Teorias de gravidade modificadas aplicadas a estrelas compactas

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Outline

- teoria da relatividade geral (RG)
- teoria de Rastall
- estrelas de nêutrons na teoria de Rastall
- estrelas com rotação na teoria de Rastall
- relações de dispersão modificadas / equação de estado (EoS) com relações de dispersão modificadas
- gravidade Rainbow / estrelas de nêutrons nas teorias de Rastall e Rainbow combinadas

RG

- estende o princípio da relatividade para referenciais não-inerciais
- generaliza teoria de gravitação Newtoniana
- Princípio da Equivalência
- gravidade pode ser explicada por uma curvatura no espaço-tempo

RG

- solução de Schwarzschild $\rightarrow$ testes clássicos
- anos 1950 $\rightarrow$ objetos astronômicos altamente energéticos: quasares, fontes compactas de raios-X
- 1967 $\rightarrow$ descoberta dos pulsares
- atualmente $\rightarrow$ ondas gravitacionais, buraco negro

RG

Problemas em aberto:

- matéria escura
- energia escura
- inflação cósmica
- entre outros

$$\Downarrow$$

Teorias de gravidade alternativas à RG

Equações de campo de Einstein

- ação para a RG

$$S = S_G + S_M,$$

- ação de Einstein-Hilbert

$$S_G = \frac{1}{16\pi}\int R\sqrt{-g}\, dx^4$$

- ação relativa ao conteúdo de matéria e energia

$$S_M = \int \mathcal{L}_M \sqrt{-g}\, dx^4$$

Equações de campo de Einstein

- princípio da mínima ação

$$\delta S = 0$$

- equações de campo de Einstein

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu}$$

- onde, tensor de energia-momento

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta \left(\mathcal{L}_M \sqrt{-g} \right)}{\delta g^{\mu\nu}}$$

Equações de campo de Einstein

- temos que

$$\nabla_\mu G^{\mu\nu} = \nabla_\mu T^{\mu\nu} = 0$$

- traço das equações de campo de Einstein

$$R = -8\pi T$$

- forma alternativa das equações de campo de Einstein

$$R_{\mu\nu} = 8\pi \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right)$$

- vácuo

$$R_{\mu\nu} = 0$$

Teoria de Rastall

- proposta de Peter Rastall: $T^{\nu}{}_{\mu;\nu} \propto R_{,\mu}$

$$T^{\nu}{}_{\mu;\nu} = \frac{\lambda}{k\left(4\lambda - 1\right)} R_{,\mu}$$

- consistente com equação de campo

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = k \left(T_{\mu\nu} - \frac{\lambda}{k\left(4\lambda - 1\right)} g_{\mu\nu} R \right)$$

- traço da equação de campo

$$R = k\left(4\lambda - 1\right) T$$

- equação de campo

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = k \tau_{\mu\nu}$$

Teoria de Rastall

- onde

$$\tau_{\mu\nu} = T_{\mu\nu} - \lambda g_{\mu\nu} T$$

- vácuo

$$R_{\mu\nu} = 0$$

- tomando o divergente da equação de campo

$$\tau^{\nu}{}_{\mu;\nu} = 0$$

- limite newtoniano

$$k = \frac{8\pi}{2\lambda + 1}$$

Estrelas de nêutrons

- raio aproximadamente $10Km$
- massa entre $1-2M_{\odot}$
- densidade central $5-10\rho_{sat}$
- campo magnético intenso $10^{12}G$
- rotação até $716Hz$

- tensor de energia-momento para fluido perfeito

$$T_{\mu\nu} = pg_{\mu\nu} + (p+\varepsilon)u_{\mu}u_{\nu}$$

- tensor de energia-momento efetivo

$$\tau_{\mu\nu} = p_{ef}\, g_{\mu\nu} + (p_{ef} + \varepsilon_{ef})u_{\mu}u_{\nu}$$

