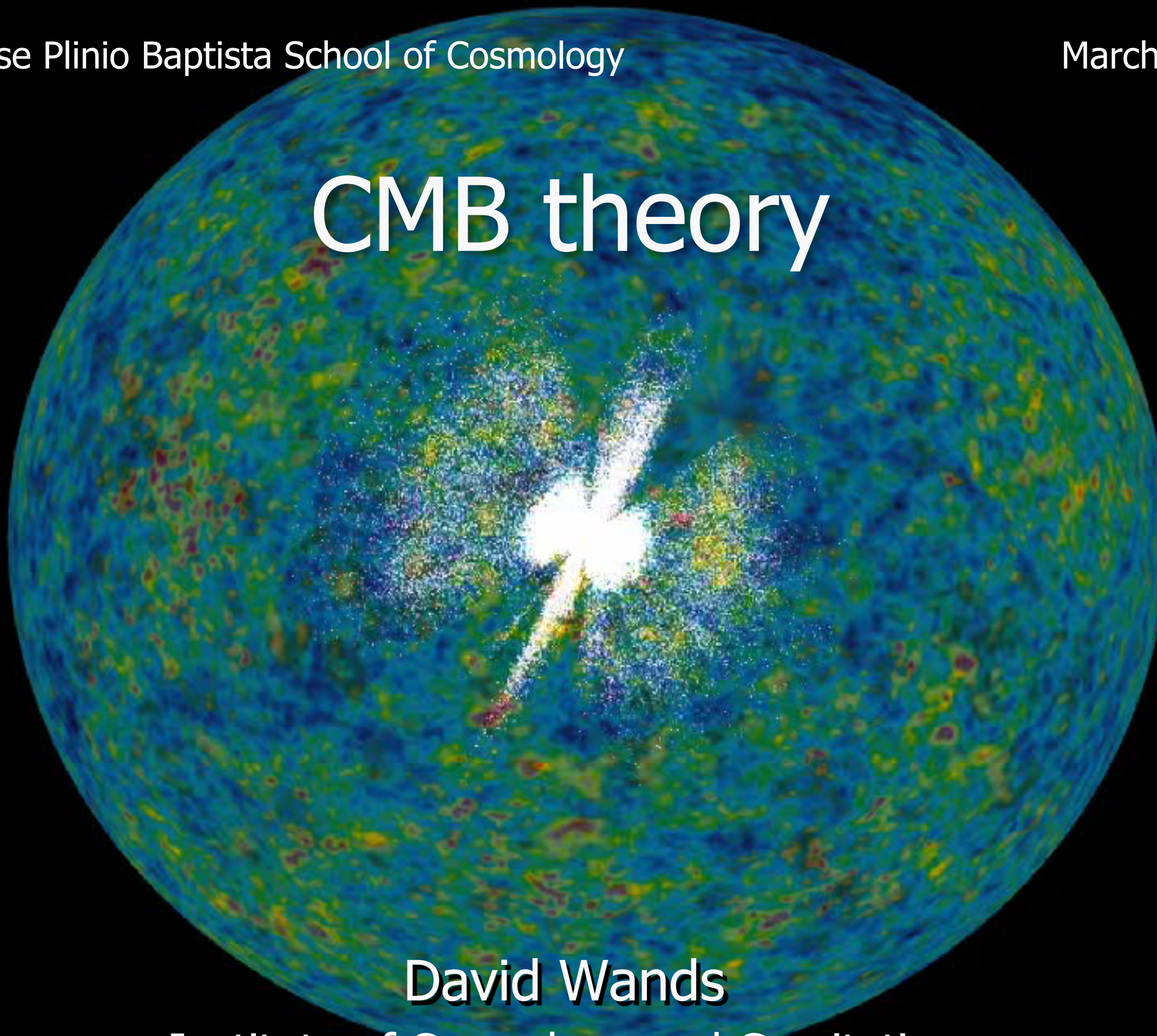
II Jose Plinio Baptista School of Cosmology

March 2014



CMB theory

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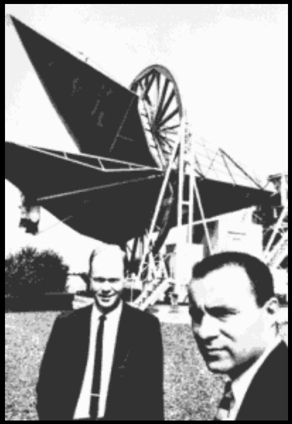
University of Portsmouth

