Boson stars

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Summary

PART I. Introducing boson stars: Stars from fundamental Fields

PART II. Presenting the equations: Spherically symmetric case

PART III. Hands-on! Integrating the equations

Introducing ***boson stars.***
How do fields self-gravitate?

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

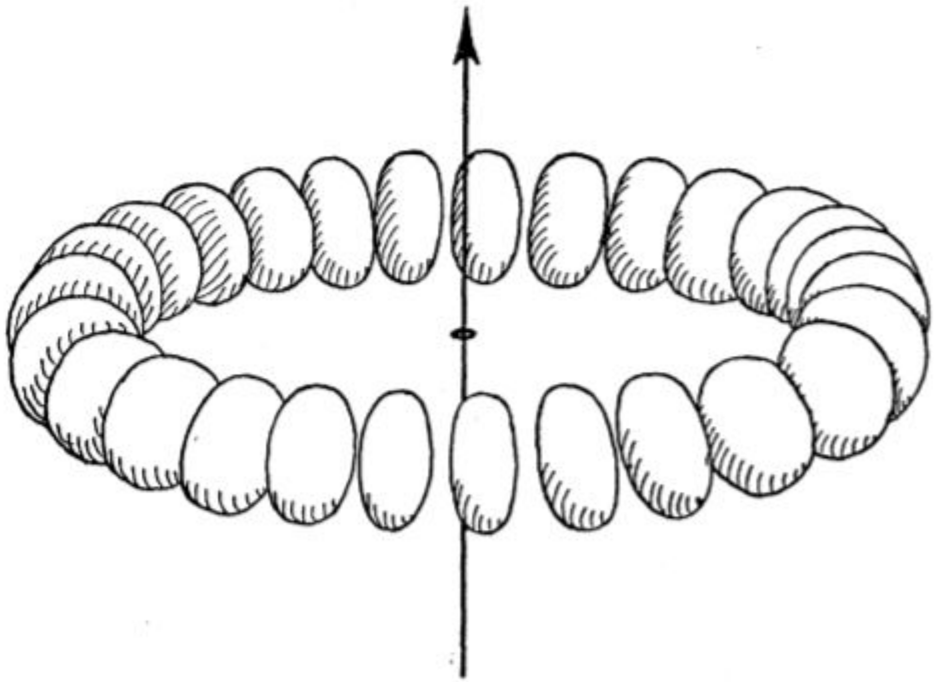
Gravity + ***Fundamental fields***

???????

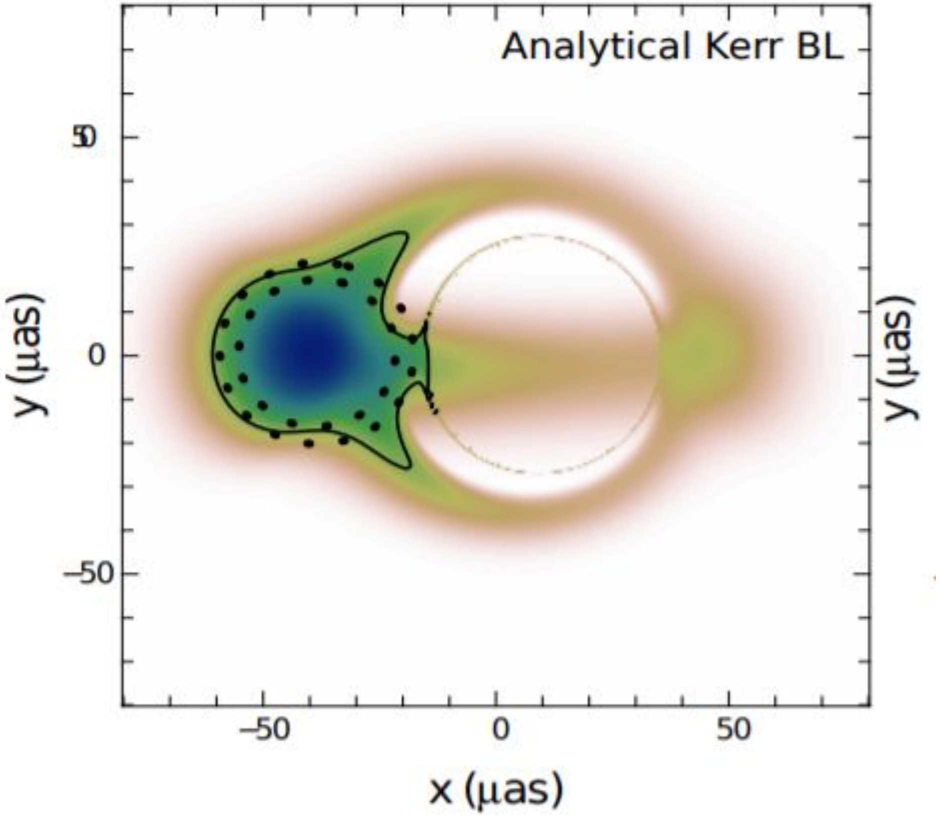
Introducing ***boson stars: Some highlights***

