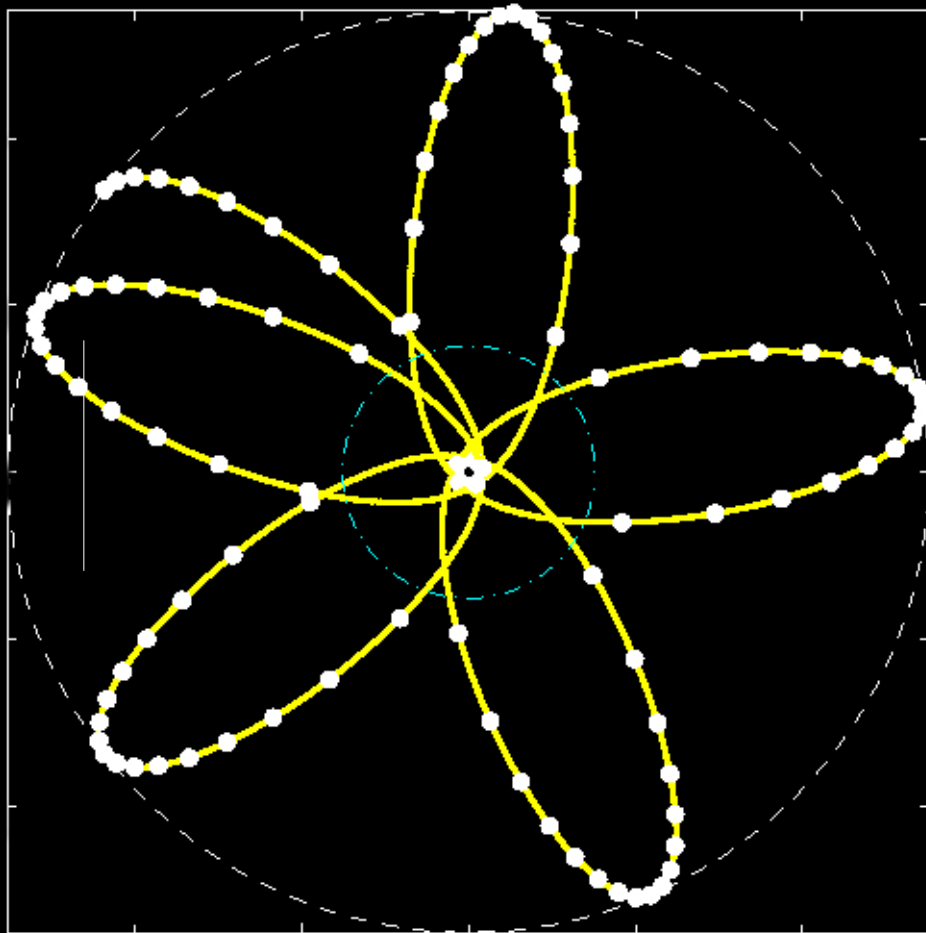


Test particle motion in boson star space-times



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Ubu 14/4/2015

Black holes and their analogues
100 years of General Relativity

Contents

- Motivation: What are *boson stars*?
Why study *boson stars*?
- Results: Typical mass and radius of a boson star
Probing the space-time of a *boson star*
- Conclusions & Outlook

Motivation ...

...for studying boson stars

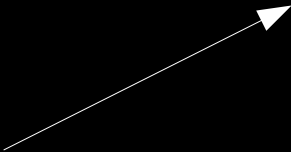
Self-gravitating scalar fields

$$S = \int \sqrt{-g} d^4 x \left(\frac{R}{16\pi G} + L_{\text{matter}} \right) \quad \text{action}$$

Matter Lagrangian
density

$$L_{\text{matter}} = -(\partial_\mu \Phi)(\partial^\mu \Phi)^* - V(|\Phi|)$$

Scalar potential



The scalar potential

Example 1:

Compact
boson stars

$$V(|\Phi|) = 2\lambda|\Phi|^4$$

Arodz, Lis

Phys. Rev. D 77 (2008) 107702

BH, Kleihaus, Kunz, Schaffer,
Phys. Lett. B 714 (2012)

Example 2:

Non-compact
boson stars

$$V(|\Phi|) = m^2 \eta^2 \left[1 - \exp(-|\Phi|^2/\eta^2) \right]$$

Scalar boson mass

Energy scale....

... e.g. SUSY breaking scale:
Copeland, Tsumagari
Phys. Rev. D 80 (2009)

Properties of boson stars

L_{matter} has global U(1) symmetry $\Phi \rightarrow \Phi \exp(i\chi)$

Globally conserved Noether charge Q

$$Q = -i \int \sqrt{-g} (\Phi^* \partial^0 \Phi - \Phi \partial^0 \Phi^*) d^3x$$

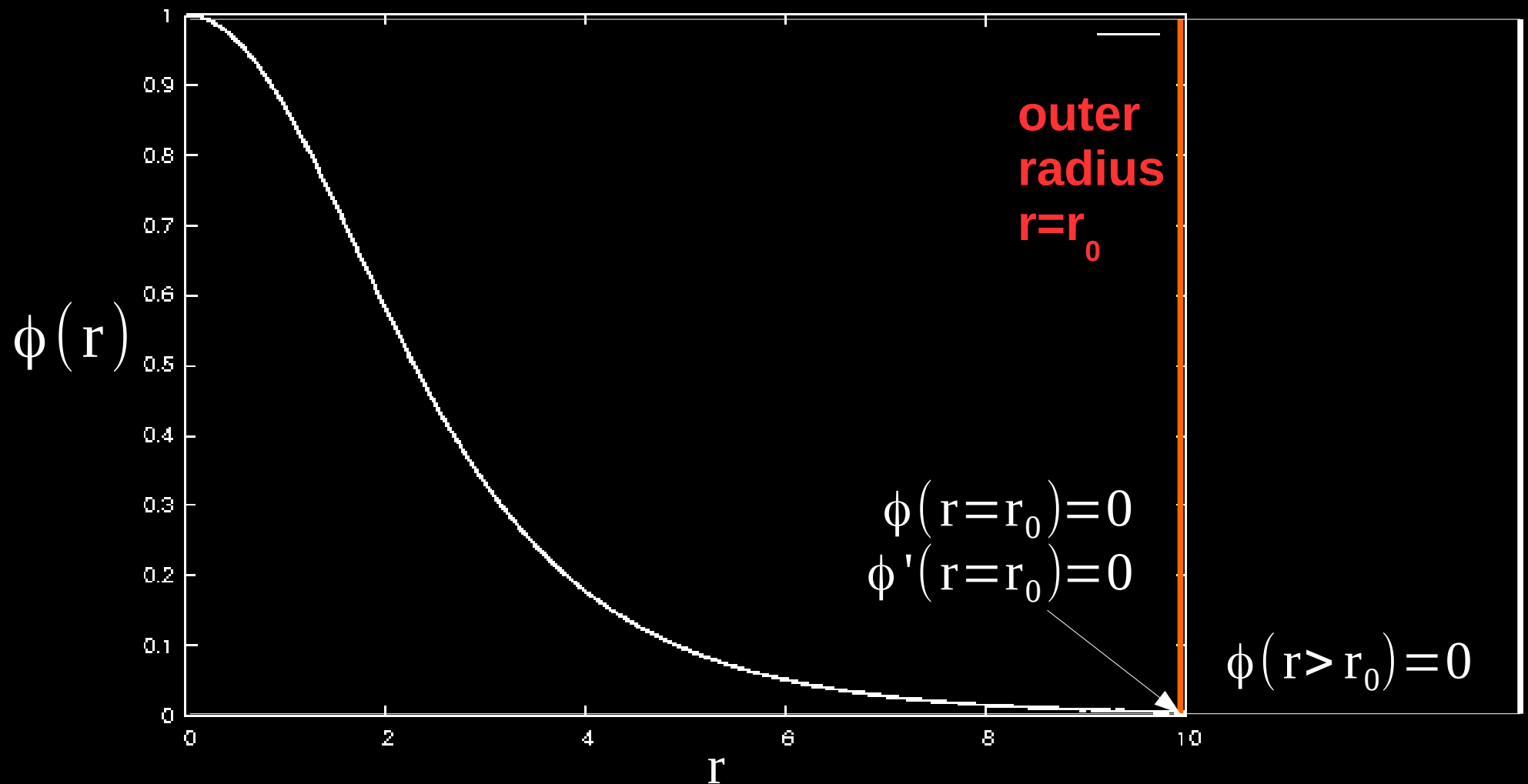
can be interpreted as number of scalar particles