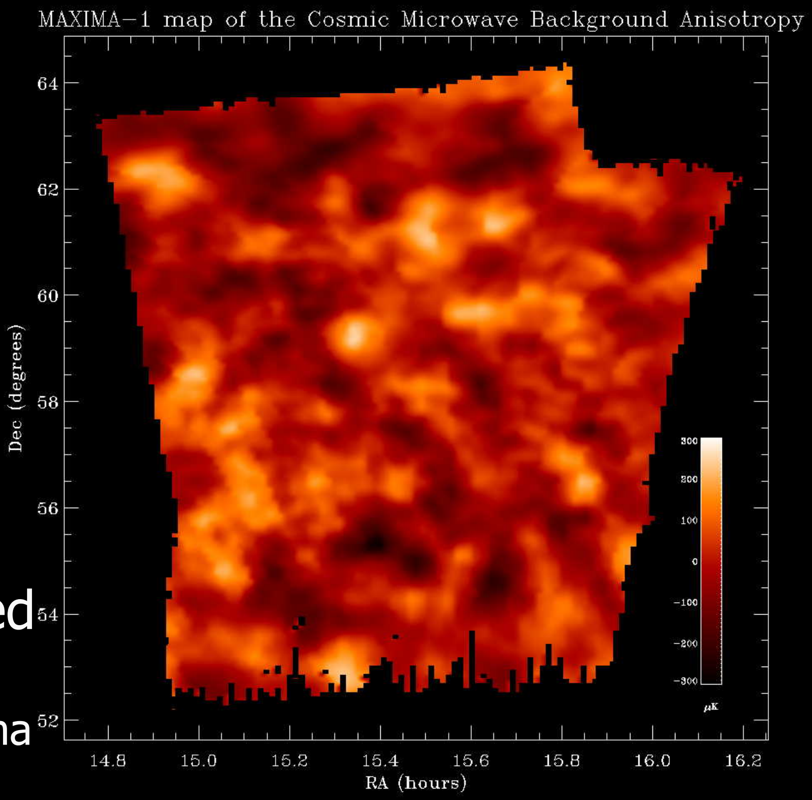
Part 1: overview

- introduction
- cosmological parameters
- a brief thermal history

Cosmic Microwave Background radiation



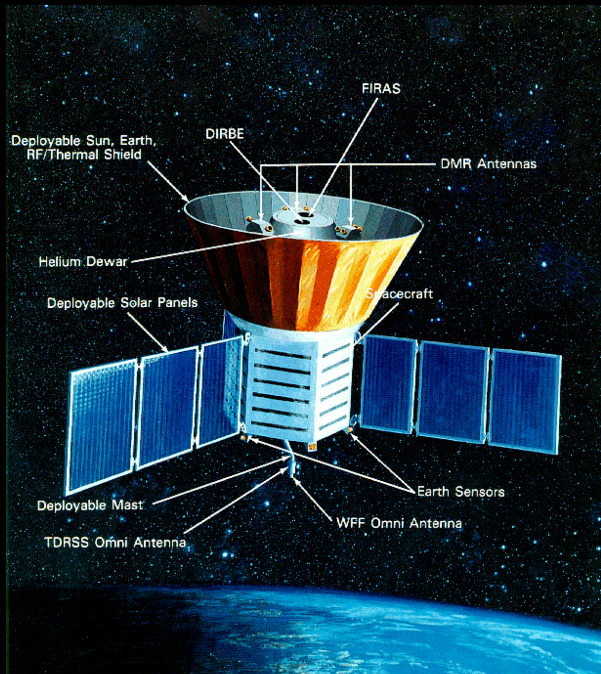
- discovered in 1965 by
  Arno Penzias and Robert Wilson

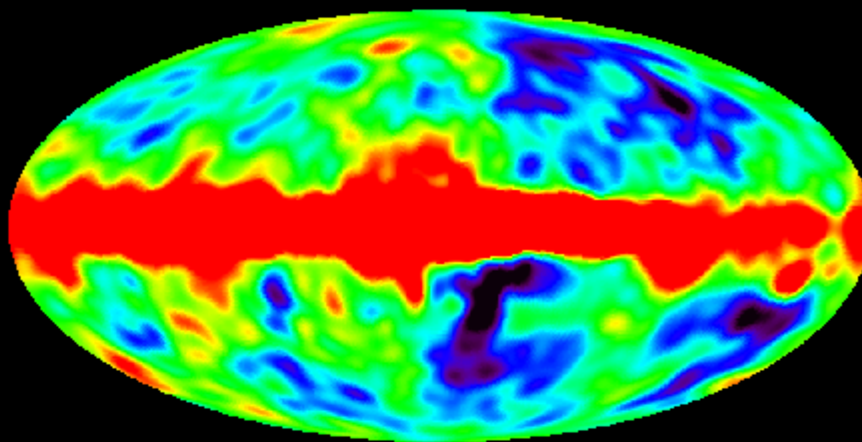


- relic thermal radiation
  from the hot big bang
- 3 Kelvin, just three degrees
  above absolute zero
- Many ground- and balloon-based experiments since
  - e.g., South Pole Telescope, Atacama Cosmology Telescope

CoBE satellite

launched by NASA in 1990





2.7 K in all directions

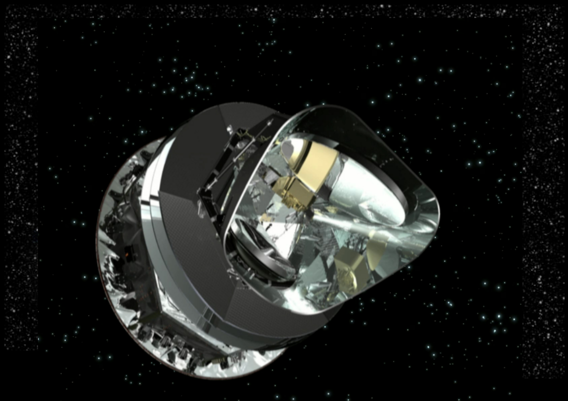
+/- 3.3 mK Doppler shift due local motion
(at 1 million miles per hour)

