中国科学技术大学
University of Science and Technology of China

Primordial Cosmology:
What you need to know as a beginner

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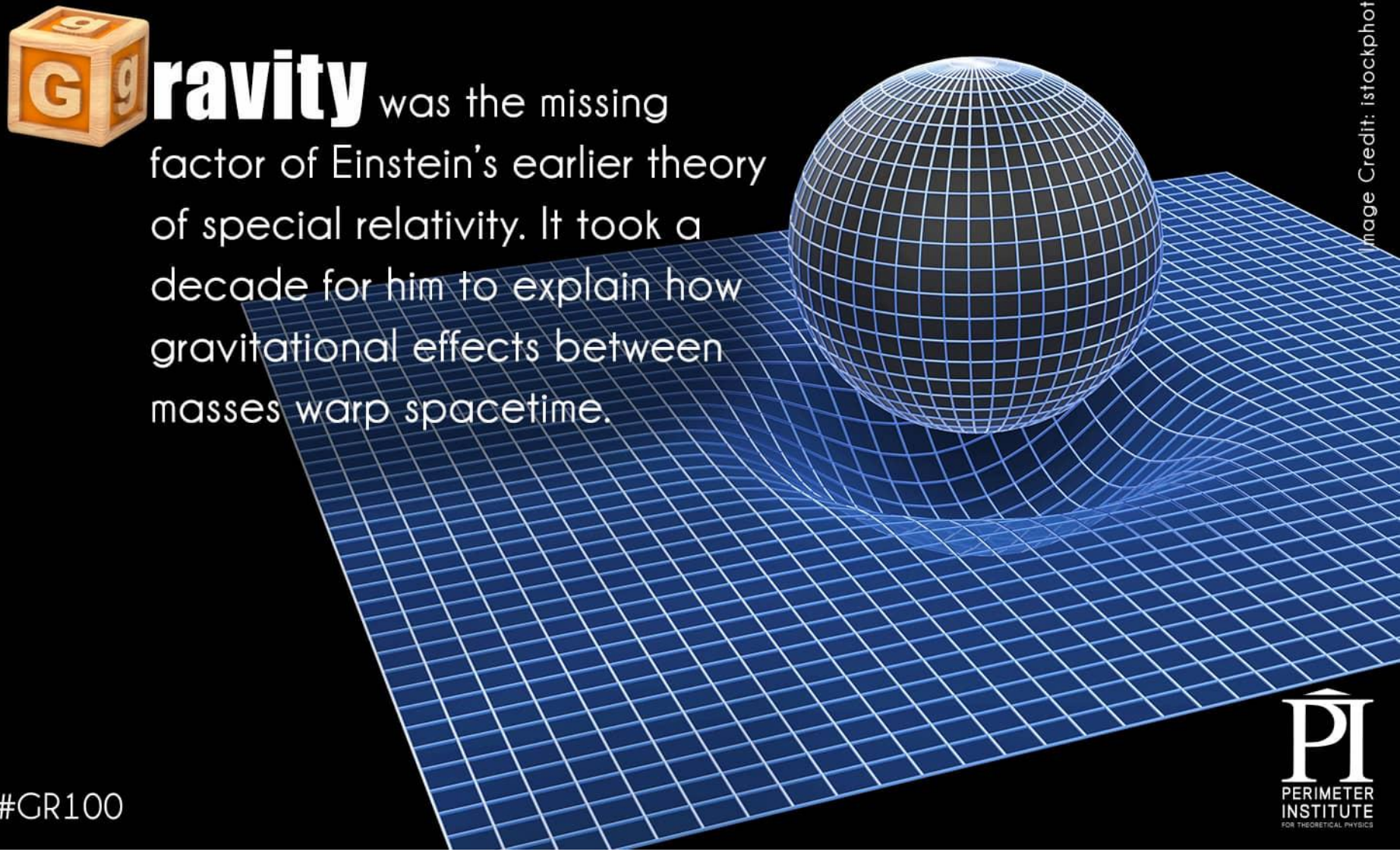
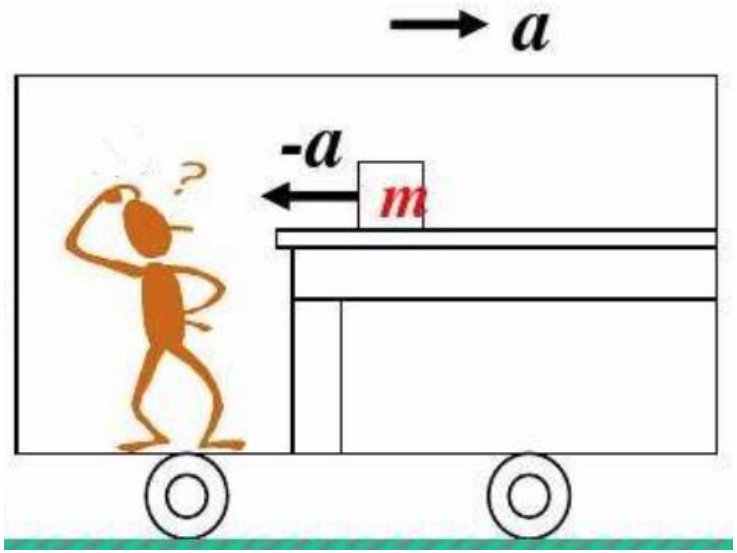
Outline

- Basics of General Relativity
- Basics of Modern Cosmology
- A Brief History of Our Universe
- Origin of the Universe: Inflation or Bounce?

Basics of General Relativity

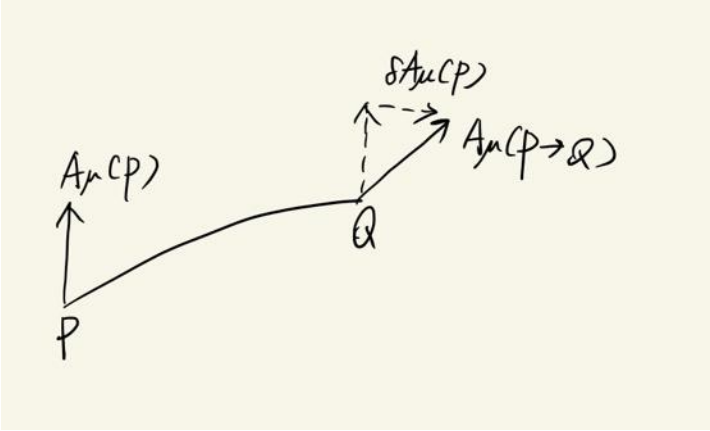

Gravity was the missing factor of Einstein's earlier theory of special relativity. It took a decade for him to explain how gravitational effects between masses warp spacetime.
#GR100
Image Credit: istockphoto
PERIMETER INSTITUTE
FOR THEORETICAL PHYSICS

➢Equivalence Principle



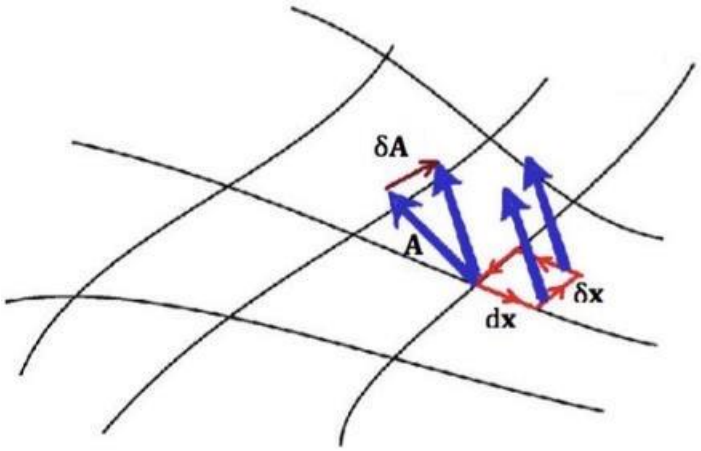
$$\frac{d^2x^\mu}{d\tau^2} = 0 \longrightarrow \frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\sigma}\frac{dx^\nu}{d\tau}\frac{dx^\sigma}{d\tau} = 0$$

Connection



$\Gamma^{\lambda}_{\mu\nu}$ is a geometrical quantity

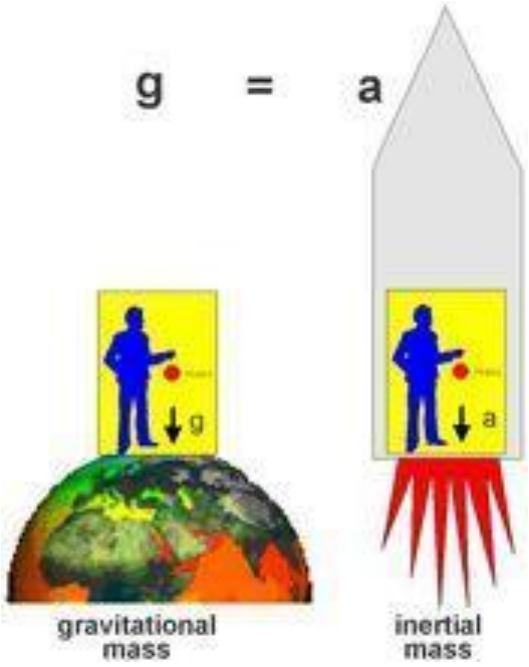
$$\delta A_\mu(P) \equiv A_\mu(P \to Q) - A_\mu(P) = \Gamma^{\lambda}_{\mu\nu} A_\lambda dx^\nu$$

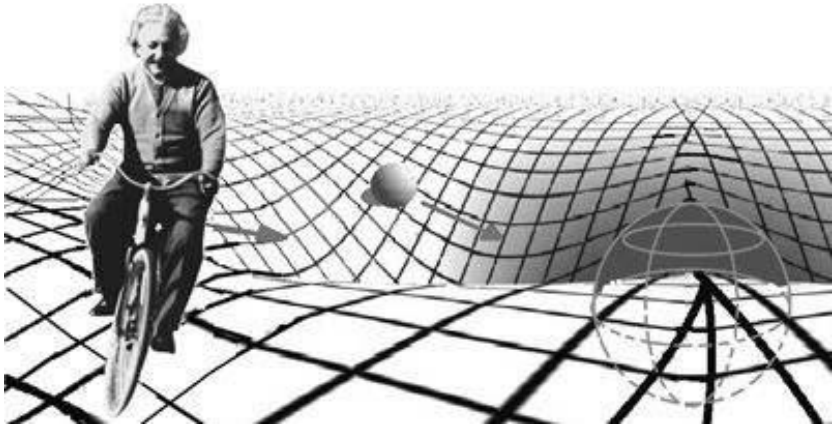


$$\Delta A^\mu = -R^{\mu}_{\alpha\beta\gamma} A^\alpha dx^\beta \delta x^\gamma$$

Riemann curvature:

$$R^{\rho}_{\mu\nu\sigma} = \partial_\nu \Gamma^{\rho}_{\mu\sigma} - \partial_\mu \Gamma^{\rho}_{\nu\sigma} + \Gamma^{\lambda}_{\sigma\mu}\Gamma^{\rho}_{\nu\lambda} - \Gamma^{\lambda}_{\sigma\nu}\Gamma^{\rho}_{\mu\lambda}$$



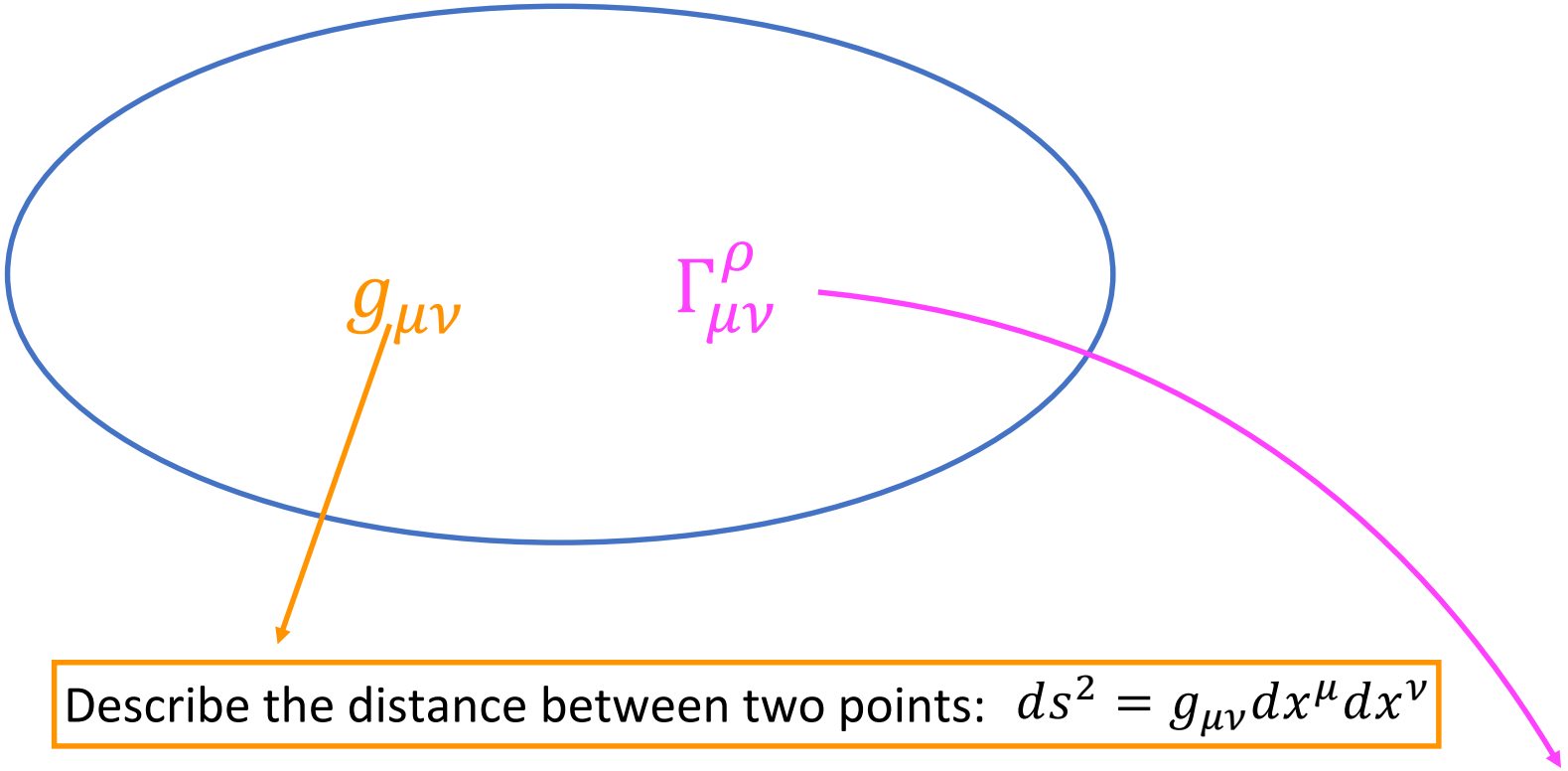


Gravity can be described by geometry!

"Gravity force" is a consequence of the curved spacetime!