Ansatz for spherically symmetric, non-rotating solutions

$$\Phi(r, t) = \phi(r) \exp(i\omega t)$$

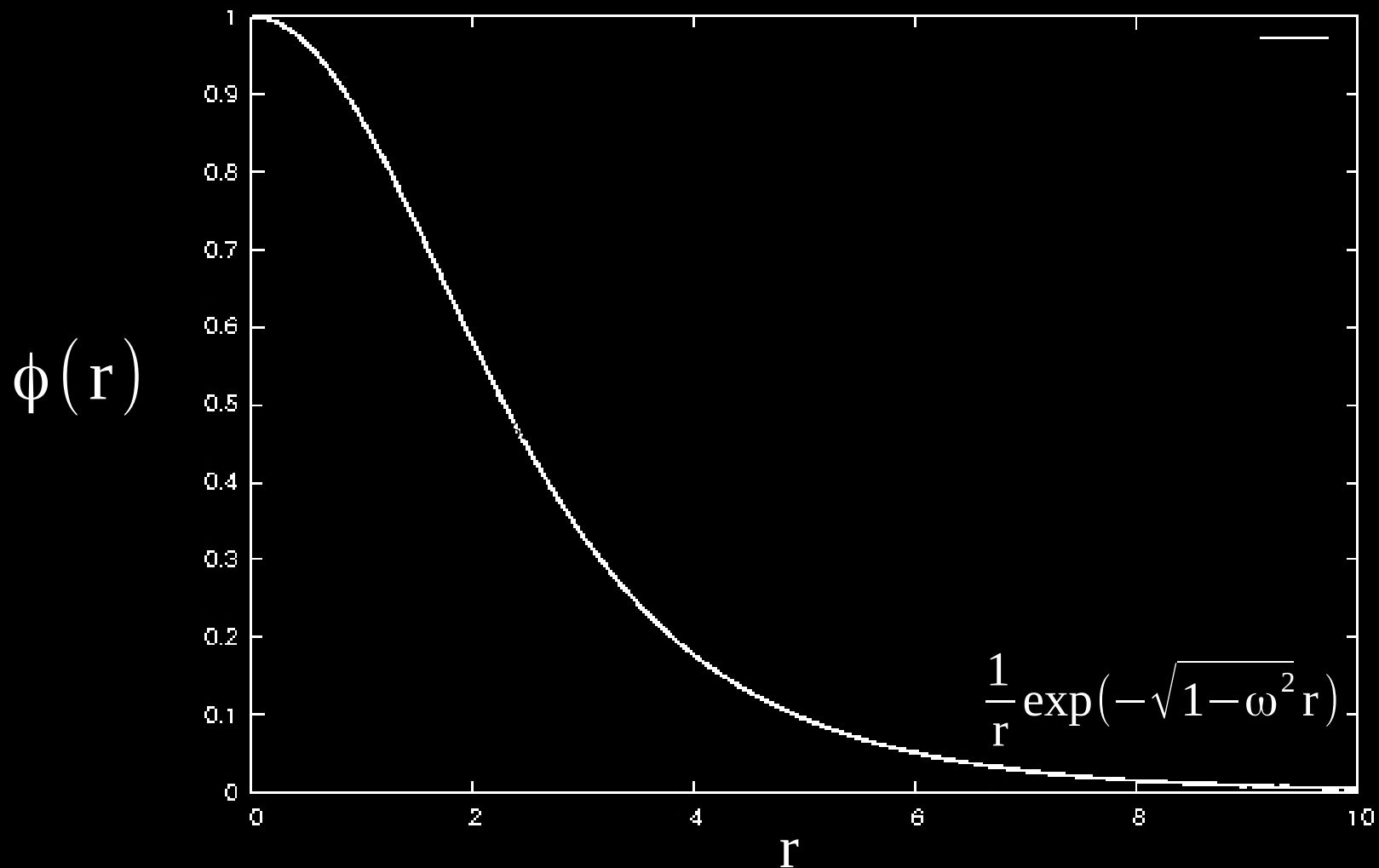
Harmonic time
dependence

Compact boson stars



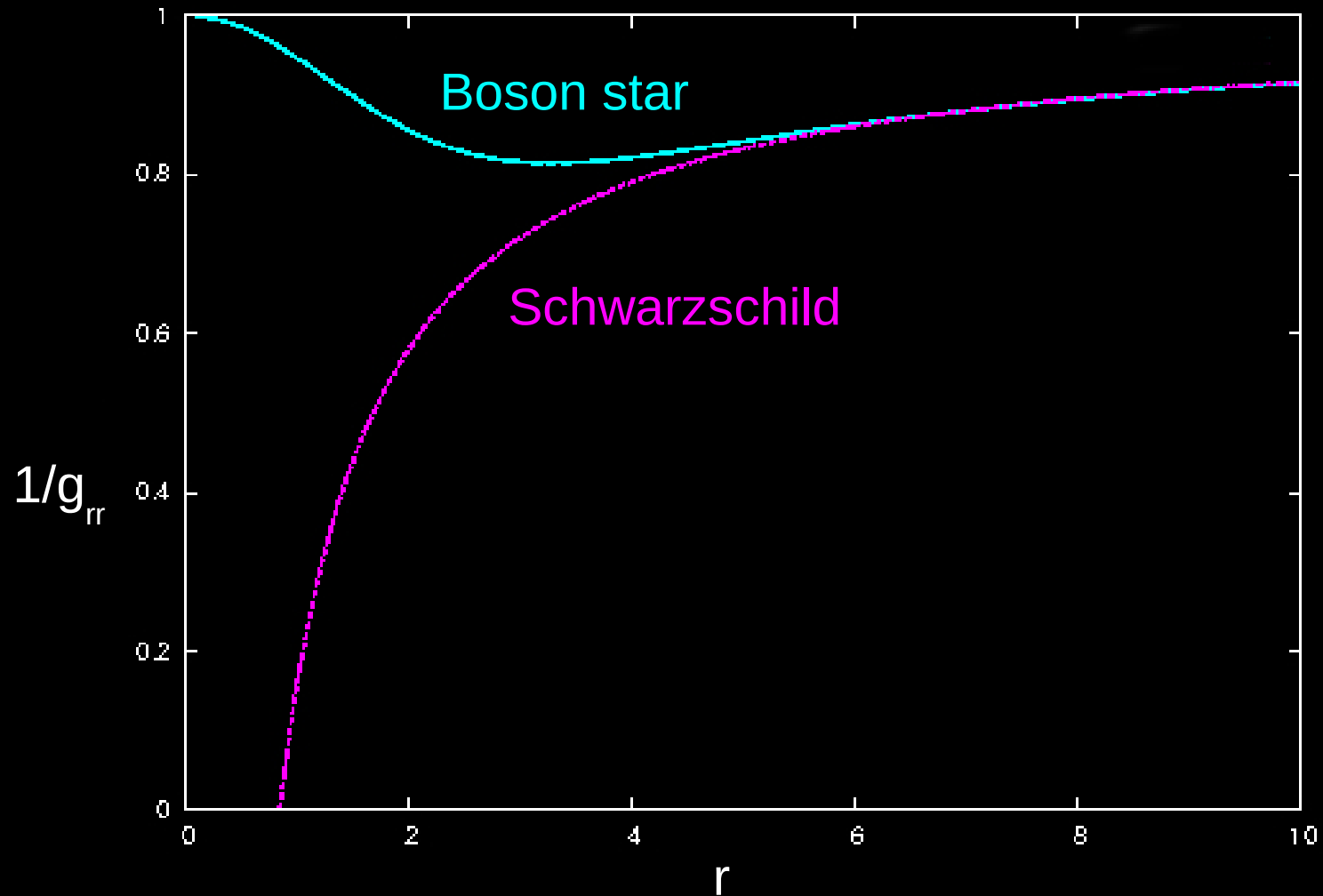
Boson star has a definite surface ("hard core") at $r=r_0$

