Cosmic acceleration, dark energy, gravity

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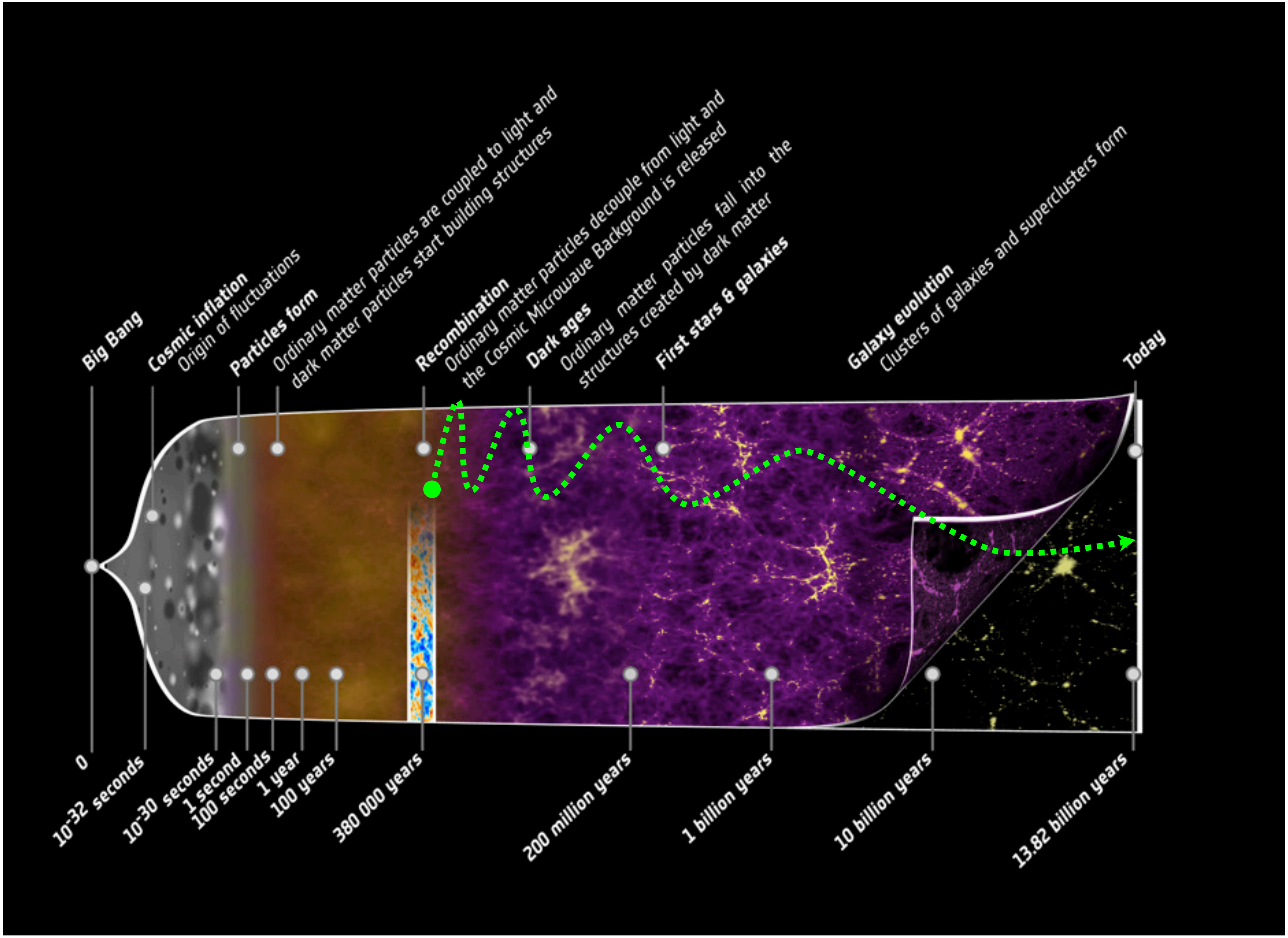


Big Bang
Cosmic inflation
Origin of fluctuations
Particles form
Ordinary matter particles are coupled to light and dark matter particles start building structures
Recombination
Ordinary matter particles decouple from light and the Cosmic Microwave Background is released
Dark ages
Ordinary matter particles fall into the structures created by dark matter
First stars & galaxies
Galaxy evolution
Clusters of galaxies and superclusters form
Today
0
10^{-32} seconds
10^{-30} seconds
1 second
100 seconds
1 year
100 years
380 000 years
200 million years
1 billion years
10 billion years
13.82 billion years

Cosmology in one equation

$$ds^2 = (1 + 2\Psi)dt^2 - \frac{\exp \int H(t')dt'}{(1 - \frac{1}{4}\Omega_k r^2)^2}(1 - 2\Phi)dx_i dx^i$$

The geometry of space-time at all epochs
depends at first order on the functions H(t), $\Psi(t, x_i)\Phi(t, x_i)$
and the parameter Ω_k

Big questions

$$ds^2 = (1 + 2\Psi)dt^2 - \frac{\exp \int H(t)dt}{(1 - \frac{1}{4}\Omega_k r^2)^2}(1 - 2\Phi)dx_i dx^i$$

Is space flat?

How is the universe expanding?

How are perturbations growing?

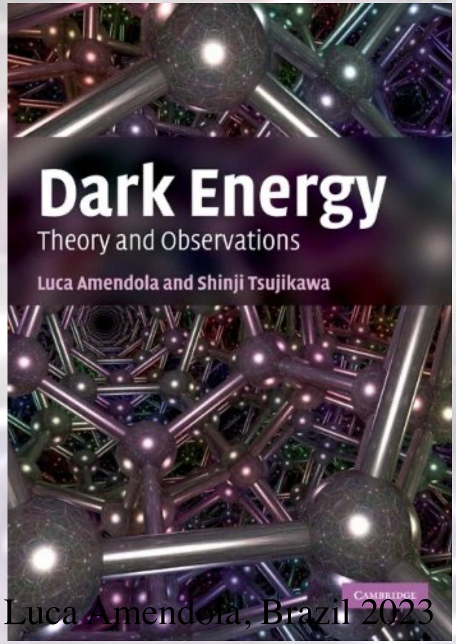
Is GR correct at all scales and epochs?

Outline

- Lecture 1 Introduction to Dark Energy
- Lecture 2 Beyond Einstein's Gravity
- Lecture 3 Testing gravity and dark energy

Texts

- **Dodelson**, Modern Cosmology
- **Amendola & Tsujikawa**, Dark Energy. Theory and Observations, Cambridge UP (rev. ed. 2015)
- **Euclid Theory WG**, Cosmology and Fundamental Physics with the Euclid Satellite, arXiv 1206.1225+1606.00180



Cosmology Executive Summary

Dark matter 27%

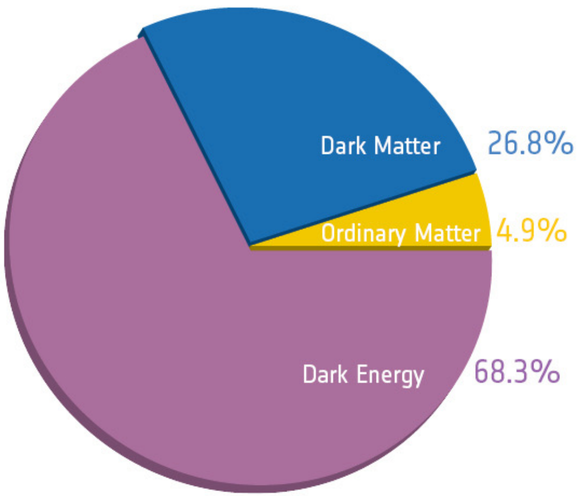
Baryons 5%

Massive neutrinos: 0.1%

Photons: 0.01%

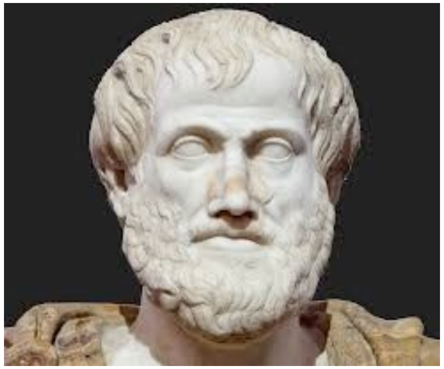
Humans: 10^{-39} %

Spatial curvature: very close to 0



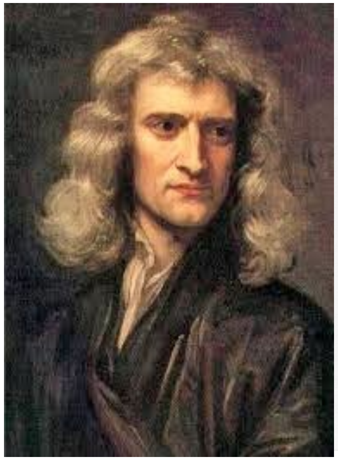
Something else: ≈ 70%

350 BCE



- Gravity is always attractive: how to avoid that the **sky** falls on our head?
- **Aristotle**'s answer: quintessence

1700



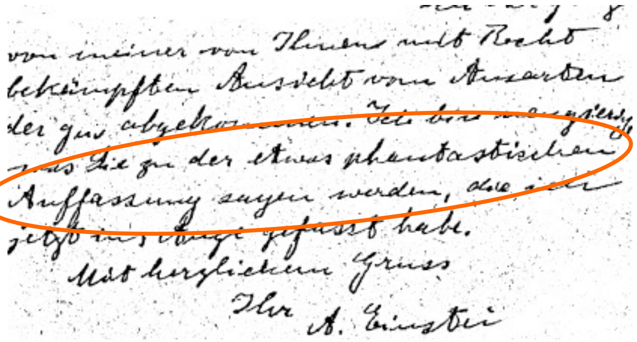
- Gravity is always attractive: how to avoid that the **stars** fall on our head?
- **Newton**’s answer: initial conditions

1900

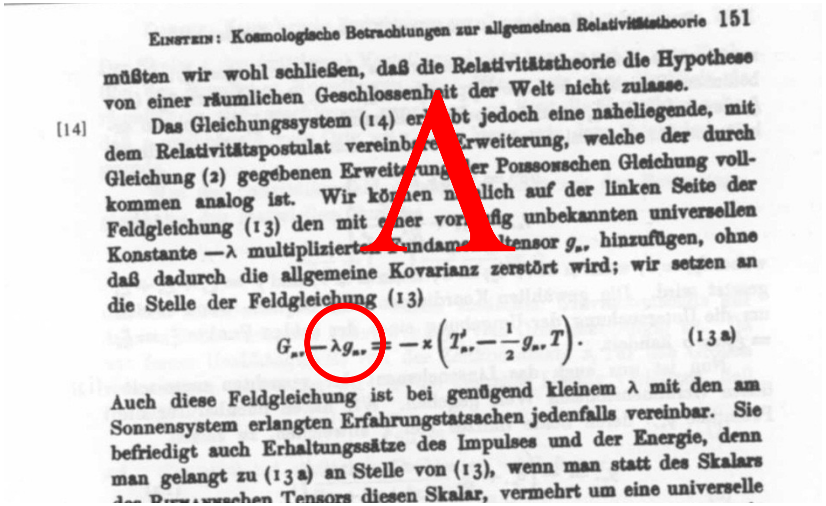
- Gravity is always attractive: how to avoid that the **Universe** fall on our head?
- Einstein's answer: modify GR introducing a form of repulsive gravity

Einstein’s equations

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$



von meiner von Ihnen mit Recht
bekämpften Ansicht vom Ausarten
der $g_{\mu\nu}$ abgekommen. Ich bin neugierig,
was Sie zu der etwas phantastischen
Auffassung sagen werden, die ich
jetzt ins Auge gefasst habe.
Mit herzlichem Gruss
Ihr A. Einstein



Einstein: Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie 151

müßten wir wohl schließen, daß die Relativitätstheorie die Hypothese von einer räumlichen Geschlossenheit der Welt nicht zulasse.

[14] Das Gleichungssystem (14) erlaubt jedoch eine naheliegende, mit dem Relativitätspostulat vereinbare Erweiterung, welche der durch Gleichung (2) gegebenen Erweiterung der Poissonschen Gleichung vollkommen analog ist. Wir können nämlich auf der linken Seite der Feldgleichung (13) den mit einer vorläufig unbekannten universellen Konstante $-\lambda$ multiplizierten Fundamentaltensor $g_{\mu\nu}$ hinzufügen, ohne daß dadurch die allgemeine Kovarianz zerstört wird; wir setzen an die Stelle der Feldgleichung (13)

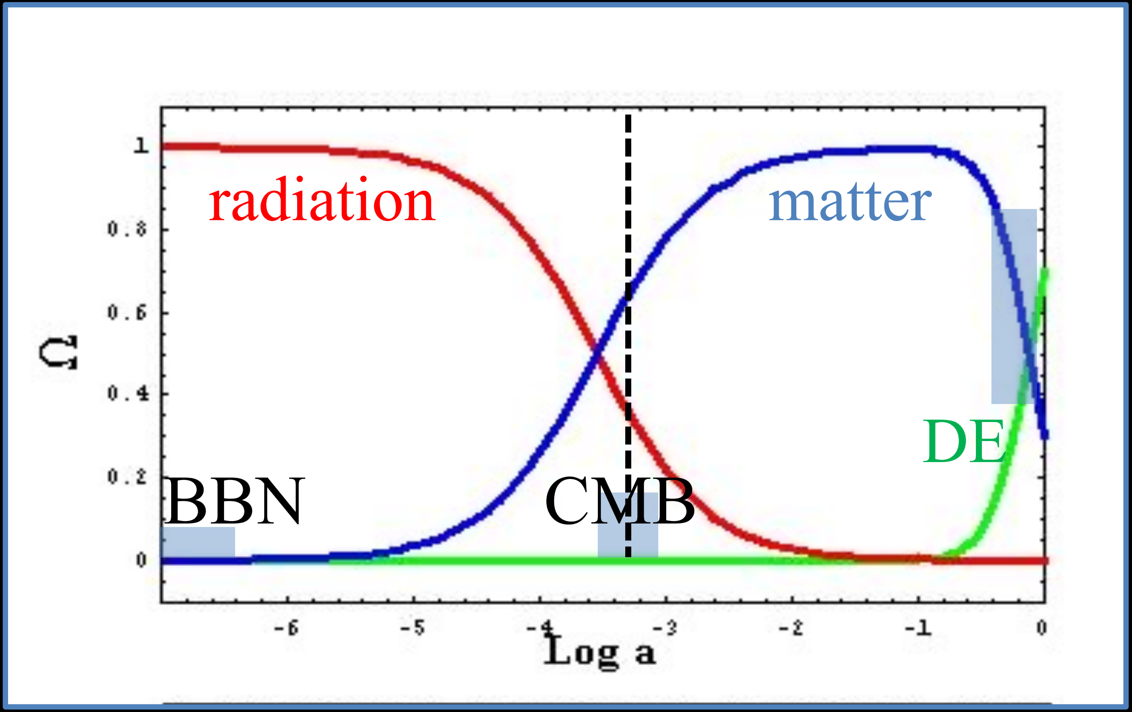
$$G_{\mu\nu} - \lambda g_{\mu\nu} = -\varkappa\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right). \qquad (13a)$$

Auch diese Feldgleichung ist bei genügend kleinem λ mit den am Sonnensystem erlangten Erfahrungstatsachen jedenfalls vereinbar. Sie befriedigt auch Erhaltungssätze des Impulses und der Energie, denn man gelangt zu (13a) an Stelle von (13), wenn man statt des Skalars des Riemannschen Tensors diesen Skalar, vermehrt um eine universelle

Einstein 1917

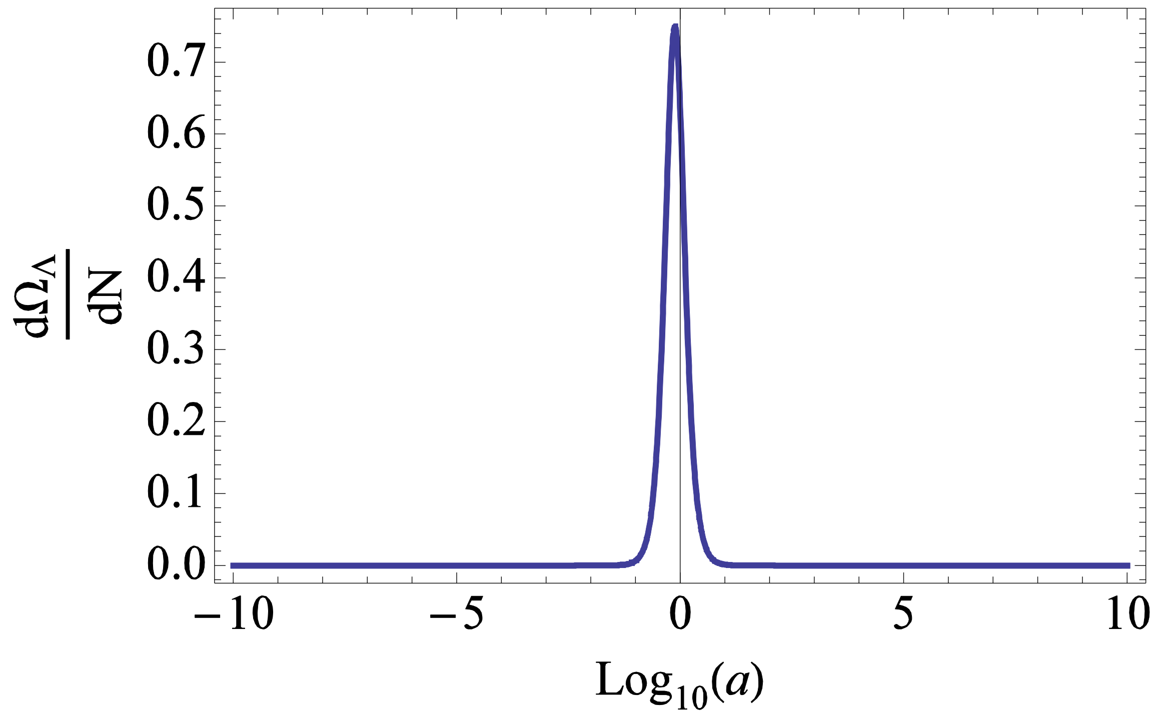
Time view

We know so little about the evolution of the universe!



Why now?