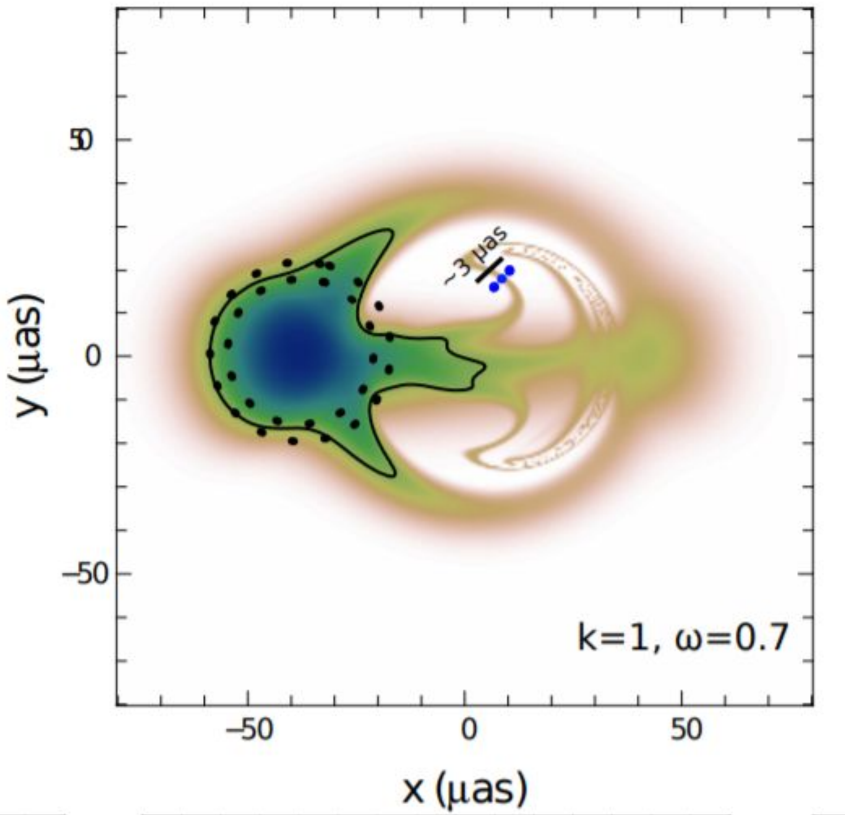
- 1955 - Wheeler: **Geon**
- 1964 - Derrick's Theorem. **Why does this work?**
- 1968 - Kaup: ***Klein-Gordon Geons***
- 1969 - Ruffini&Bonazzola.

  Quantum version
- 1986 - Colpi *et al.*: ***Self-interactions***
- 1989 - Jetzer: ***Charged scalar fields***
- 1991 - Seidel&Suen: ***Oscillatons***
- 2015 - Brito *et al.*: ***Vector fields***



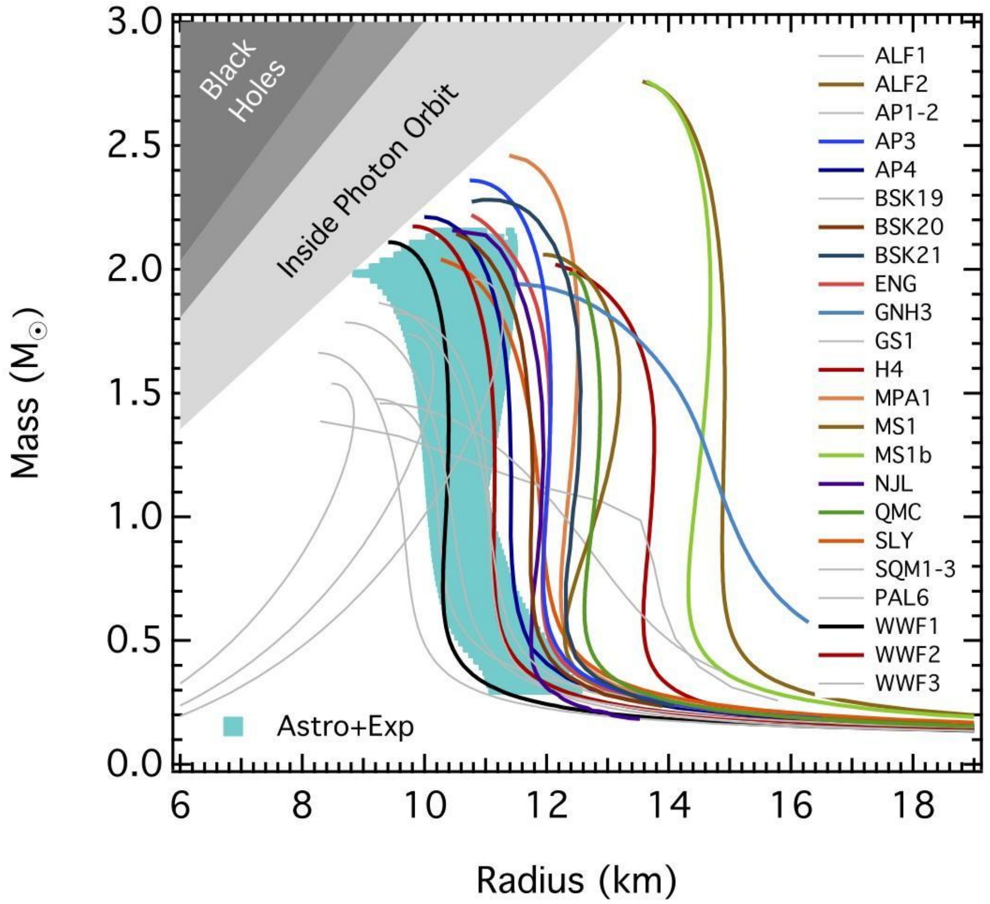
Introducing ***boson stars: What is it good for?***
Black hole alternatives
(Vincent *et al.* 2015)