➢Einstein Equations

For a Riemann manifold



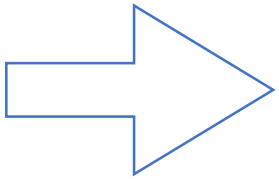
Describe the distance between two points: $ds^2 = g_{\mu\nu}dx^{\mu}dx^{\nu}$

Describe translation properties of vectors: $\delta A_{\mu}(P) \equiv A_{\mu}(P \to Q) - A_{\mu}(P) = \Gamma^{\lambda}_{\mu\nu}A_{\lambda}dx^{\nu}$

We expect:

Torsion-less: $\Gamma^{\rho}_{\mu\nu} = \Gamma^{\rho}_{\nu\mu}$

Metric conservation: $\nabla_{\sigma} g_{\mu\nu} = \partial_{\sigma} g_{\mu\nu} - g_{\alpha\nu}\Gamma^{\alpha}_{\mu\sigma} - g_{\mu\alpha}\Gamma^{\alpha}_{\nu\sigma} = 0$

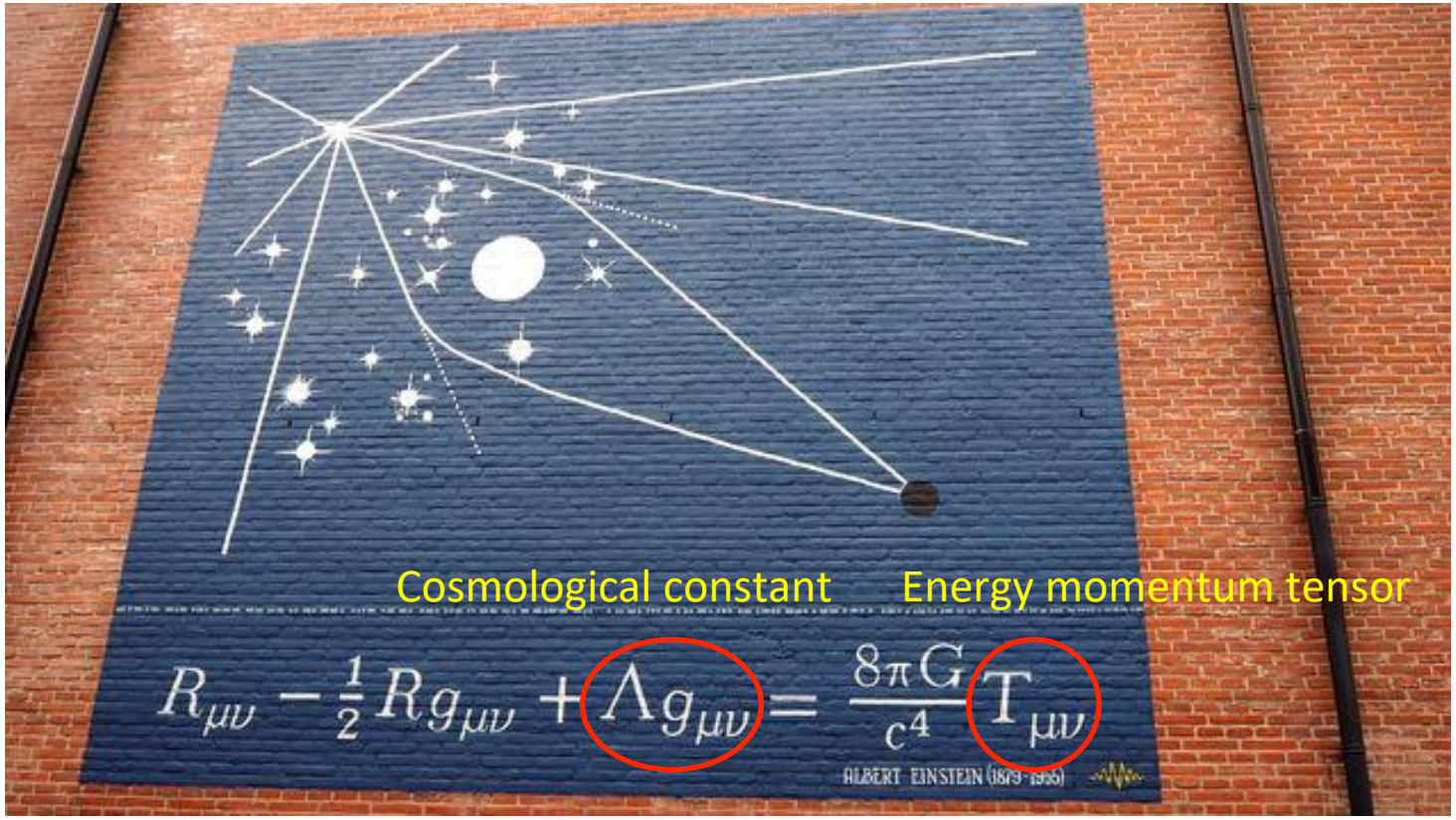


$$\Gamma^{\alpha}_{\sigma\mu} = \frac{g^{\alpha\nu}}{2}\left(\partial_{\sigma} g_{\mu\nu} + \partial_{\mu} g_{\nu\sigma} - \partial_{\nu} g_{\sigma\mu}\right)$$

The only tensor that can be constructed by: $\Gamma^{\rho}_{\mu\nu}$

$$R^{\rho}_{\mu\nu\sigma} = \partial_{\nu}\Gamma^{\rho}_{\mu\sigma} - \partial_{\mu}\Gamma^{\rho}_{\nu\sigma} + \Gamma^{\lambda}_{\sigma\mu}\Gamma^{\rho}_{\nu\lambda} - \Gamma^{\lambda}_{\sigma\nu}\Gamma^{\rho}_{\mu\lambda}$$

Einstein field equations:

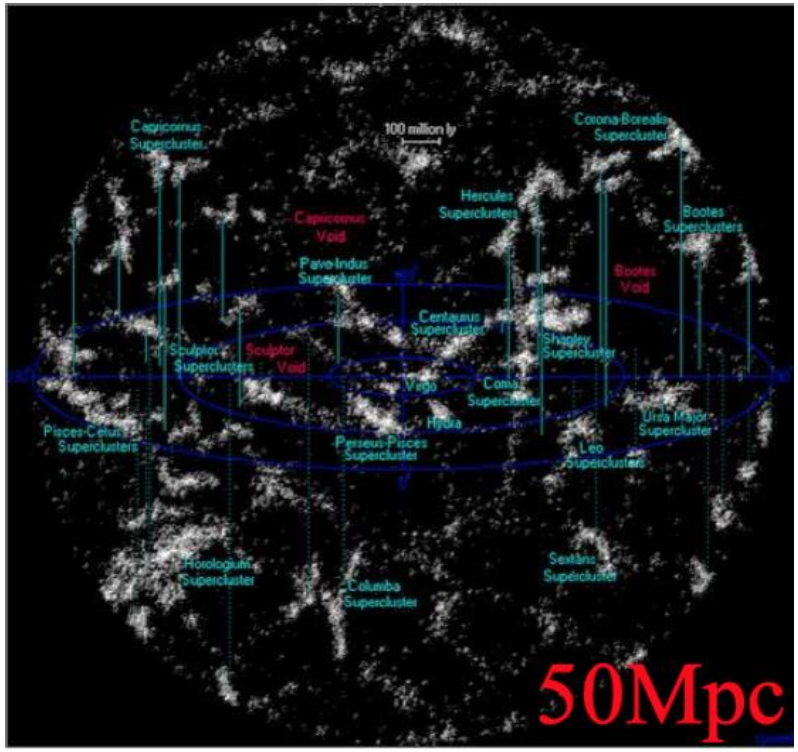


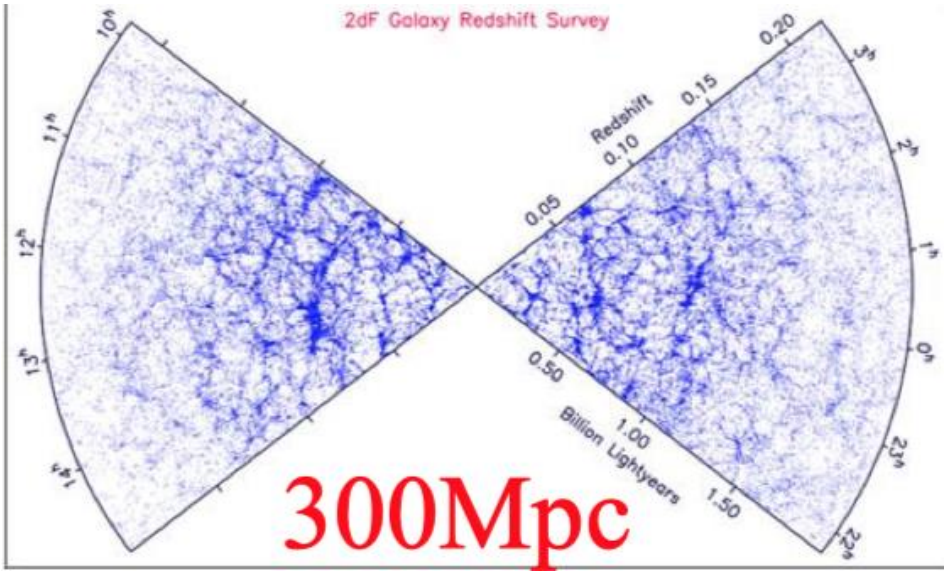
"*Spacetime tells matter how to move; matter tells spacetime how to curve.*"

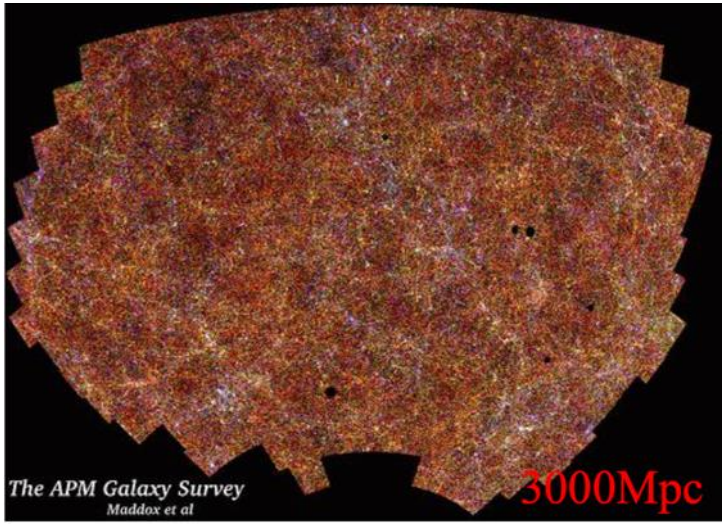
- John Wheeler

Basics of Modern Cosmology

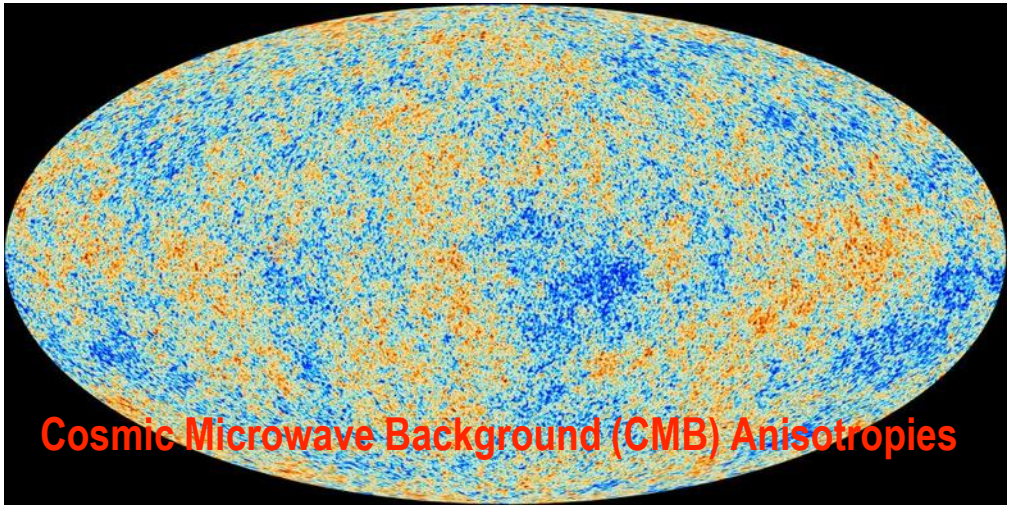
➢Cosmological Principle







Cosmological principle:
Our Universe is **homogenous** and **isotropic** on **large scales**.

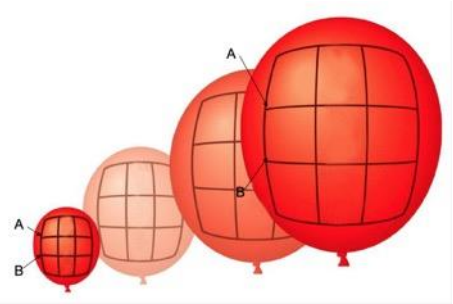


Cosmological principle:
Our Universe is **homogenous** and **isotropic** on **large scales**.

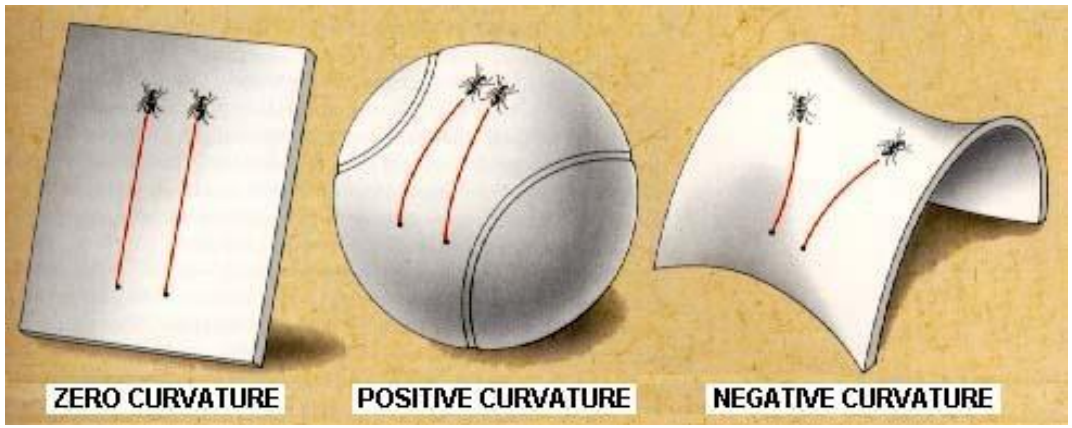
Friedmann–Lemaître–Robertson–Walker metric:

$$ds^2 = c^2dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right]$$

Scale factor

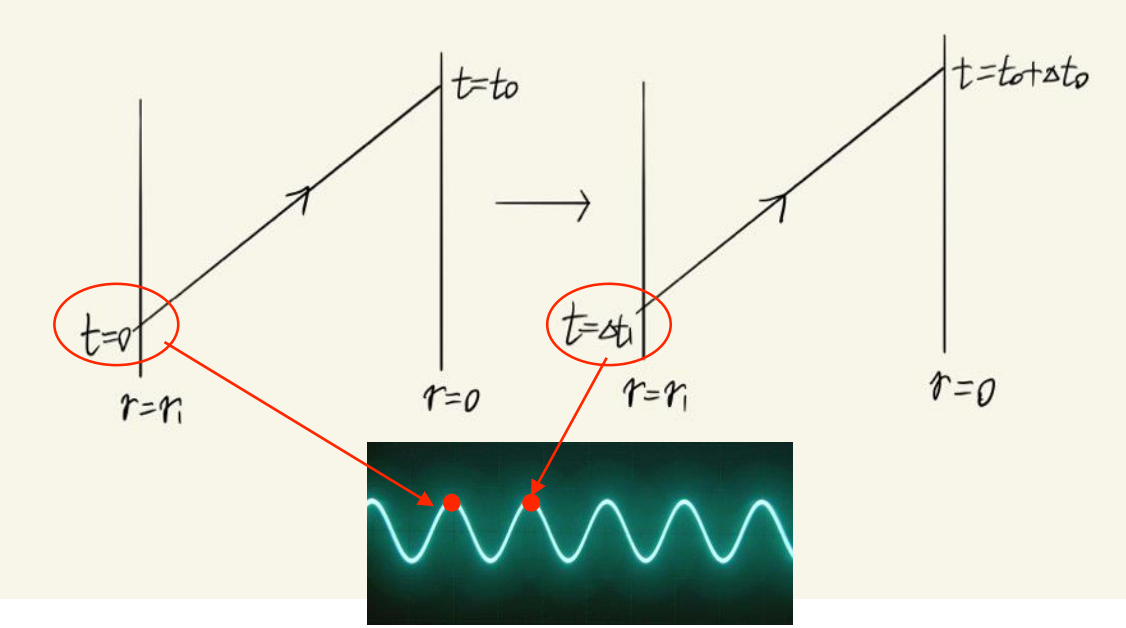


$k = 0$ $k = 1$ $k = -1$



The observations support $k = 0$

➢Cosmological redshift



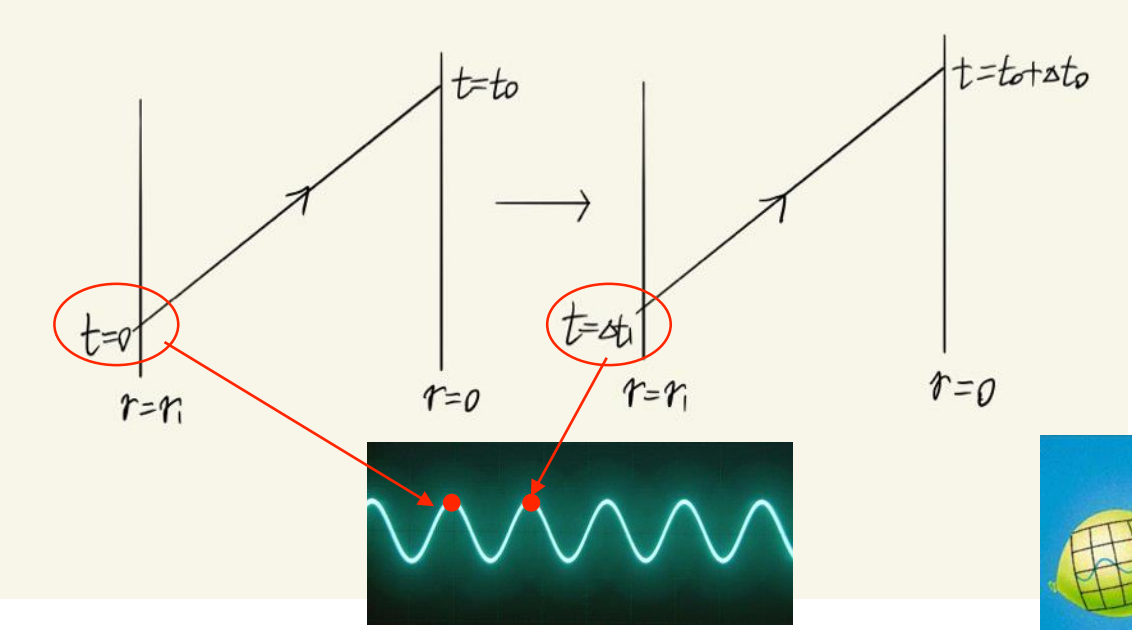
$$ds^2 = c^2dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right]$$

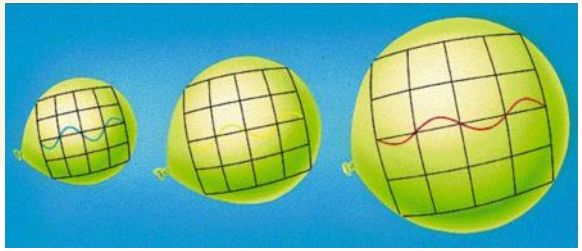
For photons, $ds^2 = 0$.