The coincidence problem



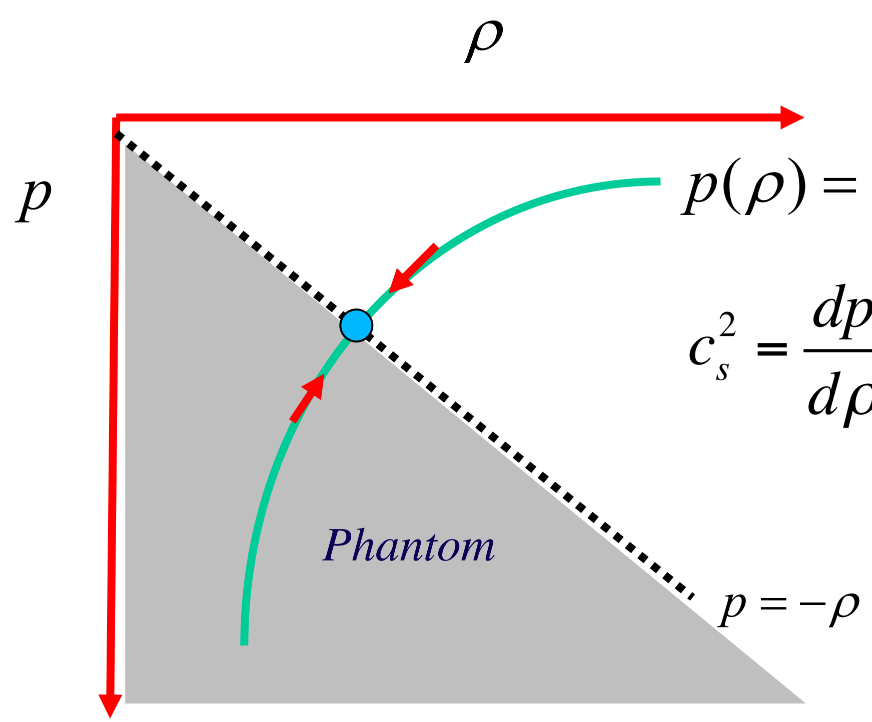
The twofold way

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

modify the
theory of gravity

modify the
matter component

Beyond the cosmological constant

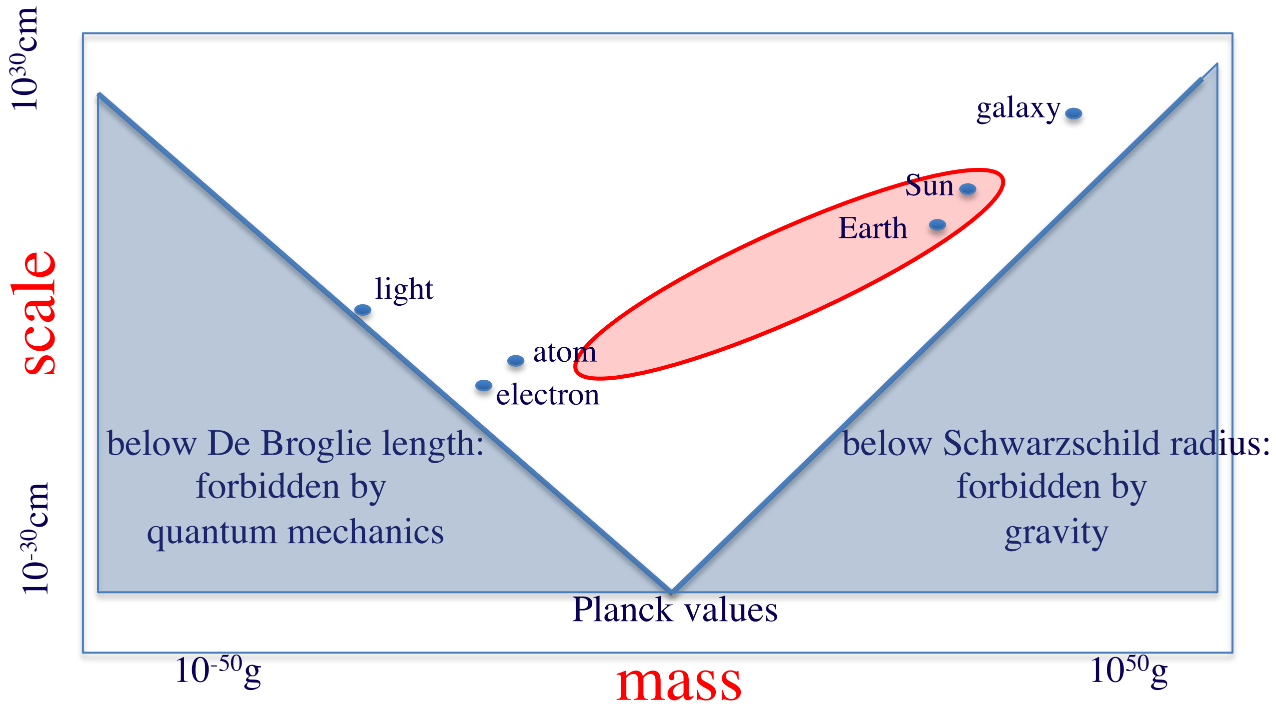


$$p(\rho) = -\frac{A}{\rho}$$

$$c_s^2 = \frac{dp}{d\rho} > 0$$

$$\dot{\rho} = -3H(p+\rho)$$

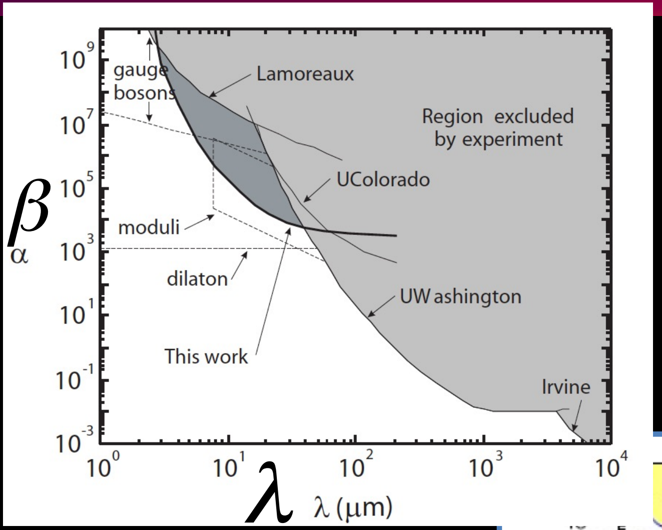
Knowledge diagram



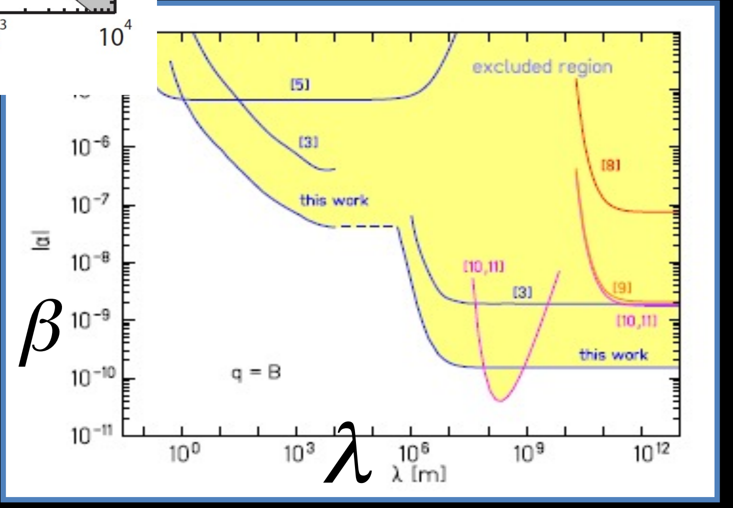
Inspired to a plot in *On Space and Time* Edited by Shahn Majid

we only directly test gravity within the solar system,
at the present time, and with "baryons"

Testing Gravity



Smullin et al. 2004



Schlamminger et al 2008

$$\frac{GM}{r} \to \frac{GM}{r}\left(1+\beta e^{-r/\lambda}\right)$$

?

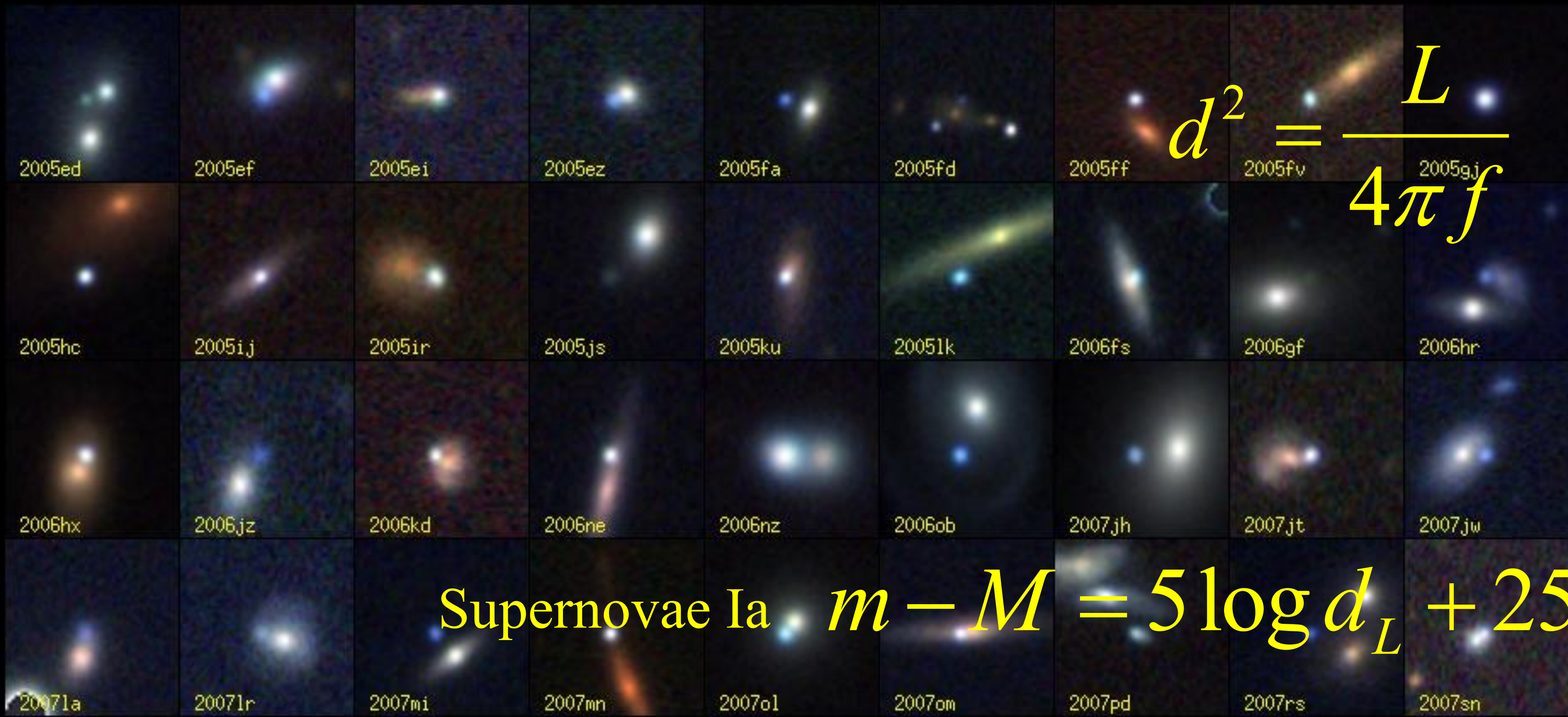
The fourfold way out of local gravity

$$\Psi = -\frac{MG^*}{r}(1+\beta\ e^{-m_\varphi r})$$

$$m_\phi, \beta \left\{ \begin{array}{l} \text{depends on time} \\ \text{depends on space} \\ \text{depends on local density} \\ \text{depends on species} \end{array} \right.$$

First task: measure expansion and curvature

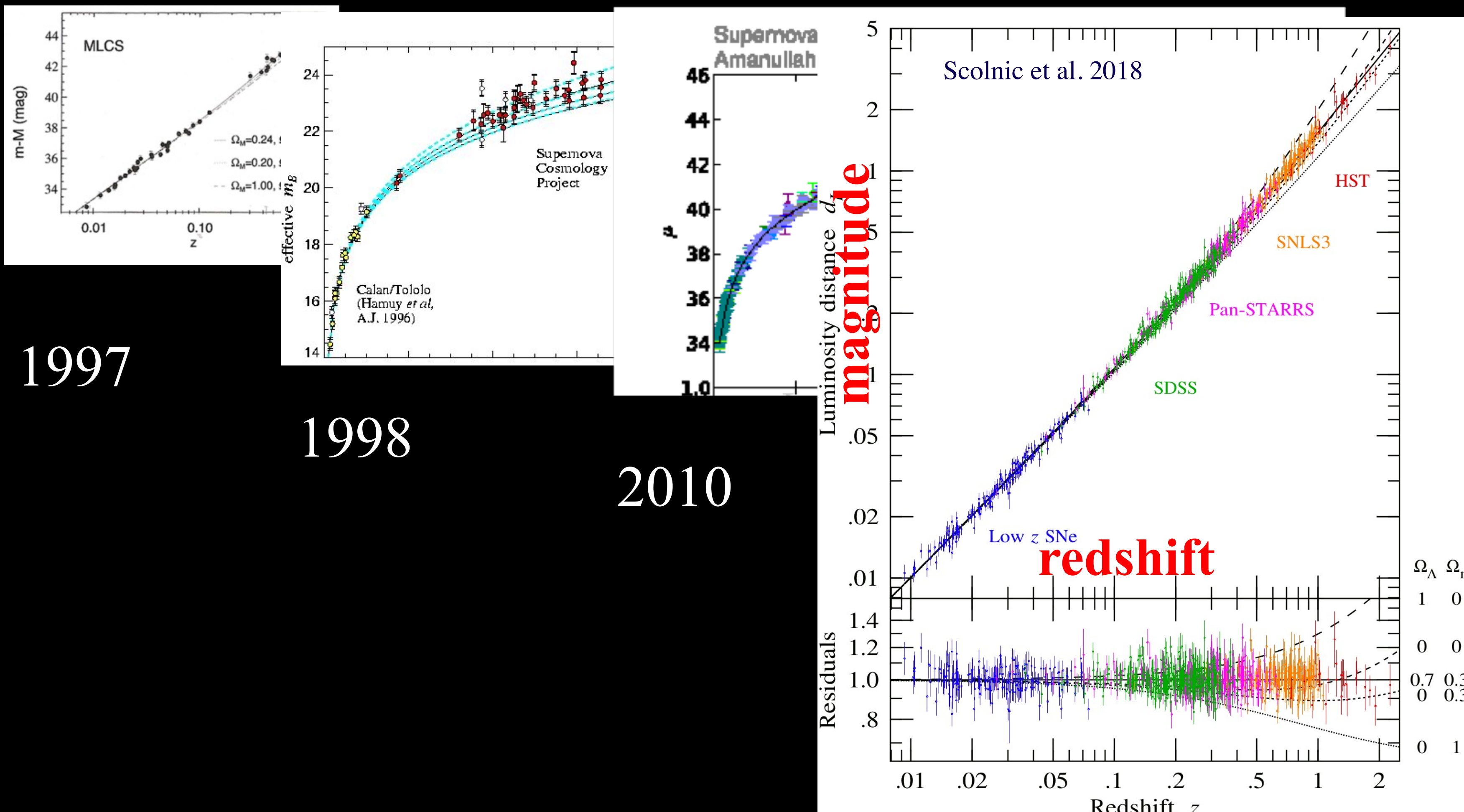
Lighthouses in the dark



$$d^2 = \frac{L}{4\pi f}$$

Supernovae Ia $\quad m - M = 5\log d_L + 25$

Hubble diagram



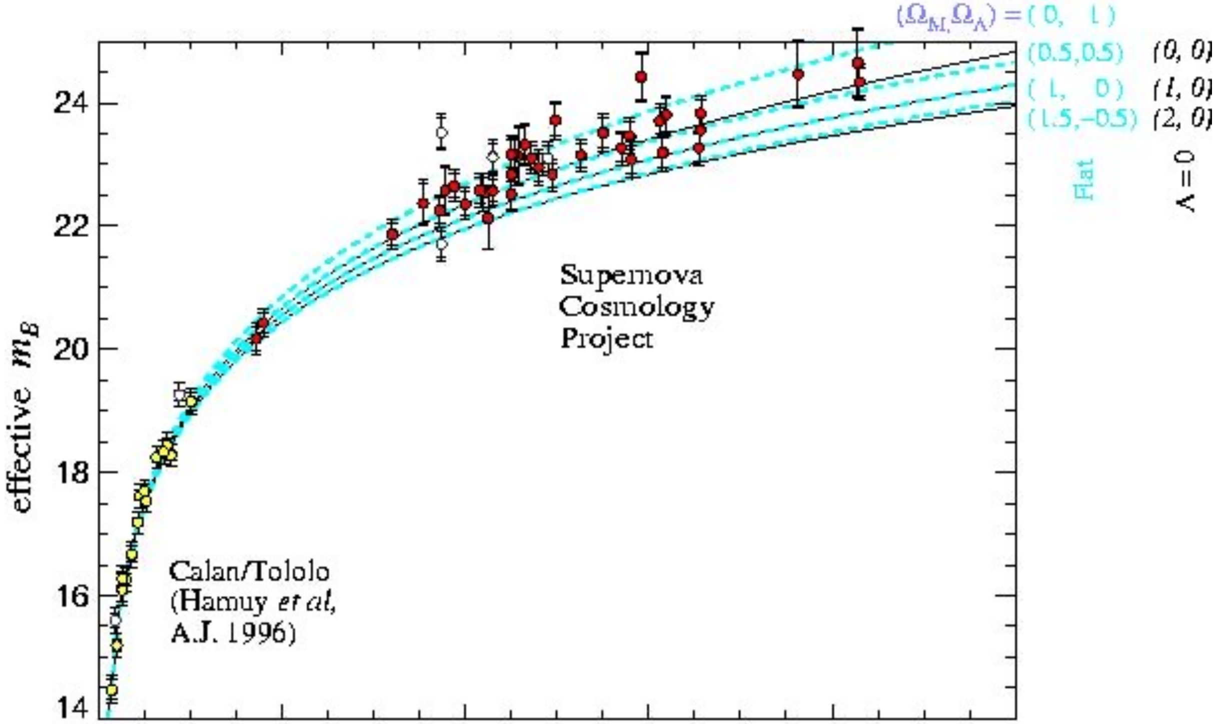
1997

1998

2010

2018

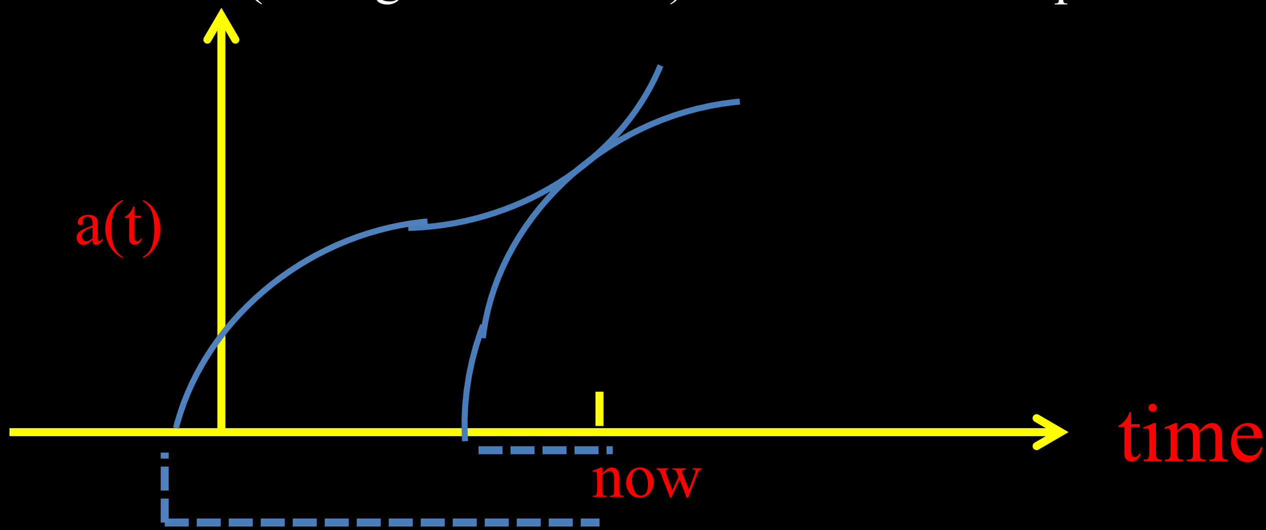
SNIa are dimmer than expected!