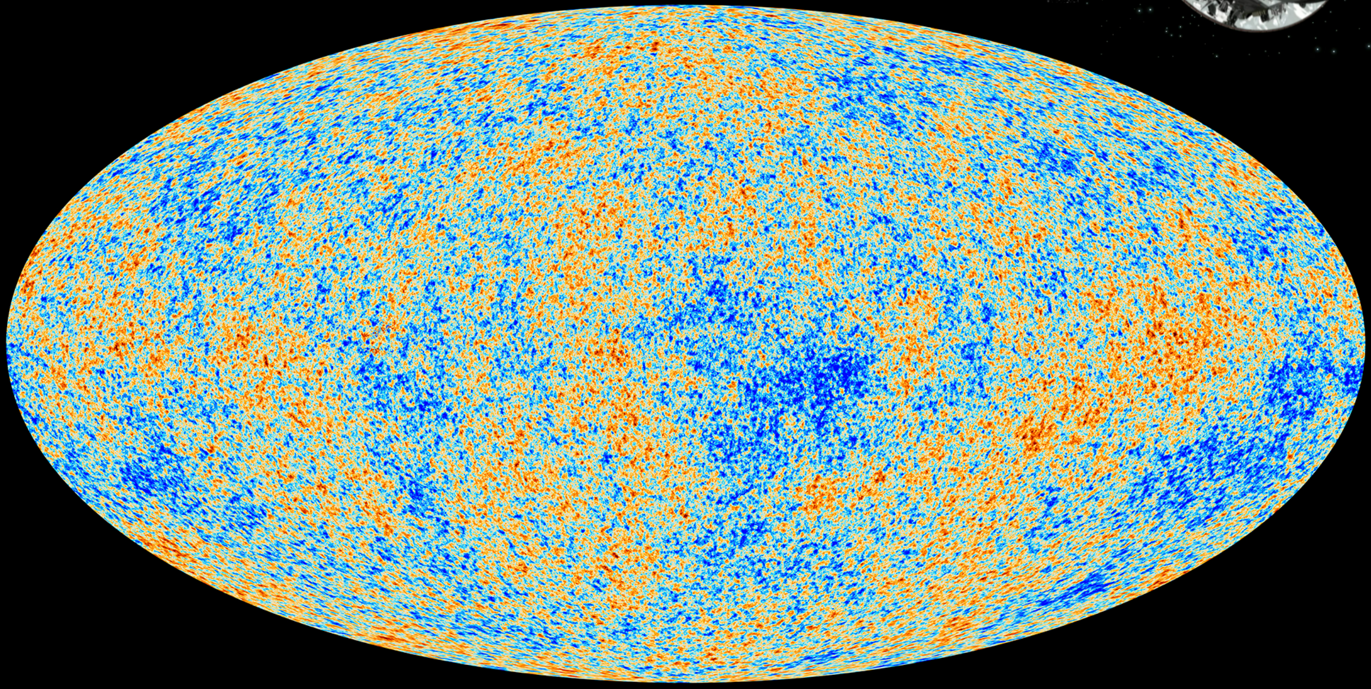
+/- 18 μK intrinsic anisotropies

COBE launched 1990

WMAP launched 2001, final data 2012

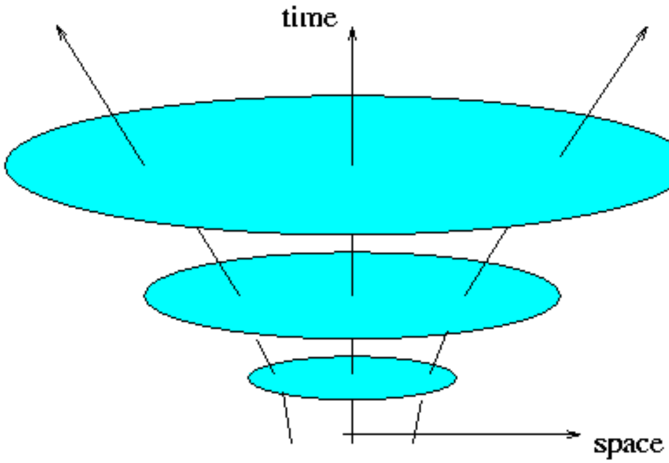
Planck launched 2009, first data 2013





Friedmann's dynamic cosmology

- slice up 4D spacetime into expanding 3D space
  with uniform matter density and spatial curvature



scale factor $a(t)$

Hubble rate $H \equiv \dot{a} / a$

$$H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$$

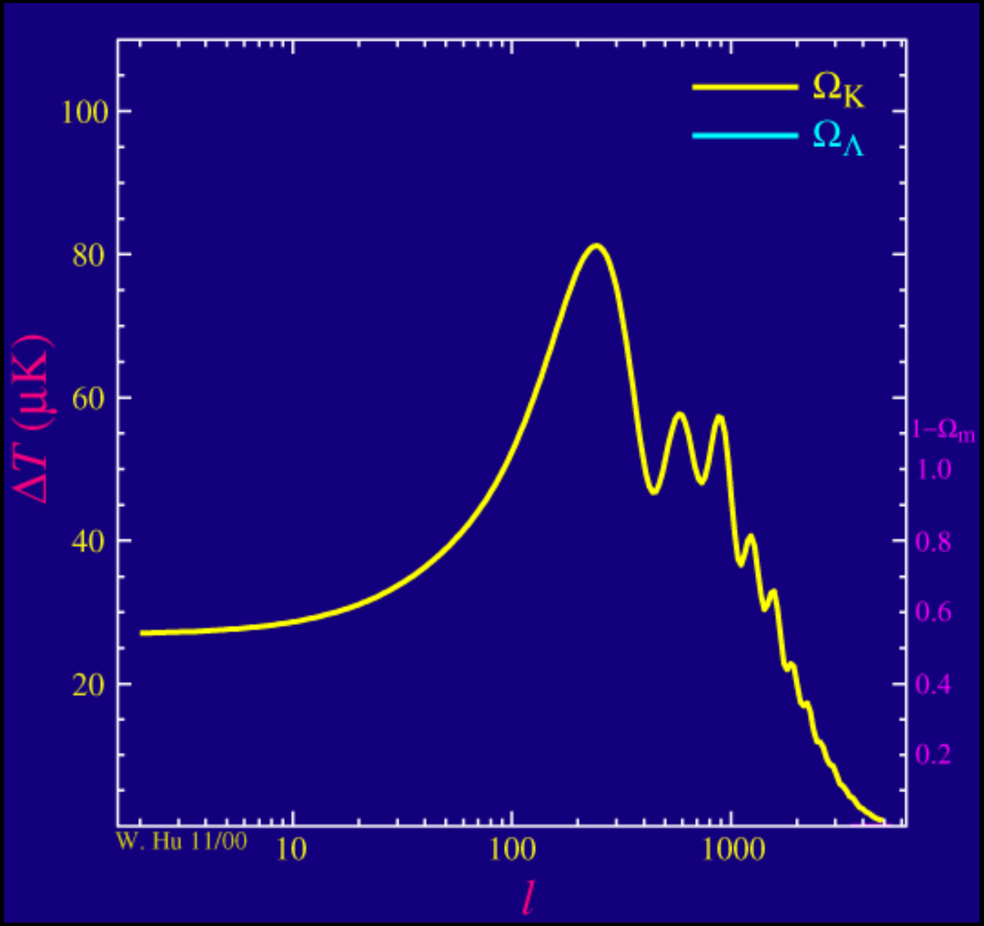
$$h \approx 0.7$$

- Friedmann equation
  - from Einstein's energy constraint:

$$H^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{\kappa}{a^2}$$

$$\Rightarrow 1 = \Omega_m + \Omega_\Lambda + \Omega_\kappa$$

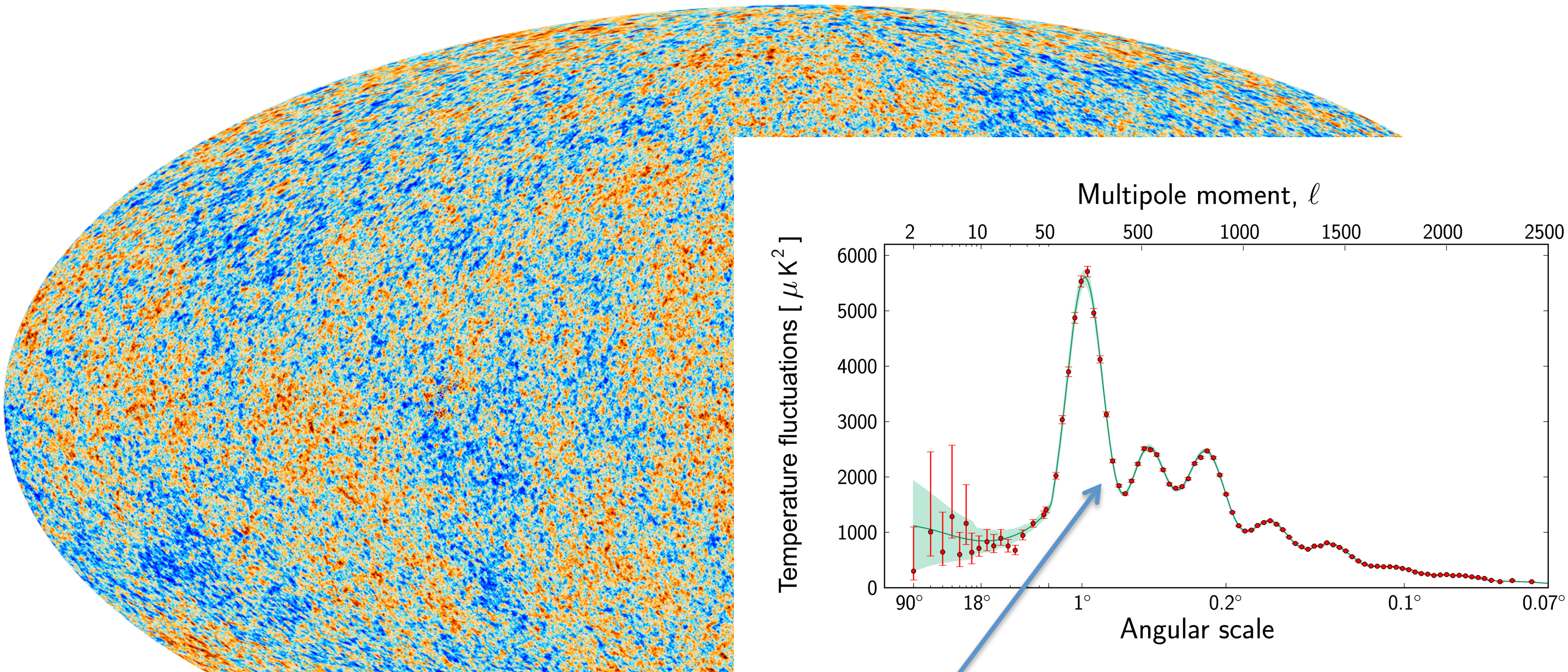
Precision cosmology from angular power spectrum



background.uchicago.edu/~whu

angular scale indicates flat space geometry, but also depends on nature of energy density in the universe

Planck - new standard model of primordial cosmology



Cosmological parameters (Ade *et al*: Planck 2013 results. XVI)