Set $d\theta = d\phi = 0$

& $c = 1$

➢Cosmological redshift





$$\int_0^{t_0} \frac{dt}{a(t)} = -\int_{r_1}^{0} \frac{dr}{\sqrt{1-kr^2}}$$

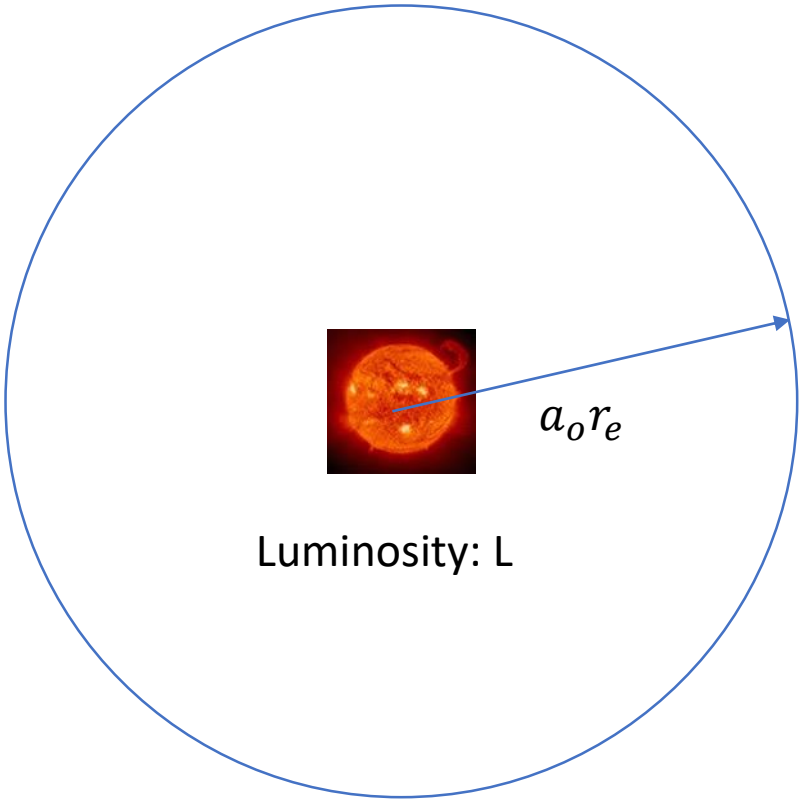
$$\int_0^{\Delta t_1} \frac{dt}{a} = \int_{t_0}^{t_0+\Delta t_0} \frac{dt}{a}$$

$$\int_{\Delta t_1}^{t_0+\Delta t_0} \frac{dt}{a(t)} = -\int_{r_1}^{0} \frac{dr}{\sqrt{1-kr^2}}$$

$$\frac{a(t_0)}{a(0)} = \frac{\Delta t_0}{\Delta t_1} = \frac{\lambda_0}{\lambda_1} = \frac{\lambda_0 - \lambda_1}{\lambda_1} + 1 \equiv z + 1$$

z is the redshift

➢Luminosity distance



The observed flux: F

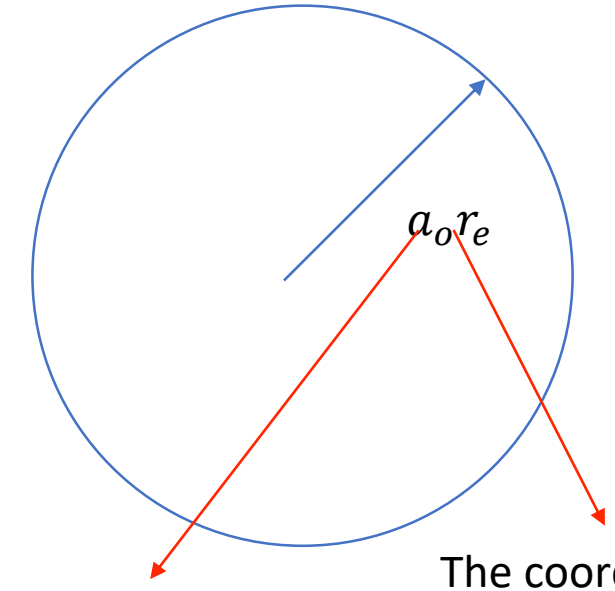
Luminosity distance:

$$d_L = \left(\frac{L}{4\pi F}\right)^{\frac{1}{2}}$$

Cosmic expansion

$$\text{Frequency} \propto \frac{1}{a}$$

$$\text{Number of photons per unit time} \propto \frac{1}{a}$$



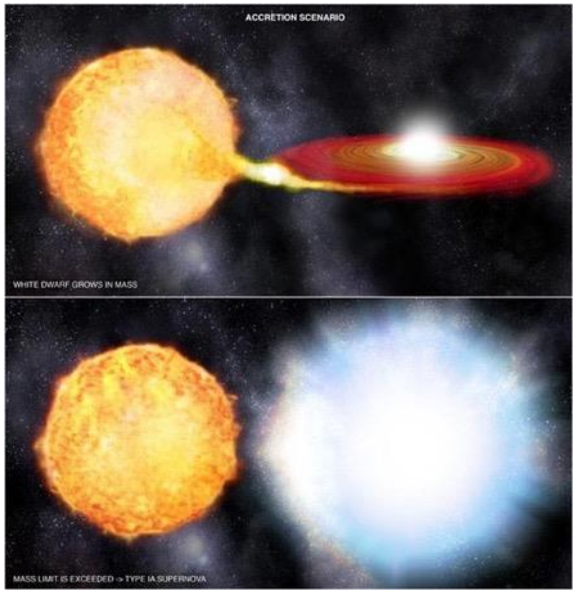
Today's scale factor

The coordinate of the source

$$\frac{F}{L} = \frac{1}{4\pi(a_0 r_e)^2}\frac{1}{(1+z)^2}$$

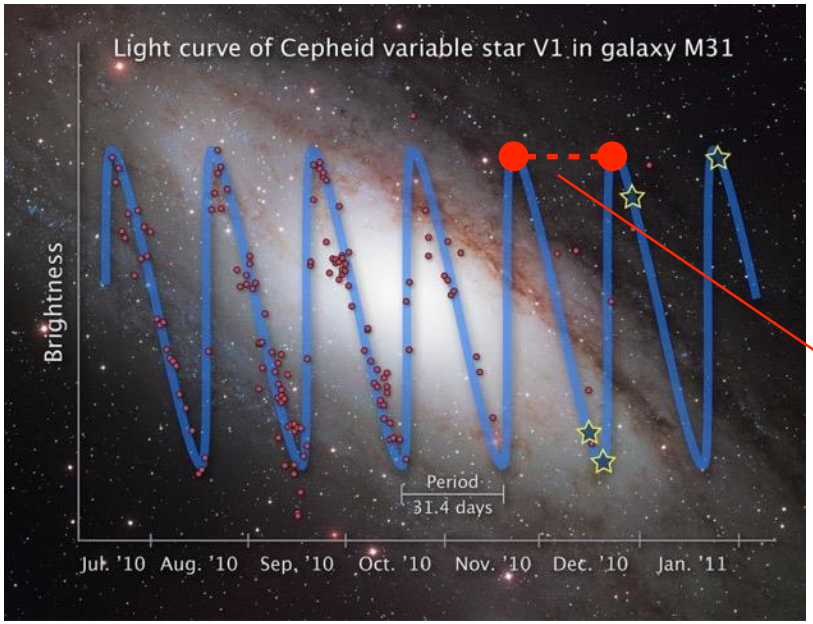
$$d_L = a_0 r_e (1+z)$$

"Standard Candle"



Type Ia supernova

2011, Nobel Prize, Saul Perlmutter, Brian P. Schmidt, and Adam G. Riess: the expansion of the Universe is **accelerating.**

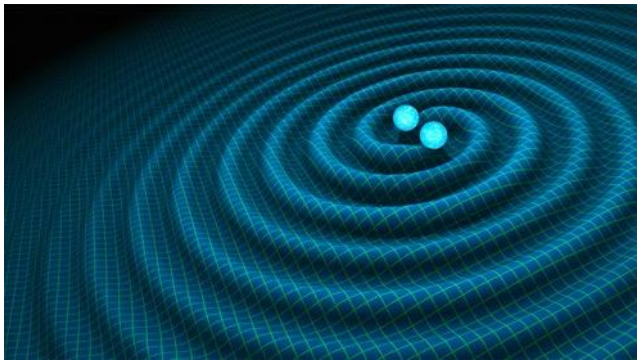


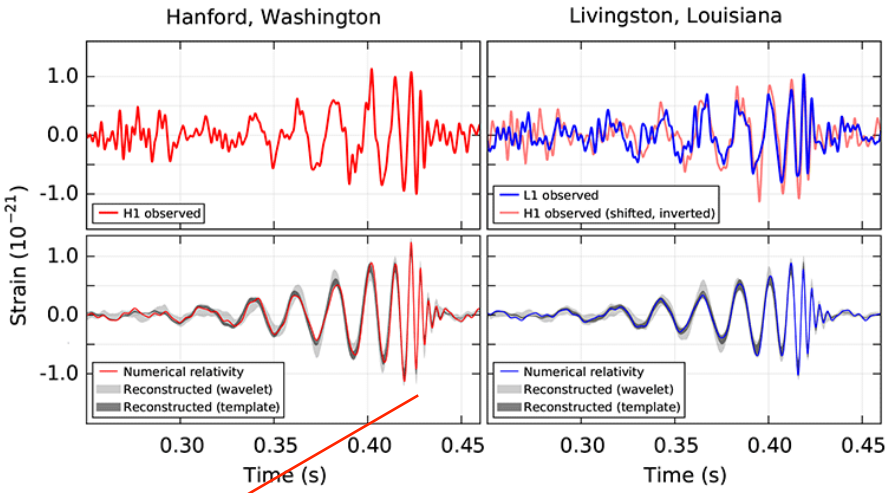
Cepheid variable star

Period ⟶ Luminosity

"Standard Siren”

Light ⟶ Gravitational waves



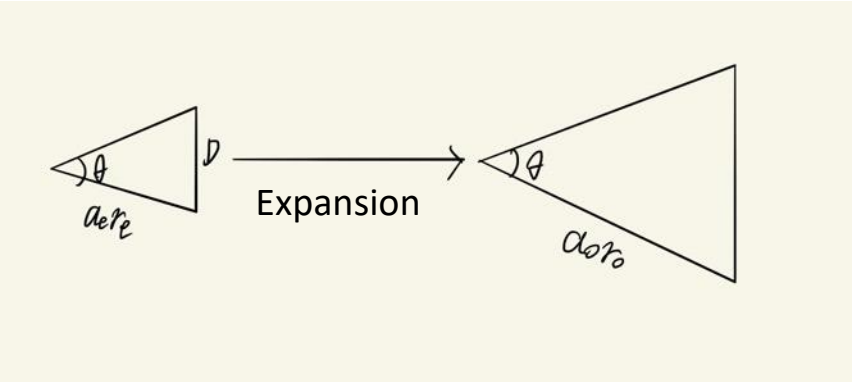


According to the waveform, we can obtain the emission power.

Luminosity

➢Angular distance

"Standard Ruler"

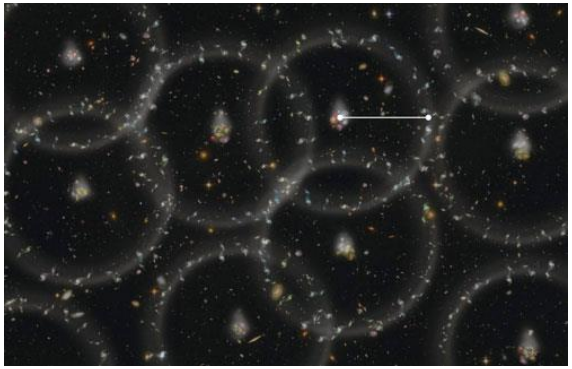


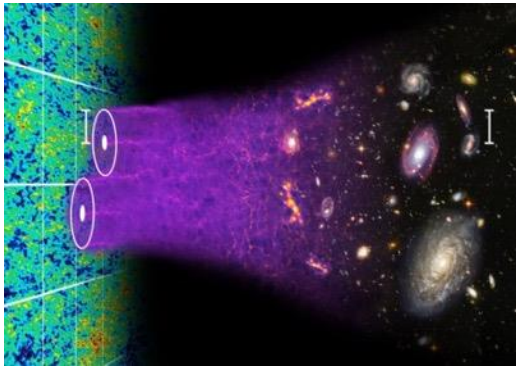
D is known.

We can get θ from observations.

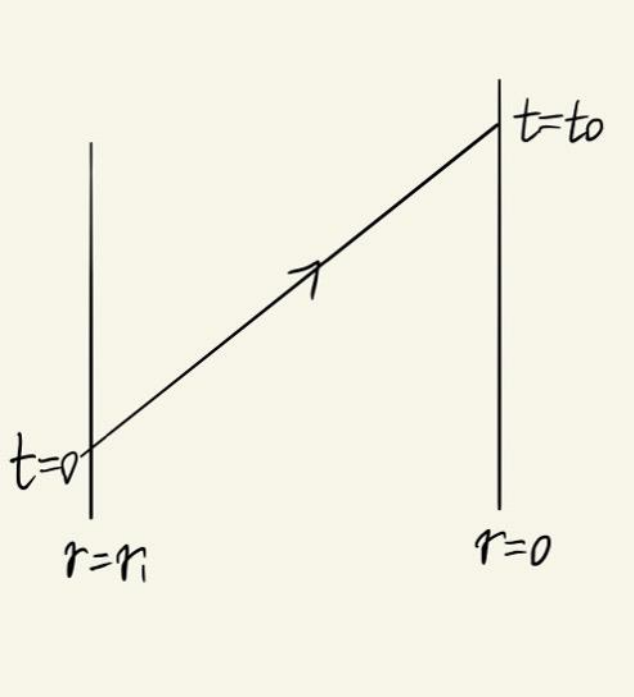
Angular distance: $d_A = \frac{D}{\theta}$ $d_A = a_e r_e = \frac{a_0}{1+z} r_e$

Baryon acoustic oscillations (BAO)





➢Hubble law



$$\int_0^{t_0} \frac{dt}{a(t)} = -\int_{r_1}^{0} \frac{dr}{\sqrt{1-kr^2}}$$

$$t_0 \longrightarrow 0, r_1 \longrightarrow 0, \sqrt{1-kr^2} \longrightarrow 1$$

$$\frac{t_0}{a(t_0)} \approx r_1$$

Luminosity distance $D_1 \simeq r_1 a(t_0) \simeq t_0$

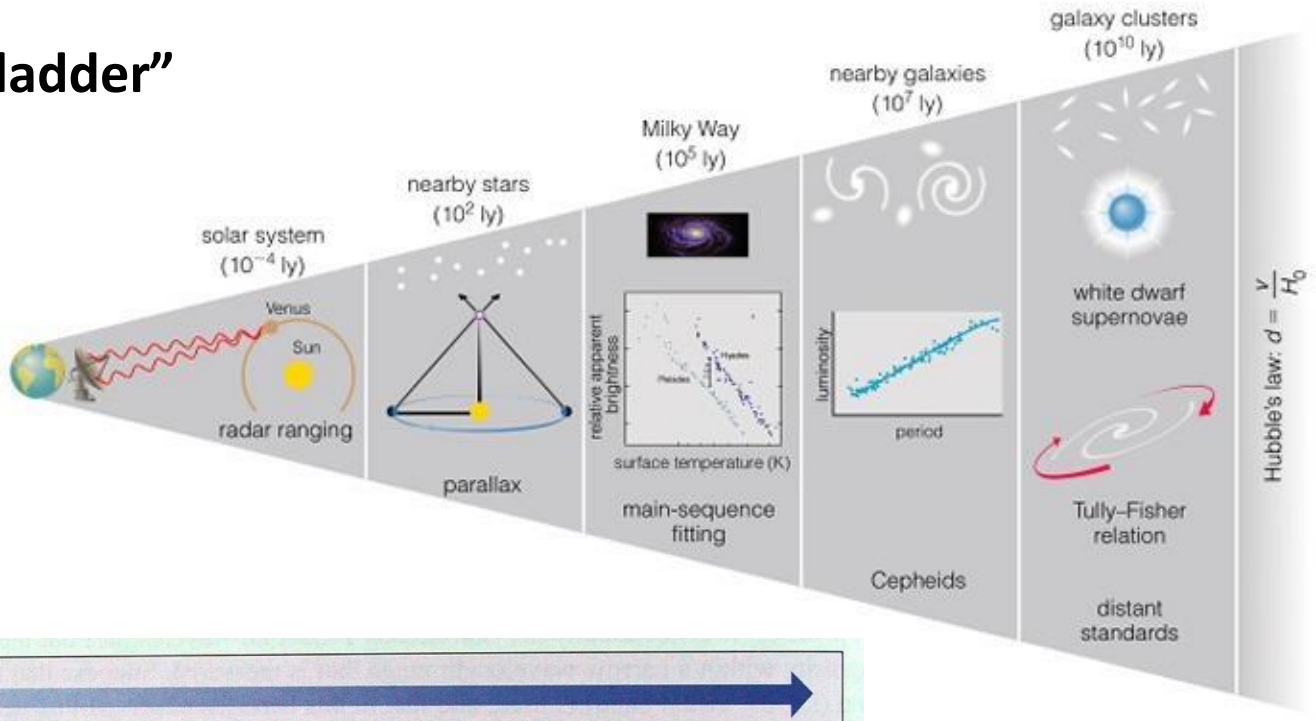
$$a(t_0) \simeq a(0) + \dot{a}(0)t_0 \simeq a(0) + \dot{a}(t_0)t_0$$