Non-compact boson stars



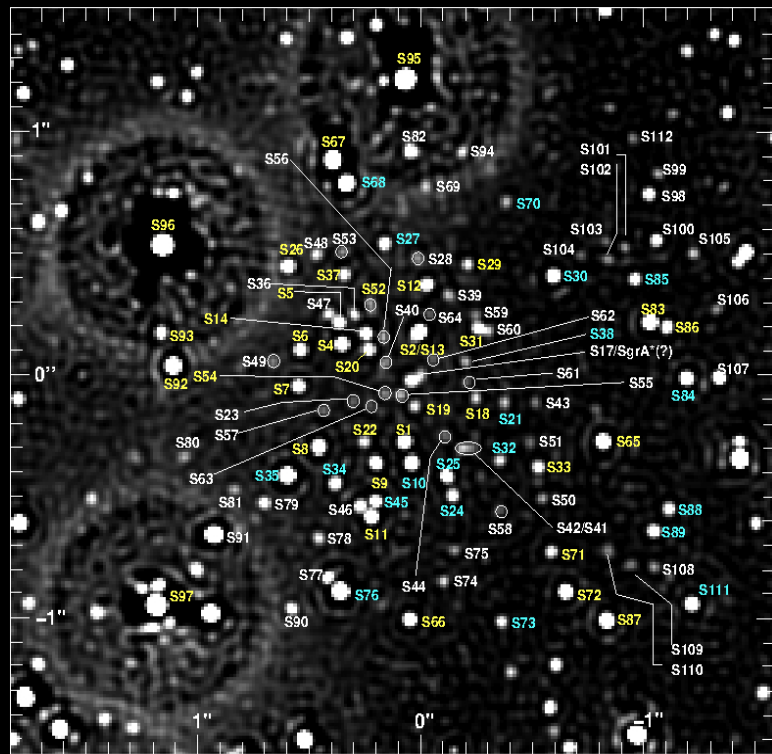
Boson star has no definite surface (no “hard core”)