Basic property 1

Local Hubble law $r(z)=\frac{z}{H_0}$ ⟶ $r(z)=\int\frac{dz}{H(z)}$ Global Hubble law

If *H(z)* in the past is smaller (i.e. acceleration), then *r(z)* is larger: larger distances (for a given redshift) make dimmer supernovae



LambdaCDM

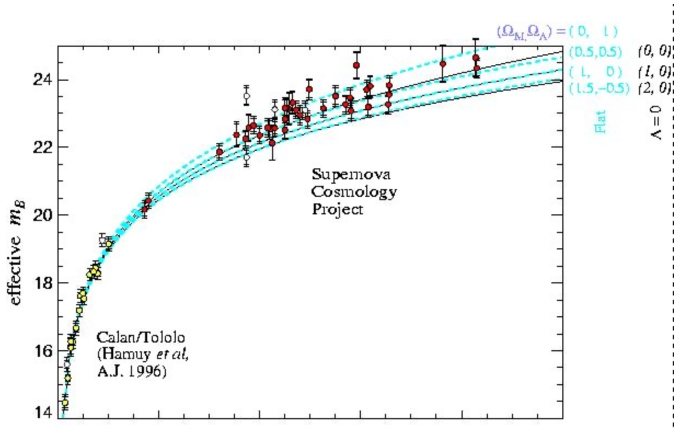
$$E^2 = \frac{H^2}{H_0^2} = \Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0} + \left(1 - \Omega_{m,0} - \Omega_{\Lambda,0}\right)(1+z)^2$$

Only two parameters!

$$d_L \sim \int \frac{dz}{E(z; \Omega_{i,0}, w_i)}$$

$$d_L\left(z; \Omega_{m,0}, \Omega_{\Lambda,0}\right) \sim \int \frac{dz}{[\Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0} + \left(1 - \Omega_{m,0} - \Omega_{\Lambda,0}\right)(1+z)^2]^{1/2}}$$

Statistics in three steps



1: take a model

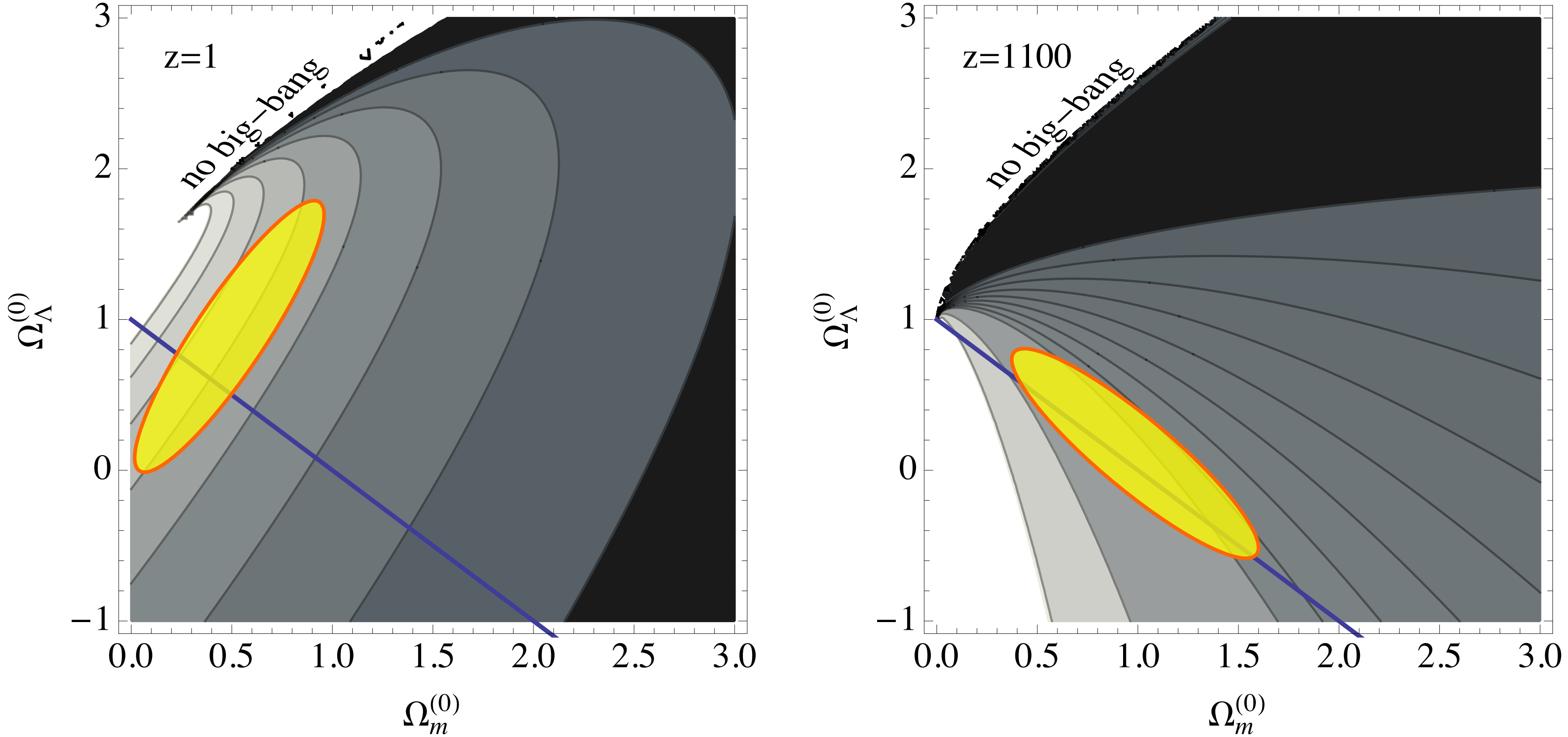
$$H^2 = H_0^2 E^2(z; params)$$

2: find the distance

$$d_L(z; params\ ) \sim \int \frac{dz}{E(z)}$$

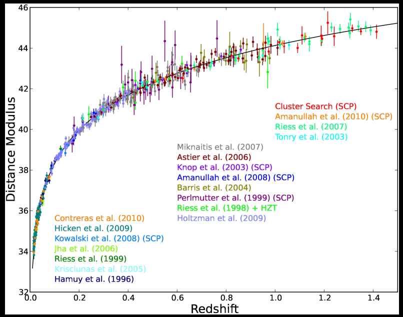
3: vary the parameters and minimize the chi-squared with data

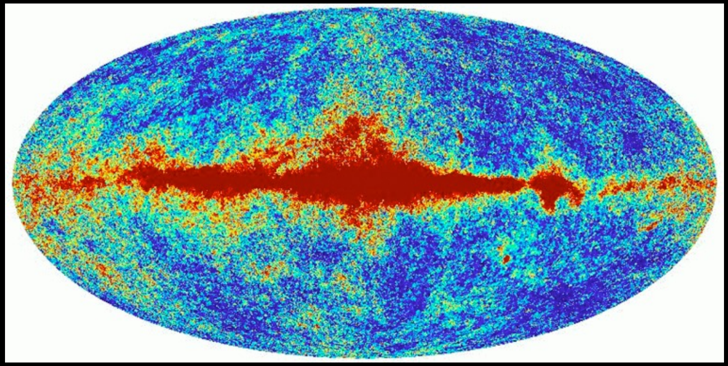
Basic property 2

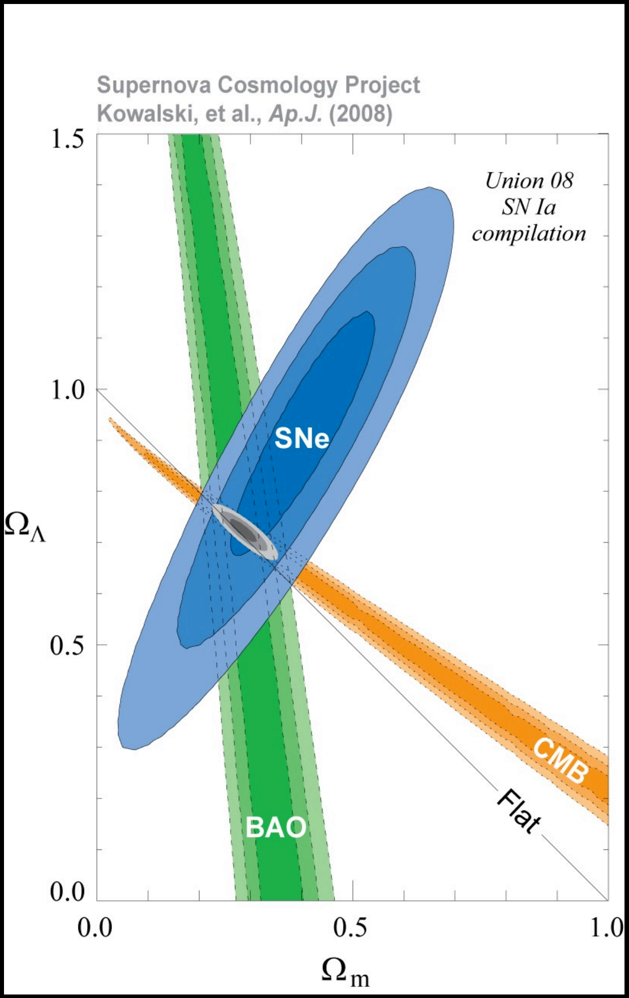


Curves of constant luminosity distance

Properties of Dark Energy



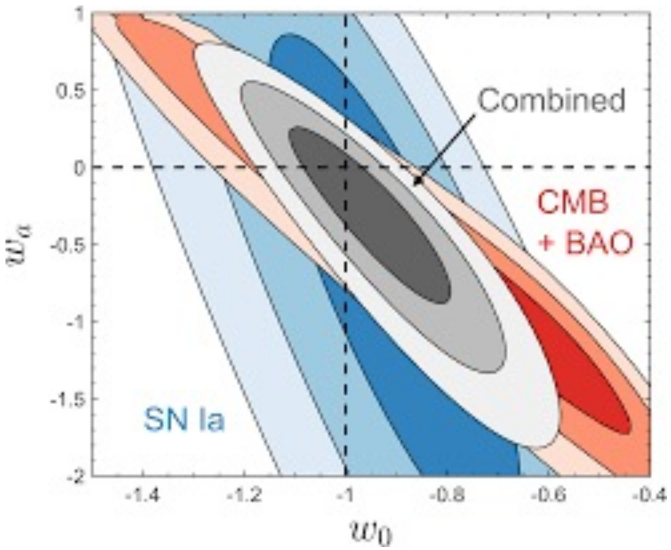




XCDM

Three parameters?

$$E^2 = \Omega_{m,0}(1+z)^3 + \Omega_{X,0}(1+z)^{3(1+w_X)} + \left(1-\Omega_{m,0}-\Omega_{\Lambda,0}\right)(1+z)^2$$



Huterer, Shafer 1709.01091

Four parameters?

$$w(z) = w_0 + w_a \frac{z}{1+z}$$

Chevalier-Polarski-Linder param.

Etc. etc. etc.

Problems

Parametrizations of *w(z)* are simple but arbitrary (strong model-dependence)

They only specify the background, not the perturbations

They do not correspond to fundamental degrees of freedom (fields)

We measure $d_L(z)$ but we want *E(z)* and Ω_k!

$$d(z) = \frac{1}{H_0\sqrt{\Omega_K^{(0)}}}\sinh\left(\sqrt{\Omega_K^{(0)}}\int_0^z \frac{\mathrm{d}\tilde{z}}{E(\tilde{z})}\right) ,$$

Space curvature

To measure Ω_k we need both $H(z)$ and $d_L(z)$

$$\Omega_K^{(0)} = \frac{[H(z)d_{,z}(z)]^2 - 1}{[H_0 d(z)]^2},$$

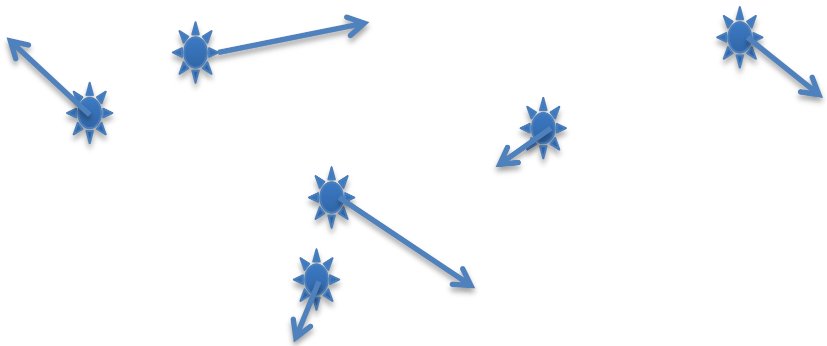
Multi-purpose standard candles

The luminosity of standard candles gives us d_L

But we measure also position and scatter around the mean!

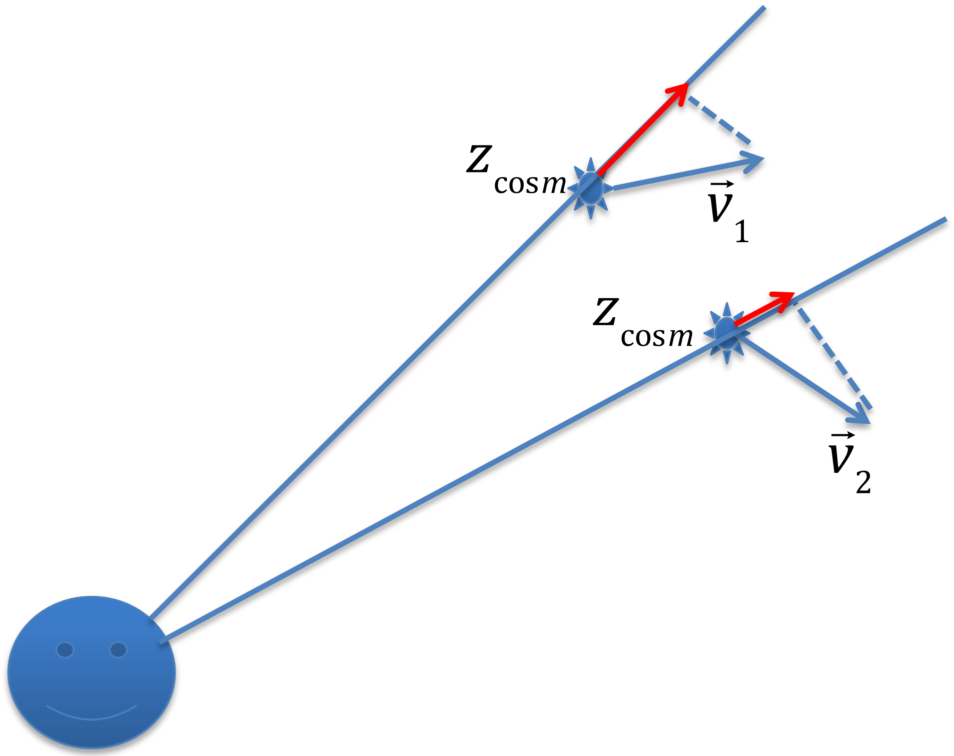
Their position gives us a tracer of LSS

The scatter around the mean gives us their peculiar velocity



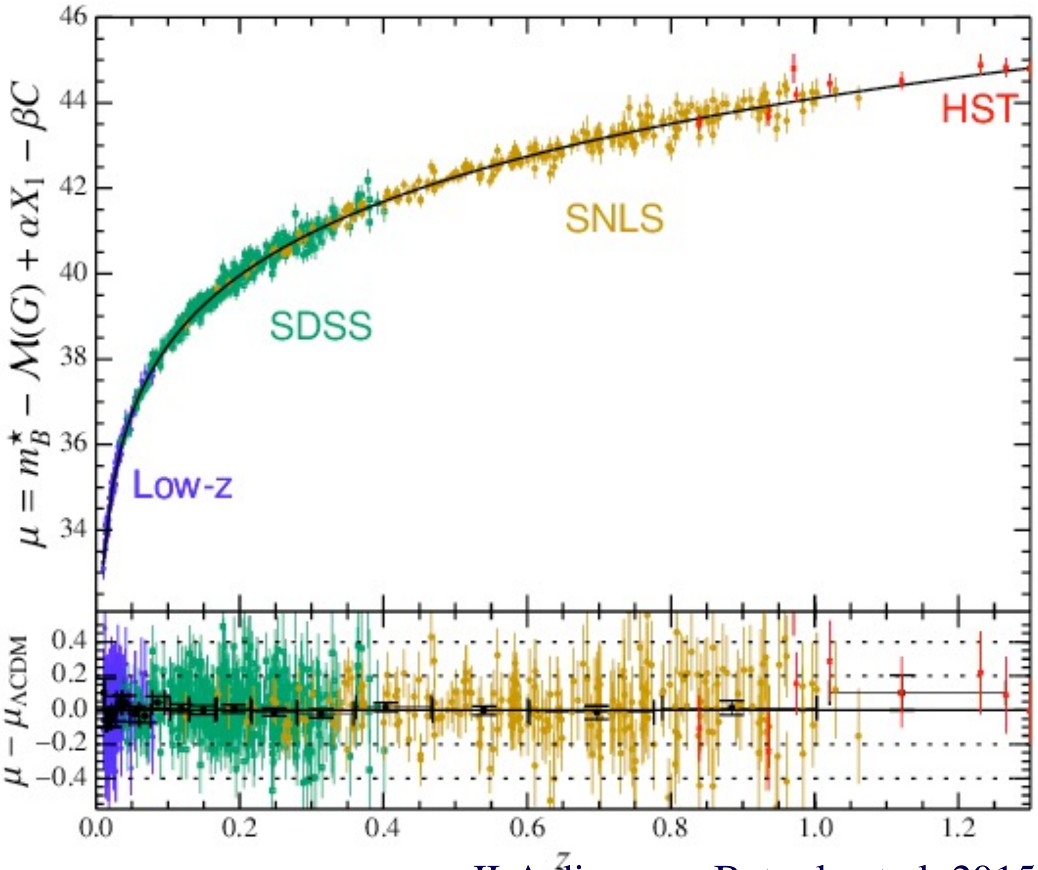
Measuring H(z)

Standard candles give us d_L
But their scatter around z also give us their peculiar velocity:



SNIa scatter

$$d_L(z) = d_L(z_{\cos m} + z_{pec})$$
$$\approx d_L(z_{\cos m}) + \Delta d_L(z_{pec})$$



JLA diagram, Betoule et al. 2015

From luminosity to velocity

$$\frac{\delta D_L}{D_L} = v_s \left[2 - \frac{d\log D_L}{d\log(1+z)}\right]$$

Hui & Greene 2006
astro-ph/0512159
Davis et al. 2011
1012.2912

$$\frac{\delta D_L}{D_L} = \frac{\log 10}{5}\delta m$$

new observable!

$$v_s = \frac{\log 10}{5}\delta m \left[2 - \frac{d\log D_L}{d\log(1+z)}\right]^{-1}$$

Measuring H(z)

What is the variance of *v*?

$$\sigma^2_{v,\mathrm{eff}} \equiv \left[\frac{\log 10}{5}\sigma_{\mathrm{int}}\right]^2 \left[2 - \frac{d\log D_L}{d\log(1+z)}\right]^{-2} + \frac{\sigma^2_{v,\mathrm{nonlin}}}{c^2}.$$

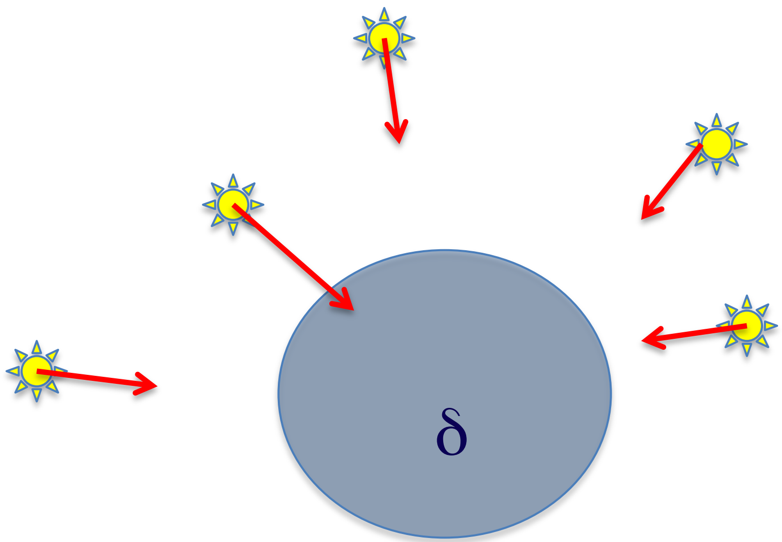
Hui & Greene 2006
astro-ph/0512159
Davis et al. 2011
1012.2912

scatter in magnitude

conversion

non-linear scatter

Peculiar velocity field



correlation between linear velocity field and density contrast

Measuring H(z)

In the linear regime:

cosine of angle
between k and r

$$v_{\parallel} = i\frac{H}{k(1+z)}\beta\frac{\boldsymbol{k}\cdot\boldsymbol{r}}{kr}\delta_g = i\frac{H\mu}{k(1+z)}\beta\delta_g\,,$$

$$\beta = \frac{f}{b} = \frac{growth}{bias}$$

Measuring H(z)

So given a large number of SNIa we can measure the correlations of their positions and of their peculiar velocity field for every *k*,*μ*

$$\bar{n}V\langle\delta\delta^*\rangle = \bar{n}\left(1+\beta\mu^2\right)^2 S_\delta^2 b^2 P_m + 1 = B^2\, PN_\delta\,,$$

$$i\bar{n}V\langle\delta v^*\rangle = \frac{\bar{n}H\mu}{k(1+z)}\left(1+\beta\mu^2\right)S_\delta S_v\,\beta b^2 P_m = ABP,$$

$$\bar{n}V\langle vv^*\rangle = \bar{n}\left[\frac{H\mu}{k(1+z)}\right]^2 S_v^2\beta^2 b^2 P_m + \sigma_{v,\mathrm{eff}}^2 = A^2 PN_v,$$

D. Burkey and A. N. Taylor, Mon. Not. Roy. Astron. Soc. **347**, 255 (2004), arXiv:astro-ph/0310912 [astro-ph].
C. Howlett, A. S. G. Robotham, C. D. P. Lagos, and A. G. Kim, Astrophys. J. **847**, 128 (2017), arXiv:1708.08236 [astro-ph.CO].
M. Quartin, L.A., B. Moraes, 2111.05185

3x2 Covariance matrix

two random fields (density contrast and peculiar velocity), three correlation functions

$$L = [(2\pi)^N C]^{-1/2} \exp(-\frac{1}{2} x_i C_{ij}^{-1} x_j)$$

$$x = \{\delta, v\}$$

$$C_{ij} = \begin{pmatrix} B^2 P N_\delta & ABP \\ ABP & A^2 P N_v \end{pmatrix}$$

All together: Clustering of Standard Candles (CSC)

M. Quartin, L.A., B. Moraes, 2111.05185

observables

$$\bar{n}V\langle\delta\delta^*\rangle = \bar{n}(1+\beta\mu^2)^2 S_\delta^2 b^2 P_m + 1 = B^2 PN_\delta\,,$$

$$i\bar{n}V\langle\delta v^*\rangle = \frac{\bar{n}H\mu}{k(1+z)}(1+\beta\mu^2)S_\delta S_v\,\beta b^2 P_m = ABP,$$

$$\bar{n}V\langle vv^*\rangle = \bar{n}\left[\frac{H\mu}{k(1+z)}\right]^2 S_v^2\beta^2 b^2 P_m + \sigma_{v,\mathrm{eff}}^2 = A^2 PN_v,$$

depend entirely on the following parameters

$$H_0 D(z),\quad E(z),\quad \beta(k,z),\quad \sigma_\delta(z), \sigma_v(z),\quad P(k,z)$$

non-linear velocity smoothing (fixed)

galaxy power spectrum

They completely specify the correlations at any (linear) k,μ,z without assuming any cosmology !