Introducing ***boson stars: What is it good for?***
As compact objects themselves

- ***Neutron stars and microphysics***
- ***Problems in collisions***
- ***Everything is well-posed***



xtreme.as.arizona.edu/NeutronStars/

Introducing ***boson stars: What is it good for?***

For DM modelling

Hui, (2021)* 2101.11735; *Guerra et. al (2019)

- QCD-axion and fuzzy dark matter.

Reviews on boson stars

- Jetzer (1992)
- Lee and Pang (1992)
- Schunk and Mielke (2003)
- Liebling and Palenzuela (2017)
- Visinelli (2021)

PART II.

Presenting the equations: Spherically symmetric case

Action and equations of motion

Action

EH | EM | Vector | scalar

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{\kappa} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}\bar{B}^{\mu\nu}B_{\mu\nu} - \frac{1}{2}g^{\mu\nu}\bar{D}_\mu\bar{\Phi}D_\nu\Phi - \frac{1}{2}V_\Phi(|\Phi|^2) - \frac{1}{2}V_V(|C|^2)\right].$$

Self-interactions

(Add whatever you want)

where $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$, $B_{\mu\nu} = \nabla_\mu C_\nu - \nabla_\nu C_\mu$, and $D_\mu = \nabla_\mu - iqA_\mu$

Now we proceed the usual way...

Action and equations of motion

$$G_{\mu\nu} = \frac{\kappa}{2} T_{\mu\nu},$$

$$D_\mu D^\mu \Phi = V'_\Phi(|\Phi|^2)\Phi,$$

$$\nabla_\mu F^{\mu\nu} = i\frac{q}{2}\left(\bar{\Phi} D^\nu \Phi - \Phi \bar{D}^\nu \bar{\Phi}\right),$$

$$\nabla_\mu B^{\mu\nu} = V'_V(|C|^2) C^\nu,$$

$$T_{\mu\nu} = T_{\Phi\,\mu\nu} + T_{F\,\mu\nu} + T_{B\,\mu\nu}$$

$$T_{\Phi\mu\nu} = \frac{1}{2}\left(\bar{D}_\mu \bar{\Phi} D_\nu \Phi + D_\mu \Phi \bar{D}_\nu \bar{\Phi}\right) - \frac{1}{2} g^{\mu\nu}\left(\bar{D}_\alpha \bar{\Phi} D^\alpha \Phi + V_\Phi\right),$$

$$T_{F\,\mu\nu} = F_{\mu\alpha} F_\mu{}^\alpha - \frac{1}{4} g_{\mu\nu} |F|^2,$$

$$T_{B\,\mu\nu} = \frac{1}{2}\left(B_{\mu\alpha} \bar{B}_\mu{}^\alpha + \bar{B}_{\mu\alpha} B_\mu{}^\alpha\right) - \frac{1}{4} g_{\mu\nu} |B|^2 + V'_V C_{(\mu} \bar{C}_{\nu)} - \frac{1}{2} g_{\mu\nu} V_V$$

$$\nabla_\mu \left[V'_V(|C|^2) C^\mu\right] = 0.$$

Currents and conserved quantities

$$J^{\mu}_{\Phi} = -iq\left(\bar{\Phi}D^{\nu}\Phi - \Phi\bar{D}^{\nu}\bar{\Phi}\right),$$
$$J^{\mu}_{V} = \frac{i}{2}\left[\bar{B}^{\mu\nu}C_{\nu} - B^{\mu\nu}\bar{C}_{\nu}\right].$$

$\longrightarrow$

$$Q = -\int d^3x\sqrt{-g}J^{t}_{\Phi}, \quad N_{\Phi} = Q/q,$$
$$N_{V} = -\int d^3x\sqrt{-g}J^{t}_{V}.$$