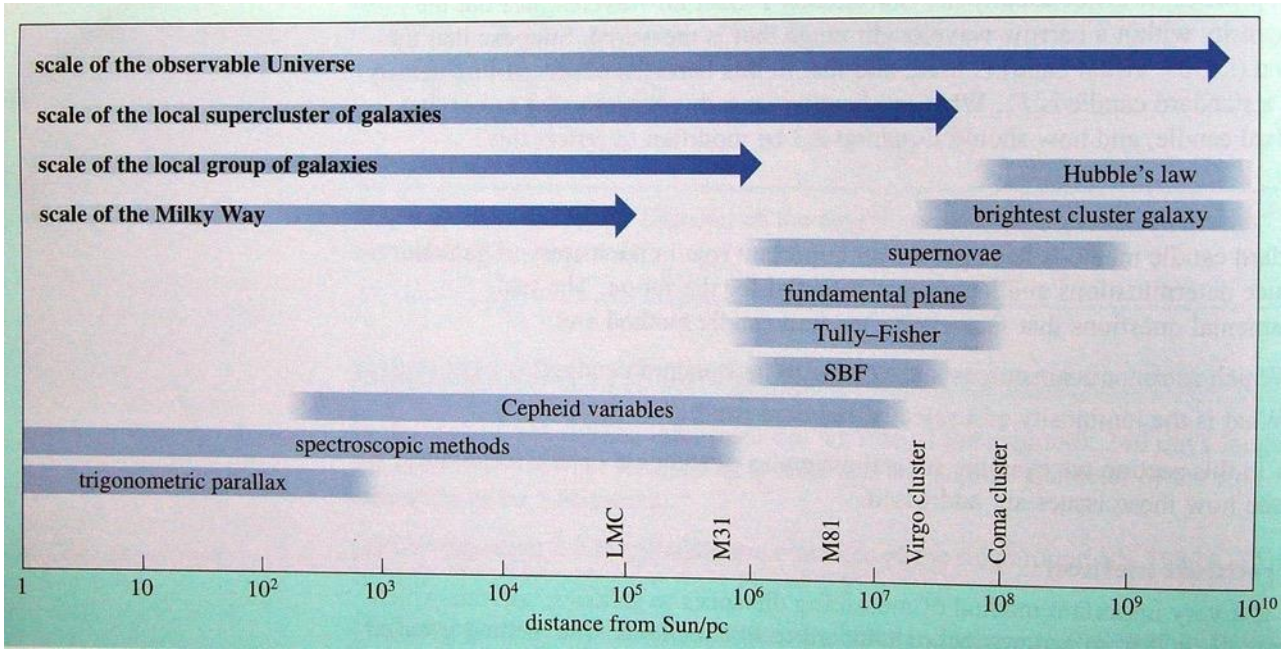
$$1 = \frac{a(0)}{a(t_0)} + \frac{\dot{a}(t_0)}{a(t_0)}t_0 = \frac{1}{1+z} + H_0 t_0 \simeq 1 - z + H_0 t_0$$

$$z \simeq H_0 t_0 \simeq H_0 D_1$$

$H \equiv \frac{\dot{a}}{a}$, Hubble parameter

“Cosmic distance ladder”





➢The evolution of scale factor

Cosmological principle:
Our Universe is **homogenous** and **isotropic** on **large scales**.

$$ds^2 = c^2dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right]$$

$$(T_{\mu\nu}) = \begin{pmatrix} \rho & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix}.$$

$p = w\rho$

w: equation of state parameter

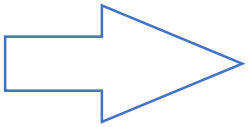
$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho$$

$$\frac{\ddot{a}}{a} + \frac{1}{2}\left(\frac{\dot{a}}{a}\right)^2 + \frac{1}{2}\frac{k}{a^2} = -4\pi G p$$

$$k = 0$$



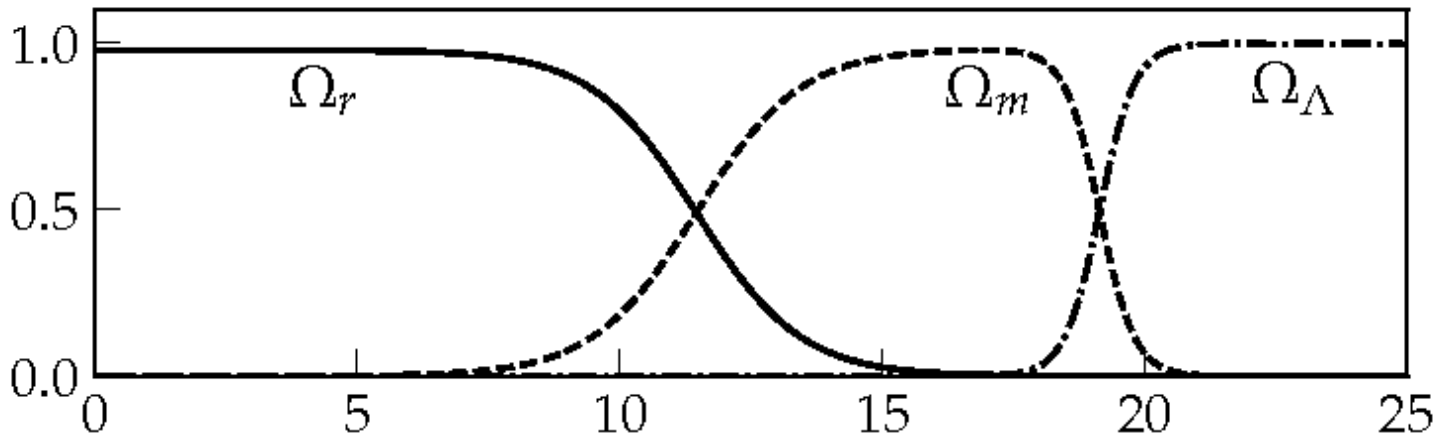
- **Energy momentum tensor conservation:**

$$\frac{d}{da}\left(a^3\rho\right) + 3a^2 p = 0 \qquad \rho \propto a^{-3(1+\omega)}$$

Dust, $w = 0, \rho \propto a^{-3}$

Radiation, $w = \frac{1}{3}, \rho \propto a^{-4}$

Cosmological constant $\Lambda, w = -1, \rho = constant$



- **Deceleration parameter:** $q(t) \equiv -\frac{\ddot{a}}{aH^2}$ $H \equiv \frac{\dot{a}}{a}$, Hubble parameter

$$q(t) = \frac{4\pi G}{3}\frac{\rho + 3p}{H^2} = \frac{\rho_r + \rho_m + \rho_\Lambda + 3(p_r + p_m + p_\lambda)}{2\rho_c} = \frac{1}{2}\Omega_m + \Omega_r - \Omega_\Lambda$$

$$\rho_c \equiv \frac{3H^2}{8\pi G} \qquad \Omega_i \equiv \frac{\rho_i}{\rho_c}$$

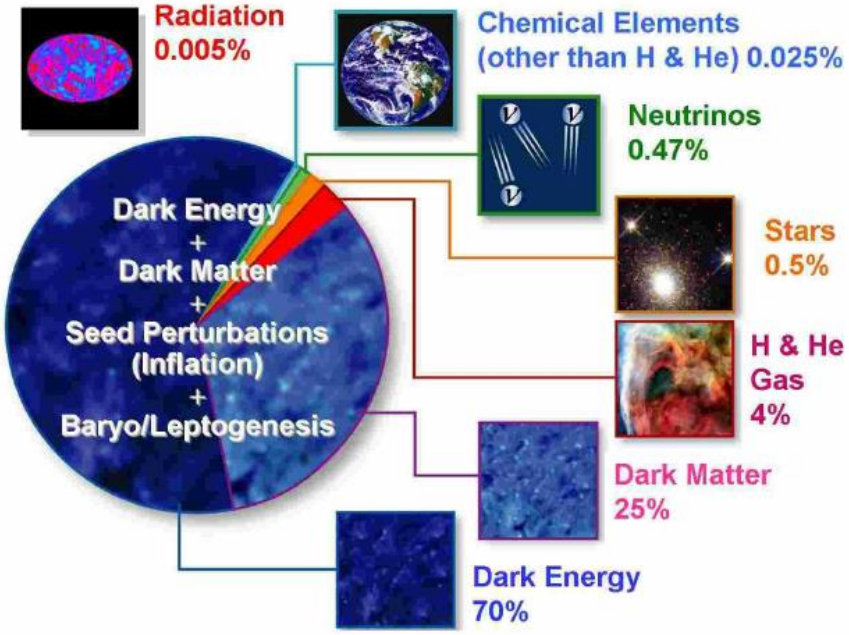
Today $q < 0$, accelerating expansion

- **Dark energy**

$$\frac{d^2a/dt^2}{a} = -\frac{4\pi G}{3}(\rho + 3p) > 0$$

$$\Rightarrow \quad p < -\frac{\rho}{3}$$

For example, cosmological constant Λ, $w = -1$, can be a candidate



A brief history of our universe

Quasi-adiabatic Expansion

➢From the Friedmann equations, one reads that cosmic expansion of the background is a quasi-adiabatic process:

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = 0$$

$$\mathrm{d}U = -P\mathrm{d}V$$

$$U = \rho V$$
$$V \propto a^3$$

Adiabatic Expansion

➢ **Temperature evolution**

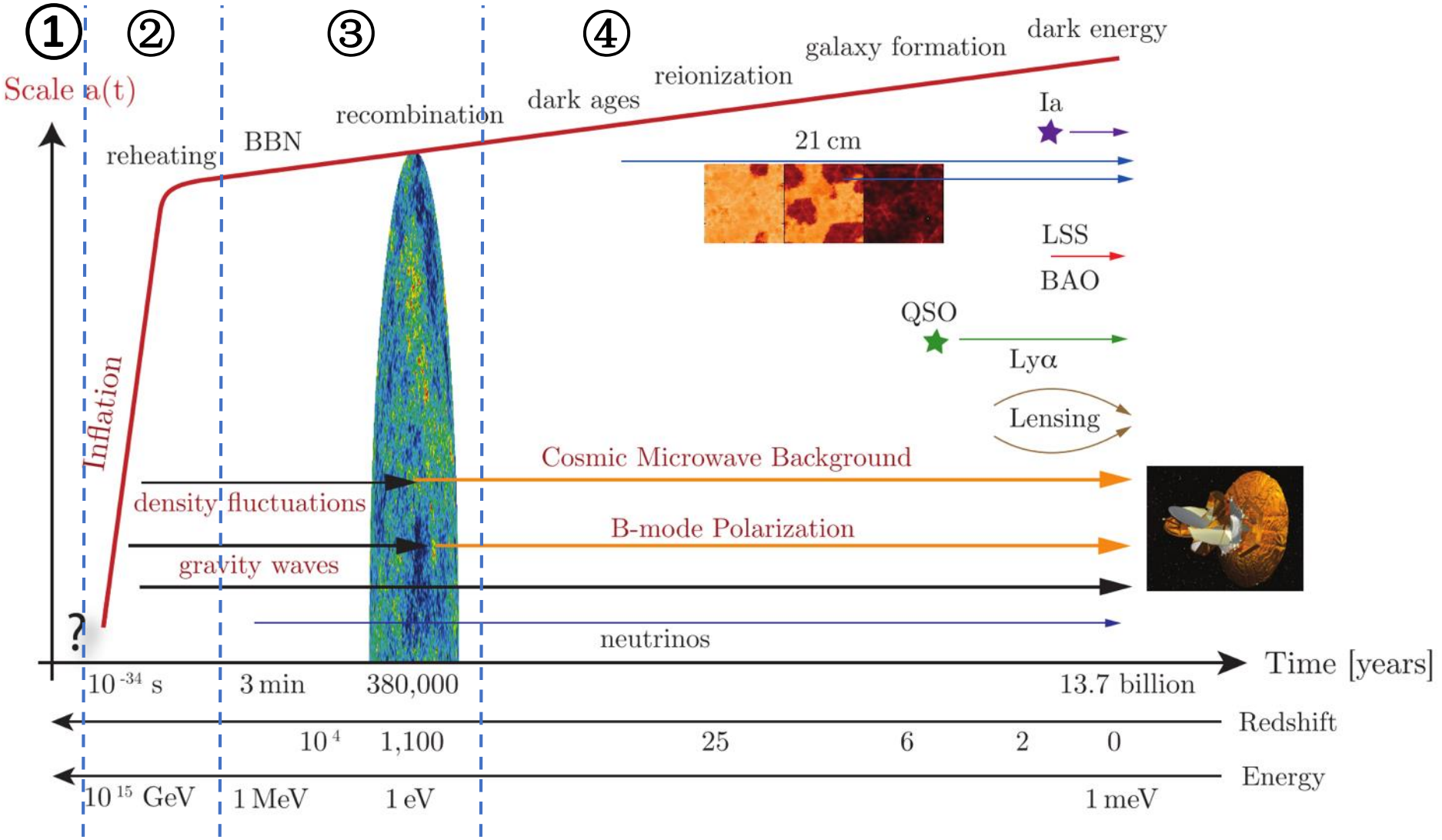
Translating the adiabatic expansion into the temperature evolution of particle gas, we find that they cool down as the Universe expands

$$p \propto \rho^{\gamma} \longrightarrow TV^{\gamma-1} = const$$

➢ **Trace back**

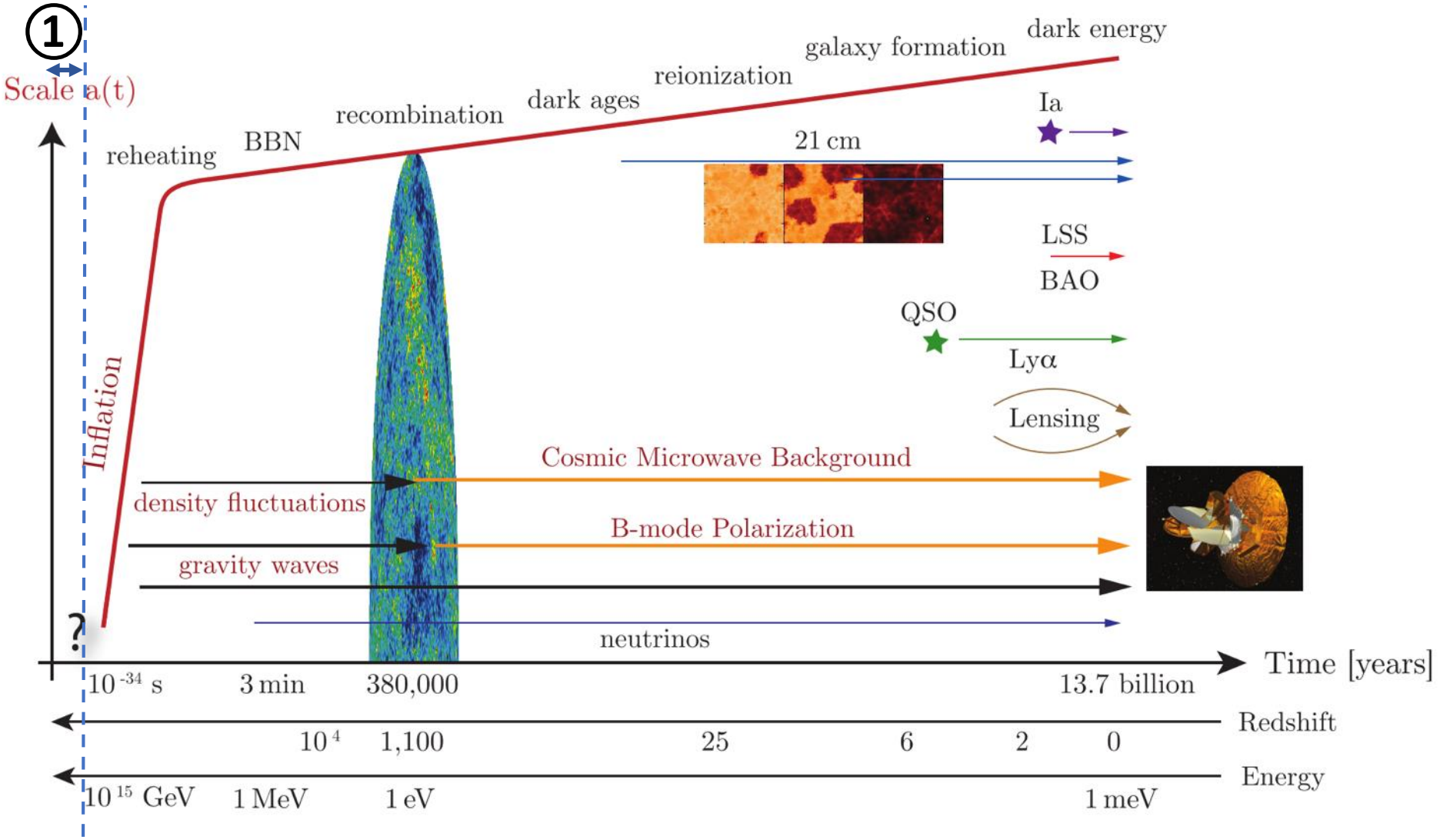
Hot Big Bang

A Brief History of Time



From: D. Baumann

Planck Epoch



Planck Epoch

➢ **Scales:** s, GeV

Typical Planck scales:

$$m_{Pl} \equiv \sqrt{\frac{\hbar c}{G}} \sim 10^{19}\,\mathrm{GeV}$$

$$l_{Pl} \equiv \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-35}\,\mathrm{m}$$