Even more: 6x2 covariance matrix

We can use SN and galaxies as two different tracers of LSS

$$P_{\rm gg}(k,\mu,z) = \left[1+\beta_{\rm g}\mu^2\right]^2 b_{\rm g}^2\, S_{\rm g}^2\, D_+^2 P_{\rm mm}(k) + \frac{1}{n_{\rm g}}, \quad (7)$$

$$P_{\rm ss}(k,\mu,z) = \left[1+\beta_{\rm s}\mu^2\right]^2 b_{\rm s}^2\, S_{\rm s}^2\, D_+^2 P_{\rm mm}(k) + \frac{1}{n_{\rm s}}, \quad (8)$$

$$P_{\rm gs}(k,\mu,z) = \left[1+\beta_{\rm g}\mu^2\right]\left[1+\beta_{\rm s}\mu^2\right] b_{\rm g}\, b_{\rm s}\, S_{\rm g}\, S_{\rm s}\, D_+^2 P_{\rm mm}(k) + \frac{n_{\rm gs}}{n_{\rm g} n_{\rm s}}, \quad (9)$$

$$P_{\rm gv}(k,\mu,z) = \frac{H\mu}{k(1+z)}\left[1+\beta_{\rm g}\mu^2\right] b_{\rm g}\, S_{\rm g}\, S_{\rm v}\, f D_+^2 P_{\rm mm}(k), \quad (10)$$

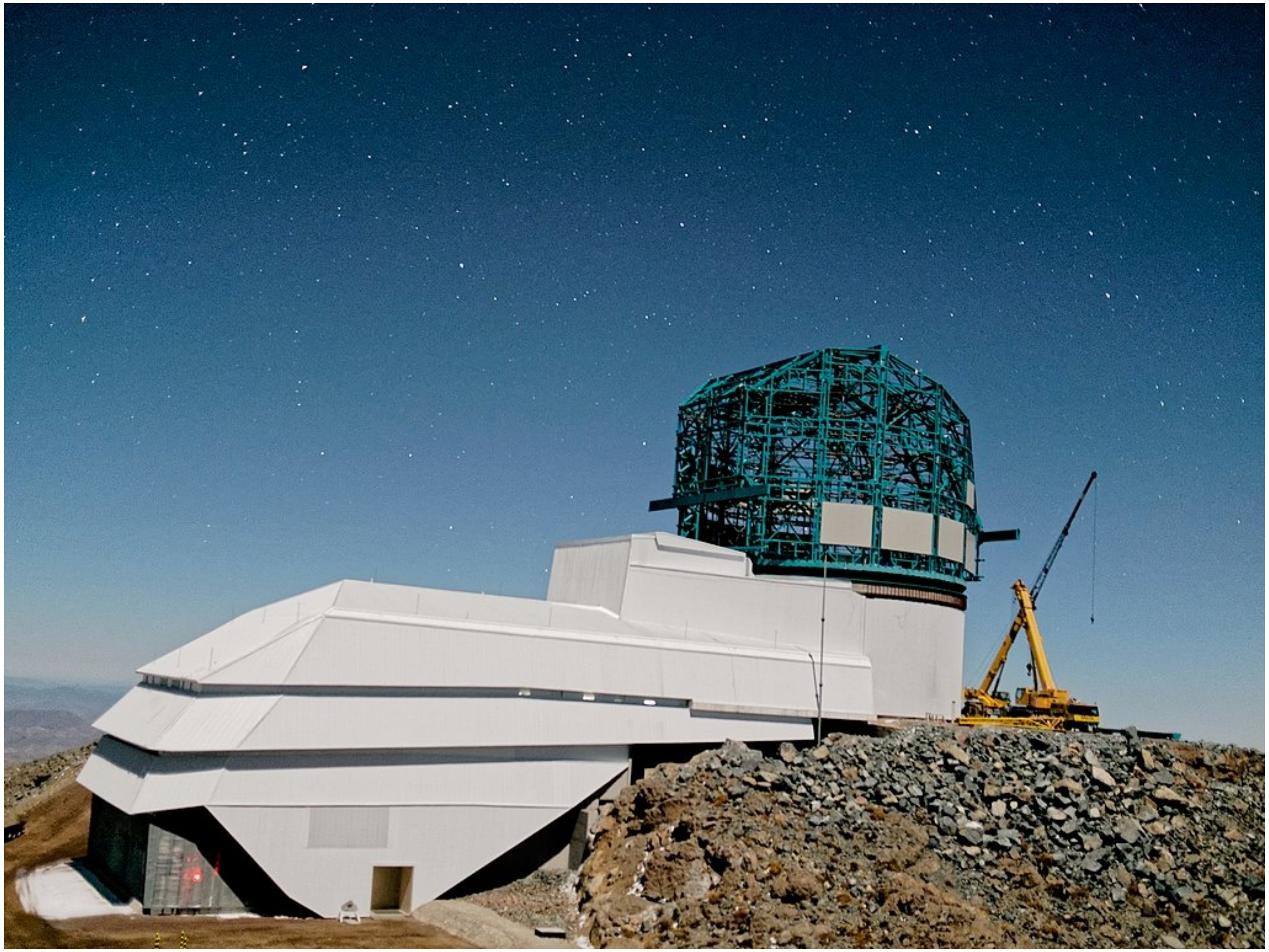
$$P_{\rm sv}(k,\mu,z) = \frac{H\mu}{k(1+z)}\left[1+\beta_{\rm s}\mu^2\right] b_{\rm s}\, S_{\rm s}\, S_{\rm v}\, f D_+^2 P_{\rm mm}(k), \quad (11)$$

$$P_{\rm vv}(k,\mu,z) = \left[\frac{H\mu}{k(1+z)}\right]^2 S_{\rm v}^2\, f^2\, D_+^2 P_{\rm mm}(k) + \frac{\sigma_{v,\rm eff}^2}{n_{\rm s}}, \quad (12)$$

$$\Rightarrow \quad \mathbf{C} = \begin{pmatrix} P_{\rm gg} & P_{\rm gs} & P_{\rm gv} \\ P_{\rm gs} & P_{\rm ss} & P_{\rm sv} \\ P_{\rm gv} & P_{\rm sv} & P_{\rm vv} \end{pmatrix}.$$

M. Quartin, L.A., B. Moraes,
2111.05185

LSST at Vera Rubin Observatory



By Wil O'Mullane 2019-09-11,
https://commons.wikimedia.org/w/index.php?curid=97813768

The rest is Fisherology

Garcia K., Quartin M., Siffert B. B., 2020, Phys. Dark Univ., 29, 100519, 1905.00746

M. Quartin, L.A., B. Moraes, 2111.05185

$z_{\rm bin}$	$\Delta H/H$ (%)					
	Conservative			Aggressive		
	only g	3×2pt	6×2pt	only g	3×2pt	6×2pt
0.05	83	28	28	39	9.1	9.0
0.15	26	14	13	15	6.1	6.0
0.25	15	9.8	9.5	9.4	5.2	5.0
0.35	14	9.1	8.2	8.5	4.7	4.0
0.45	-	-	-	7.0	4.5	3.6
0.55	-	-	-	6.1	4.4	3.3
0.65	-	-	-	5.5	4.3	3.2

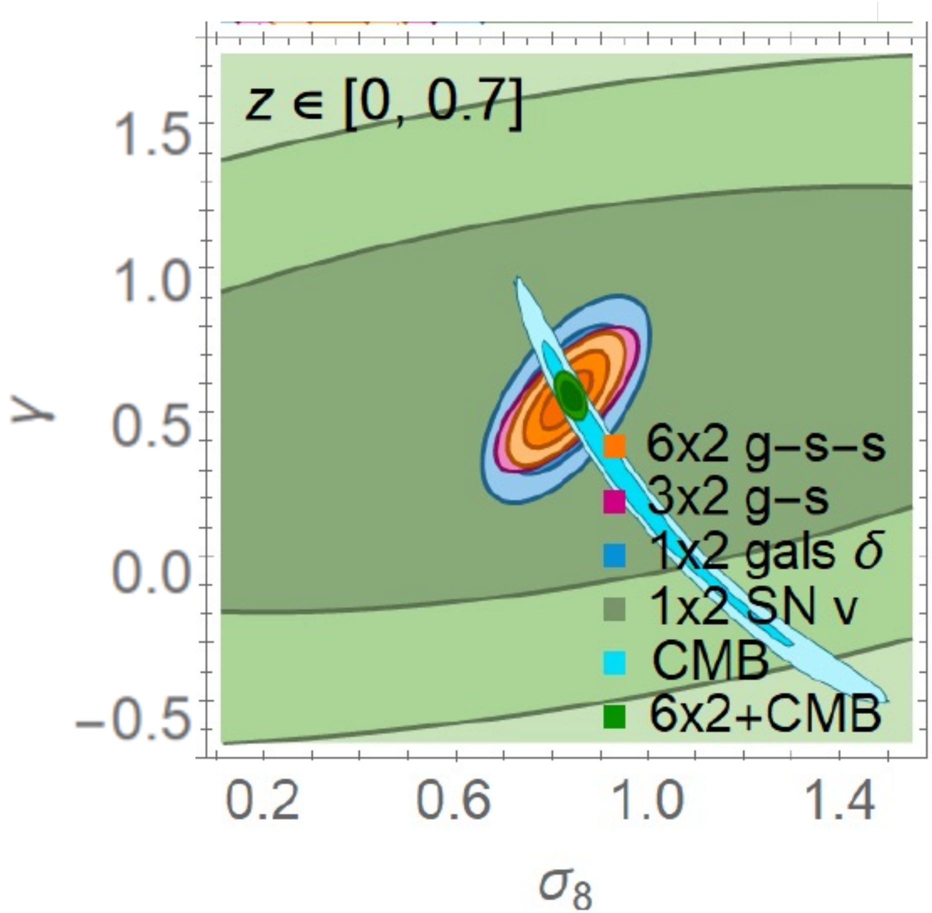
expected 10^5-10^6 SNIa

up to 3% errors in LSST

We can measure *H(z)* and *D(z)* to good accuracy without assuming a cosmological model !

Finally, you can project onto your favorite cosmology!

$f \equiv d \log \delta_m / d \log a \equiv \Omega_m(z)^\gamma$



- a 4MOST-like spectroscopic survey (7500 deg^2) +
  LSST SNe detections "aggressive" case ($0 < z < 0.7$)
- one pair of bias (nuisance) parameters $\{b_g, b_s\}$ per redshift bin
- 3 global non-linear RSD parameters

Constraints are orthogonal to Planck-CMB

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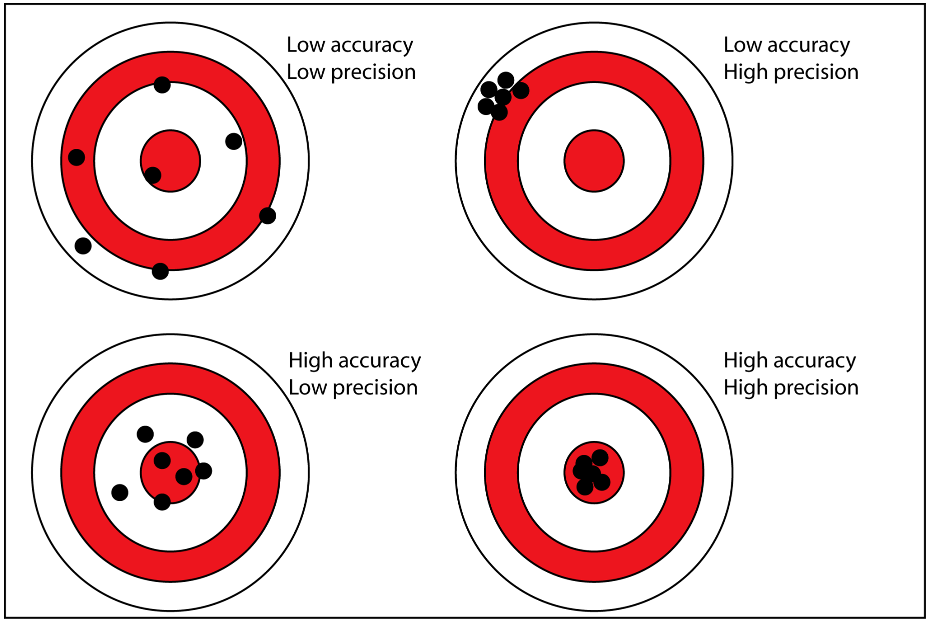
Accuracy *and* precision

$$\boldsymbol{\theta} = \{P^{k_i,z_j}, \beta_{\rm g}^{k_i,z_j}, \beta_{\rm s}^{k_i,z_j}, \eta^{z_j}, \sigma_{\rm g}, \sigma_{\rm s}, \sigma_{\rm v}\},$$

Low precision (weaker constraints)
High accuracy (minimal assumptions)



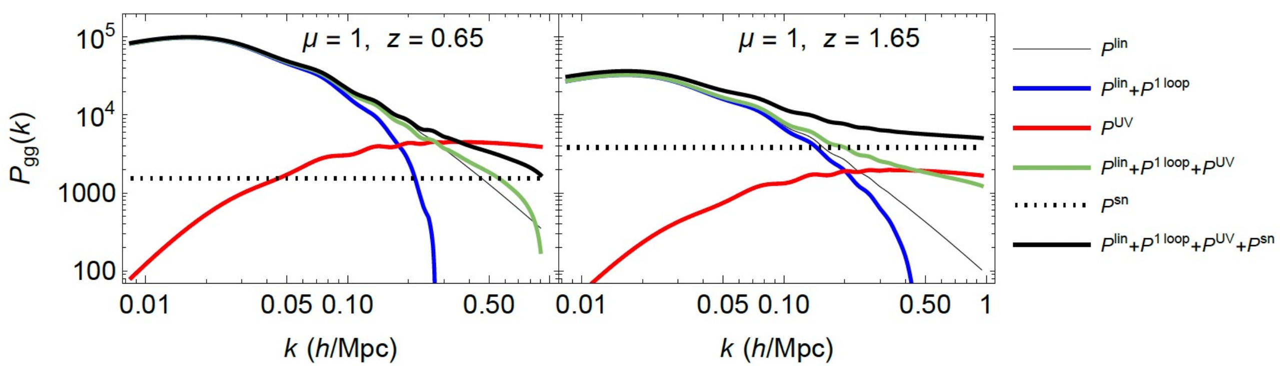
Can we have both?

Go non-linear!

$$P^{\rm lin}(k,\mu,z) = \left(1+\mu^2\beta(k,z)\right)^2 b_1(z)^2 P(k,z)\,,$$

$$\Downarrow$$

$$P_{gg}(k,\mu,z) = S_{\rm g}(k,\mu,z)^2\left[P^{\rm lin}(k,\mu,z)+P^{\rm 1loop}(k,\mu,z)+P^{\rm UV}(k,\mu,z)\right]+P^{\rm sn}(z)\,,$$



Amendola, Pietroni & Quartin (2205.00569)

Go non-linear!

$$L = [(2\pi)^N C]^{-1/2} \exp(-\frac{1}{2} x_i C_{ij}^{-1} x_j)$$ (sum over k-modes)

$$x = \{\delta, v\}$$

More k-modes, more information (grow as k^3)!

But we need model-agnostic non-linear corrections!

Higher-order PT

$$\boxed{P_{gg}(k,\mu,z) = S_{\rm g}(k,\mu,z)^2 \left[P^{\rm lin}(k,\mu,z) + P^{\rm 1loop}(k,\mu,z) + P^{\rm UV}(k,\mu,z)\right] + P^{\rm sn}(z)\,,}$$

$$P^{\rm 1loop}(k,\mu,z) = P_{13}(k,\mu,z) + P_{22}(k,\mu,z)\,, \tag{A.1}$$

Where, omitting the redshift dependence,

$$P_{13}(k,\mu) \equiv 6\, Z_1(\mathbf{k}) P(k) \int_{\mathbf{q}} Z_3(\mathbf{k},\mathbf{q},-\mathbf{q})\, P(q)\,, \tag{A.2}$$

$$P_{22}(k,\mu) \equiv 2 \int_{\mathbf{q}} Z_2^2(\mathbf{q},\mathbf{k}-\mathbf{q})\, P(q) P(|\mathbf{k}-\mathbf{q}|)\,, \tag{A.3}$$

and where we have defined

$$\int_{\mathbf{q}} \equiv \int \frac{d^3 q}{(2\pi)^3}\,. \tag{A.4}$$

The biased and RSD-corrected kernels are (see e.g. [41])

$$Z_1(\mathbf{k}) = \left(1 + \mu^2 \beta(k)\right) b_1\,, \tag{A.5}$$

$$\begin{aligned} Z_2(\mathbf{k}_1,\mathbf{k}_2) = b_1 \Big\{ & F_2(\mathbf{k}_1,\mathbf{k}_2) + \beta(k)\mu^2 G_2(\mathbf{k}_1,\mathbf{k}_2) \\ & + b_1 \frac{\beta(k)\mu k}{2} \left[\frac{\mu_1}{k_1}(1+\beta(k_2)\mu_2^2) + \frac{\mu_2}{k_2}(1+\beta(k_1)\mu_1^2)\right] \Big\} + \frac{b_2}{2} + b_{\mathcal{G}_2} S_1(\mathbf{k}_1,\mathbf{k}_2)\,, \end{aligned} \tag{A.6}$$

(already symmetrized) and

$$\begin{aligned} Z_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = b_1 \Big\{ & F_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) + \beta(k)\mu^2 G_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \\ & + b_1 \beta(k)\mu k \frac{\mu_3}{k_3}\left[F_2(\mathbf{k}_1,\mathbf{k}_2) + \beta(k_{12})\mu_{12}^2 G_2(\mathbf{k}_1,\mathbf{k}_2)\right] \\ & + b_1 \beta(k)\mu k (1+\beta(k_1)\mu_1^2) \frac{\mu_{23}}{k_{23}} G_2(\mathbf{k}_2,\mathbf{k}_3) + b_1^2 \frac{[\beta(k)\mu k]^2}{2}(1+\beta(k_1)\mu_1^2)\frac{\mu_2}{k_2}\frac{\mu_3}{k_3} \Big\} \\ & + 2 b_{\mathcal{G}_2} S_1(\mathbf{k}_1,\mathbf{k}_2+\mathbf{k}_3) F_2(\mathbf{k}_2,\mathbf{k}_3) + b_1 b_{\mathcal{G}_2} \beta(k)\mu k \frac{\mu_1}{k_1} S_1(\mathbf{k}_2,\mathbf{k}_3) \\ & + 2 b_{\Gamma_3} S_1(\mathbf{k}_1,\mathbf{k}_2+\mathbf{k}_3)(F_2(\mathbf{k}_2,\mathbf{k}_3) - G_2(\mathbf{k}_2,\mathbf{k}_3))\,, \end{aligned} \tag{A.7}$$

$$P^{\rm UV}(k,\mu,z) = -2P(k)\left(c_0^2 k^2 + \frac{3}{2} c_2^2 \mu^2 k^2 - \frac{1}{2}\tilde{c}\, b_1^6 \beta^4 \mu^4 k^4 (1+\beta\mu^2)^2\right).$$

Heroic efforts starting from the 80's:
Fry, Bernardeau,
Scoccimarro, Pietroni,
Crocce, Zaldarriaga, Senatore
and (apologies) many others

"counterterms"

And the other spectra?

$$\boxed{P_{\rm gg}(k,\mu,z) = \left[1+\beta_{\rm g}\mu^2\right]^2 b_{\rm g}^2\, S_{\rm g}^2\, D_+^2 P_{\rm mm}(k) + \frac{1}{n_{\rm g}}, \qquad (7)}$$

$$P_{\rm ss}(k,\mu,z) = \left[1+\beta_{\rm s}\mu^2\right]^2 b_{\rm s}^2\, S_{\rm s}^2\, D_+^2 P_{\rm mm}(k) + \frac{1}{n_{\rm s}}, \qquad (8)$$

$$P_{\rm gs}(k,\mu,z) = \left[1+\beta_{\rm g}\mu^2\right]\left[1+\beta_{\rm s}\mu^2\right] b_{\rm g}\, b_{\rm s}\, S_{\rm g}\, S_{\rm s}\, D_+^2 P_{\rm mm}(k) + \frac{n_{\rm gs}}{n_{\rm g}n_{\rm s}}, \qquad (9)$$

$$P_{\rm gv}(k,\mu,z) = \frac{H\mu}{k(1+z)}\left[1+\beta_{\rm g}\mu^2\right] b_{\rm g}\, S_{\rm g}\, S_{\rm v}\, f D_+^2 P_{\rm mm}(k), \qquad (10)$$

$$P_{\rm sv}(k,\mu,z) = \frac{H\mu}{k(1+z)}\left[1+\beta_{\rm s}\mu^2\right] b_{\rm s}\, S_{\rm s}\, S_{\rm v}\, f D_+^2 P_{\rm mm}(k), \qquad (11)$$

$$P_{\rm vv}(k,\mu,z) = \left[\frac{H\mu}{k(1+z)}\right]^2 S_{\rm v}^2\, f^2\, D_+^2 P_{\rm mm}(k) + \frac{\sigma_{v,\rm eff}^2}{n_{\rm s}}, \qquad (12)$$

…work in progress!

