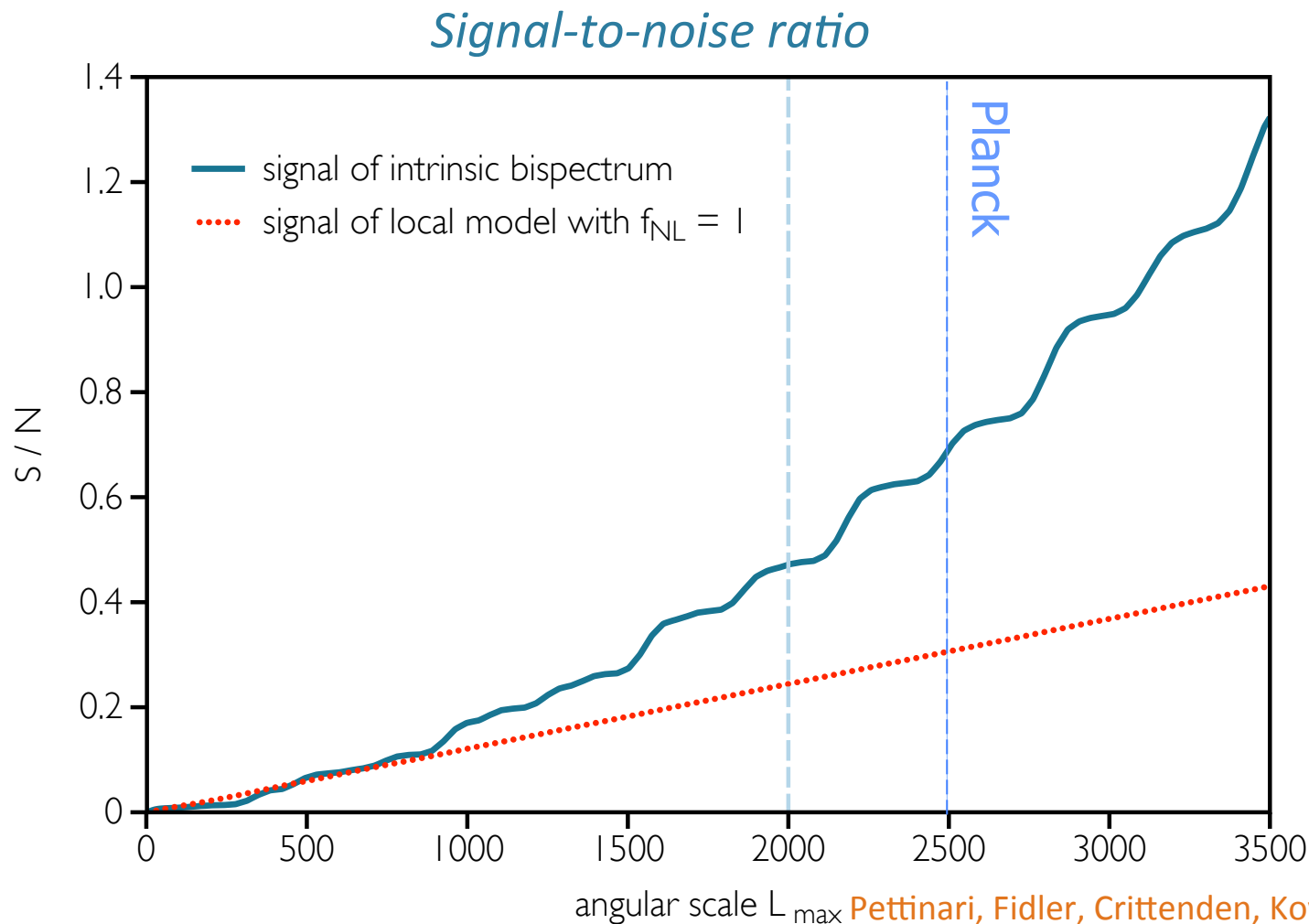
Intrinsic bias on local non-Gaussianity

Effects at recombination small compared to Planck sensitivity



Detecting the intrinsic signal

Even using the perfect template, it is hard to see the intrinsic signal with Planck.



outlook

- Standard Λ CDM cosmology provides good “base” model
 - Basic cosmological parameters tightly constrained
 - No reason to believe this is the final theory of everything!
- Powerful constraints on extended models and very early universe
 - especially combination with other data sets: galaxy and HI (SKA) survey data
- Future CMB observations:
 - Planck polarisation data coming November 2014
 - B-mode experiments ($r \sim 10^{-3}$) on ground and in space
 - Japanese LiteBird
- Europe and US waiting on next space satellite
 - PRISM and CMBpol (2025?)
- Lots of time for theoretical developments...

webliography:

- WMAP, the Universe 101 tutorial
 - <http://wmap.gsfc.nasa.gov/universe/>
- ESA Planck mission
 - <http://sci.esa.int/planck/>
- COSMUS animations, University of Chicago
 - <http://astro.uchicago.edu/cosmus/>
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 - <http://background.uchicago.edu/~whu/>
- Clem Pryke, University of Minnesota
 - <http://find.spa.umn.edu/~pryke/>
- Scott Dodelson textbook
 - Modern Cosmology
- Reviews
 - Hu, <http://arxiv.org/abs/0802.3688>
 - Dodelson & Hu, <http://arxiv.org/abs/astro-ph/0110414>
 - Komatsu, IUCAA review 2011, <http://www.mpa-garching.mpg.de/~komatsu/>
- Institute of Cosmology and Gravitation, University of Portsmouth
 - <http://www.icg.port.ac.uk/>

what we can learn by looking at the sky!



THE END