Taxonomy for scalar boson stars

Compact object	Self-interaction $U(\|\Phi\|^2)$ (α, β, λ, $\lambda_{(2n+2)}$, Φ_0 are constants)	Year, publication
Mini-BS	$U_K = m^2\|\Phi\|^2$	1968, Kaup [159]
Newtonian BS	$U_N = m^2\|\Phi\|^2$	1969, Ruffini–Bonazzola [237]
Self-interacting BS	$U_{HKG} = m^2\|\Phi\|^2 - \alpha\|\Phi\|^4 + \beta\|\Phi\|^6$	1981, Mielke–Scherzer [210]
BS	$U_{CSW} = m^2\|\Phi\|^2 + \lambda\|\Phi\|^4/2$	1986, Colpi–Shapiro–Wasserman [58]
Non-topol. soliton star	$U_{NTS} = m^2\|\Phi\|^2 \left(1 - \|\Phi\|^2/\Phi_0^2\right)^2$	1987, Friedberg–Lee–Pang [99, 184]
General BS	$U_{HKL} = U_{CSW} + \cdots + \lambda_{(2n+2)}\|\Phi\|^{2n+2}$	1999, Ho–Kim–Lee [131]
Sine-Gordon BS	$U_{SG} = \alpha m^2[\sin(\pi/2[\beta\sqrt{\|\Phi\|^2} - 1]) + 1]$	2000, Schunck–Torres [262]
Cosh-Gordon BS	$U_{CG} = \alpha m^2[\cosh(\beta\sqrt{\|\Phi\|^2}) - 1]$	2000, Schunck–Torres [262]
Liouville BS	$U_L = \alpha m^2[\exp(\beta^2\|\Phi\|^2) - 1]$	2000, Schunck–Torres [262]

Spherically symmetric ansatz

For the metric

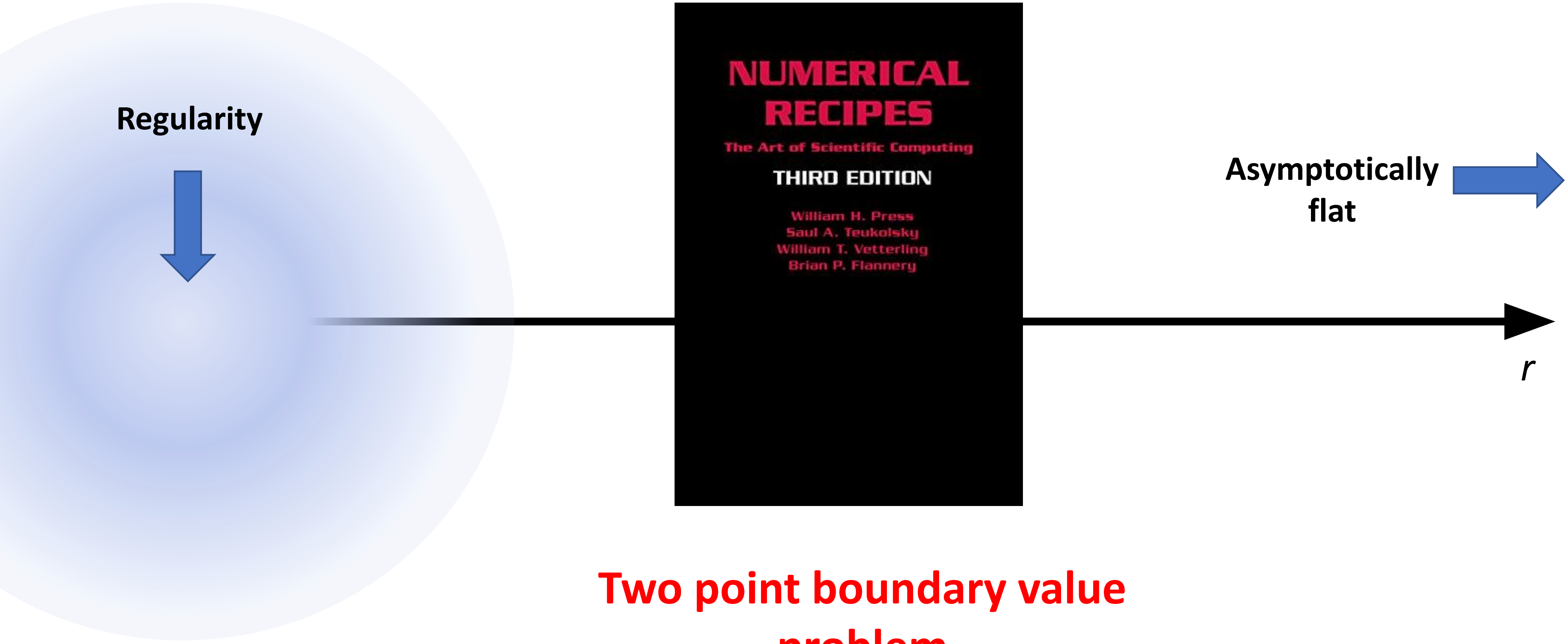
$$ds^2 = -A(t,r)dt^2 + B(t,r)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right).$$

For the fields

$$\Phi = \Phi(t,r),$$
$$A_\mu = \{A_0(t,r), 0, 0, 0\},$$
$$B_\mu = \{f(t,r), ig(t,r), 0, 0\},$$

The equations usually leads to some fundamental frequencies of oscillation (ω_s, ω_v).