- onde

$$\begin{aligned} p_{ef} &= p\,(1-3\lambda) + \lambda\varepsilon, \\ \varepsilon_{ef} &= \varepsilon\,(1-\lambda) + 3\lambda p \end{aligned}$$

Estrelas estáticas na RG

- métrica estática e esfericamente simétrica

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2 d\theta^2 + r^2 \sin\theta^2 d\phi^2$$

- vácuo: métrica de Schwarzschild

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin\theta^2 d\phi^2$$

- interior estelar com $T^{\alpha\beta}$ de fluido perfeito

$$m(r) = 4\pi \int_0^r \rho(r') r'^2 dr'$$

- equação de Tolman–Oppenheimer–Volkoff (TOV)

$$p' = -\frac{m}{r^2}\rho \left(1 + \frac{p}{\rho}\right)\left(1 + \frac{4\pi r^3 p}{m}\right)\left(1 - \frac{2m}{r}\right)^{-1}$$

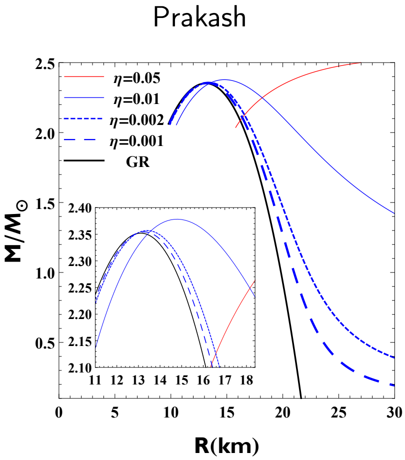
- interior estelar com $T^{\alpha\beta}$ de fluido perfeito

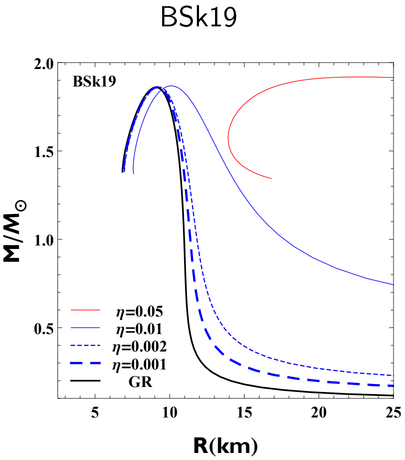
$$m(r) = 4\pi \int_0^r \rho_{ef}(r')r'^2 dr'$$

- equação de Tolman–Oppenheimer–Volkoff (TOV)

$$p'_{ef} = -\frac{m}{r^2}\rho_{ef}\left(1 + \frac{p_{ef}}{\rho_{ef}}\right)\left(1 + \frac{4\pi r^3 p_{ef}}{m}\right)\left(1 - \frac{2m}{r}\right)^{-1}$$

Estrelas estáticas na Rastall





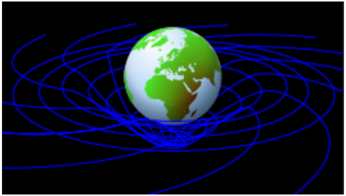
Oliveira, A. M., Velten, H. E. S., Fabris, J. C., Casarini, L. (2015). Neutron stars in Rastall gravity. Physical Review D, 92(4), 044020.

Métrica para estrela com rotação

- estrela relativística rotacionando em equilíbrio $\Rightarrow$ métrica estacionária e axialmente simétrica
- forma da métrica que vamos utilizar

$$ds^2 = -e^{2\nu} dt^2 + e^{2\alpha} \left(dr^2 + r^2 d\theta^2 \right) + e^{2\beta} r^2 \sin^2\theta \left(d\phi - \omega dt \right)^2$$

- $\omega \Rightarrow$ efeito Lense-Thirring



Fluido perfeito com rotação uniforme

- quadri-velocidade

$$u^\alpha = \frac{dx^\alpha}{d\tau} = \frac{e^{-\nu}}{\sqrt{1 - v^2}}\,(1, 0, 0, \Omega)$$