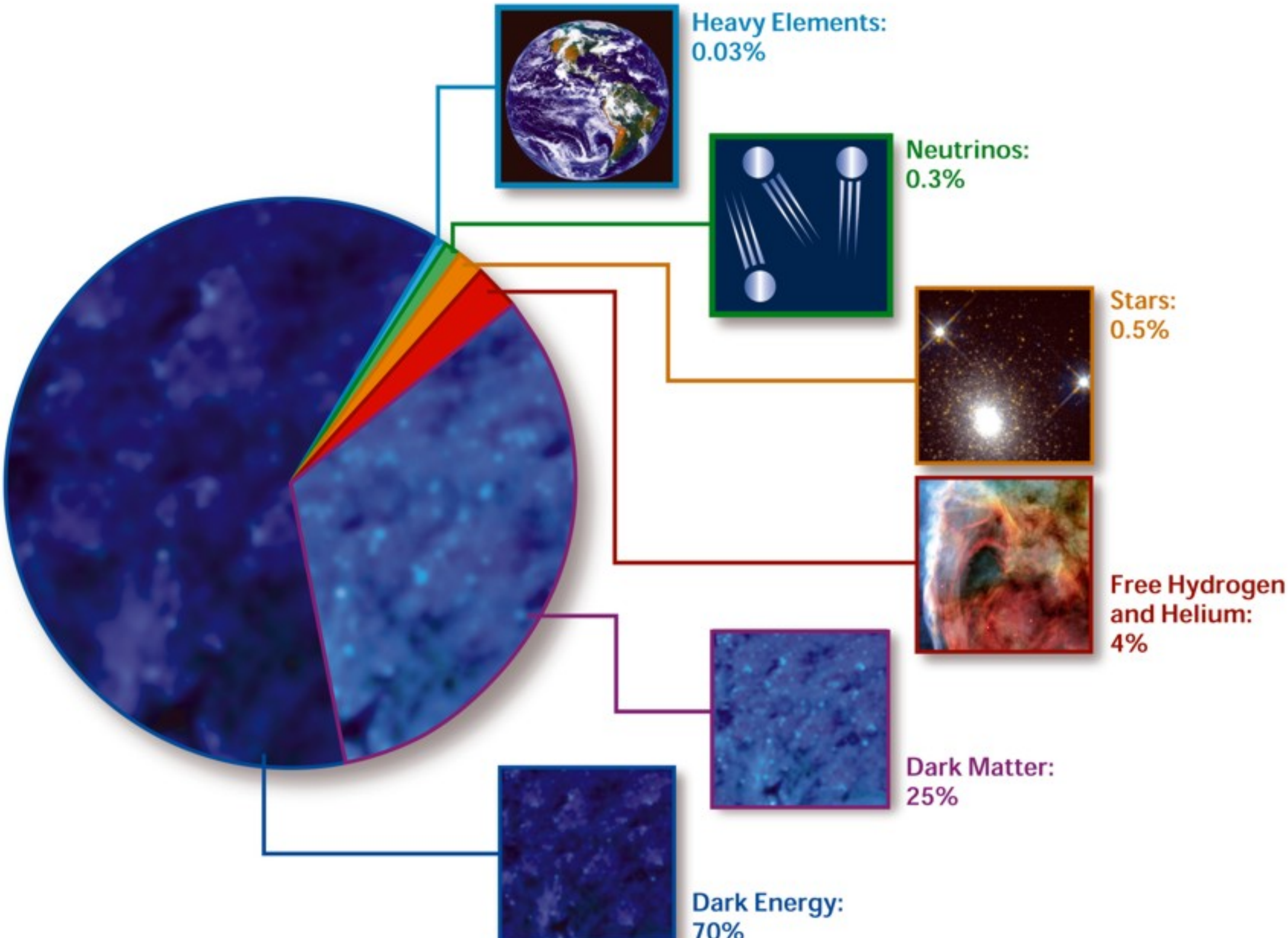
$$h = 0.674 \pm 0.014$$

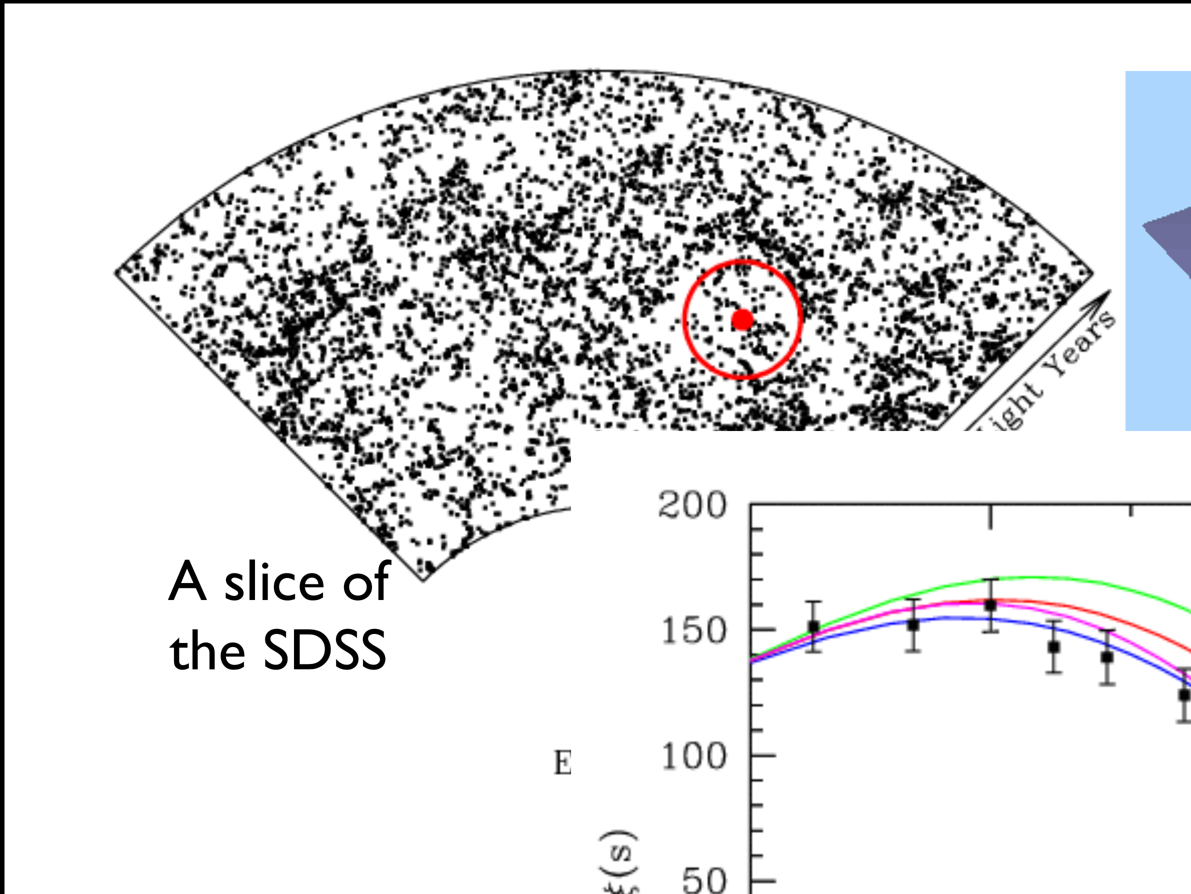
$$\Omega_\Lambda = 0.686 \pm 0.020$$

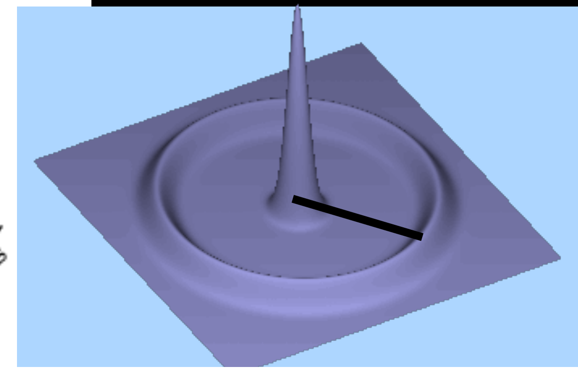
$$\Omega_{matter} = 0.314 \pm 0.020$$