Boson star vs Black hole



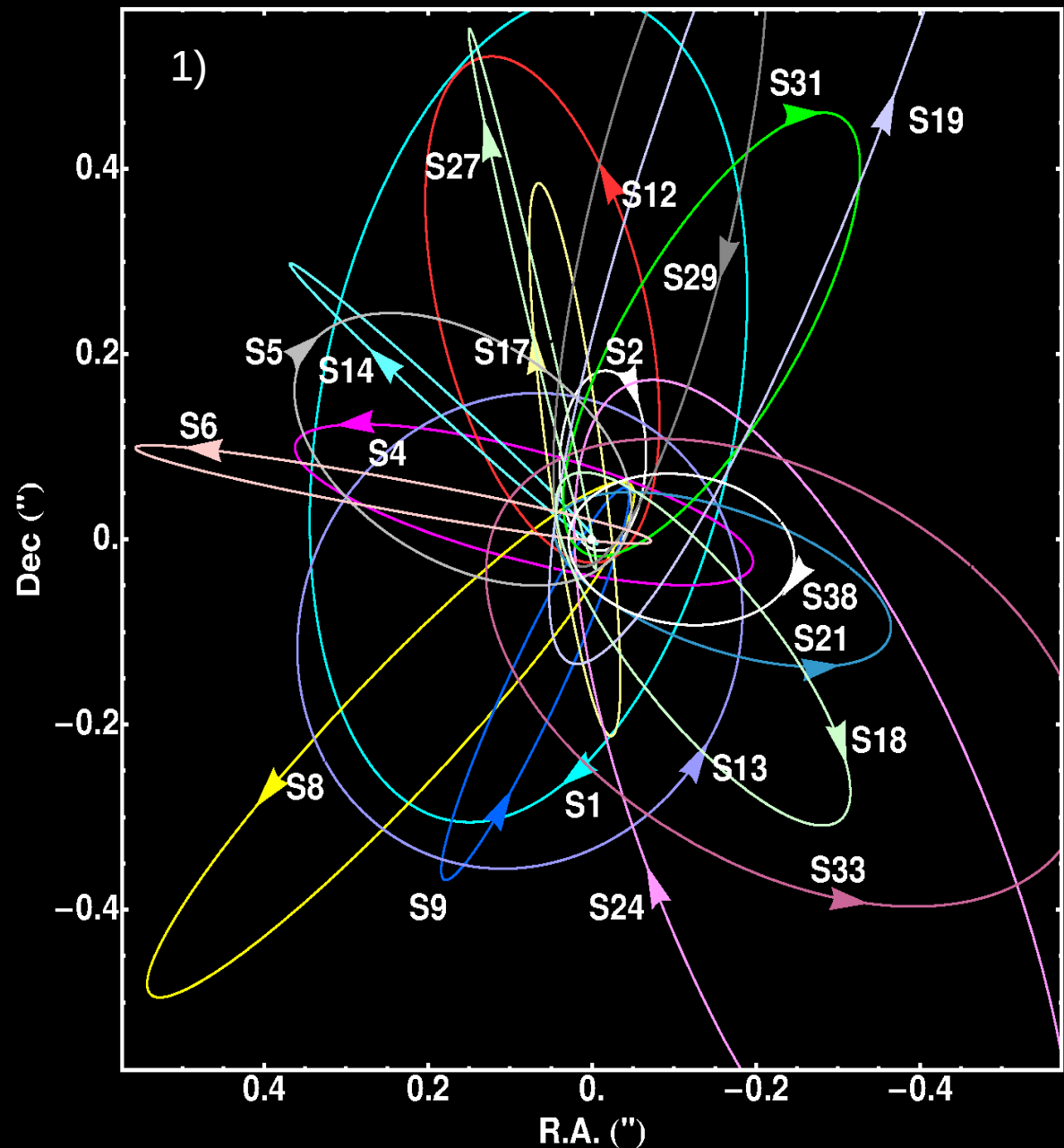
Boson star is globally regular

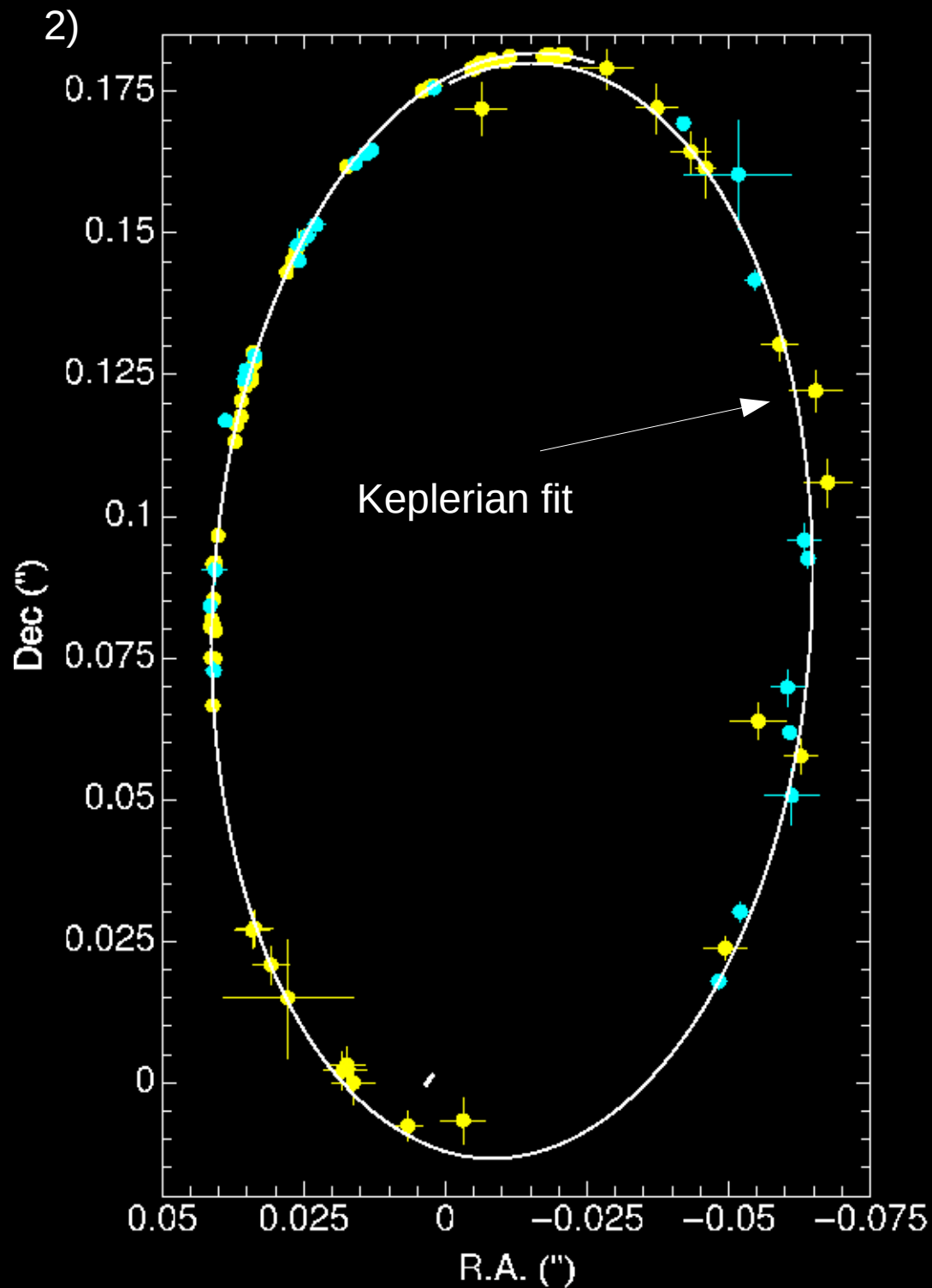
Supermassive black holes (?!?)



Monitoring O(100) stars
around Sagittarius A*
since 1992

1) from:
Gillessen, Eisenhauer, Trippe,
Alexander, Genzel, Martins, Ott
Astrophys. J. 692 (2009)





Orbit of S2

Period $\approx 15.56 \pm 0.35$ years
(first full orbit completed in 2008)

Eccentricity ≈ 0.87

$$R_{\text{SgA}^*} \approx 2.2 \times 10^7 \text{ km}$$

$$M_{\text{SgA}^*} \approx 4.5 \times 10^6 M_{\text{sun}}$$

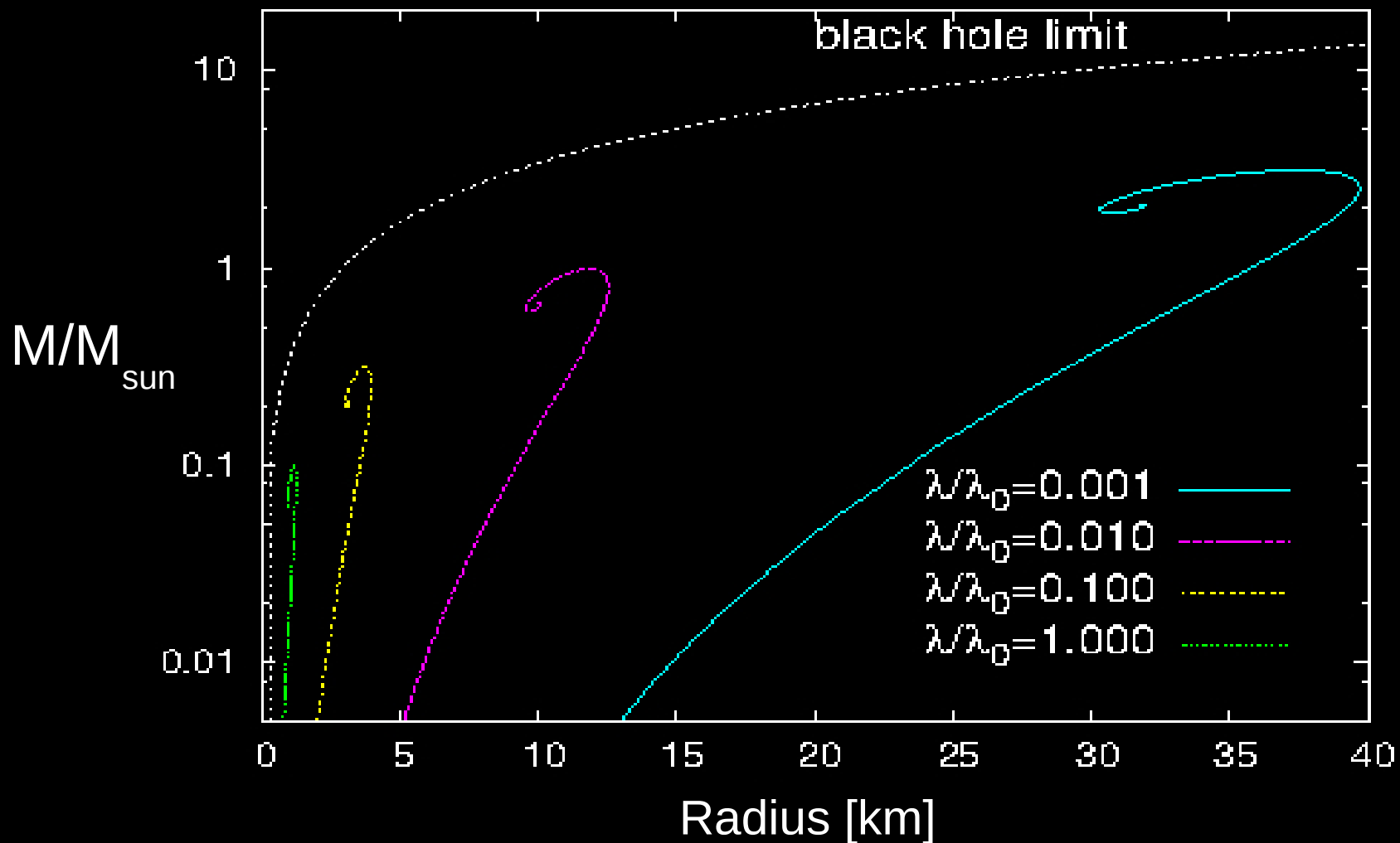
SgA*

Best “standard” explanation:
Supermassive black hole

2) From:
Gillessen, Eisenhauer, Fritz, Bartko,
Dodds-Eden, Pfuhl, Ott, Genzel
Astroph. J. 707 (2009)