(see e.g. F. Qin, C. Howlett, and L. Staveley-Smith arXiv:1906.02874)

Go non-linear!

$$P_{gg}(k,\mu,z) = S_{\rm g}(k,\mu,z)^2 \left[P^{\rm lin}(k,\mu,z) + P^{\rm 1loop}(k,\mu,z) + P^{\rm UV}(k,\mu,z)\right] + P^{\rm sn}(z)\,,$$

Many more *z*-dependent parameters:

$$\{\log P(k), \log \beta(k), \log \eta, \underbrace{\log b_1, b_2, b_{\mathcal{G}_2}, b_{\Gamma_3}}_{\text{NL bias}}, \underbrace{c_0}_{\text{UV}}, \log \sigma_f, P_{\rm shot}\}\,.$$

More modes 😀, more parameters 😒: who wins?

An important side effect

$$P_{gg}(k,\mu,z) = S_{\rm g}(k,\mu,z)^2 \left[P^{\rm lin}(k,\mu,z) + P^{\rm 1loop}(k,\mu,z) + P^{\rm UV}(k,\mu,z)\right] + P^{\rm sn}(z)\,,$$

$$\{\log P(k), \log\beta(k), \log\eta, \underbrace{\boxed{\log b_1, b_2, b_{\mathcal{G}_2}, b_{\Gamma_3}}}_{\text{NL bias}}, \underbrace{\boxed{c_0}}_{\text{UV}}, \log\sigma_f, P_{\rm shot}\}\,.$$

Now we have both $\beta = f/b_1$ and b_1

so we can measure f and P_m

You cannot do this at linear level!

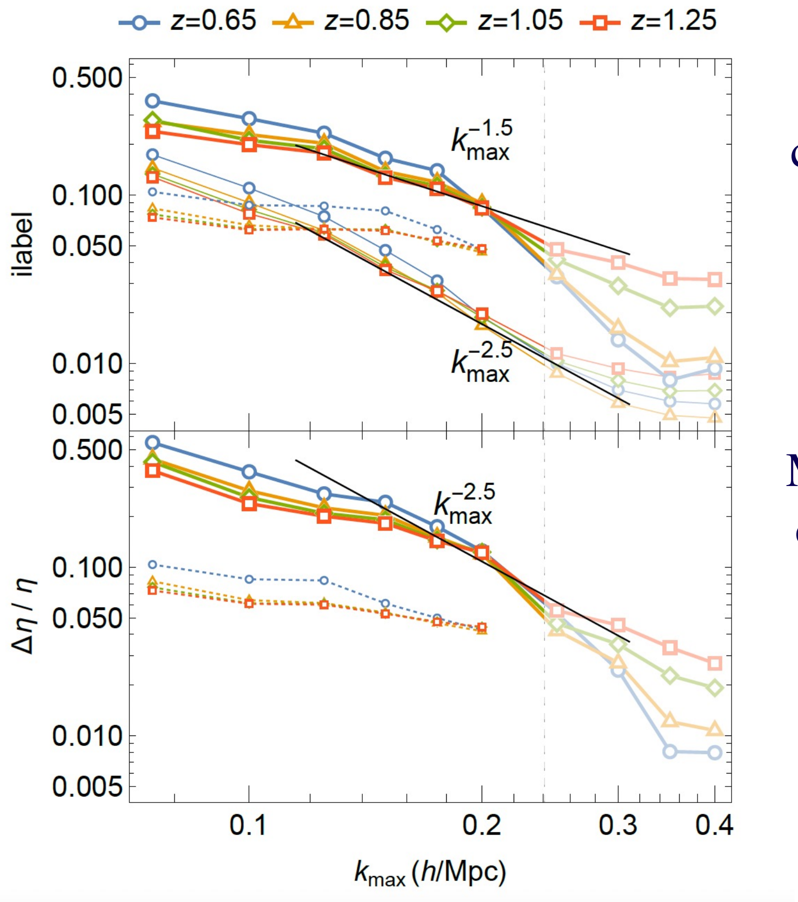
Go non-linear 1

$$P_{gg}(k,\mu,z) = S_{\rm g}(k,\mu,z)^2\left[P^{\rm lin}(k,\mu,z)+P^{\rm 1loop}(k,\mu,z)+P^{\rm UV}(k,\mu,z)\right]+P^{\rm sn}(z)\,,$$

$\frac{\Delta H}{H}$

simplified counterterms

"infinite" priors, fully marginalized



More general counterterms

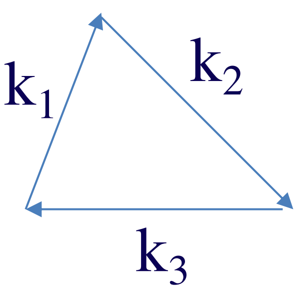
Amendola, Pietroni & Quartin (2205.00569)

Go non-linear 2

spectrum $P = <\delta(k)\ \delta(-k)>$

$$P_{gg}(k,\mu,z) = S_{\rm g}(k,\mu,z)^2 \left[P^{\rm lin}(k,\mu,z) + P^{\rm 1loop}(k,\mu,z) + P^{\rm UV}(k,\mu,z)\right] + P^{\rm sn}(z)\,,$$

(one-loop level)



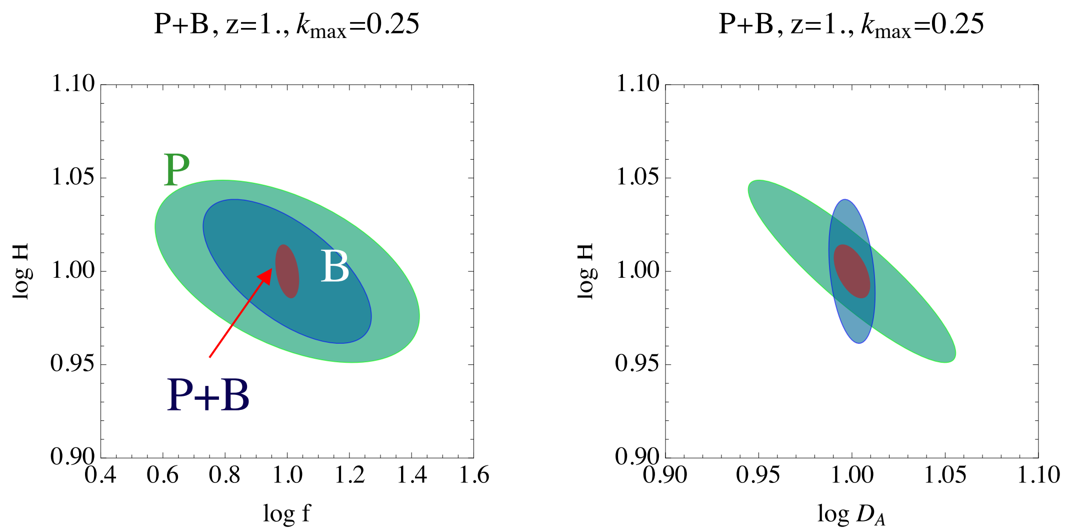
bispectrum $B = <\delta(k_1)\ \delta(k_2)\delta(k_2)>$

$$B(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = 2Z_1(\mathbf{k}_1)Z_1(\mathbf{k}_2)Z_2(\mathbf{k}_1,\mathbf{k}_2)P(\mathbf{k}_1)P(\mathbf{k}_2) + 2cycl.$$

(tree level: k<0.1 h/Mpc but no additional parameters!)

Go non-linear 2

errors on
expansion rate *H*
and growth rate *f*
around 1-2%
in $\Delta z = 0.1$
including beyond-EdS

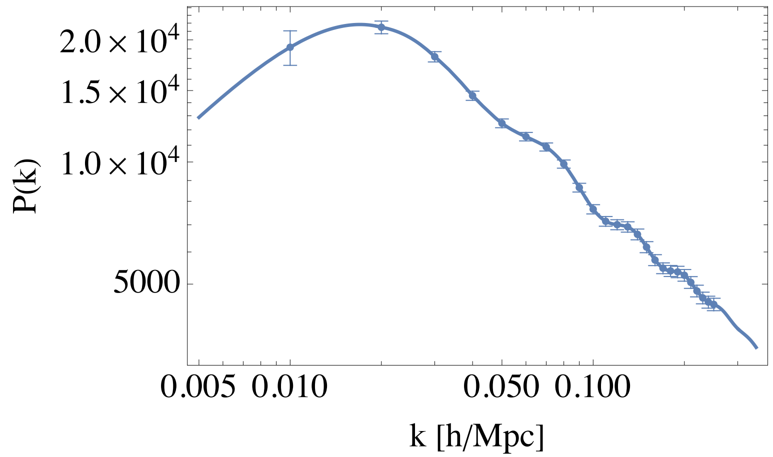


The bispectrum improves constraints by a large factor!

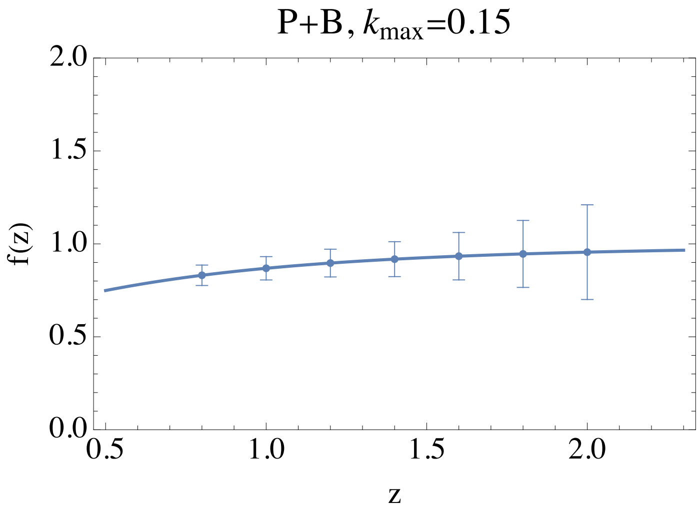
Amendola, Marinucci,
Pietroni & Quartin (in progress)

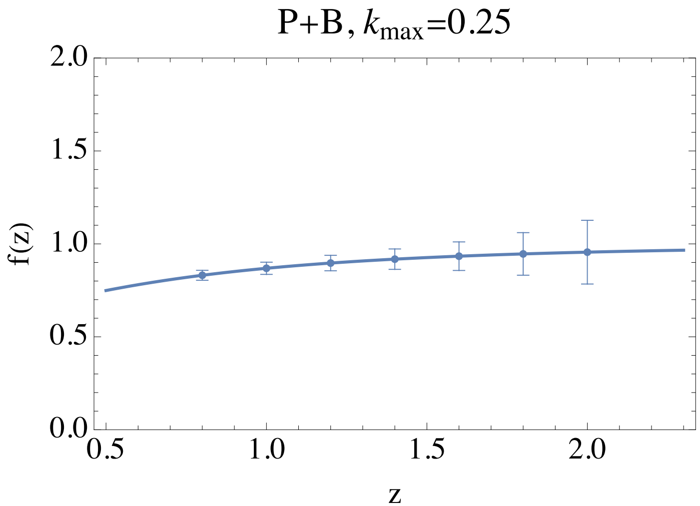
Go non-linear 2

$P_{lin,DM}(k)$
k_{max}=0.25
z=1



f(z)





Amendola, Marinucci,
Pietroni & Quartin (in progress)

Back to gravity: Observing Ψ, Φ

$$ds^2 = (1 + 2\Psi)dt^2 - \frac{\exp \int H(t')dt'}{(1 - \frac{1}{4}\Omega_k r^2)^2}(1 - 2\Phi)dx_i dx^i$$

The magic of Lagrangians

$$\int dx^4 \sqrt{-g}\left[R + L_{matter}\right] \xrightarrow{\text{variation}} R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi T_{\mu\nu}$$

The two laws of Lagrangian-cooking:
a) form a scalar: Eqs are covariant
b) no explicit functions of coords: Energy-momentum tensor is conserved

Random example:

$$\int dx^4 \sqrt{-g}\left[f(\phi)R + R_{\mu\nu}\phi^{,\nu}\phi^{,\mu} + V(\phi) + R_{\mu\nu\sigma\tau}\phi^{,\nu}\phi^{,\mu}\phi^{,\sigma}\phi^{,\tau} + R_{\mu\nu\sigma\tau}R^{,\sigma\tau}\phi^{,\nu\mu} + \ldots + L_{matter}\right]$$

The past ten years of DE research

$$\int dx^4 \sqrt{-g}\left[R + \frac{1}{2}\phi_{,\mu}\phi^{,\mu} + V(\phi) + L_{matter}\right]$$

$$\int dx^4 \sqrt{-g}\left[f(\phi)R + \frac{1}{2}\phi_{,\mu}\phi^{,\mu} + V(\phi) + L_{matter}\right]$$

$$\int dx^4 \sqrt{-g}\left[f(\phi)R + K(\frac{1}{2}\phi_{,\mu}\phi^{,\mu}) + V(\phi) + L_{matter}\right]$$

$$\int dx^4 \sqrt{-g}\left[f(\phi, \frac{1}{2}\phi_{,\mu}\phi^{,\mu})R + G_{\mu\nu}\phi^{,\nu}\phi^{,\mu} + K(\frac{1}{2}\phi_{,\mu}\phi^{,\mu}) + V(\phi) + L_{matter}\right]$$

Cosmological constant, Dark energy w=const, Dark energy w=w(z),quintessence, scalar-tensor model, coupled quintessence, k-essence, f(R), Gauss-Bonnet, Galileons, KGB,

The Horndeski Lagrangian

The most general 4D scalar field theory with second order equations of motion

$$\int dx^4 \sqrt{-g}\left[\sum_i L_i + L_{matter}\right]$$

$$\begin{aligned}
\mathcal{L}_2 =& K(\phi, X)\,, \\
\mathcal{L}_3 =& -G_3(\phi, X)\Box\phi\,, \\
\mathcal{L}_4 =& G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]\,, \\
\mathcal{L}_5 =& G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5,X}}{6}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right].
\end{aligned}$$

$$X = \frac{1}{2}\phi_{,\mu}\phi^{,\mu}$$

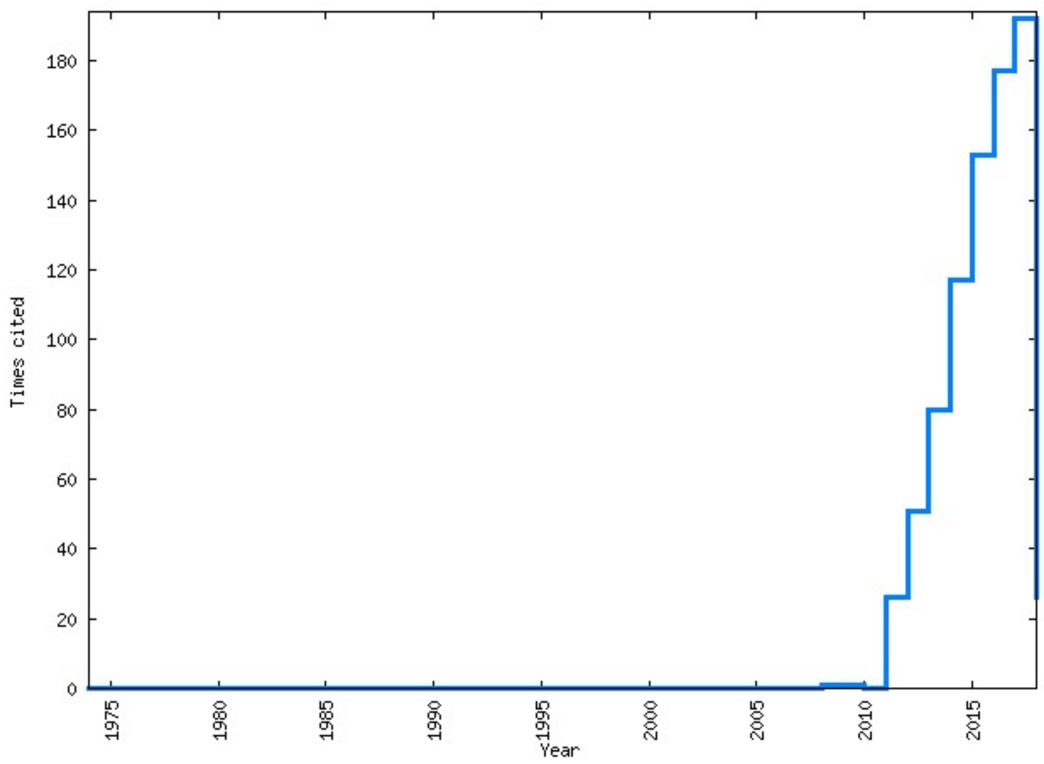
- ✓ First found by Horndeski in 1975
- ✓ rediscovered by Deffayet et al. in 2011
- ✓ no ghosts, no classical instabilities
- ✓ it modifies gravity!
- ✓ it includes f(R), Brans-Dicke, k-essence, Galileons, etc etc etc
- ✓ Invariant under conformal and disformal transformations

Horndeski…

Second-order scalar-tensor field equations in a four-dimensional space

Gregory Walter Horndeski (Waterloo U.)

Int.J.Theor.Phys. 10 (1974) 363-384



Summary (and clarification)

Acceleration versus modified gravity

Cosmological constant

Sufficient for acceleration,
but not necessary

New field

Necessary to modify
gravity, but not sufficient

The new field can drive acceleration and can modify gravity
but it does not necessarily have to do both

The Horndeski Lagrangian

The most general 4D scalar field theory with second order equations of motion

$$\int dx^4 \sqrt{-g}\left[\sum_i L_i + L_{matter}\right]$$

$$\begin{aligned}
\mathcal{L}_2 &= K(\phi, X)\,, \\
\mathcal{L}_3 &= -\,G_3(\phi, X)\Box\phi\,, \\
\mathcal{L}_4 &= G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]\,, \\
\mathcal{L}_5 &= G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5,X}}{6}\left[(\Box\phi)^3 - 3\,(\Box\phi)\,(\nabla_\mu\nabla_\nu\phi)^2 + 2\,(\nabla_\mu\nabla_\nu\phi)^3\right].
\end{aligned}$$

You can now define a (very complicated) field EMT

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi(T_{(m)\mu\nu} + T_{(\phi)\mu\nu})$$

and obtain (very complicated) field equations from the conservation

$$T_{(\phi)\mu\nu}{}^{;\mu} = 0$$

Limits of the Horndeski Lagrangian

- If $G_4 = 1/2$ and $G_5 = 0$ (it is actually sufficient $G_5 = const$) the HL reduces to ordinary gravity with a scalar field having a non-canonical kinetic sector given by $\mathcal{L}_2, \mathcal{L}_3$. The canonical form is obtained for $K = X - V(\phi)$ and $G_3 = 0$ ($G_3 = const$ is sufficient). ΛCDM is recovered for $K = -2\Lambda$.

- The "minimal" form of modified gravity within the HL is provided by $G_4 = G_4(\phi)$ and $G_5 = const$: this is then equivalent to a Brans-Dicke scalar-tensor model, again with a non-canonical kinetic sector.

- The original Brans-Dicke model is recovered assuming a kinetic sector, $K = (\omega_{BD}/\phi)X, G_3 = 0$, and $G_4(\phi) = \phi/2$.