- velocidade própria para ZAMO

$$v = (\Omega - \omega)\, r \sin\theta e^{\beta - \nu}$$

- Ω é a velocidade angular de um elemento de massa da estrela com relação a um observador estático no infinito

Equações de campo para estrela com rotação rápida na RG

- equações de campo

$$\Delta\left[\rho e^{\gamma/2}\right] = S_\rho\left(\alpha,\rho,\gamma,\omega,p,\varepsilon\right)$$

$$\left(\Delta + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}\mu\frac{\partial}{\partial\mu}\right)\gamma e^{\gamma/2} = S_\gamma\left(\alpha,\rho,\gamma,\omega,p,\varepsilon\right)$$

$$\left(\Delta + \frac{2}{r}\frac{\partial}{\partial r} - \frac{2}{r^2}\mu\frac{\partial}{\partial\mu}\right)\omega e^{(\gamma-2\rho)/2} = S_\omega\left(\alpha,\rho,\gamma,\omega,p,\varepsilon\right)$$

- onde

$$\rho = \nu - \beta, \quad \gamma = \nu + \beta \quad \text{and} \quad \mu = \cos\theta$$

$$
\begin{aligned}
&\alpha_{\prime\mu} = -\nu_{\prime\mu} - \left\{ \left(1-\mu^2\right)\left(1 + rB^{-1}B_{\prime r}\right)^2 + \left[\mu - \left(1-\mu^2\right)B^{-1}B_{\prime\mu}\right]^2 \right\}^{-1} \\
&\left[\frac{1}{2} B^{-1} \left\{ r^2 B_{\prime rr} - \left[\left(1-\mu^2\right)B_{\prime\mu}\right]_{\prime\mu} - 2\mu B_{\prime\mu} \right\} \left\{ -\mu + \left(1-\mu^2\right)B^{-1}B_{\prime\mu} \right\} \right. \\
&+ rB^{-1}B_{\prime r}\left[\frac{1}{2}\mu + \mu r B^{-1}B_{\prime r} + \frac{1}{2}\left(1-\mu^2\right)B^{-1}B_{\prime\mu}\right] + \frac{3}{2}B^{-1}B_{\prime\mu} \\
&\left[-\mu^2 + \mu\left(1-\mu^2\right)B^{-1}B_{\prime\mu}\right] - \left(1-\mu^2\right)rB^{-1}B_{\prime\mu r}(1 + rB^{-1}B_{\prime r}) \\
&- \mu r^2\left(\nu_{\prime r}\right)^2 - 2\left(1-\mu^2\right)r\nu_{\prime\mu}\nu_{\prime r} + \mu\left(1-\mu^2\right)\left(\nu_{\prime\mu}\right)^2 - 2\left(1-\mu^2\right)r^2 \\
&B^{-1}B_{\prime r}\nu_{\prime\mu}\nu_{\prime r} + \left(1-\mu^2\right)B^{-1}B_{\prime\mu}\left[r^2\left(\nu_{\prime r}\right)^2 - \left(1-\mu^2\right)\left(\nu_{\prime\mu}\right)^2\right] \\
&+ \left(1-\mu^2\right)B^2 e^{-4\nu}\left\{ \frac{1}{4}\mu r^4\left(\omega_{\prime r}\right)^2 + \frac{1}{2}\left(1-\mu^2\right)r^3\omega_{\prime\mu}\omega_{\prime r} \right. \\
&- \frac{1}{4}\mu\left(1-\mu^2\right)r^2\left(\omega_{\prime\mu}\right)^2 + \frac{1}{2}\left(1-\mu^2\right)r^4B^{-1}B_{\prime r}\omega_{\prime\mu}\omega_{\prime r} \\
&\left.\left. -\frac{1}{4}\left(1-\mu^2\right)r^2B^{-1}B_{\prime\mu}\left[r^2\left(\omega_{\prime r}\right)^2 - \left(1-\mu^2\right)\left(\omega_{\prime\mu}\right)^2\right] \right\} \right]
\end{aligned}
$$