Boundary conditions

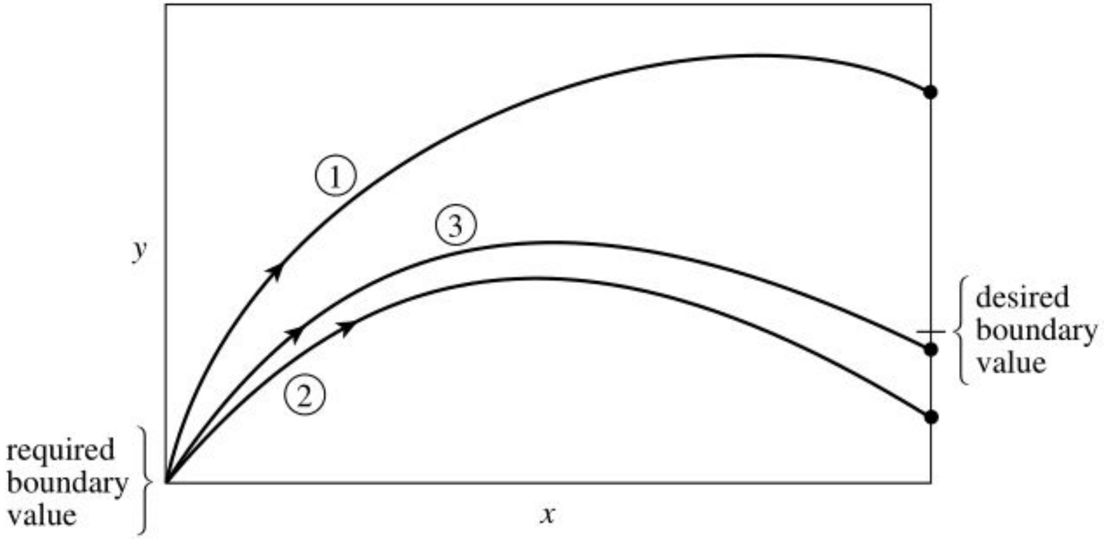


Two point boundary value problem

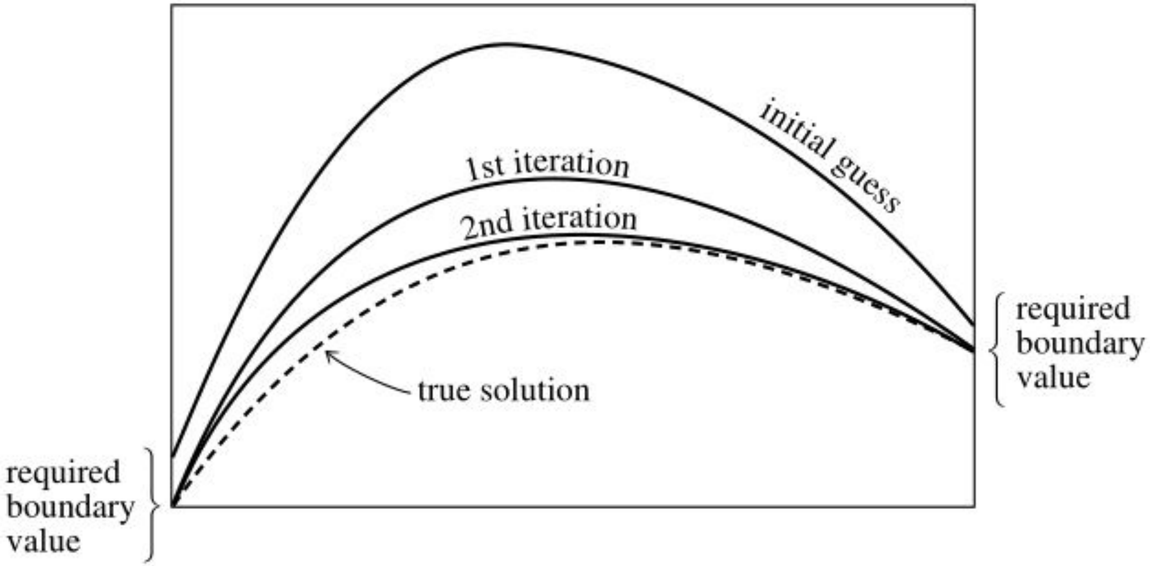
How to solve a two point BVP

(Press et al. numerical recipes)