- If the kinetic sector vanishes, $K_{,X} = G_3 = 0$, then we reduce ourselves to a $f(R)$ model [13], whose Lagrangian is $\mathcal{L}_R = (R + f(R))/2$. In fact, this model is equivalent to a scalar-tensor theory with $G_4(\phi) = e^{2\phi/\sqrt{6}}/2$ and a potential $K(\phi) = -(Rf_{,R} - f)/2$ where $\phi = \sqrt{6}/2 \log(1 + f_{,R})$. This relation should then be inverted to get $R = R(\phi)$ and used to replace R with ϕ in $K(\phi)$.

- If one sets $G_i(\phi, X) = G_i(X)$ then the Lagrangian is invariant under the shift $\phi \to \phi + c$ with $c = const$. This shift-symmetric version of the HL is connected to the Covariant Galileon when the functional dependence of the G_i is fixed [5] and is able to produce the accelerated expansion without a potential that makes the field slow roll.

The Ostrogradski theorem

Higher-than-second order equations of motion are unstable

$$L = L(q, \dot{q}, \ddot{q})$$

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} + \frac{d^2}{dt^2}\frac{\partial L}{\partial \ddot{q}} = 0$$

The Hamiltonian contains a term linear in a canonical momentum

$$H = P_1 Q_2 + P_2 a(Q_1, Q_2, P_2) - L(Q_1, Q_2, a)$$

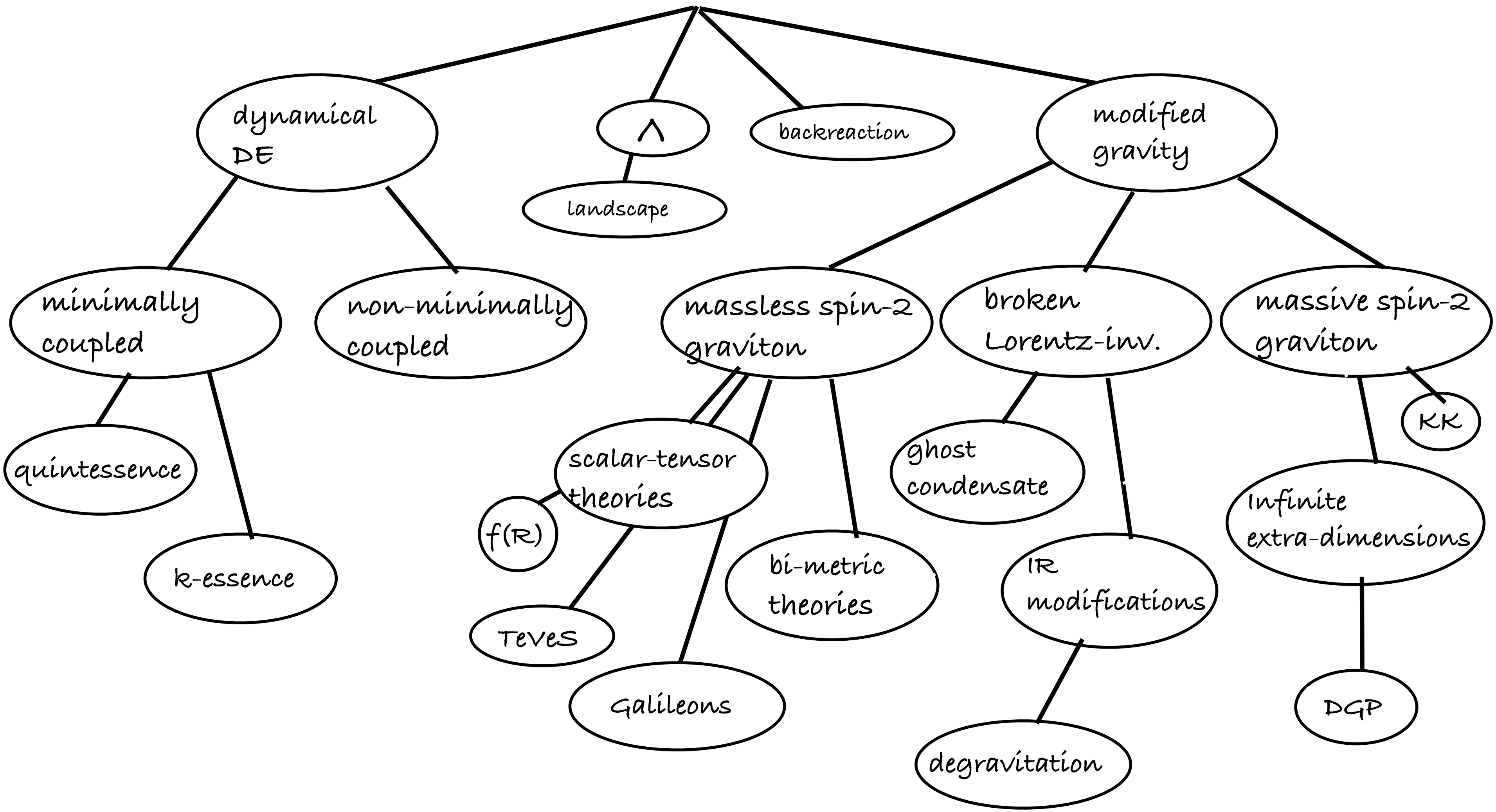
Energy states are unbounded from below!

Woodard 1506.02210

Caveats…

- The dangerous term in the Hamiltonian can actually be absent in degenerate cases
- The instability can manifest itself at very long time scales
- The proof assumes locality

The theoretical landscape



Courtesy Alessandra Silvestri

The next ten years of DE research

Combine observations of background, linear and non-linear perturbations to reconstruct as much as possible the Horndeski model

Background

As we have seen we can measure *H(z)* and

$$D(z)=\frac{1}{H_0\sqrt{\Omega_{k0}}}\sinh(H_0\sqrt{\Omega_{k0}}\int\frac{dz}{H(z)})$$

and therefore we can reconstruct the full FLRW metric

$$ds^2=dt^2-a^2(t)[\frac{dr^2}{1+\Omega_{k0}H_0^2r^2}+r^2d\Omega^2]$$

Perturbations

The most general linear, scalar metric

$$ds^2 = a^2[(1+2\Psi)dt^2 - (1+2\Phi)(dx^2 + dy^2 + dz^2)]$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi (T_{(m)\mu\nu} + T_{(\phi)\mu\nu})$$

Insert, linearize, and solve….

Two free functions

The most general linear, scalar metric

$$ds^2 = a^2[(1+2\Psi)dt^2 - (1+2\Phi)(dx^2 + dy^2 + dz^2)]$$

- Poisson's equation $$\nabla^2\Psi = 4\pi G\rho_m\delta_m$$

- anisotropic stress $$1 = -\frac{\Psi}{\Phi}$$

All the perturbation variables are root mean squares!

Modified Gravity at the linear level

The most general linear, scalar metric

$$ds^2 = a^2[(1+2\Psi)dt^2 - (1+2\Phi)(dx^2 + dy^2 + dz^2)]$$

The most general parametrization of gravity at linear level

- Poisson's equation $$\nabla^2\Psi = 4\pi G\, Y(k,a)\rho_m \delta_m$$

- anisotropic stress $$\eta(k,a) = -\frac{\Phi}{\Psi}$$

Modified Gravity at the linear level

- standard gravity

$$Y(k,a)=1$$
$$\eta(k,a)=1$$

- scalar-tensor models

$$Y(a)=\frac{G^*}{FG_{cav,0}}\frac{2(F+F'^2)}{2F+3F'^2}$$
$$\eta(a)=1+\frac{F'^2}{F+F'^2}$$

Boisseau et al. 2000
Acquaviva et al. 2004
Schimd et al. 2004
L.A., Kunz &Sapone 2007

- f(R)

$$Y(a)=\frac{G^*}{FG_{cav,0}}\frac{1+4m\frac{k^2}{a^2R}}{1+3m\frac{k^2}{a^2R}},\quad \eta(a)=1+\frac{m\frac{k^2}{a^2R}}{1+2m\frac{k^2}{a^2R}}$$

Bean et al. 2006
Hu et al. 2006
Tsujikawa 2007

- DGP

$$Y(a)=1-\frac{1}{3\beta};\quad \beta=1+2Hr_c w_{DE}$$
$$\eta(a)=1+\frac{2}{3\beta-1}$$

Lue et al. 2004;
Koyama et al. 2006

- massive bi-gravity

$$Y(a)=....$$
$$\eta(a)=....$$

F. Koennig and L. A. 2014,
Y. Akrami et al. 2014

Modified Gravity at the linear level

In the quasi-static limit, every Horndeski model is characterized at linear scales by the two functions

$$\eta(k,a) = h_2\left(\frac{1+k^2 h_4}{1+k^2 h_5}\right)$$

$$Y(k,a) = h_1\left(\frac{1+k^2 h_5}{1+k^2 h_3}\right)$$

k = wavenumber

h_i = time-dependent functions

De Felice et al. 2011; L.A. et al.PRD, arXiv:1210.0439, 2012
For a review see 1902.06978

Modified Gravity at the linear level

$$h_1 \equiv \frac{w_4}{w_1^2} = \frac{c_{\rm T}^2}{w_1}, \qquad h_2 \equiv \frac{w_1}{w_4} = c_{\rm T}^{-2},$$
$$h_3 \equiv \frac{H^2}{2XM^2}\frac{2w_1^2w_2H - w_2^2w_4 + 4w_1w_2\dot{w}_1 - 2w_1^2(\dot{w}_2 + \rho_{\rm m})}{2w_1^2},$$
$$h_4 \equiv \frac{H^2}{2XM^2}\frac{2w_1^2H^2 - w_2w_4H + 2w_1\dot{w}_1H + w_2\dot{w}_1 - w_1(\dot{w}_2 + \rho_{\rm m})}{w_1},$$
$$h_5 \equiv \frac{H^2}{2XM^2}\frac{2w_1^2H^2 - w_2w_4H + 4w_1\dot{w}_1H + 2\dot{w}_1{}^2 - w_4(\dot{w}_2 + \rho_{\rm m})}{w_4},$$

$$\begin{aligned}
w_1 &\equiv 1 + 2(G_4 - 2XG_{4,X} + XG_{5,\phi} - \dot{\phi}XHG_{5,X}),\\
w_2 &\equiv -2\dot{\phi}(XG_{3,X} - G_{4,\phi} - 2XG_{4,\phi X}) + 2H(w_1 - 4X(G_{4,X} + 2XG_{4,XX} - G_{5,\phi} - XG_{5,\phi X})) - 2\dot{\phi}XH^2(3G_{5,X} + 2XG_{5,XX}),\\
w_3 &\equiv 3X(K_{,X} + 2XK_{,XX} - 2G_{3,\phi} - 2XG_{3,\phi X}) + 18\dot{\phi}XH(2G_{3,X} + XG_{3,XX}) - 18\dot{\phi}H(G_{4,\phi} + 5XG_{4,\phi X} + 2X^2G_{4,\phi XX})\\
&\quad - 18H^2(1/2 + G_4 - 7XG_{4,X} - 16X^2G_{4,XX} - 4X^3G_{4,XXX}) - 18XH^2(6G_{5,\phi} + 9XG_{5,\phi X} + 2X^2G_{5,\phi XX})\\
&\quad + 6\dot{\phi}XH^3(15G_{5,X} + 13XG_{5,XX} + 2X^2G_{5,XXX}),\\
w_4 &\equiv 1 + 2(G_4 - XG_{5,\phi} - XG_{5,X}\ddot{\phi}). \qquad \text{(A2)}
\end{aligned}$$

De Felice et al. 2011; L.A. et al.,PRD, arXiv:1210.0439, 2012

Yukawa Potential

$$\eta(k,a) = h_2\left(\frac{1+k^2h_4}{1+k^2h_5}\right)$$

$$Y(k,a) = h_1\left(\frac{1+k^2h_5}{1+k^2h_3}\right)$$

Momentum space

$$k^2\Psi = 4\pi G Y(a,k)\rho_m(k)$$

$$k^2\Phi = 4\pi G Y(a,k)\eta(a,k)\rho_m(k)$$

$$\Psi = -\frac{GM}{r}h_2\left(1+\frac{h_4-h_5}{h_5}e^{-r/\sqrt{h_5}}\right) = -\frac{\bar{G}M}{r}(1+Qe^{-mr})$$

$$\Phi = -\frac{GM}{r}h_1\left(1+\frac{h_3-h_5}{h_5}e^{-r/\sqrt{h_5}}\right) = -\frac{\hat{G}M}{r}(1+\hat{Q}e^{-mr})$$

Real space

De Felice et al. 2011; L.A. et al.PRD, arXiv:1210.0439, 2012

2nd order perturbed Lagrangians

An elegant way to derive the perturbation equations is to
write down the perturbed Lagrangian at second order:
when you differentiate it, you get first order perturbation equations!

For instance standard EH Lagrangian gives in Minkowski

$$ds^2 = -(1+2\Psi)dt^2 + 2B_{,i}dx^i dt + ((1+2\Phi)\delta_{ij} + 2E_{,ij})dx^i dx^j$$

$$S_g = \frac{1}{2}\int d^3x dt[-8B_i\Phi_i' + 4\Phi_i\Psi_i - 4\Phi'\Box E' + 2\Phi_i^2 - 6\left(\Phi'\right)^2]$$

Definition

Propagating degree of freedom: a field with two time derivatives in the 2nd order Lagrangian, **once all the constraints have been taken into account**

Identifying the degrees of freedom

Classifying theories by their dof

Standard gravity: no scalar degree of freedom, 2 tensor dofs (beside matter)
Horndeski: one scalar dof, 2 tensor dofs (beside matter)
Massive grav: one scalar dof, 2+2 tensor dofs (beside matter)
Bimetric models: one scalar dof, 2 vector dofs, 2+2 tensor dofs (beside matter)

2nd order pert. Lagrangian shows explicitly the dofs

For Horndeski:

$$S = \int d^3x dt \{ Q_S [\dot{\varphi} - \frac{c_s^2}{a^2}(\partial_i \varphi)^2] + \sum_{\alpha=1}^{2} Q_T [\dot{h_\alpha}^2 - \frac{c_T^2}{a^2}(\partial_i h_\alpha)^2] \}$$

De Felice & Tsujikawa 2011

The magic of 2nd order perturbed Lagrangians

2nd order pert. Lagrangian also clarifies the stability conditions

$$S=\int d^3x dt\{Q_S[\dot{\varphi}-\frac{c_s^2}{a^2}(\partial_i\varphi)^2]+\sum_{\alpha=1}^{2}Q_T[\dot{h_\alpha}^2-\frac{c_T^2}{a^2}(\partial_i h_\alpha)^2]\}$$

Equation of motion in Fourier space $\ddot{\phi}-c_s^2k^2\phi=0$

Positive squared sound speeds $c_{s,T}^2\geq 0$

Positive kinetic terms $Q_{s,T}\geq 0$

Horndeski Lagrangian

$$
\begin{aligned}
\mathcal{L}_2 =& K(\phi, X)\,, \\
\mathcal{L}_3 =& -G_3(\phi, X)\Box\phi\,, \\
\mathcal{L}_4 =& G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]\,, \\
\mathcal{L}_5 =& G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5,X}}{6}\left[(\Box\phi)^3 - 3\,(\Box\phi)\,(\nabla_\mu\nabla_\nu\phi)^2 + 2\,(\nabla_\mu\nabla_\nu\phi)^3\right].
\end{aligned}
$$

Linearized Horndeski: Bellini-Sawicki parametrization

$$
\begin{aligned}
M_*^2 \equiv & 2\left(G_4 - 2XG_{4X} + XG_{5\phi} - \dot{\phi}HXG_{5X}\right) \\
HM_*^2\alpha_{\mathrm{M}} \equiv & (\dot{M_*^2}) \\
H^2M_*^2\alpha_{\mathrm{K}} \equiv & 2X\left(K_X + 2XK_{XX} - 2G_{3\phi} - 2XG_{3\phi X}\right) + \\
& + 12\dot{\phi}XH\left(G_{3X} + XG_{3XX} - 3G_{4\phi X} - 2XG_{4\phi XX}\right) + \\
& + 12XH^2\left(G_{4X} + 8XG_{4XX} + 4X^2G_{4XXX}\right) - \\
& - 12XH^2\left(G_{5\phi} + 5XG_{5\phi X} + 2X^2G_{5\phi XX}\right) + \\
& + 4\dot{\phi}XH^3\left(3G_{5X} + 7XG_{5XX} + 2X^2G_{5XXX}\right) \\
HM_*^2\alpha_{\mathrm{B}} \equiv & 2\dot{\phi}\left(XG_{3X} - G_{4\phi} - 2XG_{4\phi X}\right) + \\
& + 8XH\left(G_{4X} + 2XG_{4XX} - G_{5\phi} - XG_{5\phi X}\right) + \\
& + 2\dot{\phi}XH^2\left(3G_{5X} + 2XG_{5XX}\right) \\
M_*^2\alpha_{\mathrm{T}} \equiv & 2X\left(2G_{4X} - 2G_{5\phi} - \left(\ddot{\phi} - \dot{\phi}H\right)G_{5X}\right)
\end{aligned}
$$

sound speeds

$$S=\int d^3xdt\{Q_S[\dot{\varphi}-\frac{c_s^2}{a^2}(\partial_i\varphi)^2]+\sum_{\alpha=1}^{2}Q_T[\dot{h_\alpha}^2-\frac{c_T^2}{a^2}(\partial_i h_\alpha)^2]\}$$

$$Q_\mathrm{S}=\frac{M_*^2(2\alpha_K+3\alpha_B^2)}{(2-\alpha_\mathrm{B})^2},$$

$$c_\mathrm{S}^2=\frac{(2-\alpha_\mathrm{B})\,\alpha_1+2\alpha_2}{2\alpha_K+3\alpha_B^2},$$

$$Q_\mathrm{T}=\frac{M_*^2}{8},$$

$$c_\mathrm{T}^2=1+\alpha_\mathrm{T}$$

$$\alpha_1\equiv\alpha_B+(\alpha_B-2)\,\alpha_T+2\alpha_M$$

$$\alpha_2\equiv\alpha_B\xi+\alpha_B'-2\xi-3\tilde{\Omega}_m$$

Bellini-Sawicki parametrization

Model Class		$\alpha_{\rm K}$	$\alpha_{\rm B}$	$\alpha_{\rm M}$	$\alpha_{\rm T}$
ΛCDM		***0***	***0***	***0***	***0***
cuscuton ($w_X \neq -1$)	[71]	0	0	0	0
quintessence	[1, 2]	$(1-\Omega_{\rm m})(1+w_X)$	0	0	0
k-*essence*/perfect fluid	[45, 46]	$\frac{(1-\Omega_{\rm m})(1+w_X)}{c_{\rm s}^2}$	0	0	0
kinetic gravity braiding	[47–49]	$\frac{m^2(n_m+\kappa_\phi)}{H^2M_{\rm Pl}^2}$	$\frac{m\kappa}{HM_{\rm Pl}^2}$	0	0
galileon cosmology	[57]	$-\frac{3}{2}\alpha_{\rm M}^3H^2r_{\rm c}^2e^{2\phi/M}$	$\frac{\alpha_{\rm K}}{6}-\alpha_{\rm M}$	$\frac{-2\dot\phi}{HM}$	0
BDK	[26]	$\frac{\dot\phi^2K_{,\dot\phi\dot\phi}e^{-\kappa}}{H^2M^2}$	$-\alpha_{\rm M}$	$\frac{\dot\kappa}{H}$	0
metric $f(R)$	[3, 72]	0	$-\alpha_{\rm M}$	$\frac{B\dot H}{H^2}$	0
MSG/Palatini $f(R)$	[73, 74]	$-\frac{3}{2}\alpha_{\rm M}^2$	$-\alpha_{\rm M}$	$\frac{2\dot\phi}{H}$	0
f(Gauss-Bonnet)	[52, 75, 76]	0	$\frac{-2H\dot\xi}{M^2+H\dot\xi}$	$\frac{\dot H\dot\xi+H\ddot\xi}{H(M^2+H\dot\xi)}$	$\frac{\ddot\xi-H\dot\xi}{M^2+H\dot\xi}$

Beyond Horndeski, II

Or, we can add several Horndeski fields!

$$\int dx^4 \sqrt{-g}\left[L_{H1} + L_{H2} + \ldots + L_{matter}\right]$$

$$Y \equiv A_1 \frac{1 + A_2 k^2 + A_3 k^4}{1 + A_4 k^2 + A_5 k^4},$$

$$\eta \equiv -\frac{\Phi}{\Psi} = B_1 \frac{1 + B_2 k^2 + B_3 k^4}{1 + B_4 k^2 + B_5 k^4},$$

A. Silvestri et al. 2013
T. Baker et al. 2013
V. Vardanyan & L.A., 2015

Beyond Horndeski, III

- Torsion
- Non-metricity
- Non-universal coupling
- Palatini
- Vectors
- Tensors

Beyond Horndeski, IV

Pauli-Fierz (1939) action: the only ghost-free quadratic action for a massive spin two field

$$\int d^4x\sqrt{g}R_g + m^2\int d^4x h_{\mu\nu}h_{\alpha\beta}\left(\eta^{\mu\alpha}\eta^{\nu\beta} - \eta^{\mu\nu}\eta^{\alpha\beta}\right)$$

The three deadly sins of Pauli-Fierz theory:

- It does not reduce to massless gravity for $m \rightarrow 0$
- It violates diffeomorphism invariance
- It contains a ghost when extended to non-linear level (Boulware-Deser ghost)

Ghost-free Bigravity

- The first problem was solved by Vainshtein (1972): there exists a radius below which the linear theory cannot be applied;
  - For the Sun, this radius is larger than the solar system!