Results

Mass and radius of compact boson stars

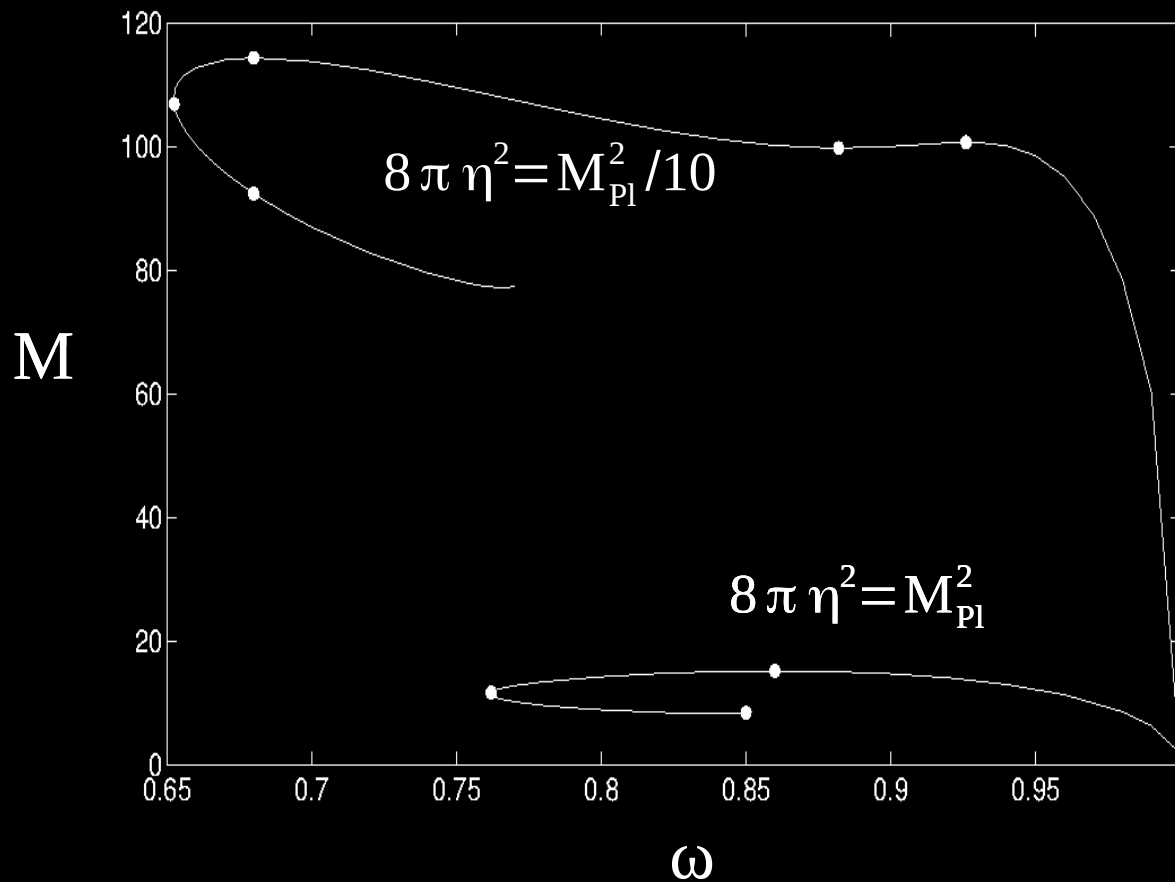


BH, Kleihaus, Kunz, Schaffer,
Phys. Lett. B 714 (2012)

$$\lambda_0 = (2 \times 10^{30} \text{ kg m}^{-3} \text{ sec}^{-2})^{1/2}$$

Mass of non-compact boson star

Diemer, Eilers, BH, Schaffer, Toma, *Phys. Rev. D*88 (2013)



Physical mass

$$M_{\text{phys}} \approx \frac{M \times 10^{20}}{m [\text{eV}]} \text{ kg}$$

Maximal physical mass

$$M_{\text{phys}}^{\text{max}} \approx \frac{7}{4 \pi} \frac{M_{\text{pl}}^4}{\eta^2 m} \approx \frac{10^{76}}{(\eta [\text{eV}])^2 m [\text{eV}]}$$

Mass & radius of compact boson star

Diemer, Eilers, BH, Schaffer, Toma, *Phys. Rev. D*88 (2013)

Example: $8 \pi \eta^2 = M_{\text{Pl}}^2 / 10$

Particle	m	M _{phys}	R ₉₉
Higgs	125 GeV/c ²	10 ¹¹ kg	10 ⁻¹⁵ meters
Pion	140 MeV/c ²	10 ¹⁴ kg	10 ⁻¹³ meters
Axion	10 ⁻⁵ eV/c ²	10 ²⁷ kg	1 meter
Dilaton	10 ⁻¹⁰ eV/c ²	10 ³² kg	100 km

R₉₉: radius in which 99% of the mass is contained ← numerics!

Mass $SgA^* \gg$ mass of S-stars

Test particles

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma_{\rho\sigma}^\mu \frac{dx^\rho}{d\tau} \frac{dx^\sigma}{d\tau} = 0$$

Geodesic equation

For spherically symmetric, static space-times:

- a) test particle motion is **planar** $\longrightarrow$ restrict to equatorial plane $\theta = \pi/2$
- b) **constants of motion**: energy E and angular momentum L

$$E \sim g_{tt} \frac{dt}{d\tau} \qquad L \sim g_{\varphi\varphi} \frac{d\varphi}{d\tau}$$

Some technicalities

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma_{\rho\sigma}^\mu \frac{dx^\rho}{d\tau} \frac{dx^\sigma}{d\tau} = 0$$