Método KEH (Komatsu, Eriguchi e Hachisu)

- utilizando funções de Green $\rightarrow$ forma integral

$$\rho = -\, e^{-\gamma/2} \sum_{n=0}^{\infty} P_{2n}(\mu) \int_0^{\infty} dr' r'^2 f_{2n}^2(r, r')$$

$$\int_0^1 d\mu' P_{2n}(\mu') \, S_\rho(r', \mu')$$

$$\gamma = -\frac{2e^{-\gamma/2}}{\pi r \sin\theta} \sum_{n=1}^{\infty} \frac{\sin(2n-1)\theta}{2n-1} \int_0^{\infty} dr' r'^2 f_{2n-1}^1(r, r')$$

$$\int_0^1 d\mu' \sin(2n-1)\theta' S_\gamma(r', \mu')$$

Método KEH

$$\omega = -\frac{e^{(2\rho-\gamma)/2}}{r\sin\theta}\sum_{n=1}^{\infty}\frac{P_{2n-1}^{1}(\mu)}{2n(2n-1)}\int_{0}^{\infty}dr' r'^{3} f_{2n-1}^{2}(r,r')$$
$$\int_{0}^{1}d\mu' P_{2n-1}^{1}(\mu') S_{\omega}(r',\mu')$$

Komatsu, H., Eriguchi, Y., Hachisu, I. (1989). Rapidly rotating general relativistic stars-I. Numerical method and its application to uniformly rotating polytropes. Monthly Notices of the Royal Astronomical Society, 237(2), 355-379.

Equilíbrio hidrostático

- equação de conservação do tensor de energia-momento

$$T^{\mu\nu}{}_{;\mu} = 0$$

- teremos

$$\frac{\nabla p}{(\varepsilon + p)} = \nabla \ln u^t - u^t u_\phi \nabla \Omega$$

- considerando: $\Omega =$ constante $=$ rotação uniforme e EoS politrópica

$$p = K\varepsilon^{1+1/N}$$

- encontraremos

$$\left(K\varepsilon^{1/N} + 1\right)^{(N+1)} e^{\nu}\sqrt{1 - v^2} = C'$$

Estrela com rotação rápida na Rastall

- equações de campo

$$\Delta\left[\rho e^{\gamma/2}\right] = S_\rho\left(\alpha, \rho, \gamma, \omega, p_{ef}, \varepsilon_{ef}\right)$$

$$\left(\Delta + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}\mu\frac{\partial}{\partial \mu}\right)\gamma e^{\gamma/2} = S_\gamma\left(\alpha, \rho, \gamma, \omega, p_{ef}, \varepsilon_{ef}\right)$$

$$\left(\Delta + \frac{2}{r}\frac{\partial}{\partial r} - \frac{2}{r^2}\mu\frac{\partial}{\partial \mu}\right)\omega e^{(\gamma-2\rho)/2} = S_\omega\left(\alpha, \rho, \gamma, \omega, p_{ef}, \varepsilon_{ef}\right)$$

Estrela com rotação rápida na Rastall

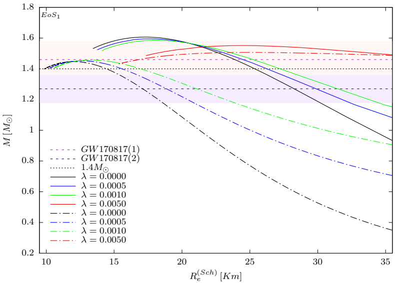
- divergente de $\tau_{\mu\nu}$

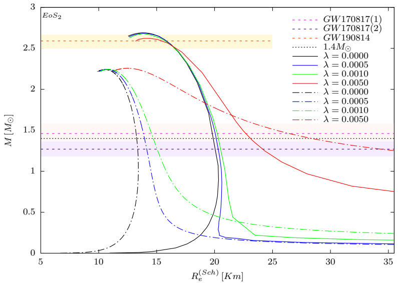
$$\frac{\nabla p_{ef}}{(\varepsilon + p)} = \nabla \ln u^t - u^t u_\phi \nabla\Omega$$