- The second and third problem can be solved introducing a second metric:

$$
\begin{aligned}
S \;=\; & -\frac{M_g^2}{2}\int d^4x\sqrt{-\det g}R(g)-\frac{M_f^2}{2}\int d^4x\sqrt{-\det f}R(f)\\
& + \; m^2M_g^2\int d^4x\sqrt{-\det g}\sum_{n=0}^{4}\beta_n e_n(\sqrt{g^{\alpha\beta}f_{\beta\gamma}})+\int d^4x\sqrt{-\det g}L_m(g,\Phi)
\end{aligned}
$$

The only ghost-free local non-linear massive gravity theory!

deRham, Gabadadze, Tolley 2010
Hassan & Rosen, 2011

Modified Gravity in bimetric gravity

In the quasi-static limit, every Horndeski model and bimetric gravity model is characterized at linear scales by the two functions

$$\eta(k,a) = h_2\left(\frac{1+k^2 h_4}{1+k^2 h_5}\right)$$

$$Y(k,a) = h_1\left(\frac{1+k^2 h_5}{1+k^2 h_3}\right)$$

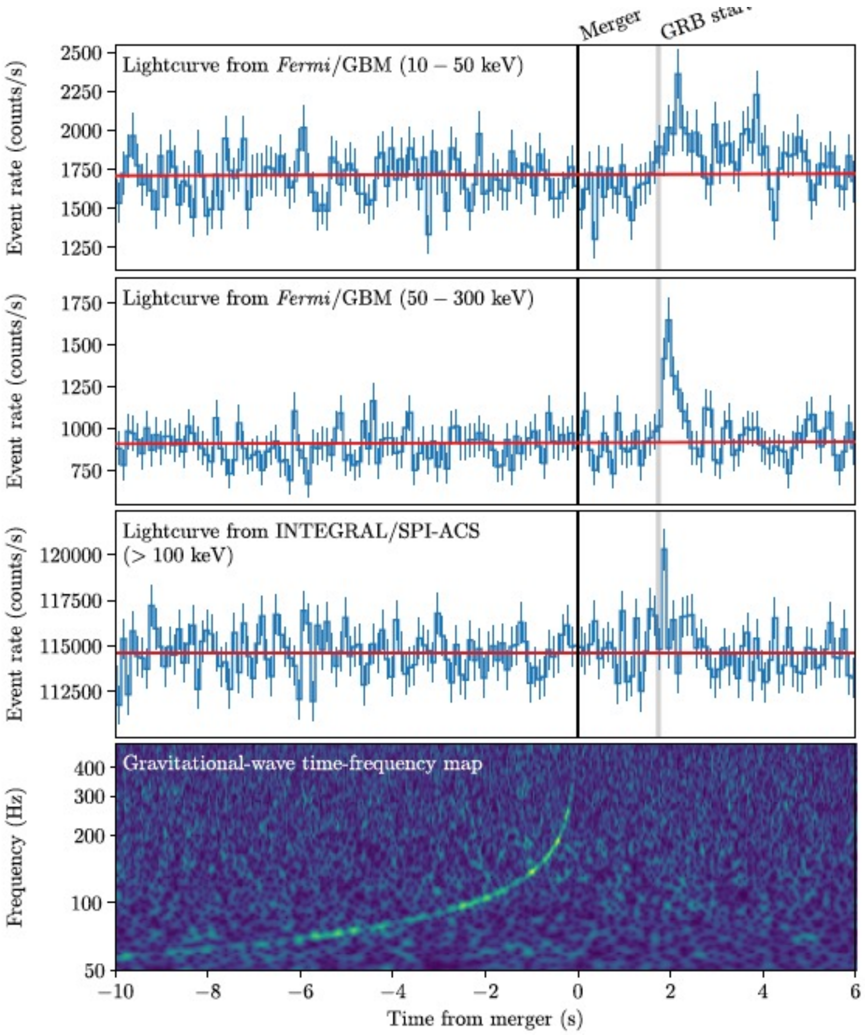
k = wavenumber

h_i = time-dependent functions

De Felice et al. 2011; L.A. et al.PRD, arXiv:1210.0439, 2012

The speed of gravitational waves

2017:
GW and gamma-ray simultaneous detection



The speed of gravitational waves

$$S = \int d^3x dt \{ Q_S [\dot{\varphi} - \frac{c_s^2}{a^2} (\partial_i \varphi)^2] + \sum_{\alpha=1}^{2} Q_T [\dot{h_\alpha}^2 - \frac{c_T^2}{a^2} (\partial_i h_\alpha)^2] \}$$

$$Q_{\rm S} = \frac{M_*^2 (2\alpha_K + 3\alpha_B^2)}{(2 - \alpha_{\rm B})^2},$$

$$c_{\rm S}^2 = \frac{(2 - \alpha_{\rm B})\, \alpha_1 + 2\alpha_2}{2\alpha_K + 3\alpha_B^2},$$

$$Q_{\rm T} = \frac{M_*^2}{8},$$

$$c_{\rm T}^2 = 1 + \alpha_{\rm T}$$

$$\alpha_1 \equiv \alpha_B + (\alpha_B - 2)\, \alpha_T + 2\alpha_M$$

$$\alpha_2 \equiv \alpha_B \xi + \alpha_B' - 2\xi - 3\tilde{\Omega}_m$$

The speed of gravitational waves

$$c^2_T = 1 + \alpha_T$$

$$M_*^2 \alpha_{\mathrm{T}} \equiv 2X \left(2G_{4X} - 2G_{5\phi} - \left(\ddot{\phi} - \dot{\phi} H\right) G_{5X}\right)$$

Only way to have $c_T = 1$ is

$$G_{4,X} = 0$$

$$G_5 = 0$$

Conformal coupling!

Horndeski Lagrangian after GWs

$$
\begin{aligned}
\mathcal{L}_2 =& K(\phi, X)\,, \\
\mathcal{L}_3 =& -G_3(\phi, X)\Box\phi\,, \\
\mathcal{L}_4 =& G_4(\phi, \cancel{X})R + \cancel{G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]}\,, \\
\mathcal{L}_5 =& \cancel{G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5,X}}{6}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]}\,.
\end{aligned}
$$

Caveats...

The speed of GW has been measured only at $z = 0$!

Test particles in linearized gravity

$$\gamma^2 \dot{\boldsymbol{v}} = \gamma^2 [2\boldsymbol{v}(\boldsymbol{v} \cdot \boldsymbol{\nabla}(\Psi - \Phi) - v^2 \boldsymbol{\nabla}(\Psi - \Phi)] - \boldsymbol{\nabla}\Psi$$

$$\gamma^2 = (1 - v^2)^{-1}$$

For small velocities, only Ψ survives

For relativistic velocities, only $\Psi - \Phi$ survives

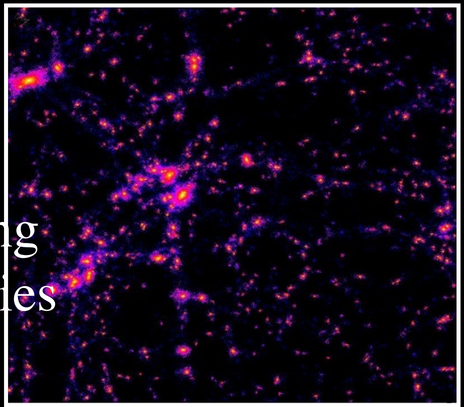
Reconstruction of the metric

$$ds^2 = a^2[(1+2\Psi)dt^2 - (1+2\Phi)(dx^2 + dy^2 + dz^2)]$$

Non-relativistic particles respond to Ψ

$$\delta'' + (1 + \frac{H'}{H})\delta' = k^2\Psi$$

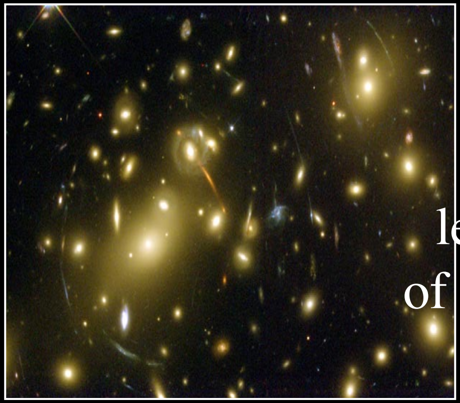
$$\delta = \frac{\rho - \hat{\rho}}{\hat{\rho}}$$



clustering of galaxies

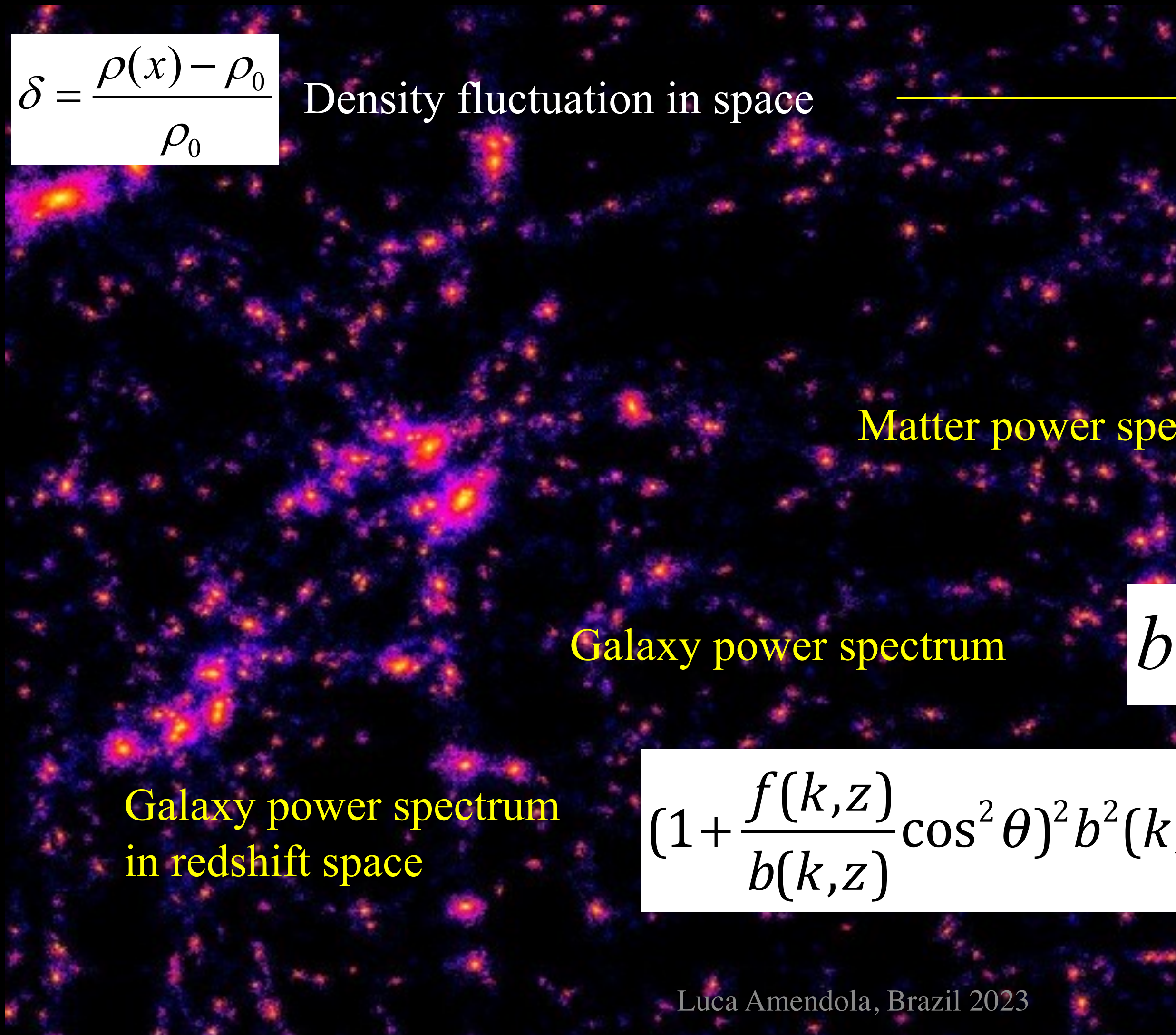
Relativistic particles respond to Φ-Ψ

$$\alpha = \int \nabla_{perp}(\Psi - \Phi)dz$$



lensing of galaxies

Reality check



$$\delta = \frac{\rho(x)-\rho_0}{\rho_0}$$ Density fluctuation in space $\longrightarrow$ $$\left\langle \delta_k{}^2 \right\rangle = P(k,z)$$

Matter power spectrum $$P_{matter}(k,z)$$

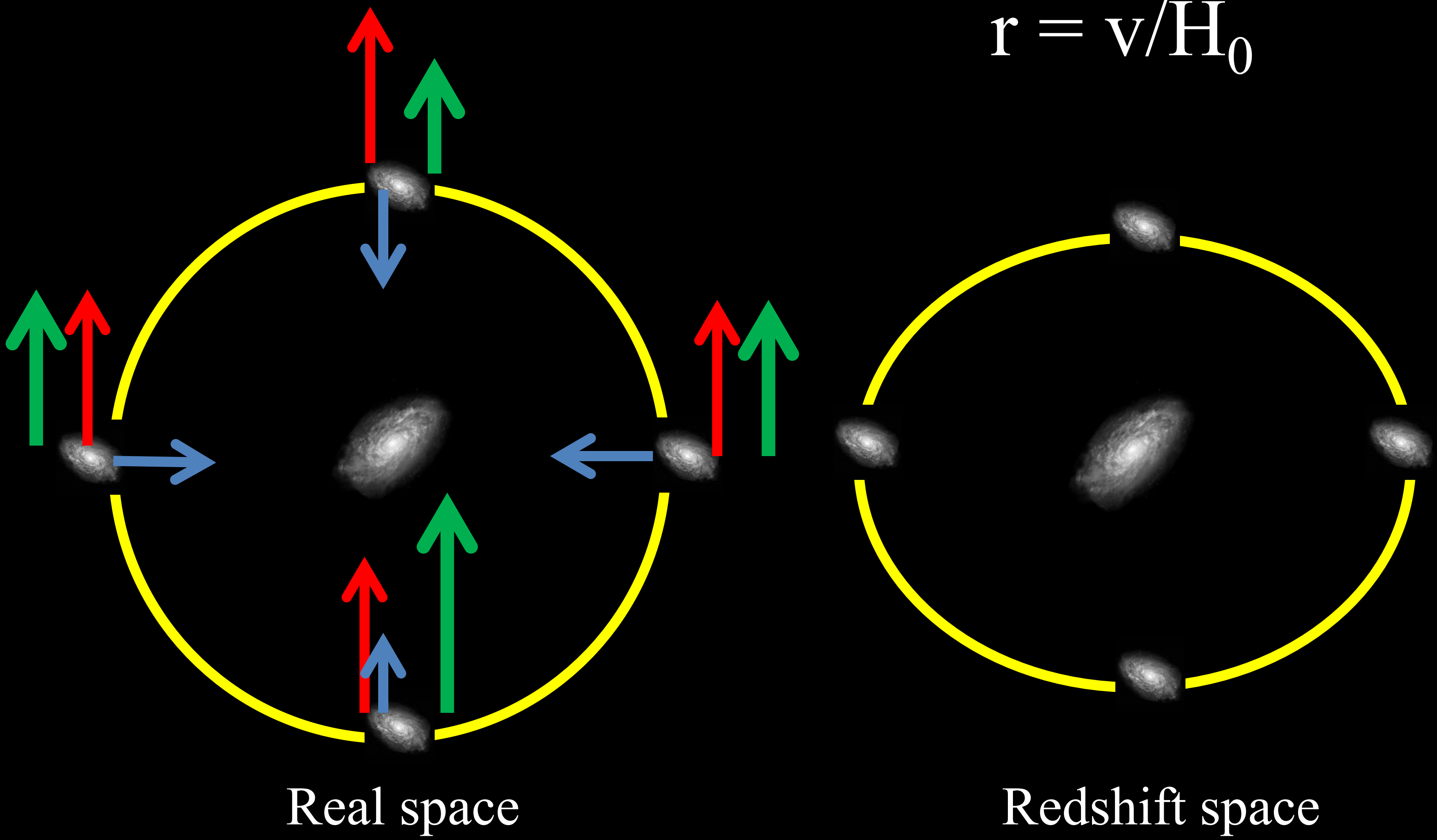
Galaxy power spectrum $$b^2(k,z)P_{matter}(k,z)$$

Galaxy power spectrum in redshift space

$$\left(1+\frac{f(k,z)}{b(k,z)}\cos^2\theta\right)^2 b^2(k,z)G^2(k,z)P^{matter}_{initial}(k,z_{in})$$

Redshift space

$r = v/H_0$



Deconstructing the galaxy power spectrum

$$P_{galaxy}(k,z,\mu=\cos\theta)=(1+\frac{f(k,z)}{b(k,z)}\cos^2\theta)^2 b^2(k,z)G^2(k,z)P_{initial}^{matter}(k,z_{in})$$

$b(k,z)$

$P_{initial}^{matter}(k,z_{in})$

do not depend on gravity

$G(k,z)$

$f(k,z)$

depend on gravity

Deconstructing the galaxy power spectrum

Line-of-sight decomposition

$$\mu \equiv \cos\theta$$

$$\delta_{galaxy}(k,z,\mu) = G(k,z)\left(1+\frac{f(k,z)}{b(k,z)}\mu^2\right)\ b(k,z)\delta_{initial}^{matter}(k,z_{in})$$

$$\equiv A + R\mu^2$$

$$\delta_{\text{lensing}}(k,z) = -\frac{3}{2}Y(1+\eta)G(k,z)\Omega_m\delta_{initial}^{matter}(k,z_{in}) \equiv L$$

Three linear observables: A, R, L

galaxy clustering

$$A = Gb\delta_{m,0}(k)$$

$$R = Gf\delta_{m,0}(k)$$

weak gravitational lensing

$$L = -\frac{3}{2}Y(1+\eta)G\Omega_m\delta_{m,0}(k)$$

The only model-independent ratios

Redshift distortion/Amplitude $$P_1 = \frac{R}{A} = \frac{f}{b}$$

Lensing/Redshift distortion $$P_2 = \frac{L}{R} = \frac{\Omega_{m0} Y(1+\eta)}{f}$$

Redshift distortion rate $$P_3 = \frac{R'}{R} = \frac{f'}{f} + f$$

Expansion rate $$E = \frac{H}{H_0}$$

How to combine them to test the theory?

Summarizing....

Matter conservation equation independent of gravity theory

$$\delta'' + (1 + \frac{H'}{H})\delta' - \frac{3}{2}\Omega_m Y\delta = 0$$

Observables

$$P_2 = \frac{L}{R} = \frac{\Omega_{m0} Y(1+\eta)}{f} \qquad P_3 = \frac{R'}{R} = \frac{f'}{f} + f \qquad E = \frac{H}{H_0}$$

Testing the entire Horndeski Lagrangian

A unique combination of observables

Independent of the bias, of the initial conditions, of the specific model

$$\frac{3P_2(1+z)^3}{2E^2(P_3+2+\frac{E'}{E})}-1=\eta=h_2\left(\frac{1+k^2h_4}{1+k^2h_5}\right)$$

Observables

Theory

L.A., M. Motta, I. Sawicki, M. Kunz, I. Saltas, 1210.0439 1305.0008

Testing the entire Horndeski Lagrangian

$$\eta_{obs} \equiv \frac{3P_2(1+z)^3}{2E^2(P_3 + 2 + \frac{E'}{E})} - 1$$

$\eta_{obs} \neq 1$ → gravity is modified

$\eta_{obs}(k) \neq \eta(Horndeski)$ → The entire Horndeski model is falsified

L.A., M. Motta, I. Sawicki, M. Kunz, I. Saltas, 1210.0439 1305.0008

Can we really measure it?

First attempt...