Note: space-time of boson star
not given analytically

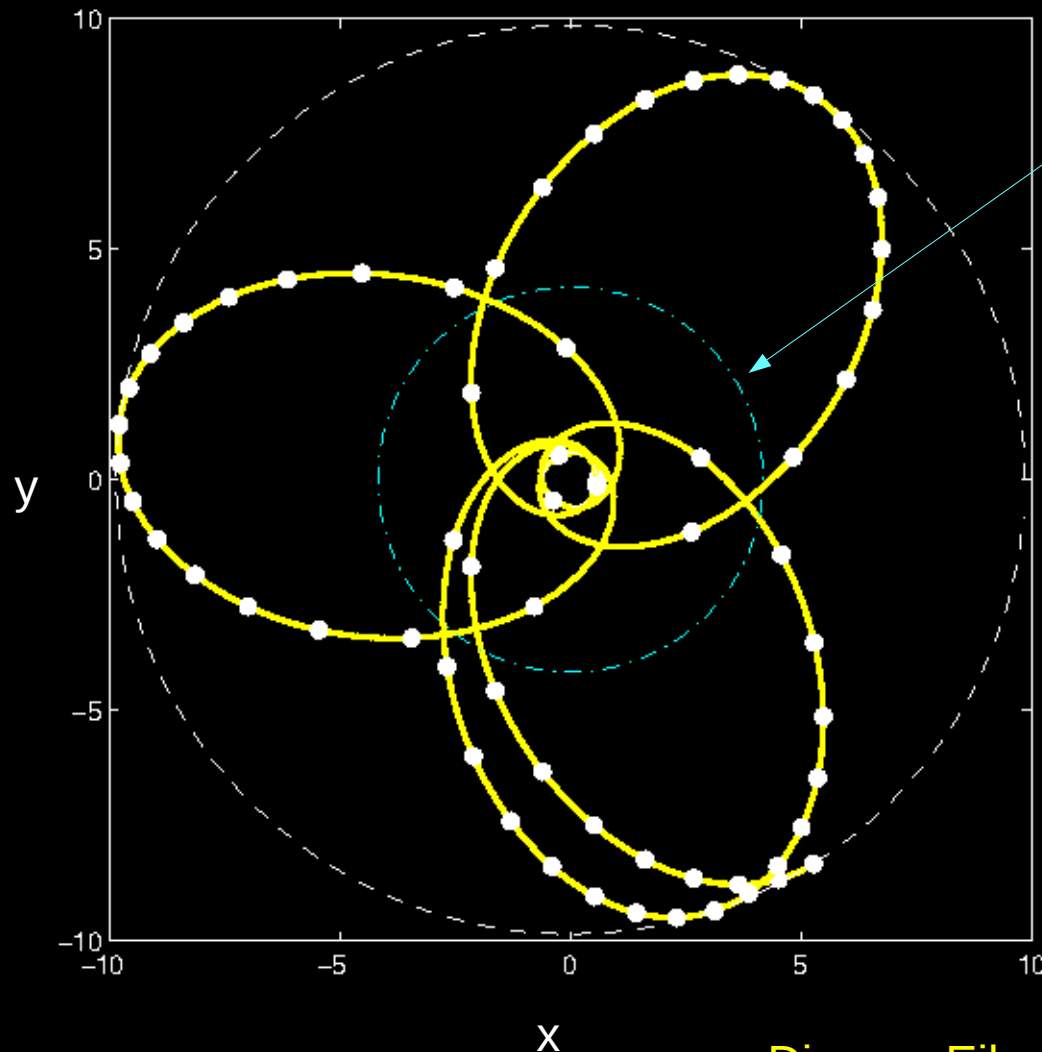
Procedure:

- 1) solve coupled scalar field and Einstein equations numerically (non-linear, coupled ODEs)
- 2) extract behaviour of metric functions
- 3) interpolate numerical data for metric functions and compute Christoffel symbols
- 4) solve geodesic equation numerically



Bound orbit of a massive test particle

$$L^2 = 1.0 \quad E^2 = 0.95 \quad 8\pi G\eta^2 = 1.0 \quad \omega = 0.85$$



corresponding
Schwarzschild radius

Perihelion shift of orbits

Particles can approach
boson star core
(arbitrarily) close &
exit that region again

Diemer, Eilers, BH, Schaffer, Toma, *Phys. Rev. D*88 (2013)

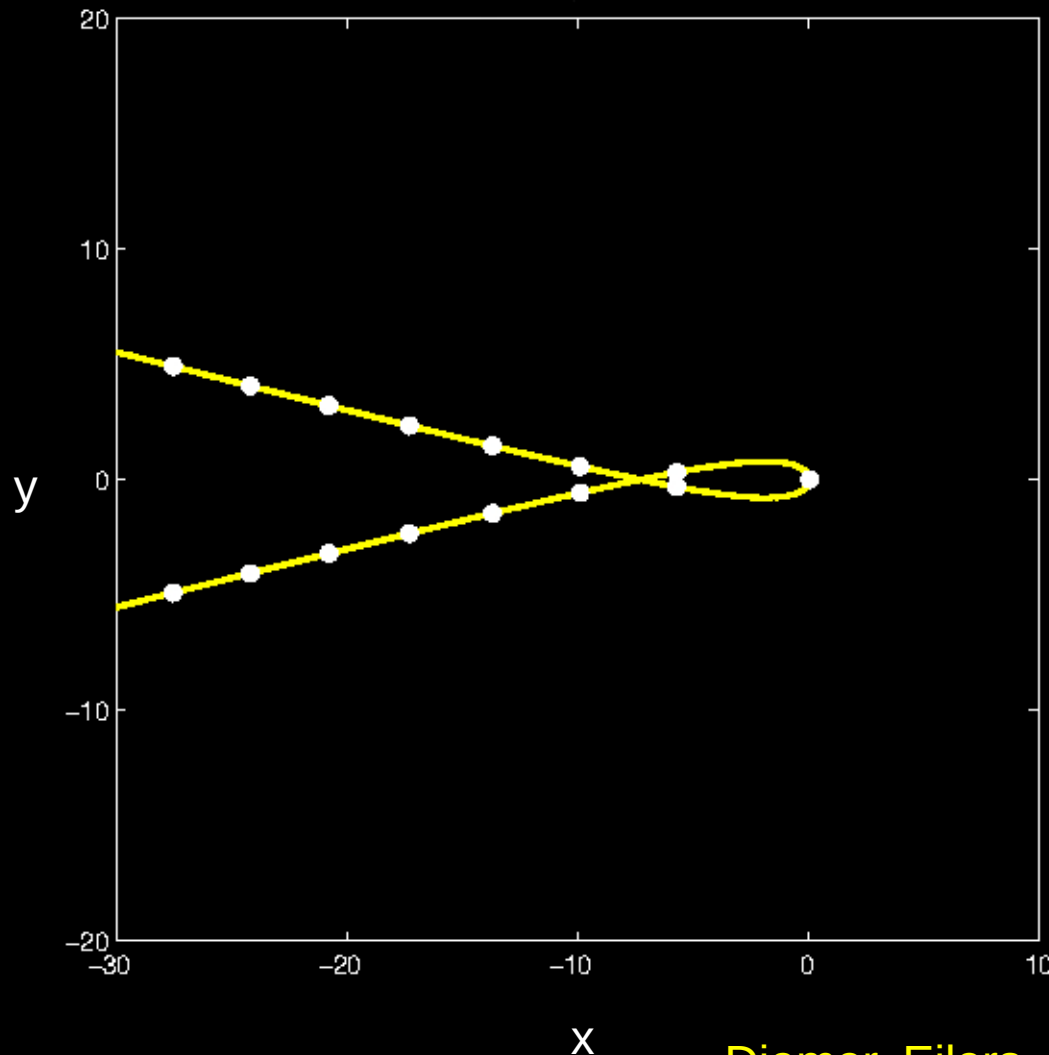
Escape orbit of a massive test particle

$$L^2 = 0.5$$

$$E^2 = 1.1$$

$$8\pi G\eta^2 = 0.1$$

$$\omega = 0.68$$



New features as
compared to
Schwarzschild
black hole

(e.g. intersecting
escape orbits)