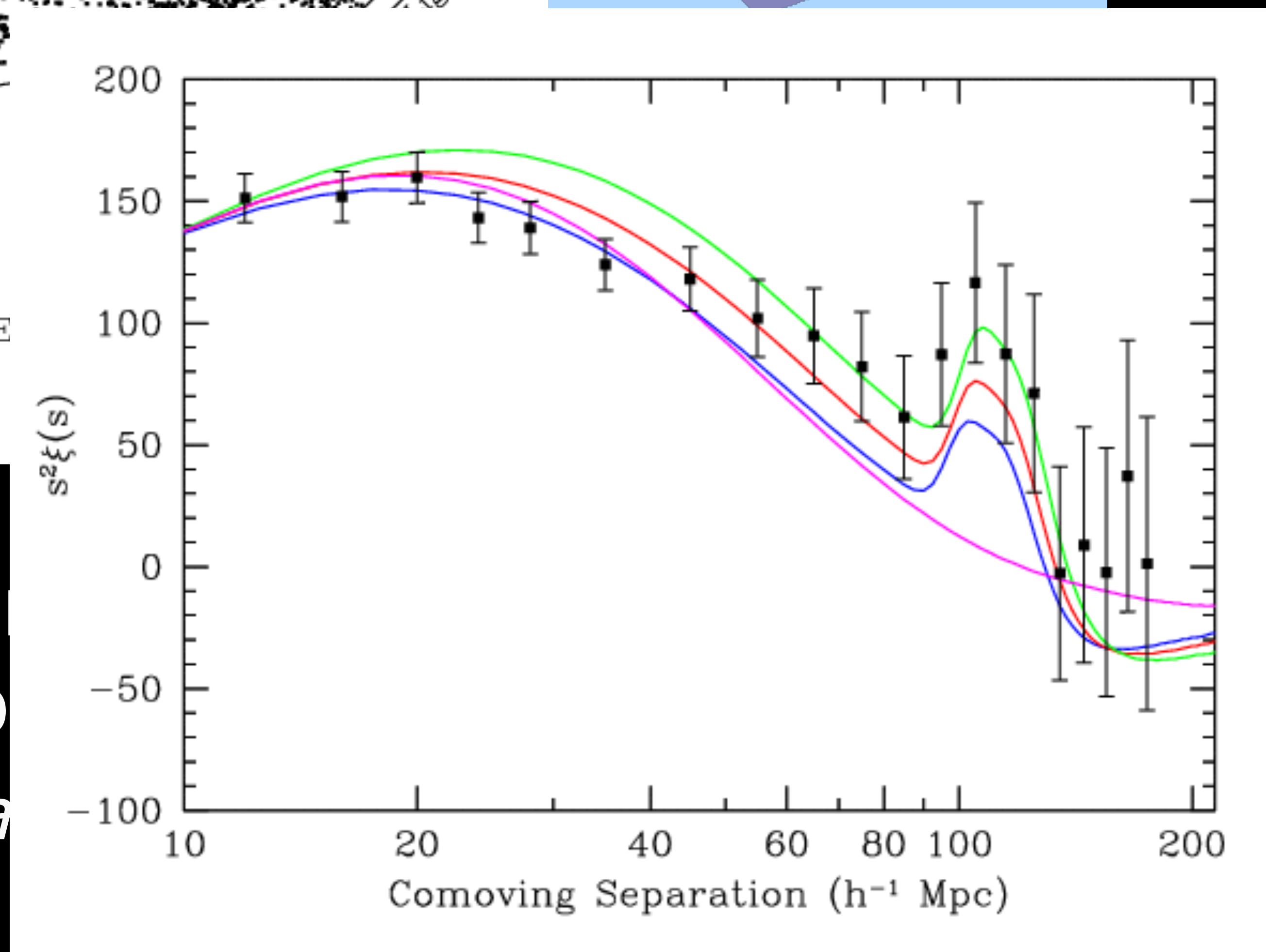
$$\Omega_\kappa = -0.04 \pm 0.05$$

COMPOSITION OF THE COSMOS









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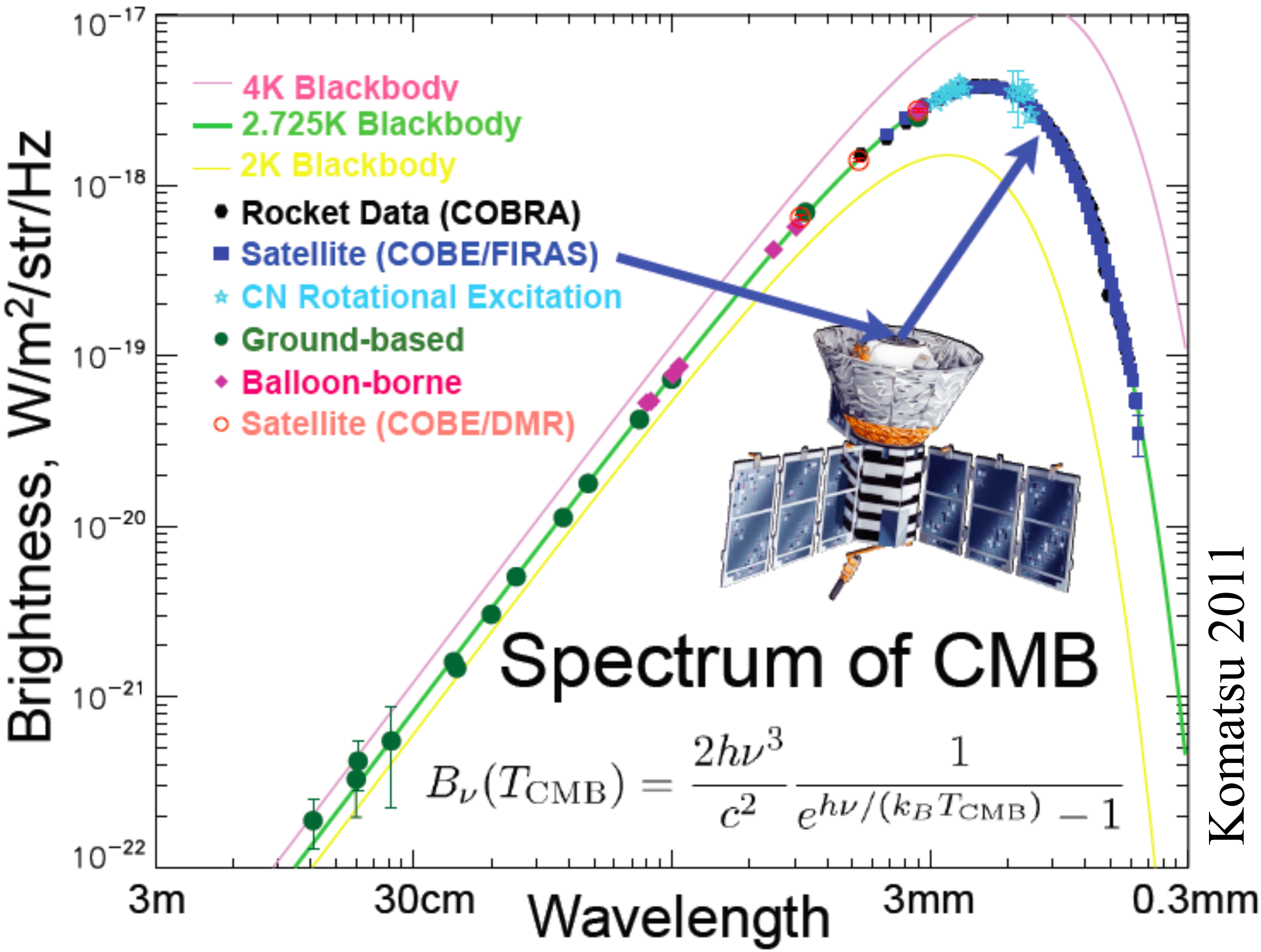
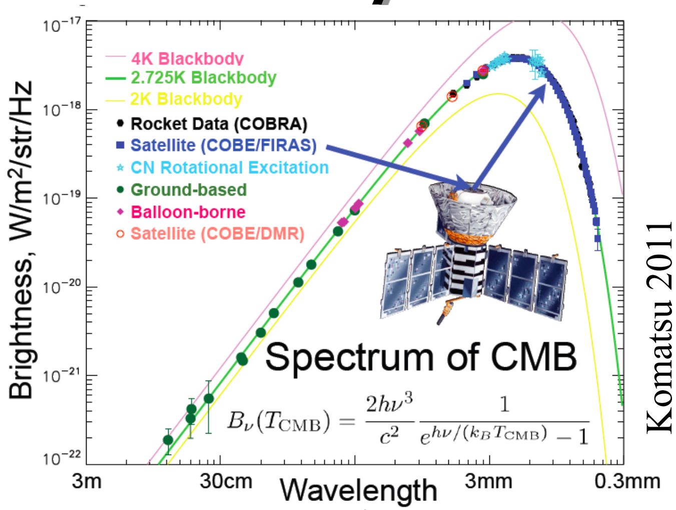
The isotropic CMB