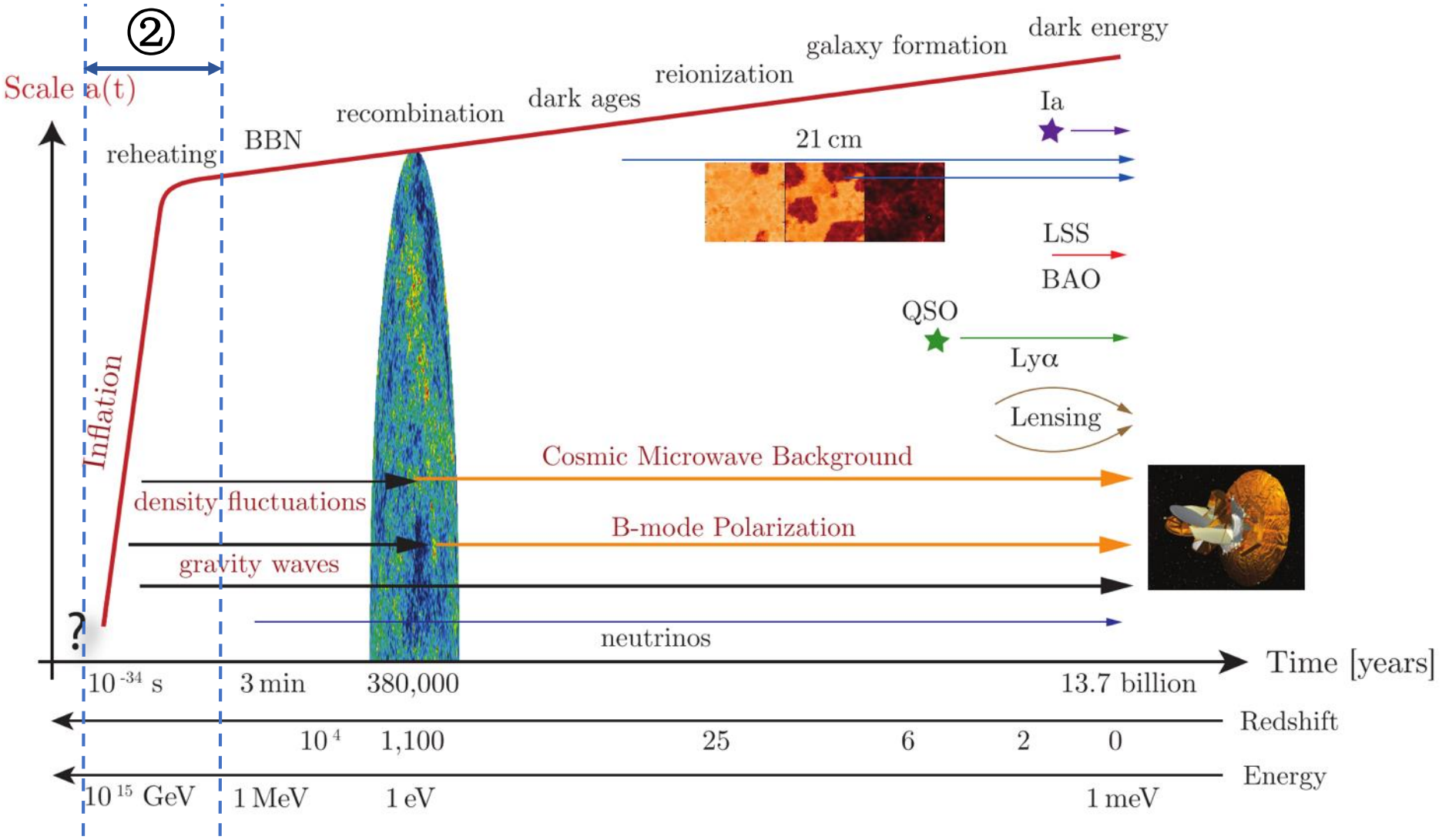
$$t_{Pl} \equiv \sqrt{\frac{\hbar G}{c^5}} \sim 10^{-43}\,\mathrm{sec}$$

de Broglie wavelength $\lambda = \frac{2\hbar\pi}{mc}$

Schwarzschild radius $R = \frac{2Gm}{c^2}$

Quantum Gravity dominant

The Very Early Universe



The Very Early Universe

- **Time scale:** s
- **Primary mission**

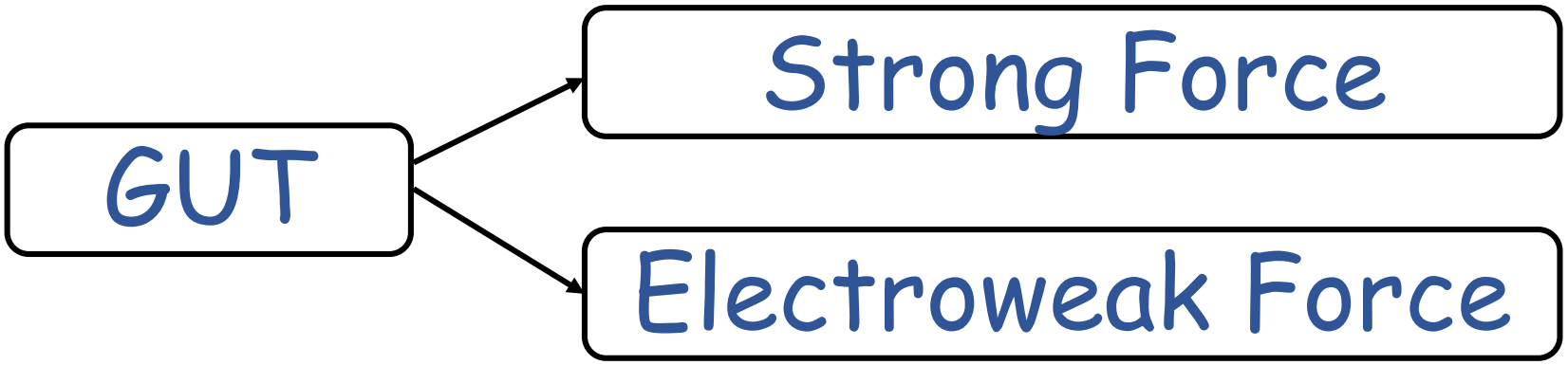
Phase Transition

During this epoch, as the universe expands and cools down, it undergoes various phase transitions, as a result of

Spontaneous Symmetry Breaking

The Very Early Universe

- GeV
- **Grand Unified Theory(GUT) Transition**



- **Inflation, Reheating**

It is conceivable that the GUT transition may be identified with inflation.

Precursor of Standard Model (SM) particles.

The Very Early Universe

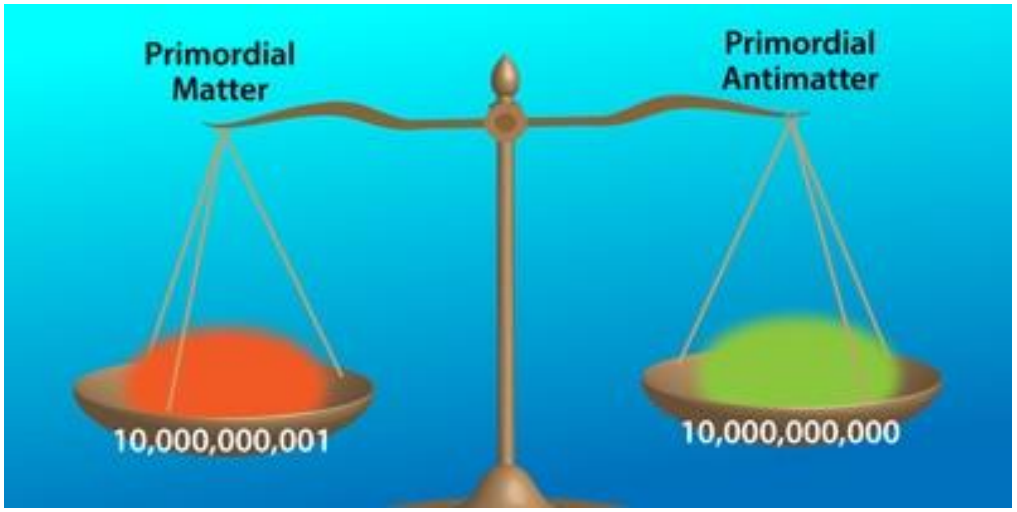
➢TeV

➢**Supersymmetry breaking**

Super partners become much heavier than particles.

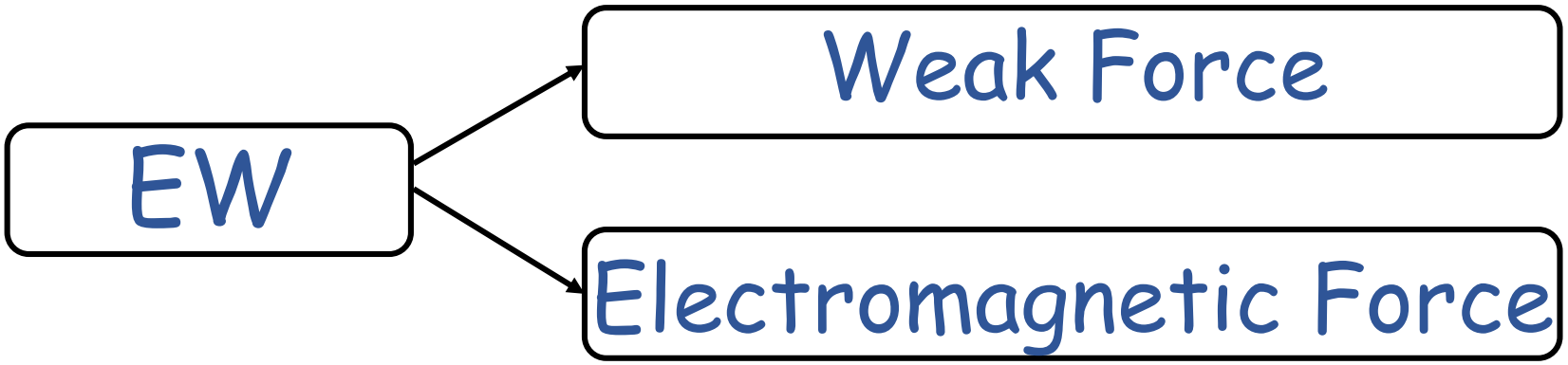
SM hierarchy problem.

➢**Baryogenesis**



The Very Early Universe

- TeV (GeV)
- **Electroweak transition**



All the leptons acquire masses.

Produce massive gauge bosons and massless photon γ.

The Very Early Universe

➢GeV

➢**Quark-Hadron transition**

Isolated Quarks → Composite Particle

QCD chiral symmetry breaking.

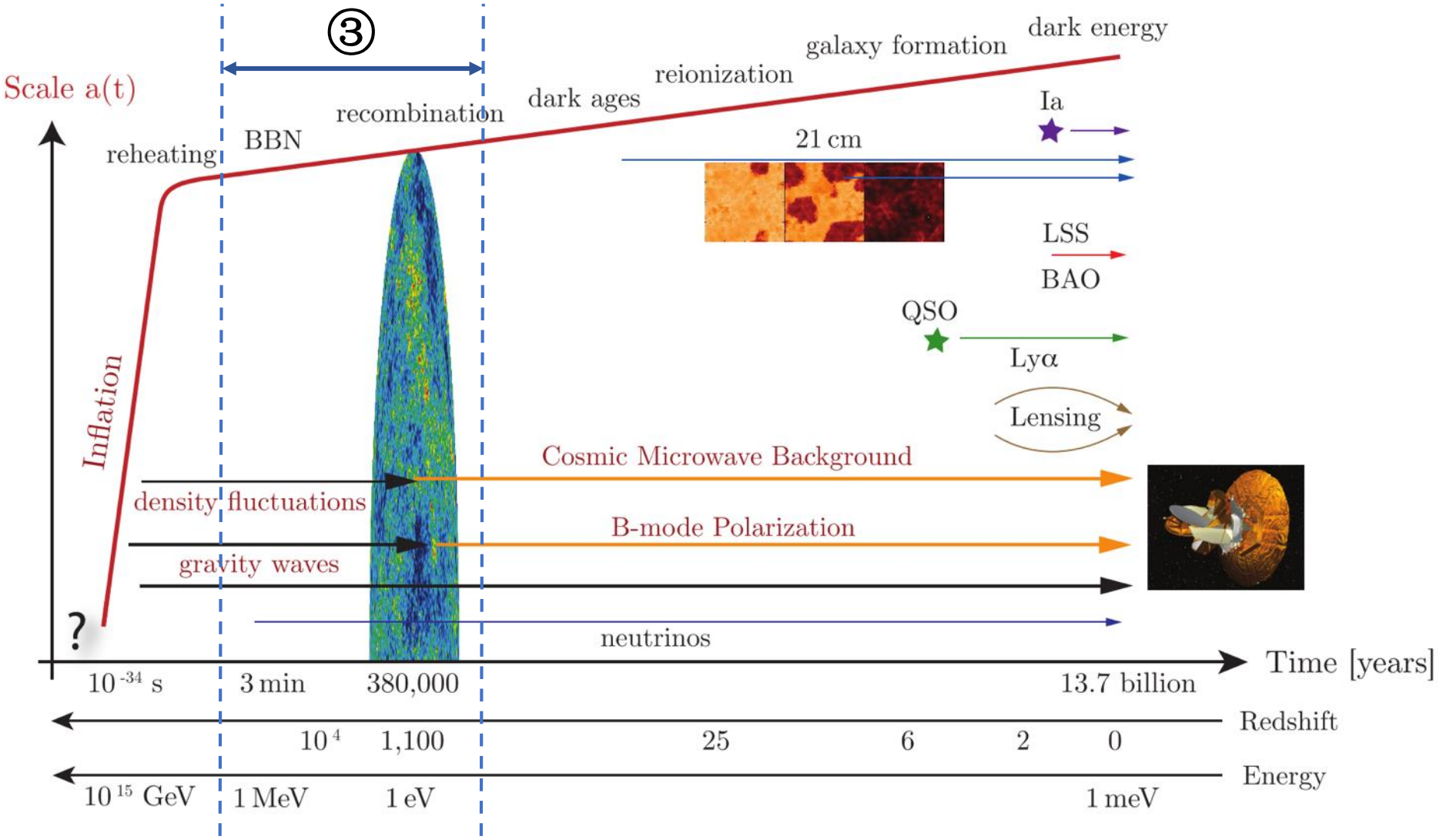
Axion acquires mass.

➢**Quark confinement (hadrons)**

Baryons ← quark triplet

Mesons ← quark-antiquark pairs

Hot Big Bang Fireball



Hot Big Bang Fireball

➢**Nucleon freeze-out (**MeV; s**)**

Collisions among the hadrons cease.

The prelude to BBN.

➢**Neutrino decoupling (**MeV; s**)**

Neutrino decouple from the rest of the matter.