- considerando: $\Omega =$ constante $=$ rotação uniforme e EoS politrópica

$$(N(1-4\lambda) + (1-3\lambda)) \ln\left(K\varepsilon^{1/N} + 1\right) + \lambda \ln(\varepsilon) + \nu$$
$$+ \frac{1}{2}\ln\left(1 - v^2\right) = C$$

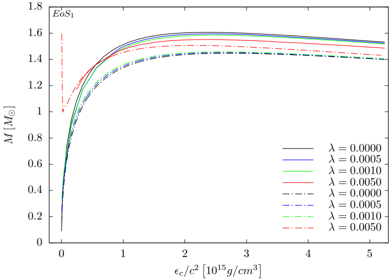
Resultados: $M \times R_e^{Schw}$

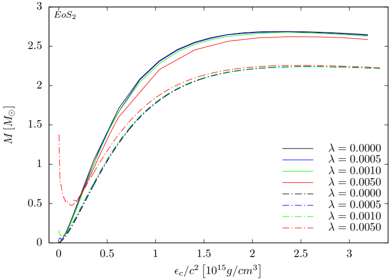




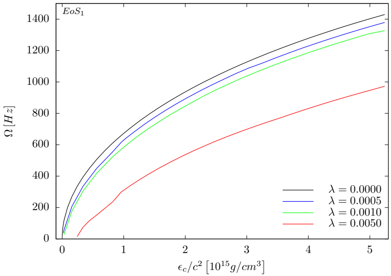
da Silva, F. M., Santos, L. C. N., Barros, C. C. (2021). Rapidly rotating compact stars in Rastall's gravity. Classical and Quantum Gravity, 38(16), 165011.

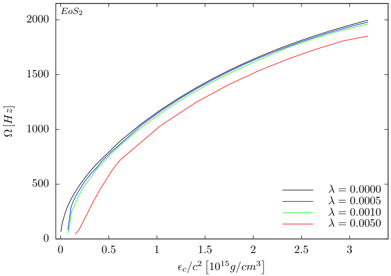
Resultados: $M \times \varepsilon_c/c^2$



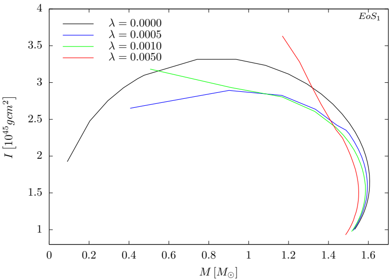


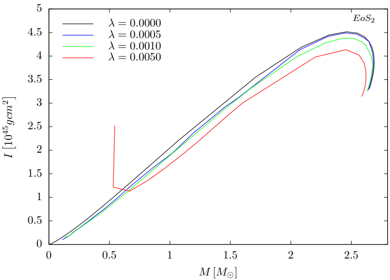
Resultados: $\Omega \times \varepsilon_c/c^2$