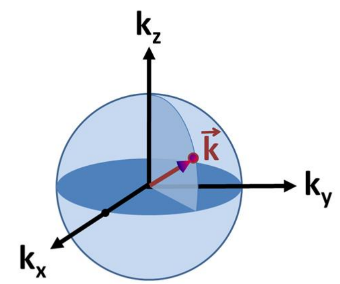
- a brief thermal history

CMB = Black-body spectrum, T_0=2.725K



Black-body <= thermal equilibrium





Photon energy: $$E = h\nu = \hbar p$$

Einstein-Boltzmann distribution: $$f(p) = \frac{1}{\exp(\hbar p/k_B T) - 1}$$

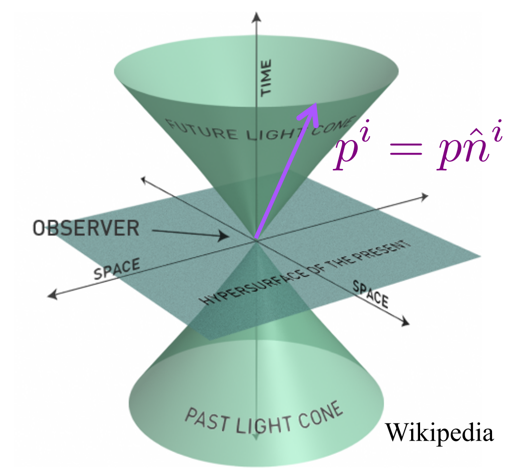
Number density: $$n_\gamma = \frac{2}{(2\pi)^3}\int 4\pi p^2 f(p)\, dp \simeq \frac{2.4}{\pi^2}\left(\frac{k_B T}{\hbar c}\right)^3$$

Energy density: $$\rho_\gamma c^2 = \frac{2}{(2\pi)^3}\int 4\pi p^2 f(p)\, \hbar p\, dp = \frac{\pi^2}{15\hbar^3 c^3}(k_B T)^4$$

but not in thermal equilibrium with matter today

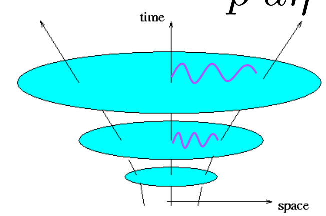
FLRW geometry

- Spatially flat metric: $ds^2 = -c^2 dt^2 + a^2 \delta_{ij} dx^i dx^j = a^2 \left[-d\eta^2 + \delta_{ij} dx^i dx^j \right]$
  - where conformal time = horizon size: $$\eta = \int \frac{c\, dt}{a}$$



Wikipedia

- Photon trajectory: $$\frac{dx^i}{d\eta} = \hat{n}^i = \text{unit vector}$$
- 4-momentum: $$P^\mu = \frac{dx^\mu}{d\lambda}$$
- 3-momentum: $$p^2 = g_{ij} P^i P^j$$
- Geodesic equation: $$\frac{dP^\mu}{d\lambda} + \Gamma^\mu_{\nu\rho} P^\nu P^\rho = 0 \quad \Rightarrow \quad \frac{1}{p}\frac{dp}{d\eta} = -\frac{1}{a}\frac{da}{d\eta}$$
- Hubble redshift: $$1 + z \equiv \frac{p}{p_0} = \frac{a_0}{a}$$



  - *preserves Einstein-Boltzmann distribution:* $$f(p) = \frac{1}{\exp(\hbar p / k_B T) - 1} = \frac{1}{\exp(\hbar p_0 / k_B T_0) - 1}$$
  - *where temperature redshift:* $$1 + z = \frac{T}{T_0} = \frac{a_0}{a}$$

natural units:

for photons, distance = time, momentum = wavenumber = energy = temperature

but by convention they have different units which we have to keep track of

much easier to use natural units, such that $\hbar = k_B = c = 1$

Leaves only one dimensional constant = Newton's constant

so only unit = Planck unit

$$M_P = \sqrt{\frac{\hbar c}{G}} = \frac{1}{\sqrt{G}} = 2.177 \times 10^{-5} \text{ g} .$$

The corresponding energy $M_P c^2 = 1.223 \times 10^{19}$GeV. The Planck temperature is, enough,

$$T_P = \frac{M_P c^2}{k_B} = \frac{1}{\sqrt{G}} = 1.42 \times 10^{32} \text{ K} ,$$

and the corresponding Compton wavelength gives the Planck length

$$L_P = \frac{\hbar}{M_P c^2} = \sqrt{G} = 1.616 \times 10^{-33} \text{ cm} ,$$

or Planck time

$$t_P = \frac{L_P}{c} = \sqrt{G} = 5.390 \times 10^{-44} \text{ sec} .$$

FLRW background dynamics

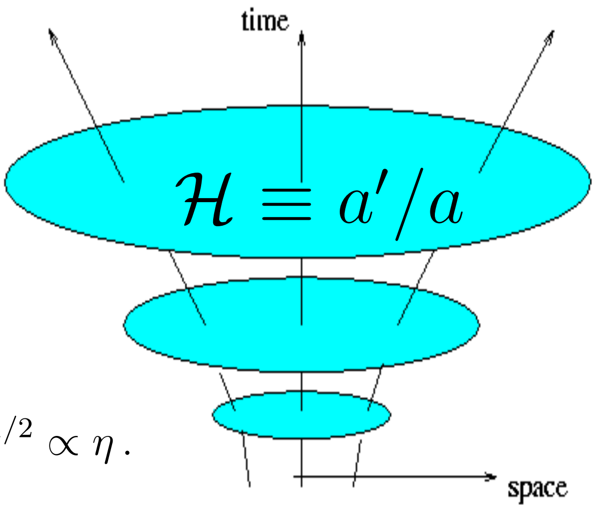
- Spatially flat metric: $ds^2 = -c^2 dt^2 + a^2 \delta_{ij} dx^i dx^j = a^2 \left[-d\eta^2 + \delta_{ij} dx^i dx^j \right]$

- Einstein energy constraint G_{00} + evolution G_{ij} equations:

$$\begin{aligned} \mathcal{H}^2 &= \frac{8\pi G}{3} a^2 \rho \,, \\ \mathcal{H}' &= -\frac{4\pi G}{3} a^2 \left(\rho + 3P \right) \end{aligned}$$

- Energy conservation

$$\rho' = -3\mathcal{H} \left(\rho + P \right)$$



Radiation domination:

$$P_\gamma = \rho_\gamma / 3 \,, \qquad \rho_\gamma \propto a^{-4} \,, \qquad a \propto t^{1/2} \propto \eta \,.$$

Matter domination:

$$P_m = 0 \,, \qquad \rho_m \propto a^{-3} \,, \qquad a \propto t^{2/3} \propto \eta^2 \,.$$

Matter-radiation equality

$$1 + z_{eq} \equiv \frac{a_0}{a_{eq}} = \frac{\rho_{m,0}}{\rho_{r,0}} = 3.4 \times 10^3 \left(\frac{\Omega_m h^2}{0.14} \right)$$

Temperature-time relation:

At sufficiently high temperatures ($k_B T \gg mc^2$, the rest mass energy of the particle) we expect all types of elementary particles to become relativistic. If they interact with the photons they will share the same equilibrium temperature and so all types of relativistic particles will have a density proportional to T^4. We can write the density as

$$\rho = g_{\mathrm{eff}} \frac{\pi^2}{30} \frac{(k_B T)^4}{\hbar^3 c^5} \tag{58}$$

where g_{eff} is a sum over the effective number of degrees of freedom of particles. g is 2 for photons alone, corresponding to the two different polarizations. All other relativistic bosons (with integer spin) counts one per degree of freedom, while for fermions (half-integer spins, e.g., neutrinos) it is 7/8 per degree of freedom due to their Fermi statistics rather than Bose-Einstein statistics [see Eq.(69)].