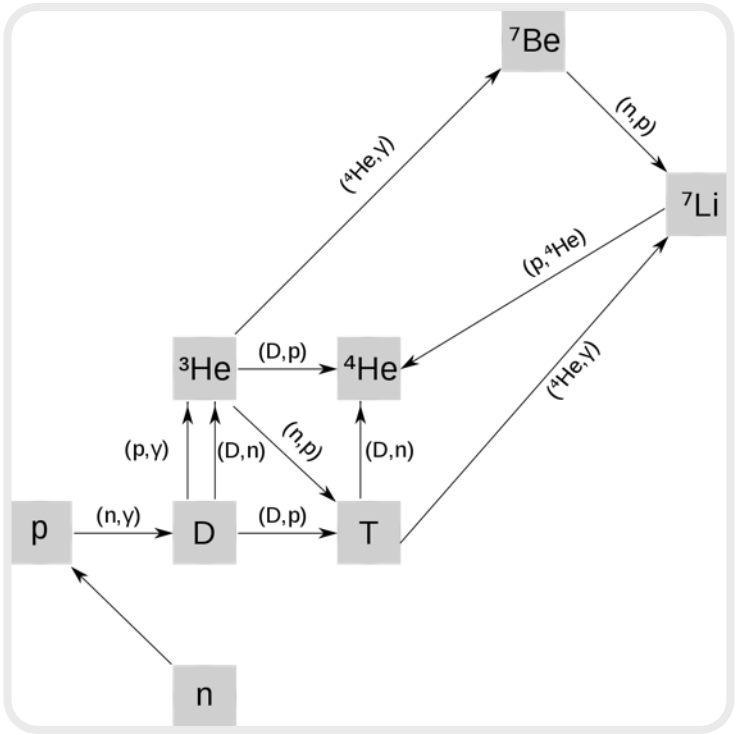
Relic neutrino background.

Hot Big Bang Fireball

➢MeV; min

➢**Big Bang Nucleosynthesis(BBN)**

- First direct probe of the Hot Big Bang.
- Very close to homogeneous.
- Strongly radiation-dominated.
- The fusion of nuclei sustained roughly 20 min.
- It was widespread, encompassing the entire observable universe.



Hot Big Bang Fireball

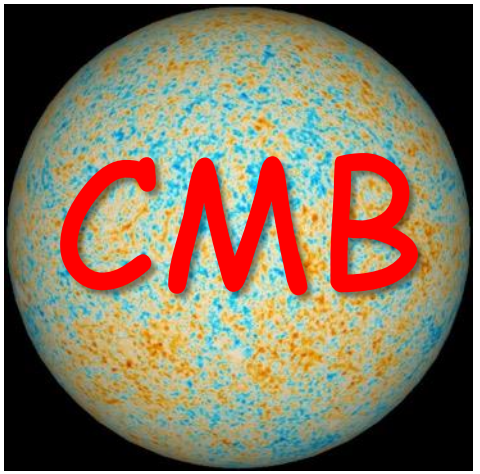
➢ K; years;

➢ **Recombination**

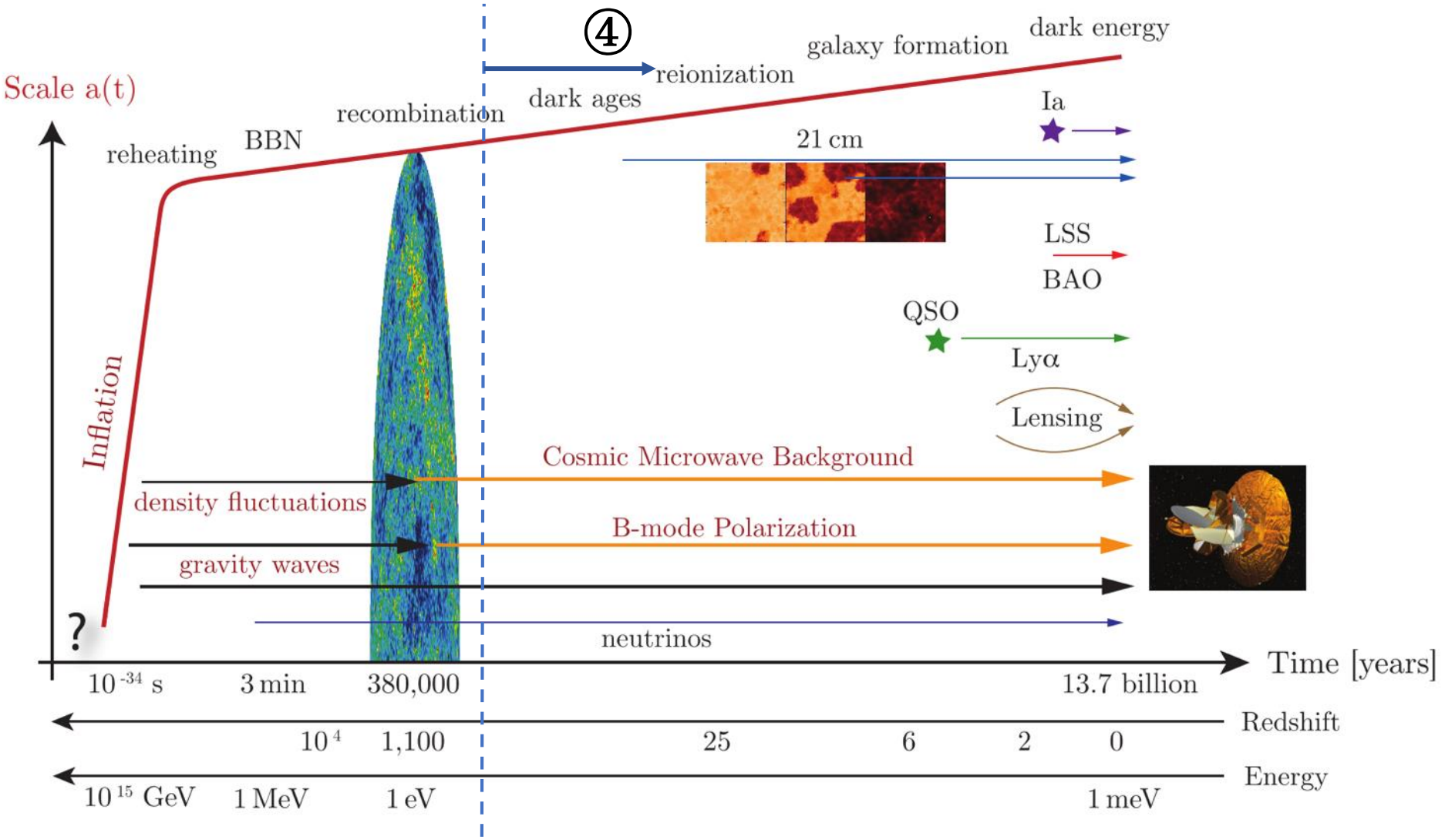
Before recombination: $e^- + \gamma \longleftrightarrow e^- + \gamma$

During recombination: $p + e^- \longleftrightarrow H + \gamma$

Major consequence of recombination:

Transparent Universe → 

Post-Recombination Era



Key Events in the Thermal History

	Time	Energy	
Planck Epoch?	$< 10^{-43}$ s	10^{18} GeV	
String Scale?	$\gtrsim 10^{-43}$ s	$\lesssim 10^{18}$ GeV	
Grand Unification?	$\sim 10^{-36}$ s	10^{15} GeV	
Inflation?	$\gtrsim 10^{-34}$ s	$\lesssim 10^{15}$ GeV	
SUSY Breaking?	$< 10^{-10}$ s	> 1 TeV	
Baryogenesis?	$< 10^{-10}$ s	> 1 TeV	
Electroweak Unification	10^{-10} s	1 TeV	
Quark-Hadron Transition	10^{-4} s	10^{2} MeV	
Nucleon Freeze-Out	0.01 s	10 MeV	
Neutrino Decoupling	1 s	1 MeV	
BBN	3 min	0.1 MeV	
			Redshift
Matter-Radiation Equality	10^{4} yrs	1 eV	10^{4}
Recombination	10^{5} yrs	0.1 eV	1,100
Dark Ages	$10^{5} - 10^{8}$ yrs		> 25
Reionization	10^{8} yrs		$25 - 6$
Galaxy Formation	$\sim 6 \times 10^{8}$ yrs		~ 10
Dark Energy	$\sim 10^{9}$ yrs		~ 2
Solar System	8×10^{9} yrs		0.5
Albert Einstein born	14×10^{9} yrs	1 meV	0

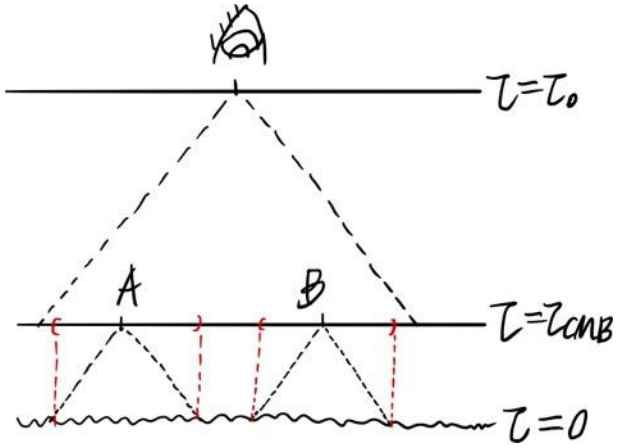
From: D. Baumann

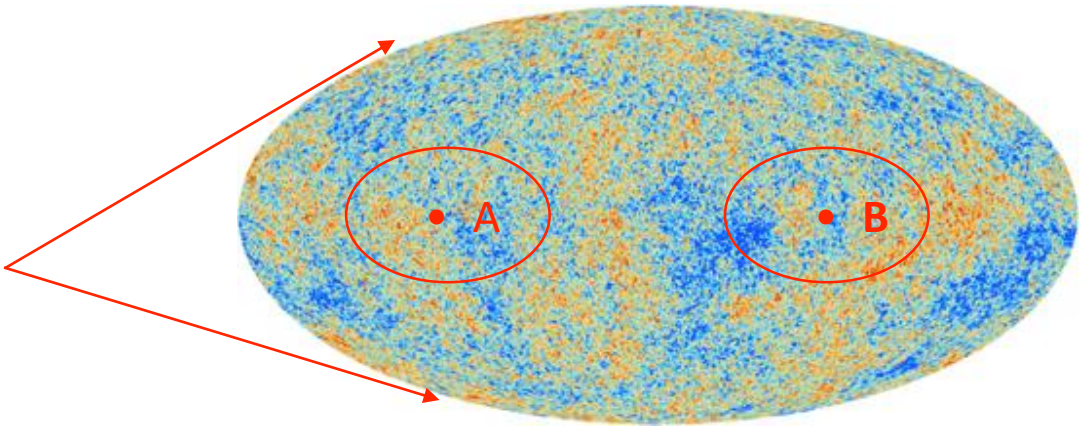
Origin of the Universe: Inflation or Bounce?

The problems of the standard cosmological model

1. Horizon problem

Comoving coordinate:





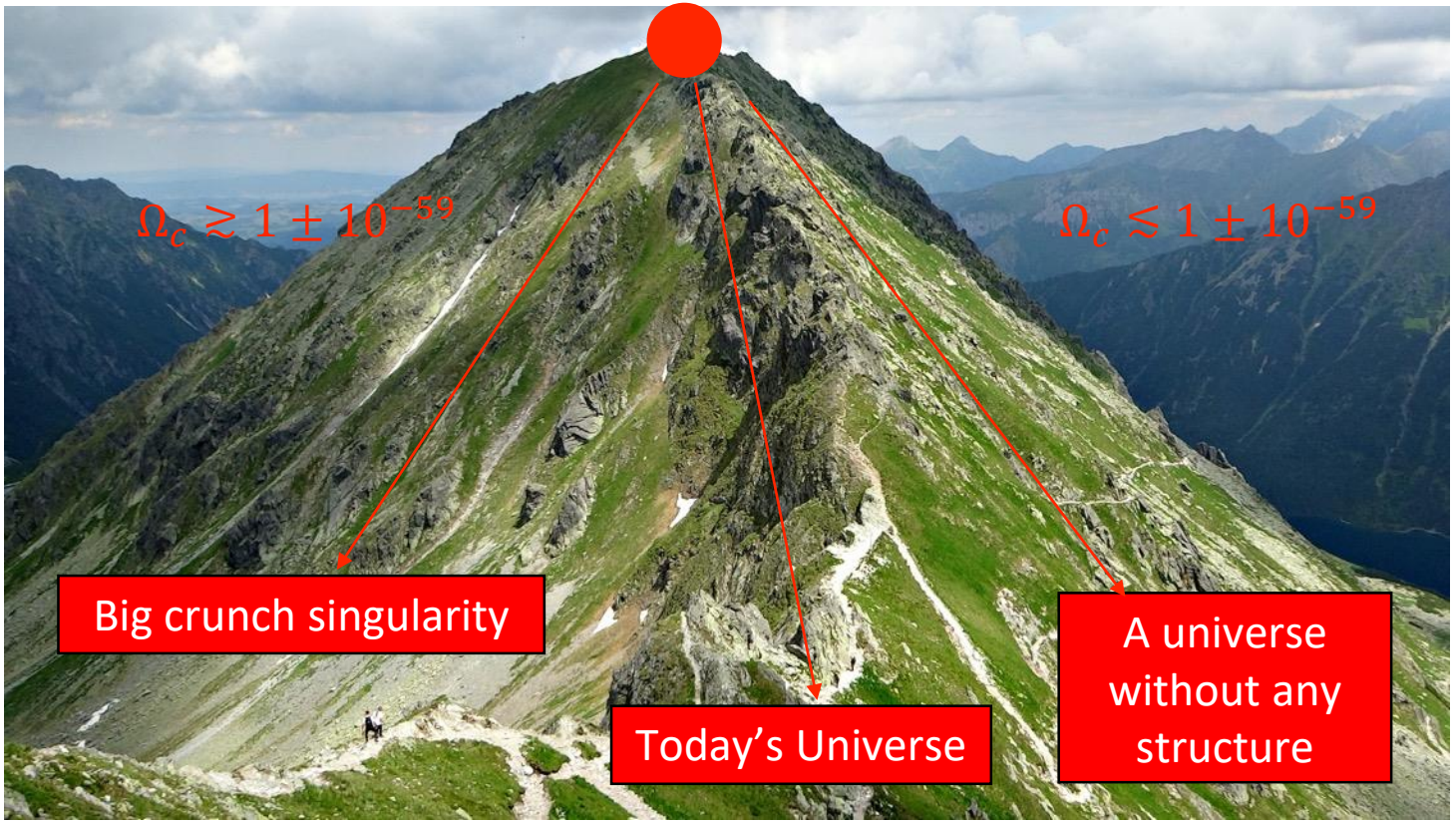
There is no causal relationship between A and B.

How the Universe become homogenous and isotropic?