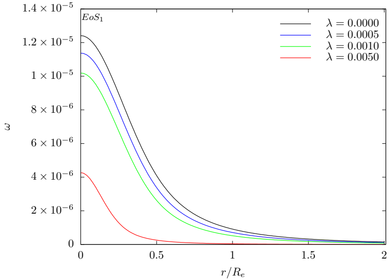


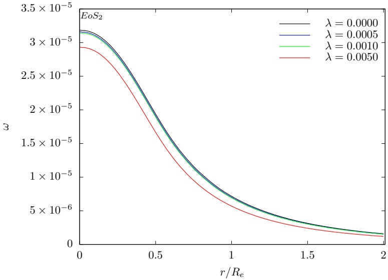
$Resultados$: $I \times M$





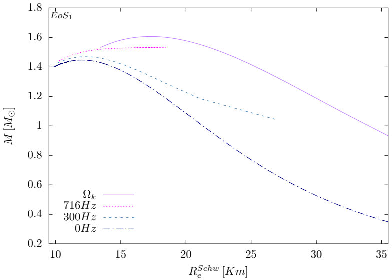
Resultados: $\omega \times r/R_e$



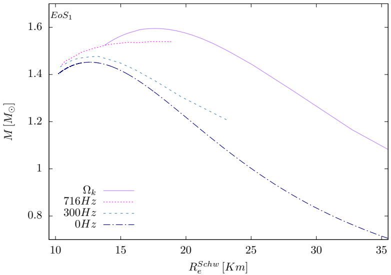


Resultados: $M \times R_e^{Schw}$ (EoS_1)

RG

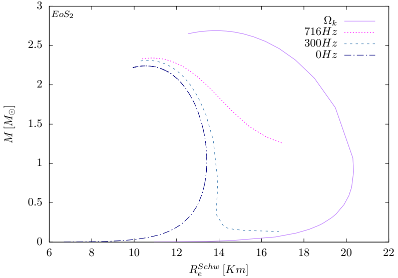


Rastall

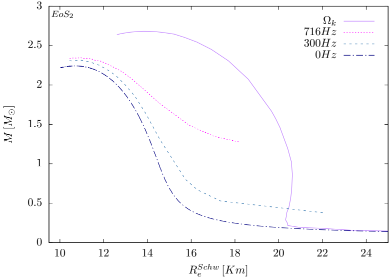


Resultados: $M \times R_e^{Schw}$ (EoS_2)

RG

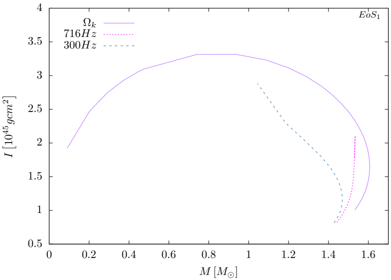


Rastall

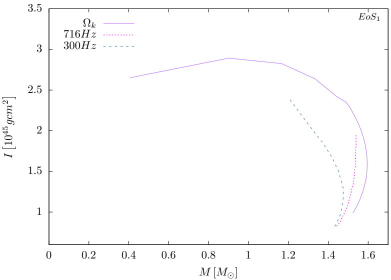


Resultados: $I \times M$ (EoS_1)

RG

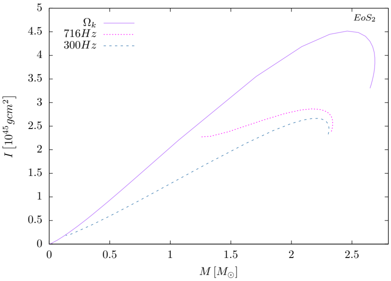


Rastall

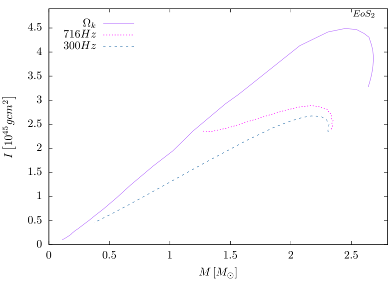


Resultados: $I \times M$ (EoS_2)

RG

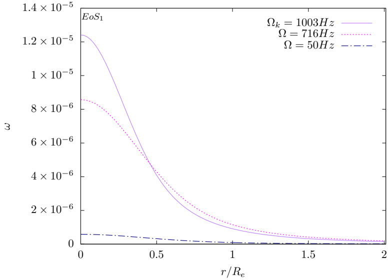


Rastall

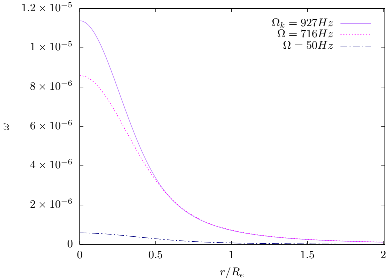


Resultados: $\omega \times r/R_e$ (EoS_1)