Shooting



Relaxation

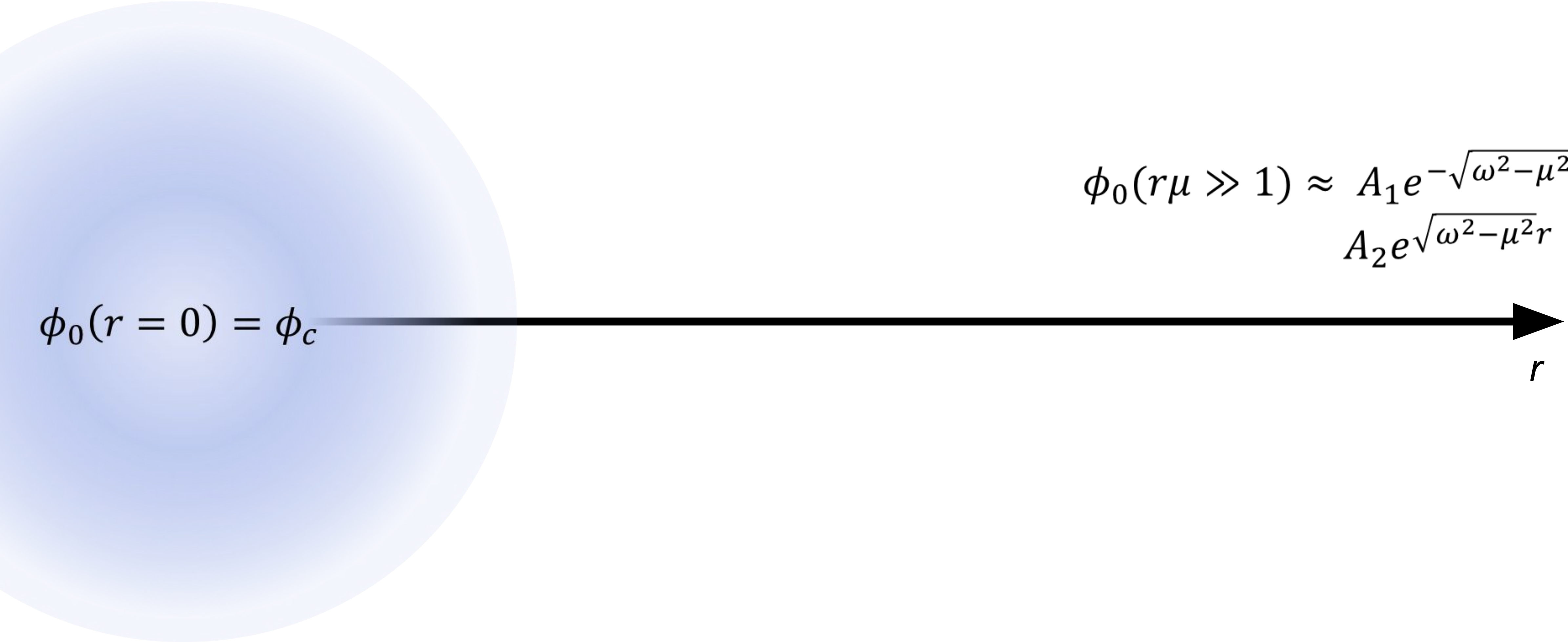


"Until you have enough experience to make your own judgment between the two methods, you might wish to follow the advice of your authors, who are notorious computer gunslingers: We always shoot first, and only then relax."

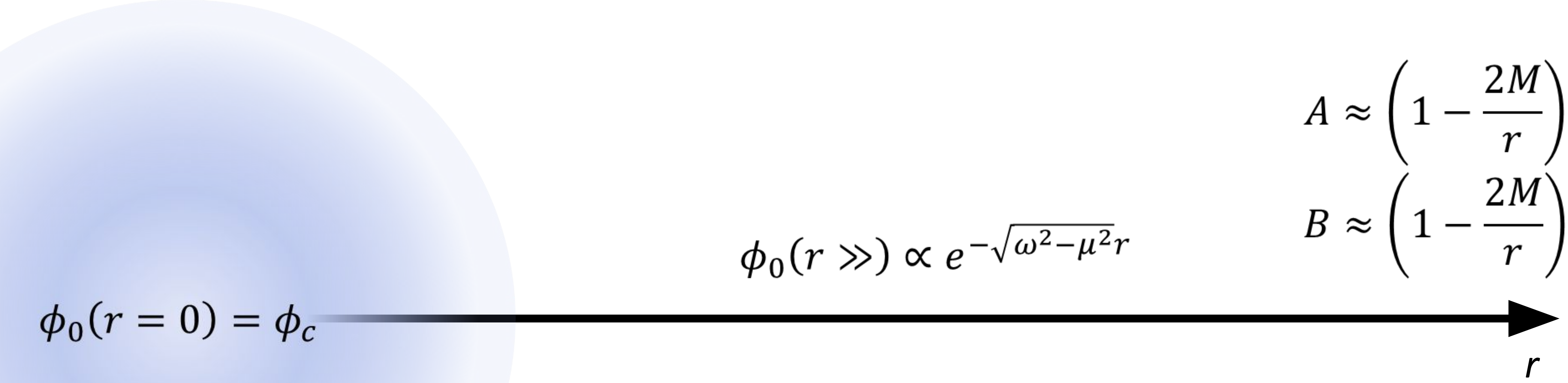
For boson stars

$$\phi_0(r\mu \gg 1) \approx A_1 e^{-\sqrt{\omega^2-\mu^2}r} + A_2 e^{\sqrt{\omega^2-\mu^2}r}$$

$$\phi_0(r=0) = \phi_c$$



For boson stars



Boson stars do not have a hard surface!

How to compute the radius?