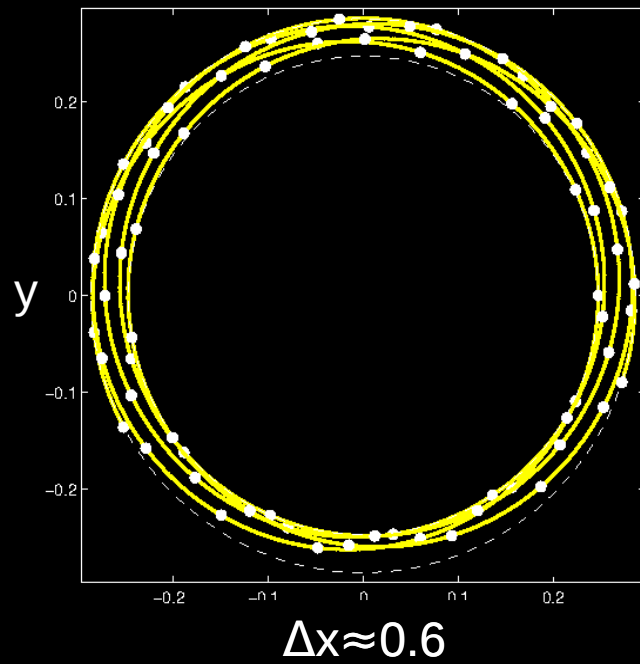
Diemer, Eilers, BH, Schaffer, Toma, *Phys. Rev. D*88 (2013)

Last stable (nearly) circular orbit

Diemer, Eilers, BH, Schaffer, Toma, *Phys. Rev. D*88 (2013)

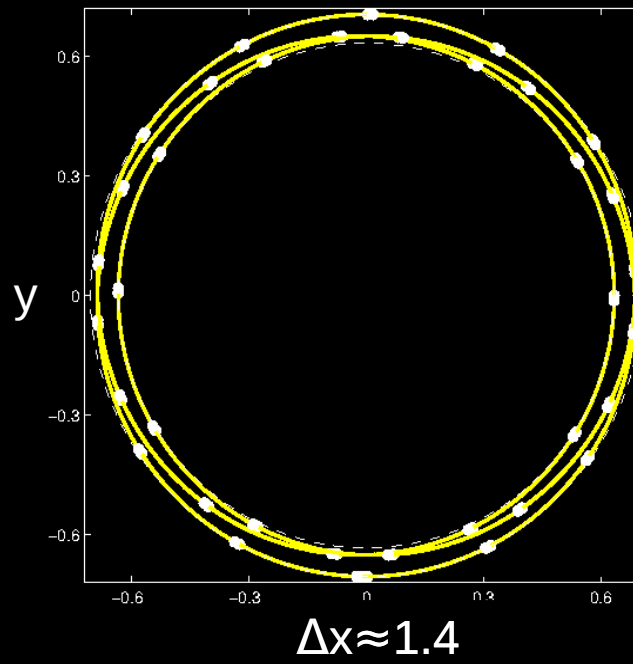
$M=92.34$

$E^2 = 0.404, L^2 = 0.5$



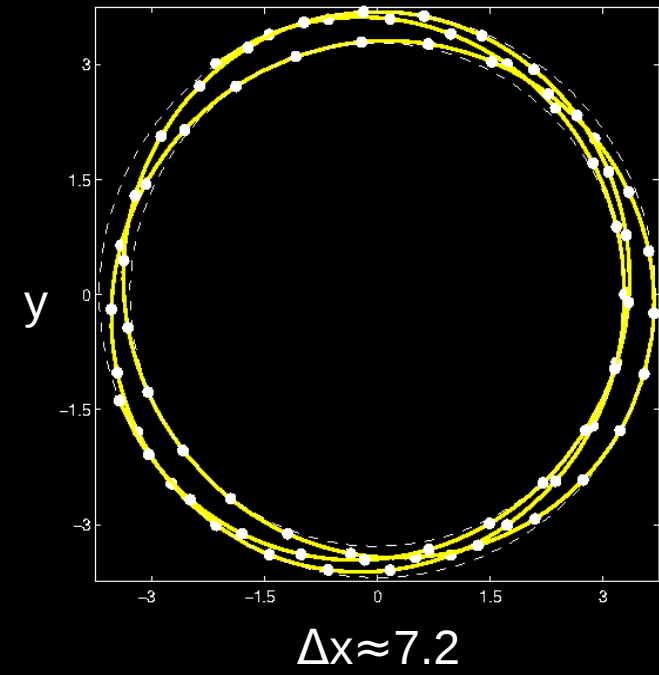
$M=106.85$

$E^2 = 0.742, L^2 = 1.5$



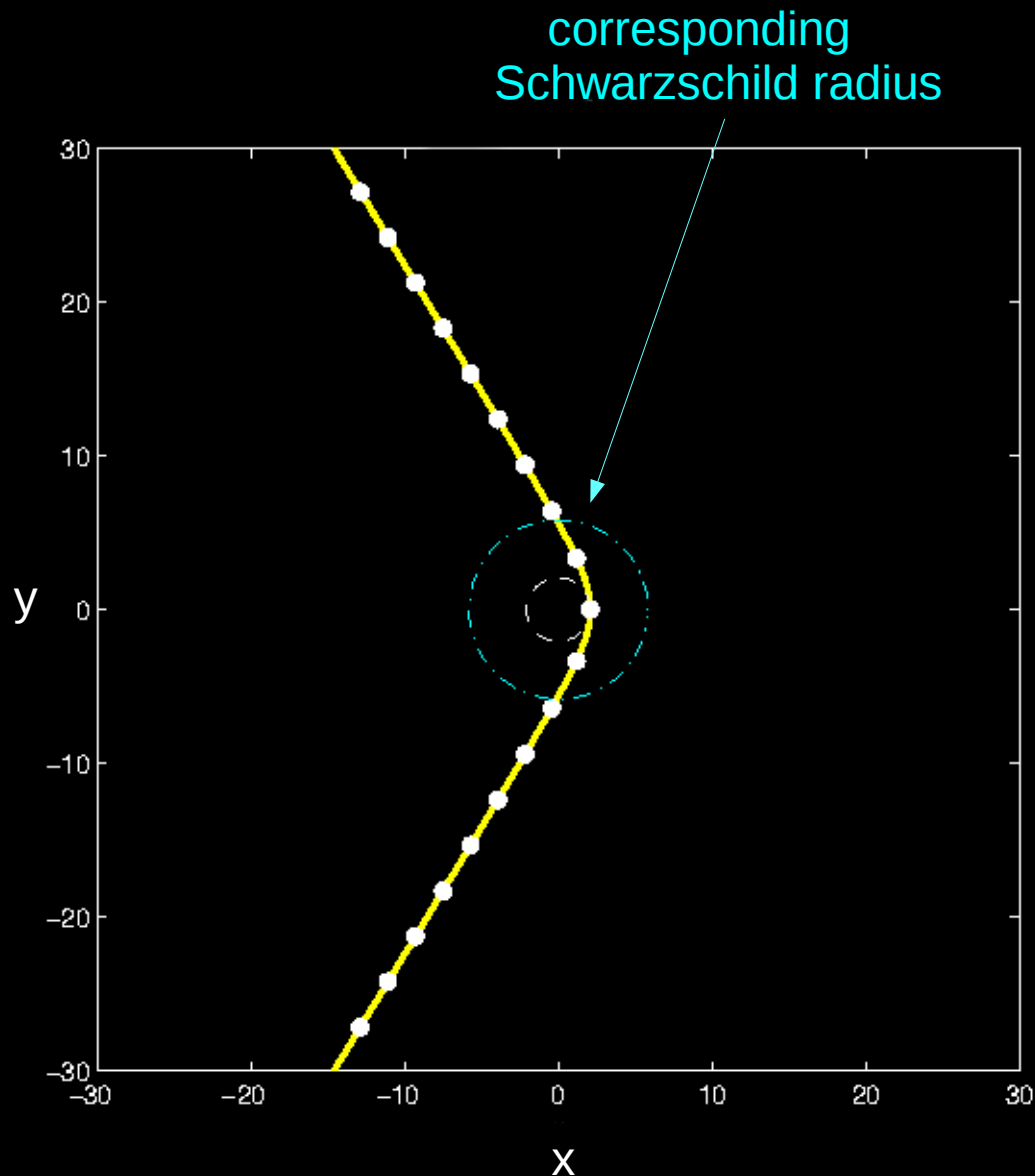
$M=100.64$

$E^2 = 0.884, L^2 = 1.0$



length scales measured in units of $1/(\text{boson mass})$

Escape orbit of massless test particle



Massless test particle
can enter into &
exit from
region beyond
assumed
Schwarzschild radius