RG

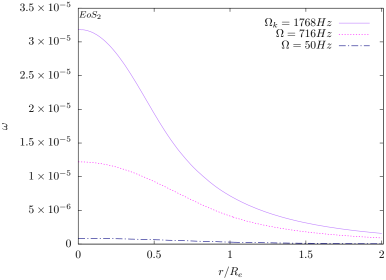


Rastall

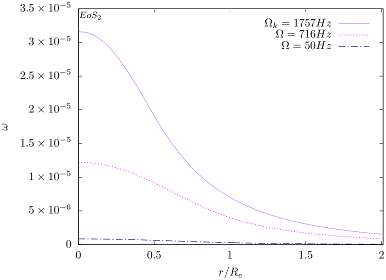


Resultados: $\omega \times r/R_e \quad (EoS_2)$

RG



Rastall



Dispersion relations

- RG $\qquad E^2 - k^2c^2 = m^2c^4$
- Doubly/Deformed special relativity

$$E^2 f(x)^2 - k^2c^2 g(x)^2 = m^2c^4$$

- where: $x = \lambda E/E_p$
- case 1

$$f(x) = \frac{1}{1-x}, \quad g(x) = \frac{1}{1-x}$$

- case 2

$$f(x) = \frac{e^x - 1}{x}, \qquad g(x) = 1$$

Fermi gas

- particle density

$$dn = f(E)\frac{g}{(2\pi\hbar)^3}d^3k$$

- energy density

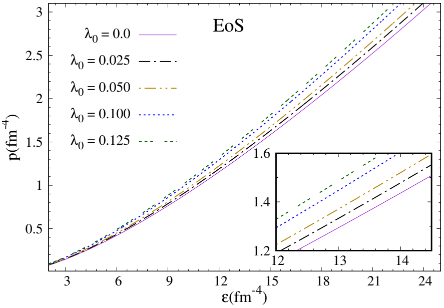
$$\varepsilon(k_f) = \frac{8\pi}{(2\pi\hbar)^3}\int_0^{k_f} E_{msr}k^2dk$$

- pressure

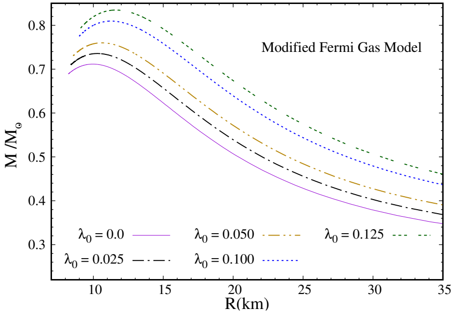
$$p(k_f) = \frac{1}{3\pi^2\hbar^3}\int_0^{k_f}\left(\frac{d}{dk}E_{msr}\right)k^3dk$$

Results, case 1

- EoS_1



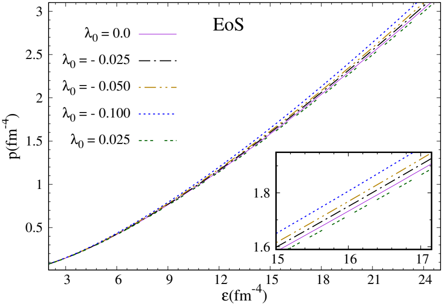
- $M \times R$



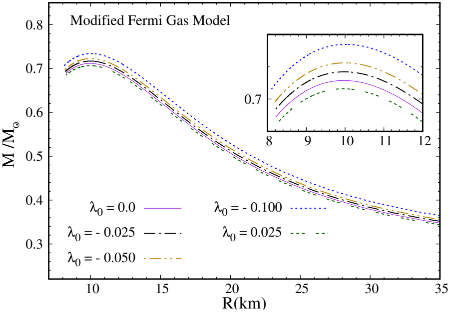
Santos, L. C., Mota, C. E., da Silva, F. M., Grams, G., Lobo, I. P. (2021). Effects of modified dispersion relations on free Fermi gas: Equations of state and applications in astrophysics. Physics Letters B, 822, 136684.