2. Flatness problem

$$\text{Friedmann equation: } 1 - \Omega_c^{-1} = \frac{3k}{8\pi G a^2 \rho}$$

$$\Omega_c \sim 1 \pm 10^{-59}$$



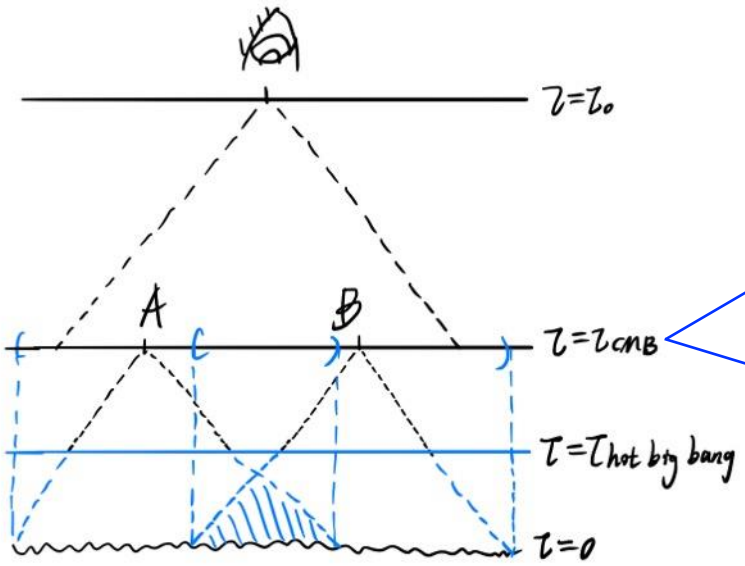
$$\Omega_c \sim 1$$

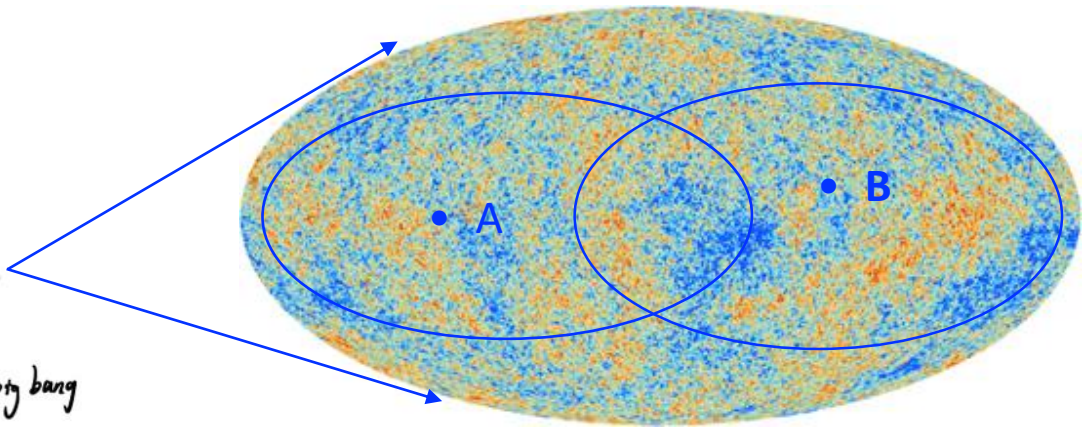
The initial critical density Ω_c must be finely tuned to 10^{-59}

Inflation

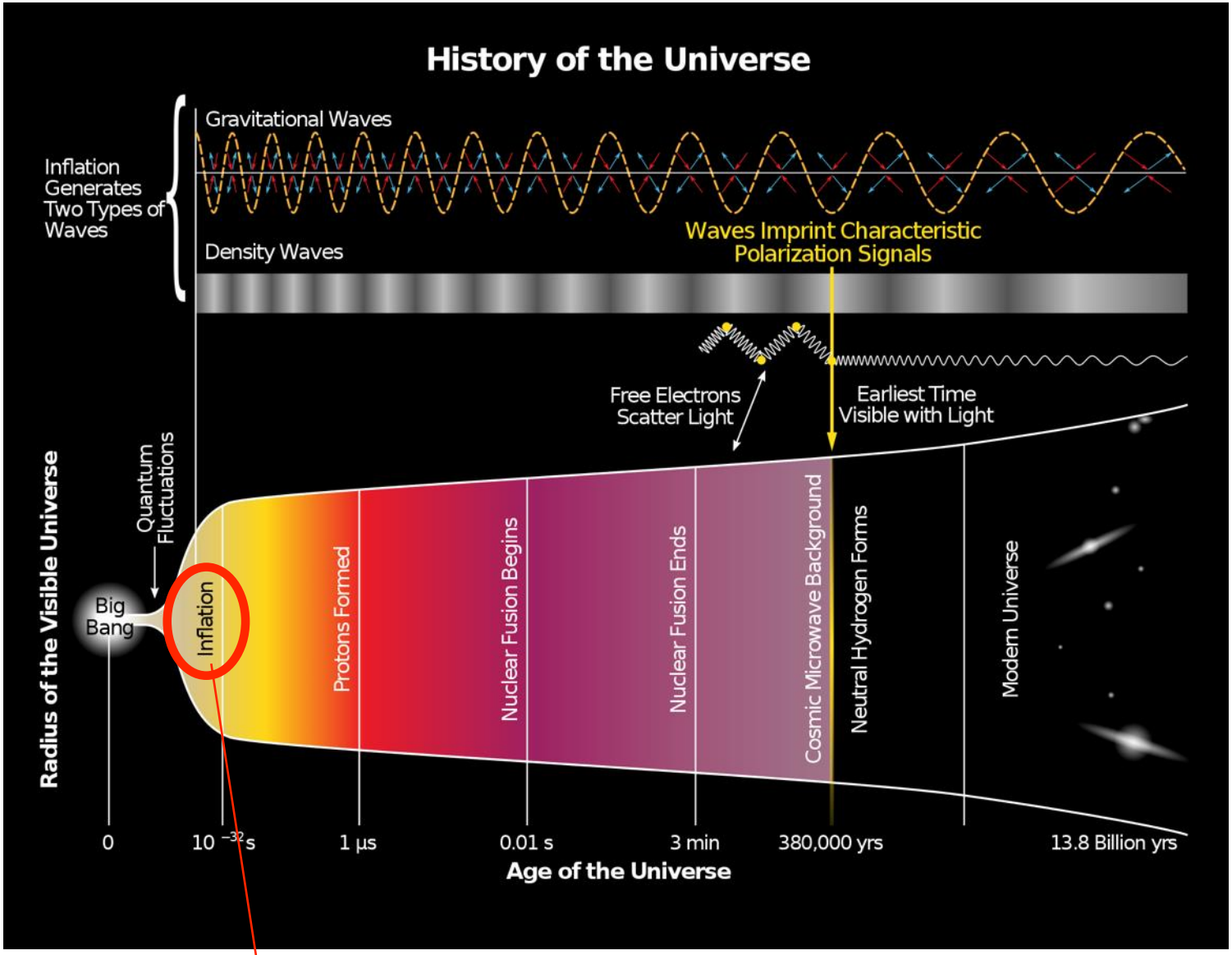
The general picture of inflation

Comoving coordinate:





The way to realize this idea is to introduce an accelerating expansion phase.



Any deviation from $\Omega_c \sim 1$ will be smoothed out.

- The basic condition for inflation

$$\ddot{a} = -4\pi a(\rho + 3p)/3.$$

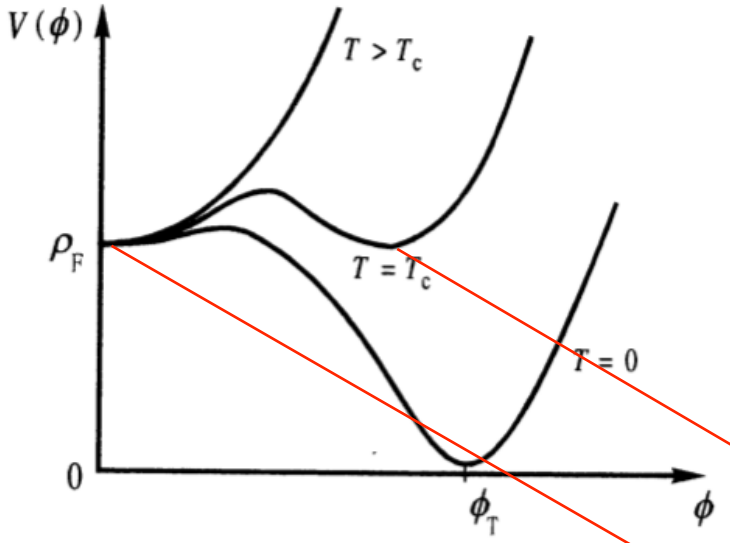
$$\ddot{a} > 0$$

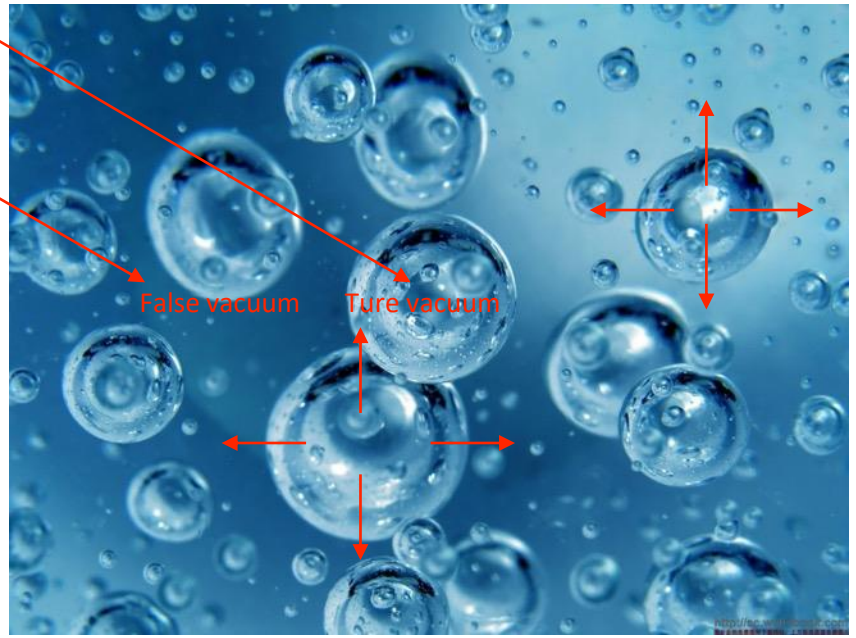
$$p < -\rho/3$$

The ordinary matter cannot realize this kind of state.

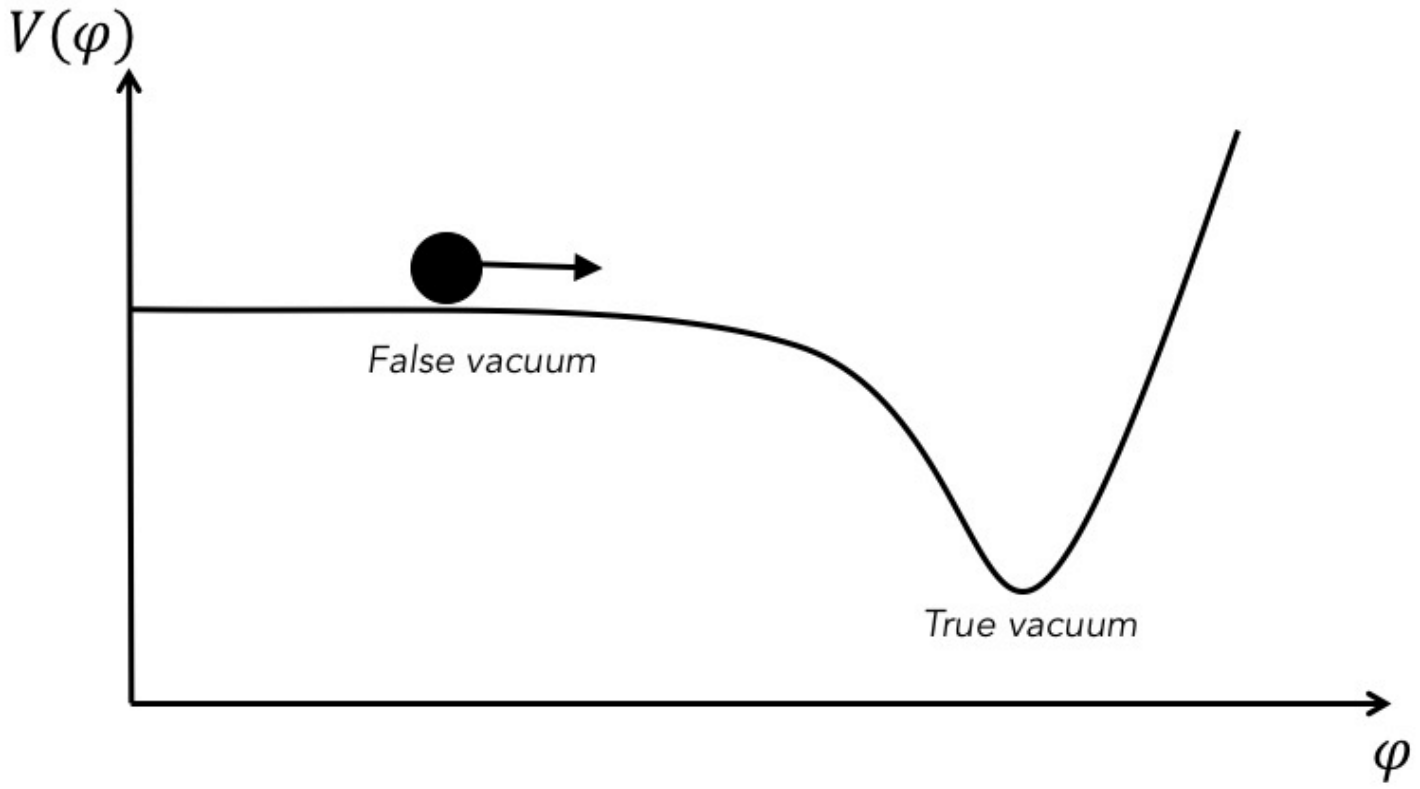
- An inflation model based on phase transition

A scalar field ϕ and its potiential V(ϕ)

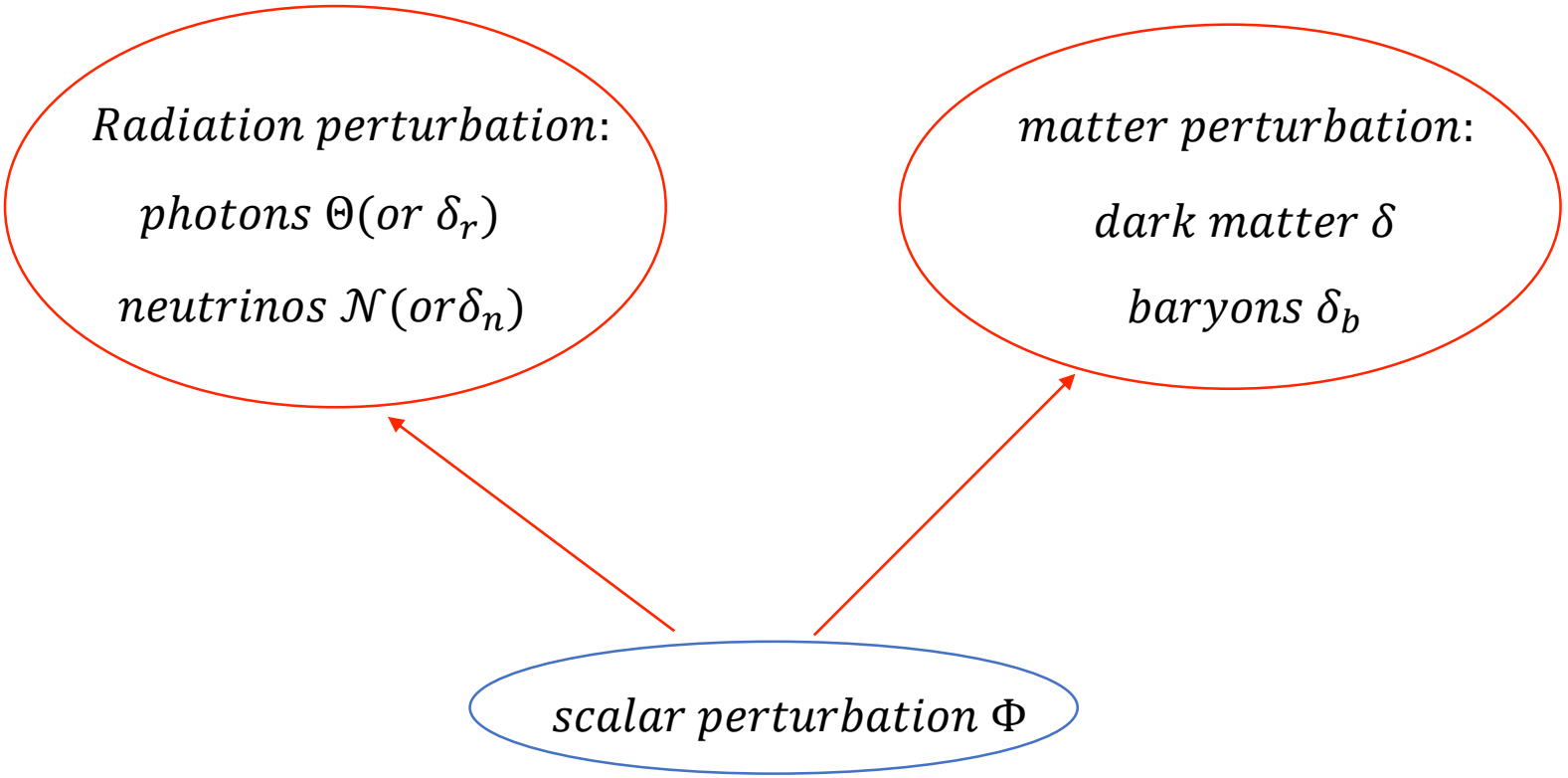




- A (small field) slow roll inflation



At the end of the inflation



In order to construct the evolution of the Universe, we need to know the **initial scalar perturbation**.

$$\Phi_{initial}?$$

If we only have the Einstein equation (only have gravity)

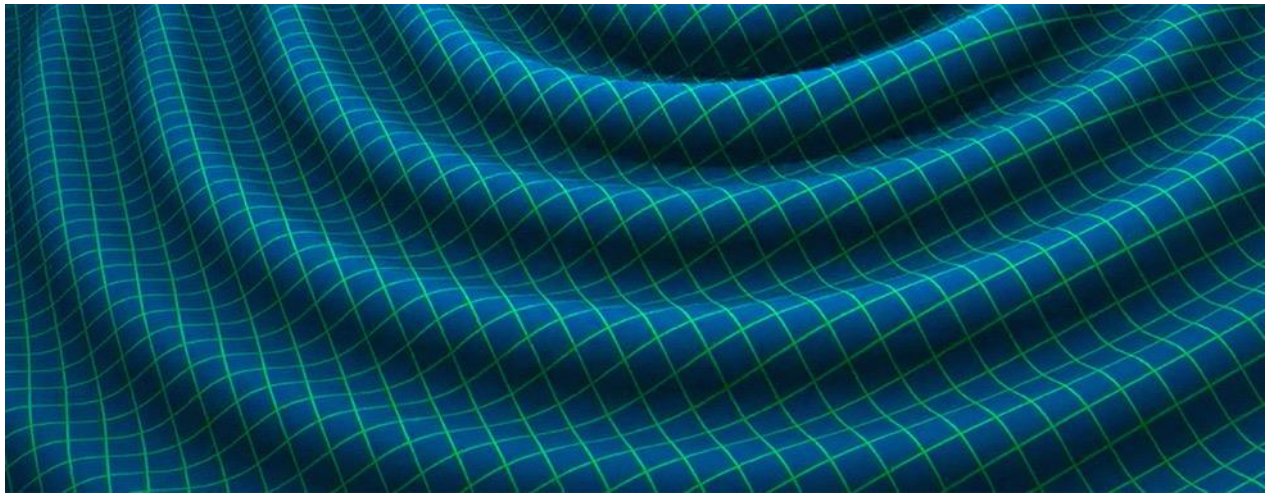
$$G_{\mu\nu} = 0$$

These equations only have two **dynamical** degrees of freedom.