Compare to observation of
emission of **radio waves** (1.3 mm)
from beyond assumed
apparent horizon of SgA*

Doeleman, Weintraub, Rogers,
Plambeck, Freund, Tilanus,
Friberg, Ziurys et al.
Nature 455 (2008)

Diemer, Eilers, BH, Schaffer, Toma,
Phys. Rev. D 88 (2013)

Extension to charged test particles

U(1) global $\rightarrow$ U(1) local

Q: charge of boson star

Equation of motion for charged test particles reads:

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma_{\rho\sigma}^\mu \frac{dx^\rho}{d\tau} \frac{dx^\sigma}{d\tau} \mp q F^{\mu\sigma} \frac{dx^\rho}{d\tau} g_{\rho\sigma}$$

charge of test particle

U(1) field strength

Force term

Conclusions...

... & Outlook

Boson stars...

... are a (not yet) excluded alternative to supermassive black holes

Objects with “hard core” ruled out (c.f. Broderick, Narayan, *Astrophys. J.* 638 (2006))

... are well motivated since at least one **fundamental scalar field** seems to exist in nature (Higgs) – maybe more (beyond the SM...)

... are highly **relativistic** and could hence be used as toy models for very compact objects, e.g. neutron stars

We have studied the motion of test particles in the space-time of a spherically symmetric, non-rotating boson star

} Compatible
with
observational
data

Need data of **relativistic** signature of gravitational field of SgA*

perihelion shift, light deflection ...

... then **boson stars can be distinguished from black holes**

To do...

... test particles in space-time of rotating boson star

$$\Phi(r, \theta, \varphi, t) = \phi(r, \theta) \exp(i\omega t + i n \varphi)$$

Space-time is axially symmetric

Volkov, Woehnert.
Phys.Rev. D66 (2002)

... test particles in extended gravity models

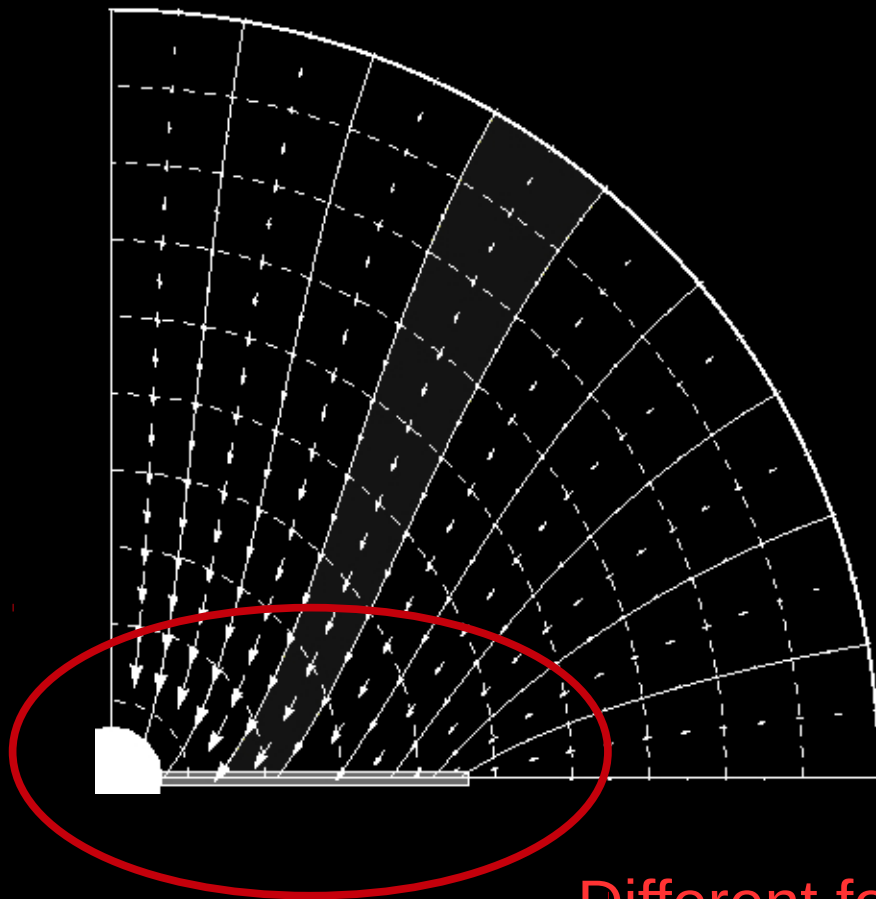
e.g. Gauss-Bonnet boson stars

Testing GR in strong
gravitational fields

BH, Riedel, Suciu,
Phys.Lett. B726 (2013) 906

And also.....

... use test particles to describe formation of **accretion discs**
in simple way



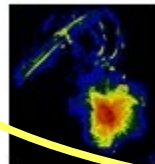
Taken from:
Tejeda, Taylor, Miller,
Mont. Not. R. Astron. Soc. 419 (2012)

Different for boson stars

Event Horizon Telescope

<http://eventhorizontelescope.org>

Primary observing targets



The black hole at the center of the Milky Way - Sgr A* - Located 25,000 light years from us in the center of our Galaxy, Sagittarius A* is our nearest massive black hole with four million times the mass of the Sun. Its proximity makes it the largest apparent black hole as viewed from the Earth.



A giant elliptical galaxy - M87 - At a distance of 53 million light years, the Messier object M87 is our second primary target. Even though it is much farther than Sgr A*, its central black hole is much more massive, 6 billion times the mass of the Sun, making it the next largest apparent black hole as viewed from the Earth.

Science requirements



High angular resolution - The nearest massive black hole is in the center of our Galaxy. Even though the size of this black hole is quite big, about 30 times the size of the Sun, it is at such a large distance from us that it appears about the same size as an orange on the Moon. Even though this is astonishingly small, the EHT will have the ability to discern things at this fine scale.



High sensitivity - Intrinsicly, the radiation from cosmic sources becomes very weak by the time it reaches the Earth. The radio emission from the nearest massive black holes is faint even by radio astronomy standards. This necessitates the use of very large telescopes and very high data rates.



Modeling and simulations - Black holes themselves are simple objects, but the details of features seen in the data are heavily dependent on the complex environment of the material surrounding them. This requires us to model the complex environment as well as its observational signatures.

**Observational data
will be available soon**

**Modeling of possible
effects extremely
important**

Thanks to my collaborators

Yves Brihaye	Université de Mons, Belgium
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Jutta Kunz	University of Oldenburg, Germany
Isabell Schaffer	University of Oldenburg, Germany
Raluca Suciù	Jacobs University Bremen, Germany
Catalin Toma	Jacobs University Bremen, Germany

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- V. Diemer, K. Eilers, BH, I. Schaffer, C. Toma, *Phys. Rev. D* 88 (2013) 044025
- BH, J. Riedel, R. Suciù, *Phys. Lett. B* 726 (2013) 906
- Y. Brihaye, V. Diemer, BH, *Phys. Rev. D* 89 (2014) 084048



Thanks for listening!