Results, case 2

- EoS_2



- $M \times R$



Rainbow gravity

- generalizes doubly special relativity to incorporate curvature
- as a consequence we have a energy dependent metric $g_{\mu\nu} = g_{\mu\nu}(x)$
- for a spherically symmetric spacetime, we have

$$ds^2 = -\frac{B(r)}{\Xi(x)^2}dt^2 + \frac{A(r)}{\Sigma(x)^2}dr^2 + \frac{r^2}{\Sigma(x)^2}(d\theta^2 + \sin\theta^2 d\phi^2)$$

Rastall-Rainbow theory

- Rastall-Rainbow field equations

$$R^{\nu}{}_{\mu}(x) - \frac{\lambda}{2}\delta^{\nu}{}_{\mu}R(x) = 8\pi G(x)T^{\nu}{}_{\mu}(x)$$

- after some simplification

$$R_{\mu\nu}(x) - \frac{1}{2}g_{\mu\nu}(x)R(x) = 8\pi G\tau_{\mu\nu}$$

- where

$$\tau_{\mu\nu} = T_{\mu\nu} - \frac{(1-\lambda)}{2(1-2\lambda)}g_{\mu\nu}(x)T$$

Neutron stars in Rastall-Rainbow

- mass

$$M(r) = \int_0^R 4\pi r'^2 \bar{\rho}(r') dr'$$

- TOV equation

$$\bar{p}' = -\frac{GM\bar{\rho}}{r^2}\left[1 + \frac{\bar{p}}{\bar{\rho}}\right]\left[1 + \frac{4\pi r^3 \bar{p}}{M}\right]\left[1 - \frac{2GM}{r}\right]^{-1}$$

- where

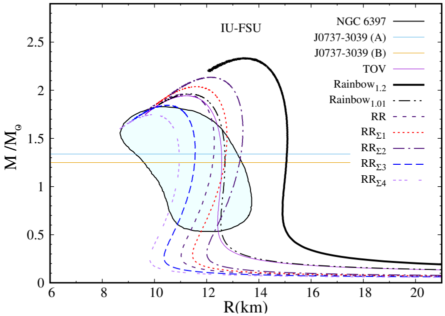
$$\bar{\rho} = \frac{1}{\Sigma(x)^2}\left[\alpha_1 \rho + 3\alpha_2 p\right],$$
$$\bar{p} = \frac{1}{\Sigma(x)^2}\left[\alpha_2 \rho + (1 - 3\alpha_2) p\right]$$

- and

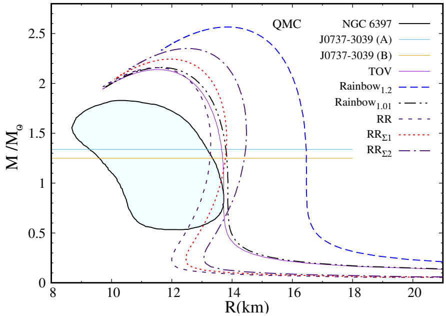
$$\alpha_1 = \frac{1 - 3\lambda}{2(1 - 2\lambda)}, \qquad \alpha_2 = \frac{1 - \lambda}{2(1 - 2\lambda)}$$

Results

- IU-FSU



- QMC



Mota, C. E., Santos, L. C., Grams, G., da Silva, F. M., Menezes, D. P. (2019). Combined Rastall and rainbow theories of gravity with applications to neutron stars. Physical Review D, 100(2), 024043.