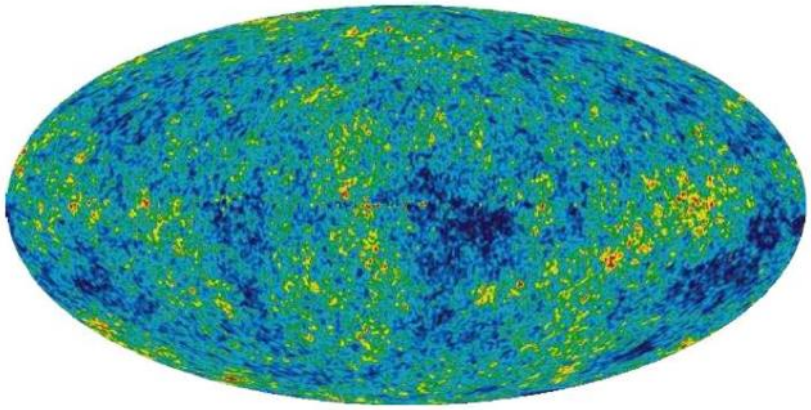
At the perturbation level

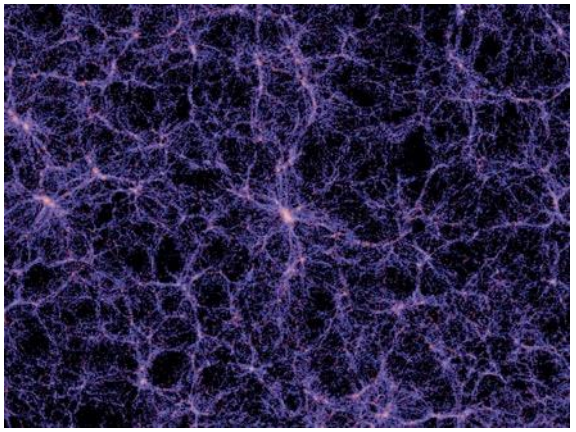
Gravitational waves: h_+ and $h_\times$

A universe only having gravitational waves



However our Universe has “structure”





Where do the **dynamical scalar degrees of freedom** come from?

$\Phi_{initial}$?

Fortunately, we have **inflation** now!

Inflation field ϕ **A dynamical scalar degree of freedom**

$$G_{\mu\nu} = T_{\mu\nu}(g_{\mu\nu}, \phi)$$

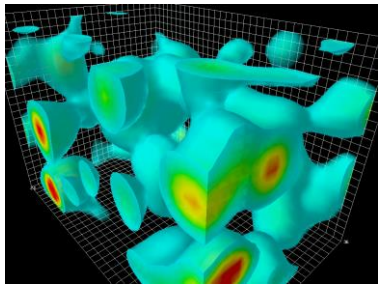
The perturbation of the scalar field

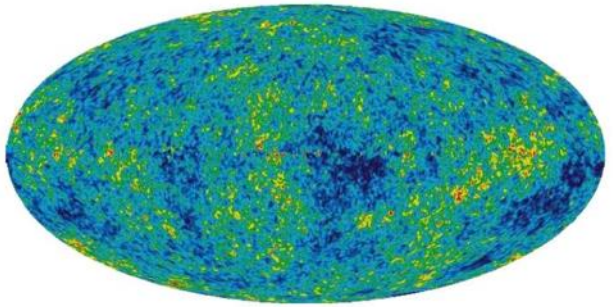
$\delta\phi$

The intial scalar perturbation Φ_{initial} of metric

So our aim is

Inflation field ϕ

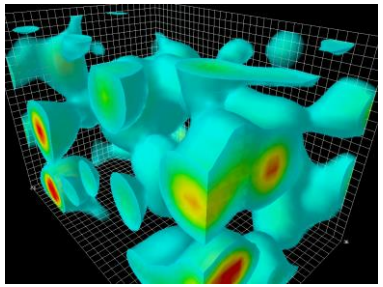


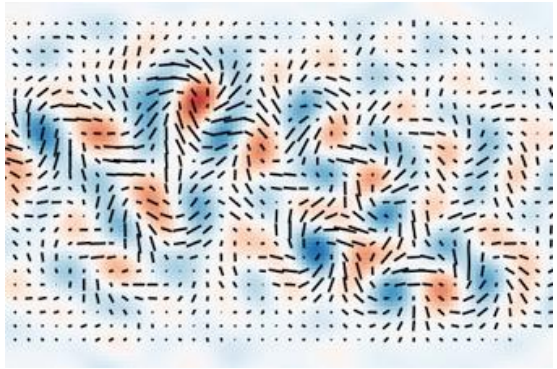


The quantum fluctuation

The classical fluctuation

Gravitational wave h_{ij}





The quantum fluctuation

The classical fluctuation

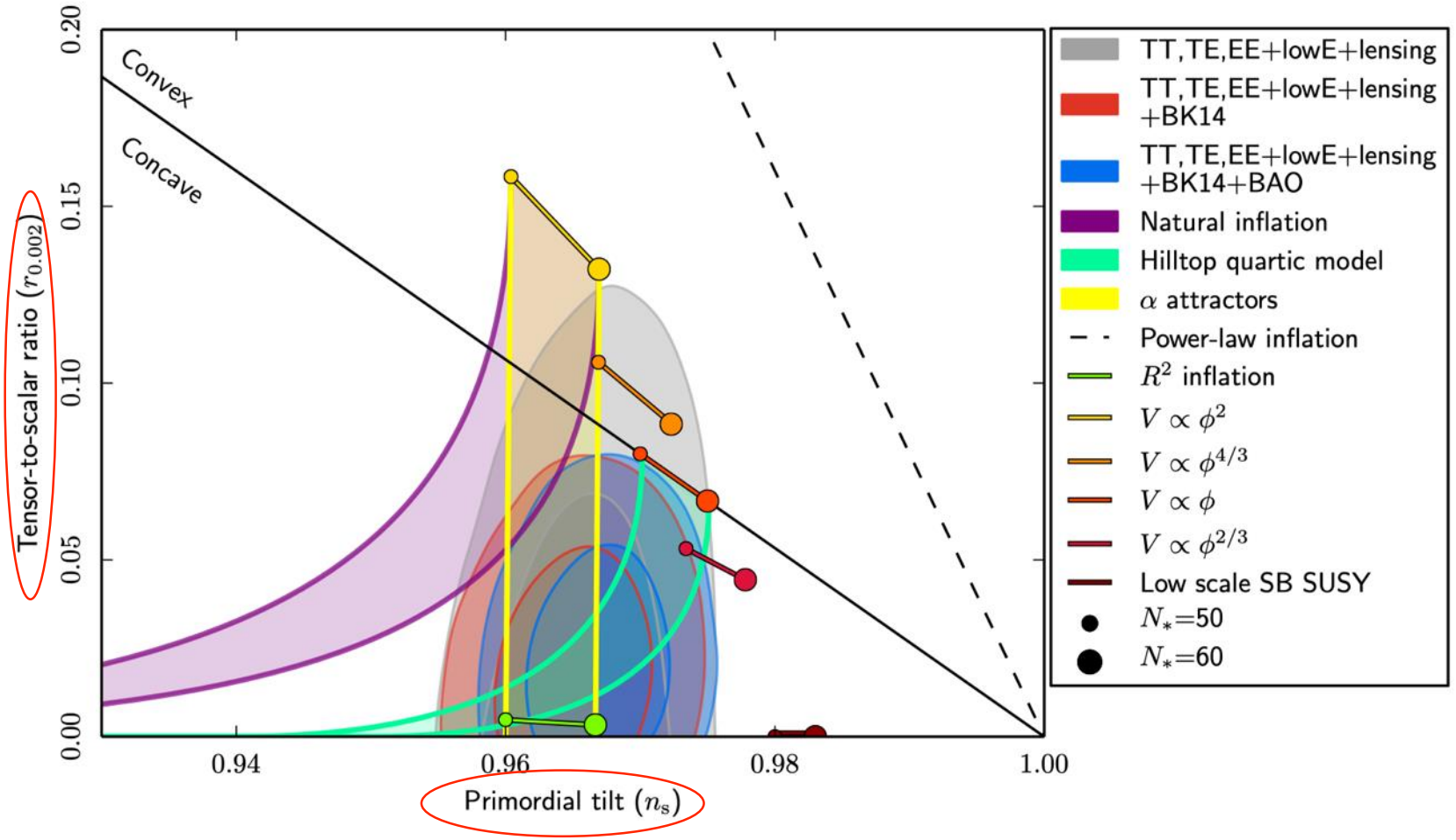
Assume that the distribution of the primordial fluctuations amplitude is **Gaussian,**

then the only object that can characterize the primordial fluctuations is the distribution of average amplitude over the wavelength **k**.

The primordial power spectrum

$$Scalar: \Delta_s^2(k) \qquad Tensor: \Delta_t^2(k)$$

Planck 2018 results: Constraints on Inflation



$Scalar: \Delta_s^2(k) \sim k^{n_s - 1}, if\, n_s = 1, scalefree.$

$Tensor: \Delta_t^2(k) \sim k^{n_t}, if\, n_t = 0, scalefree.$

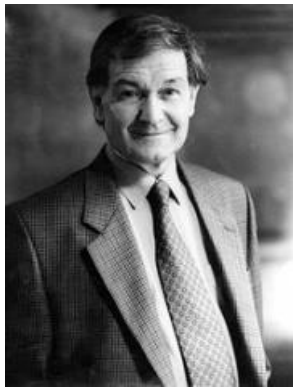
$\text{Tensor} - \text{to} - \text{scalar ratio: } r = \frac{\Delta_t^2}{\Delta_s^2}.$

Bounce

➢Why do we expect a bounce?

- **Initial big bang**

Singularity: geodesic incompleteness 1965~1970



Penrose:
Some sort of *geodesic incompleteness* occurs inside any *black hole* whenever matter satisfies reasonable *energy conditions*.

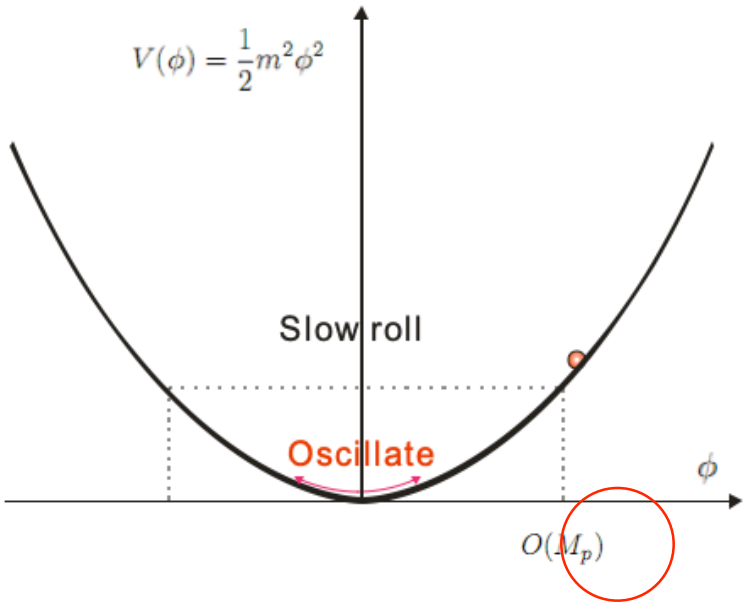
Hawking:
Singularity theorem is for *the whole universe*. It guarantees that the (classical) big bang (even combining inflation) has the singularity.

•Trans-Planckian problem

Black hole physics and inflationary cosmology:

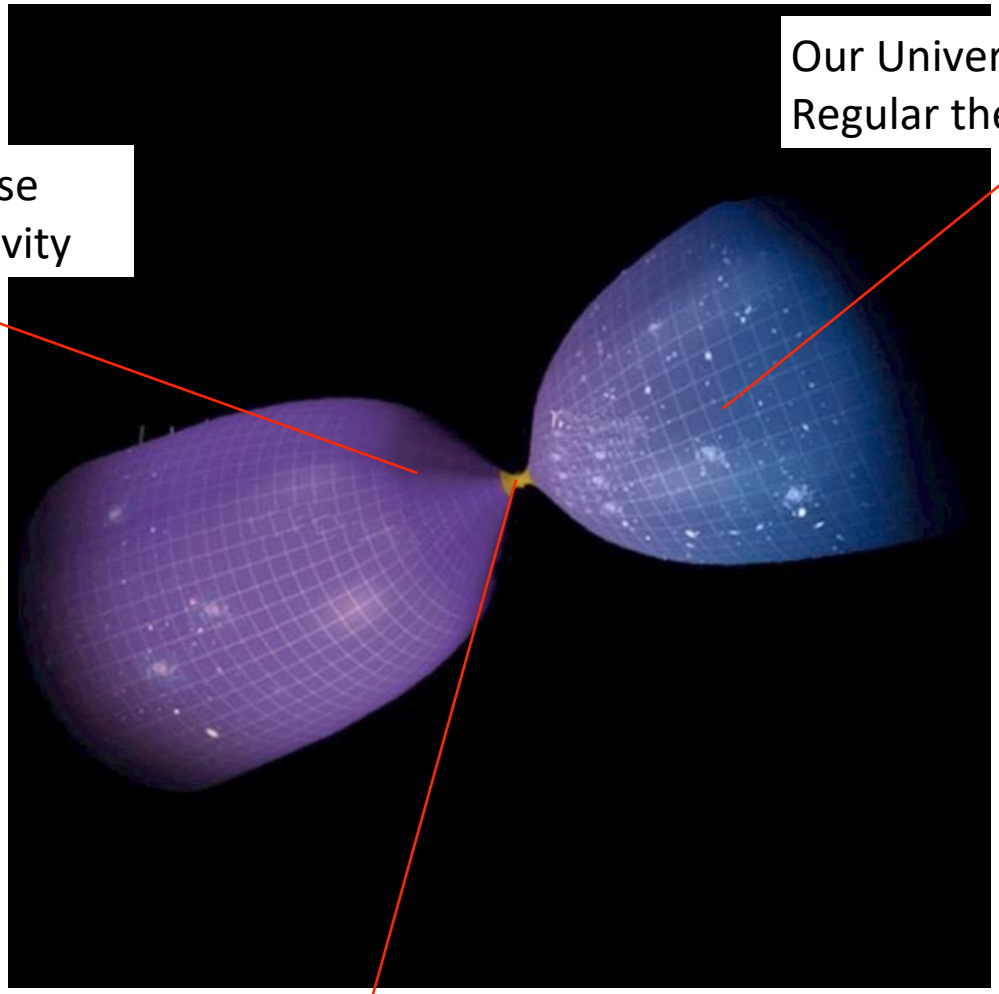
The appearance of quantities ***beyond the Planck scale*** will raise doubts on the ***physical validity*** of some results, since one expects the physical laws to suffer radical modifications beyond the Planck scale.

For example: the large field inflation



➢What is a (nonsingular) bounce?

$$w = -1 - \frac{2\dot{H}}{3H^2}$$



Our Universe
Regular thermal expansion

$H > 0, \dot{H} < 0$

$w > -1$

Pre-existing Universe
Collapse due to gravity

$H < 0, \dot{H} < 0$

$w > -1$

Big bounce $\quad H = 0, \dot{H} > 0$

➢What is used to get a bounce?

- Singular bounce
- **Nonsingular bounce**

At the ***nonsingular*** bounce point: $H = 0, \dot{H} > 0$

Friedmann equation at bounce point takes:

$$k = 0, \dot{H} = -4\pi G(\rho + p) > 0$$

$\rho + p < 0,$ violate the null energy condition (NEC)

$$k \neq 0, \dot{H} = -\frac{4\pi G}{3}(\rho + 3p) > 0$$

$\rho + 3p < 0,$ violate the strong energy condition (SEC)

An example: k-essence

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{2M_p^2} + K(\phi, X)\right), X = -\frac{1}{2}g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi$$

$$p = K$$

$$\rho = 2XK_{,X} - K$$

$$\rho + p < 0 \longrightarrow 2XK_{,X} < 0$$

The simplest case:

$$K(\phi, X) \sim -X = \frac{1}{2}g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi$$

Ghost condensate

http://dx.doi.org/10.1103/PhysRevD.66.063509.

To be continued …

Thanks!