Remember, however that the energy density in a $K = 0$ radiation dominated model can also be written in terms of the expansion rate, and thus the time since the big bang:

$$\rho = \frac{3H^2}{8\pi G} = \frac{3}{32\pi G (t - t_*)^2} \tag{59}$$

Thus we have a relation between temperature and time

$$t - t_* = \frac{1}{2H} = \sqrt{\frac{3}{32\pi G} \frac{30\hbar^3 c^5}{g_{\mathrm{eff}} \pi^2} \frac{1}{(k_B T)^2}}\ , \tag{60}$$

which can be written as

$$\frac{t - t_*}{1\ \mathrm{sec}} \approx \frac{1}{\sqrt{g_{\mathrm{eff}}}} \left(\frac{1\ \mathrm{MeV}}{k_B T} \right)^2\ . \tag{61}$$

electron-photon scattering:

- Scattering processes
  - double (radiative) Compton scattering:
    changes photon number

    $$e^- + \gamma \quad \leftrightarrow \quad e^- + \gamma + \gamma$$

  - Compton scattering:
    (relativistic electrons)

    $$e^- + \gamma \quad \leftrightarrow \quad e^- + \gamma$$

  - Thomson scattering:
    (non-relativistic electrons -> elastic)

    $$e^- + \gamma \quad \leftrightarrow \quad e^- + \gamma$$

energy

Spectral distortions:

- Full thermalization $e^- + \gamma \quad \leftrightarrow \quad e^- + \gamma + \gamma$
  - double Compton scattering maintains black-body spectrum above redshift

$$z_{th} = 2 \times 10^6 \left(\frac{\Omega_b h^2}{0.02} \right)^{-2/5}$$

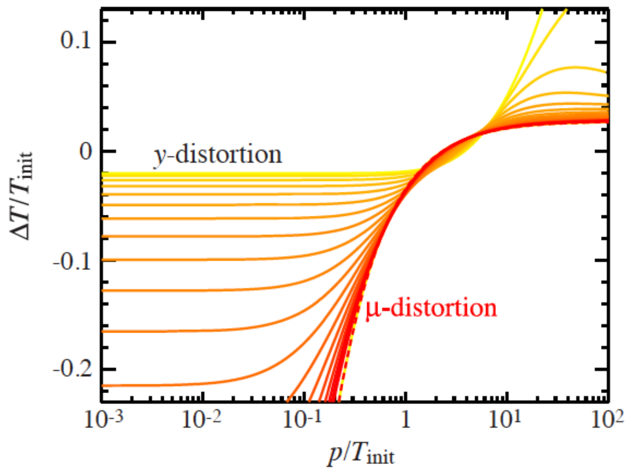
  - below this Compton scattering redistributes energy, but conserves photon number

- μ-distortion $e^- + \gamma \quad \leftrightarrow \quad e^- + \gamma$
  - Compton scattering maintains statistical equilibrium

$$f(p) = \frac{1}{\exp((\hbar p - \mu)/k_B T) - 1}$$

above redshift

$$z_\mu = 5 \times 10^4 \left(\frac{\Omega_b h^2}{0.02} \right)^{-1/2}$$



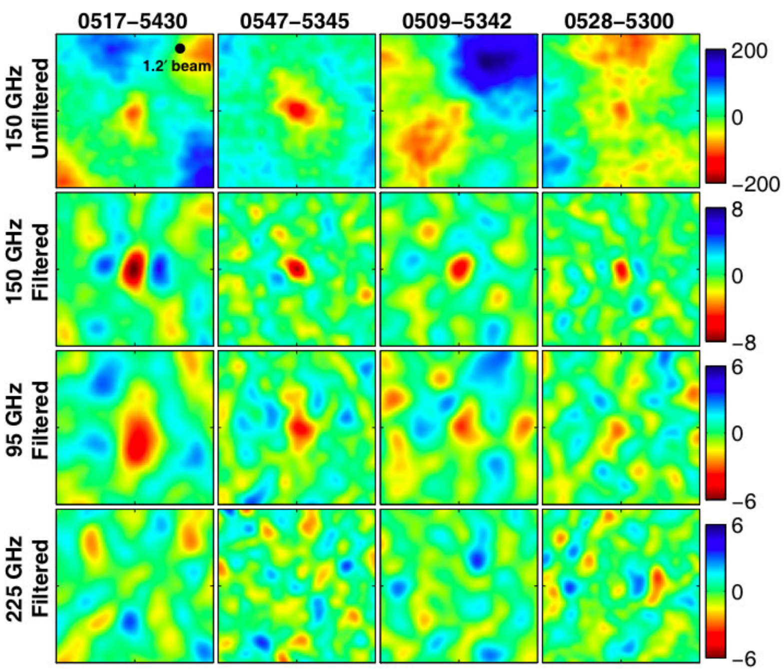
- y-distortion
  - low frequency electrons can gain energy from energetic electrons (inverse Compton)
  - leads to deficit in intensity in low energy (Rayleigh-Jeans) region of spectrum

$$\left.\frac{\Delta T}{T}\right|_{\hbar p \ll k_B T} = -2y$$

  - e.g., thermal Sunyaev-Zel'dovich from hot cluster gas along line of sight

Mechanisms leading to spectral distortions:

- particle decay or annihilation?
  - but electron-positron annihilation, for example, occurs at $z >> z_{th}$
- evaporation of primordial black holes?
- dissipation of (large?) density perturbations on small scales
- thermal Sunyaev-Zel'dovich (y-distortion) from hot cluster gas along line of sight
  - e.g., South Pole Telescope (2009)
  - Planck SZ cluster survey



All-sky upper bounds from COBE/FIRAS

$$|y| < 1.5 \times 10^{-5}$$

$$|\mu| < 9 \times 10^{-5}$$

end of part one