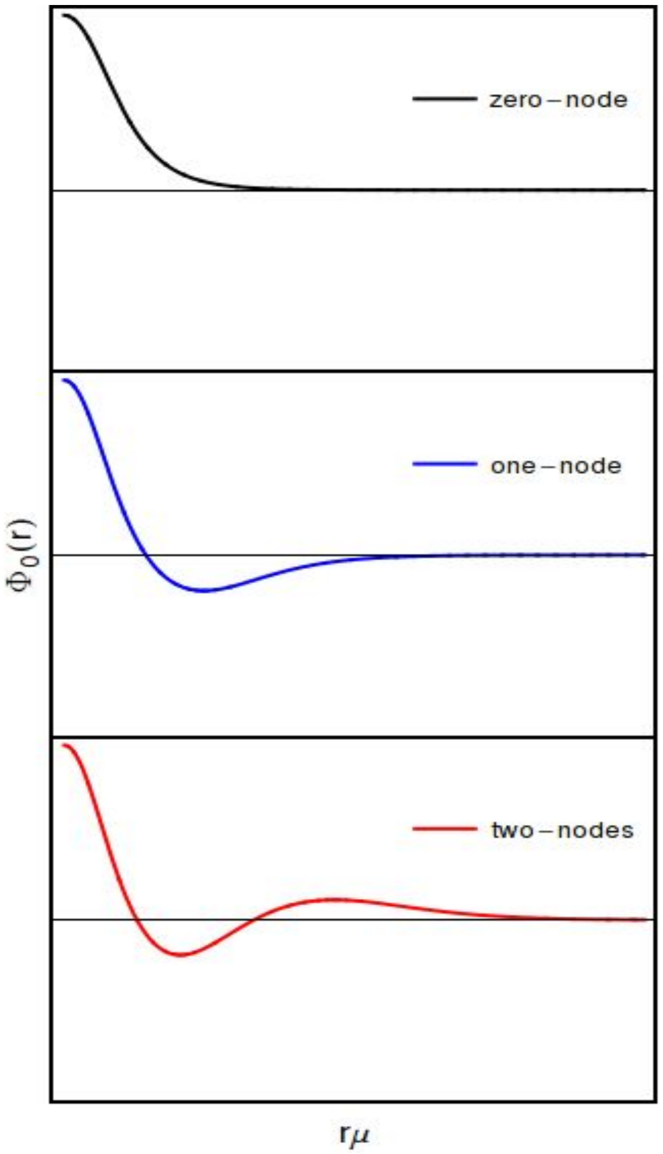
Fundamental and excited solutions

Considering, for simplicity, only complex scalar fields we have

$$\phi(t,r) = \phi_0(r)e^{-i\omega_s t}.$$

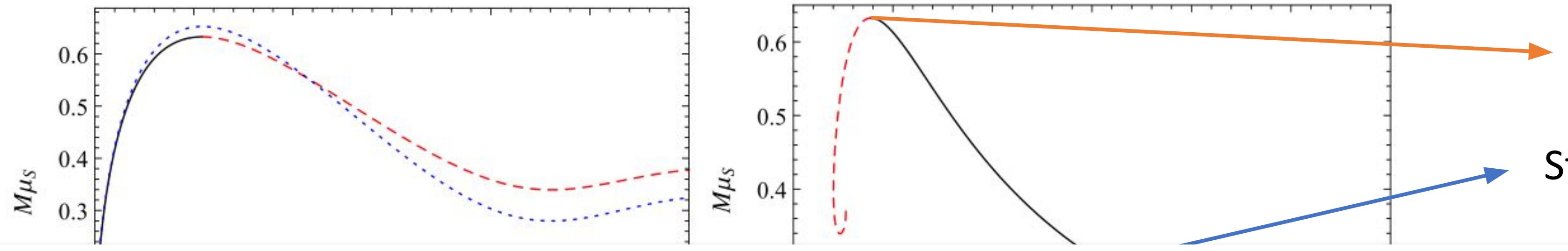
By integrating the equations, we can find different solutions for the same central field.

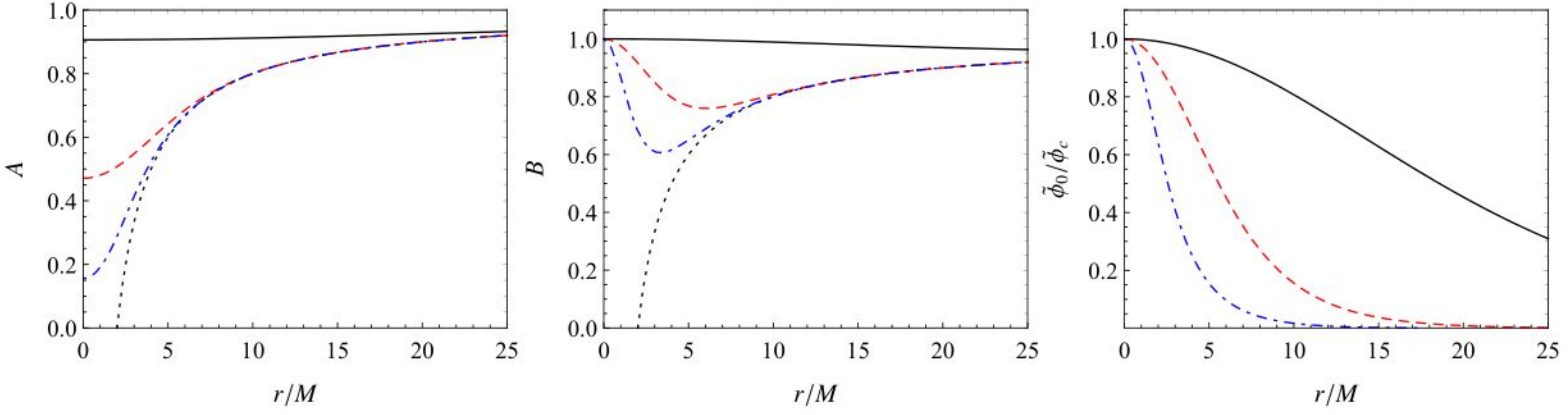
Excitated solutions → **Fundamental**



Solutions and stability properties

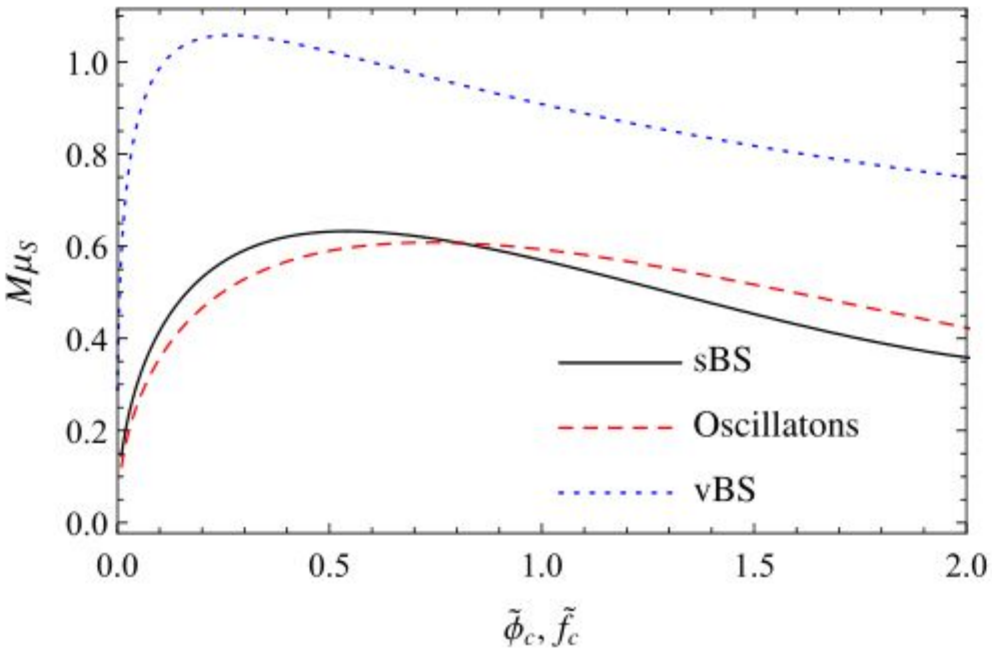
Scalar boson stars

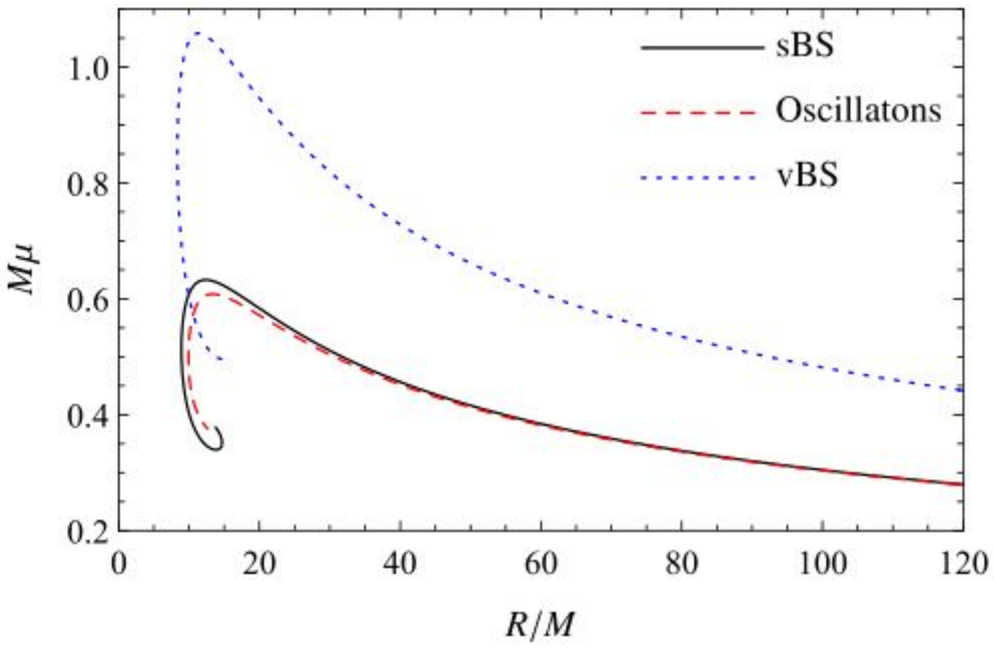




Other fields

Many faces of boson stars





PART III.

Hands-on: Integrating the equations