A compilation of all available datasets of lensing and growth

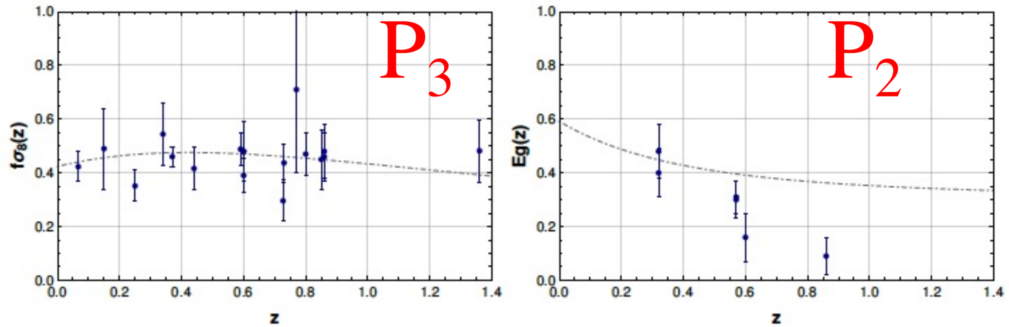


Figure 1. Left: Plot of the full $f\sigma_8$ data (table II) as a function of z with the theoretical curve in dashed dot. Right: Plot of the full E_g data (table III) as a function of z with the theoretical curve in dashed dot.

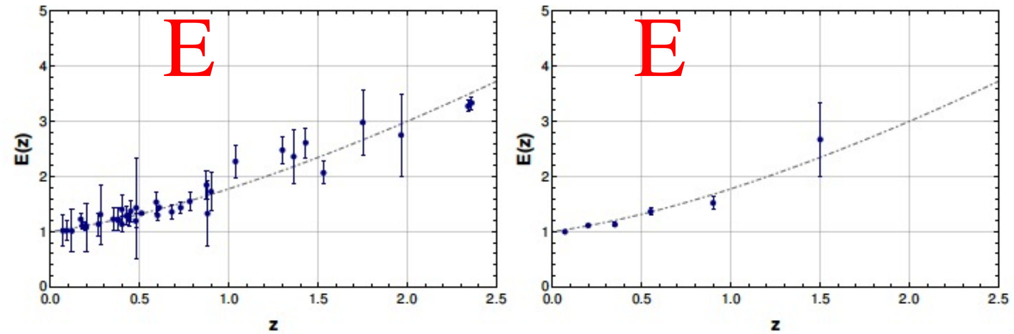
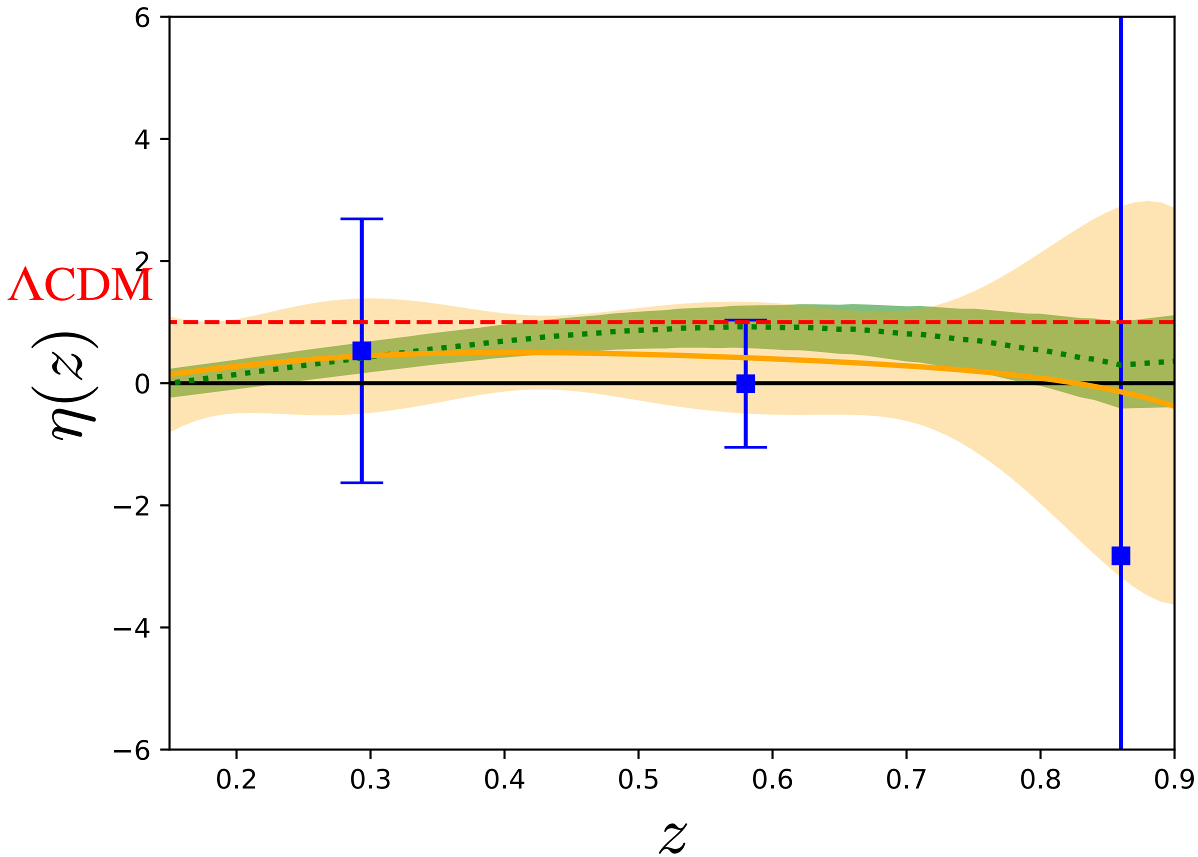


Figure 2. Right: Plot of the full $H(z)$ data (table IV) as a function of z with the theoretical curve in dashed dot. Right: Plot of the full $E(z)$ data (table II) as a function of z with the theoretical curve in dashed dot.

Collab. with Santiago Casas and Ana M. Pinho arXiv:1805.00025

The first model-independent measurement



Result compressed
in a single central bin

$$\eta = 0.40 \pm 0.60$$

Collab. with Santiago Casas
and Ana M. Pinho
arXiv:1805.00025

Euclid in a nutshell

Simultaneous (i) visible imaging (ii) NIR photometry (iii) NIR spectroscopy

15,000 square degrees

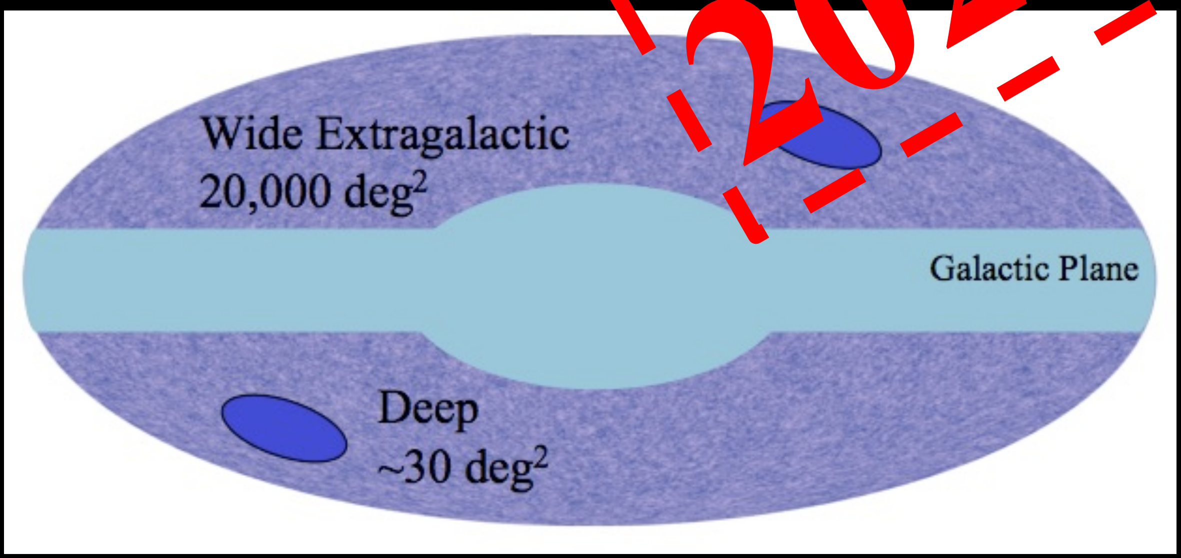
70 million redshifts, 2 billion images

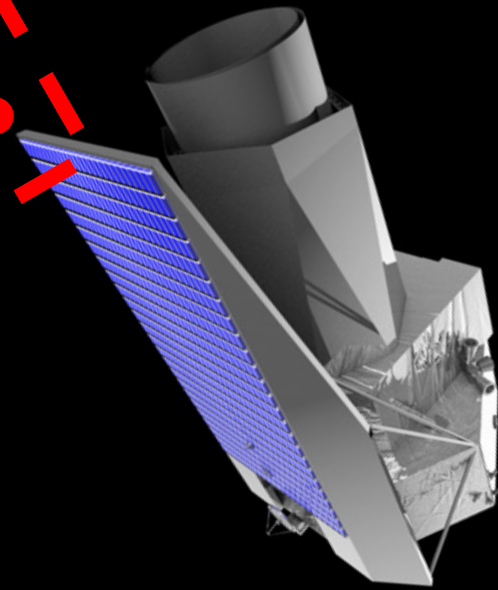
Median redshift z = 1

PSF FWHM ~0.18’’

>1000 peoples, >10 countries

2023!





Euclid satellite

arXiv Red Book 1110.3193

Results…

$$\eta(k,a) = H_2\left(\frac{1+k^2H_4}{1+k^2H_5}\right)$$

Model 1: η constant for all z, k
Error on η around 2%

Model 2: η varies in z
Error on η

η=1/2 for f(R)!
(at small scales)

TABLE X. Fiducial values and errors for the parameters P_1, P_2, P_3, E'/E and $\bar{\eta}$ for every bin. The last bin has been omitted since R' is not defined there.

$\bar{z}$	P_1	ΔP_1	$\Delta P_1(\%)$	P_2	ΔP_2	$\Delta P_2(\%)$	P_3	ΔP_3	$\Delta P_3(\%)$	(E'/E)	$\Delta E'/E$	$\Delta E'/E(\%)$	$\bar{\eta}$	$\Delta\bar{\eta}$	$\Delta\bar{\eta}(\%)$
0.6	0.766	0.012	1.6	0.729	0.013	1.8	0.134	0.13	99	-0.920	0.022	2.4	1	0.11	11
0.8	0.819	0.010	1.2	0.682	0.011	1.6	0.317	0.12	38	-1.04	0.046	4.4	1	0.091	9.1
1.0	0.859	0.0093	1.1	0.650	0.011	1.7	0.460	0.12	26	-1.13	0.099	8.7	1	0.090	9.0
1.2	0.888	0.0092	1.0	0.628	0.014	2.3	0.569	0.13	23	-1.21	0.12	10	1	0.097	9.7
1.4	0.911	0.010	1.1	0.613	0.020	3.3	0.654	0.11	16	-1.26	0.09	7.1	1	0.073	7.3

L.A, M. Kunz, A. Vollmer,
A. Guarnizo, S. Fogli, 1311.1465, 2013

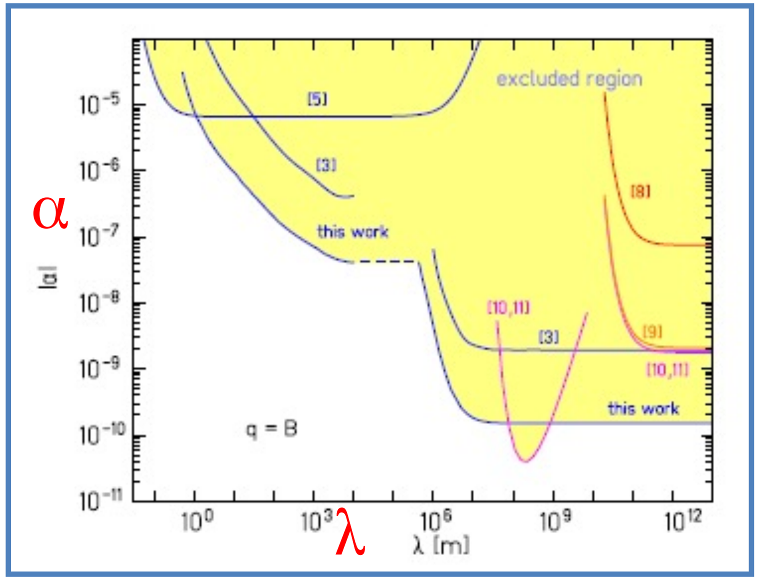
Observing Y

$$\nabla^2\Psi = 4\pi G\, Y(k,a)\rho_m\delta_m$$

In collab. with Laura Taddei and Matteo Martinelli

Observing Y

Excluded
in the Lab



$$\Psi(r) = -\frac{GM}{r}\left(1+\alpha e^{-r/\lambda}\right)$$

Observing Y

Modified-gravity
sub-horizon matter equations
$\theta = ik_j v^j$

$$\dot\delta = -\theta$$ continuity

$$\dot\theta = -\mathcal{H}\theta + k^2\Psi$$ Euler

$$k^2\Phi = 4\pi G a^2 \rho\delta Y\eta$$ Poisson

This can be written as a single equation:

$$\delta'' + \left(1 + \frac{H'}{H}\right)\delta' - \frac{3}{2}\Omega_m Y\delta = 0$$

prime is d/dlog(a)

$$\Omega_m Y = \frac{2\left[f^2E^2 + f'E^2 + fE^2\left(2 + \frac{E'}{E}\right)\right]}{3(1+z)^3},$$

measuring f, E
we can find Ω_mY!

Yukawa Potential

$$Y(k,a) = h_1 \left(\frac{1 + k^2 h_5}{1 + k^2 h_3} \right)$$

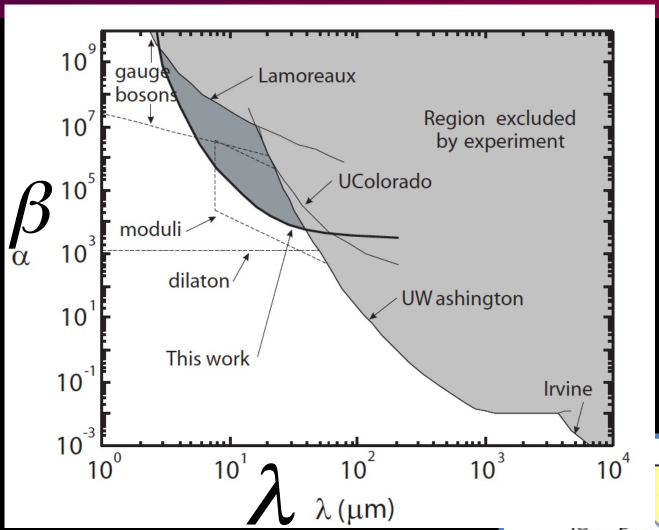
Momentum space

$$\nabla^2 \Psi = 4\pi G \, Y(k,a) \rho_m \delta_m$$

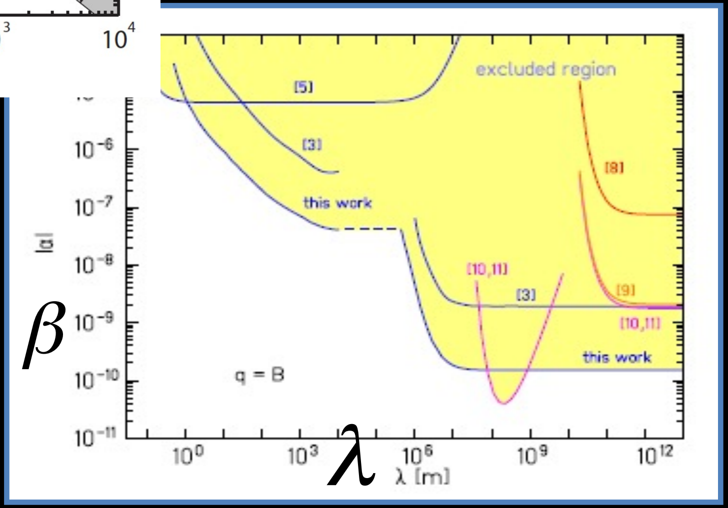
$$\Psi = -\frac{GM}{r} h_2 \left(1 + \frac{h_4 - h_5}{h_5} e^{-r/\sqrt{h_5}}\right) = -\frac{\bar{G}M}{r}\left(1 + Q e^{-mr}\right)$$

Real space

Testing Gravity



Smullin et al. 2004

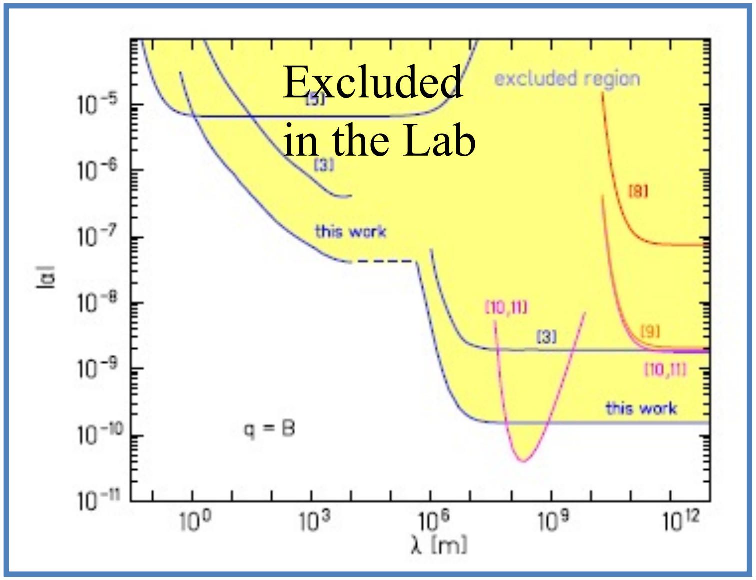


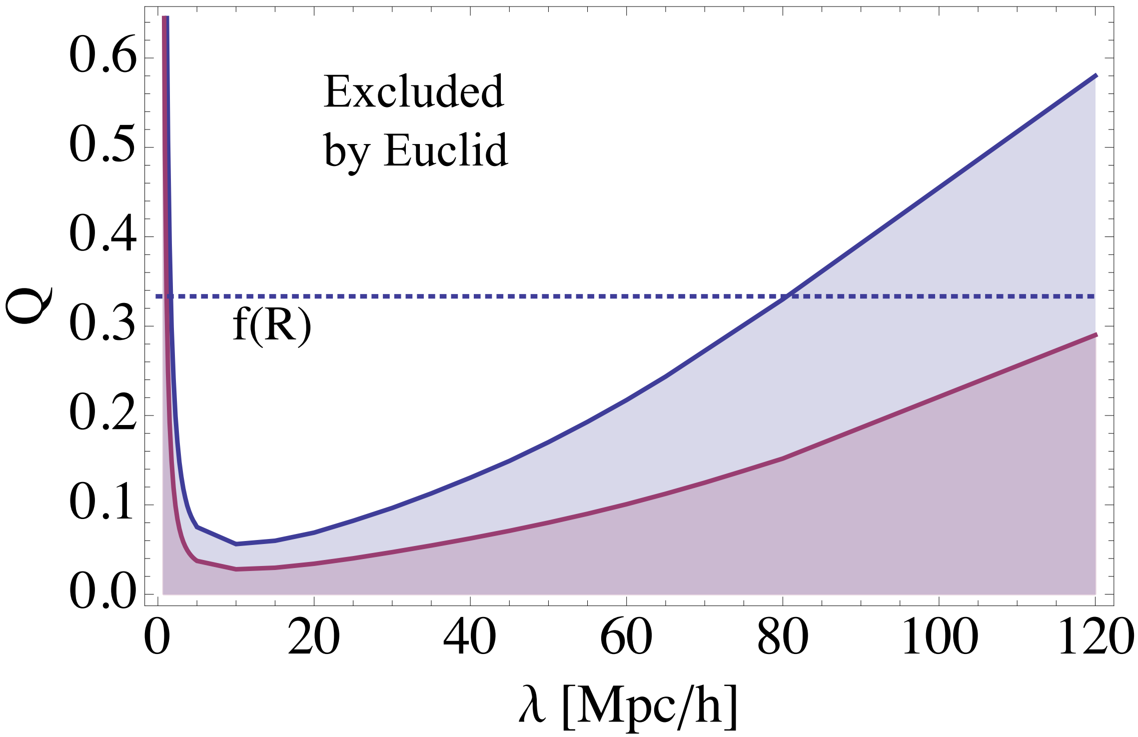
Schlamminger et al 2008

$$\frac{GM}{r} \to \frac{GM}{r}(1+\beta e^{-r/\lambda})$$

?

Cosmological exclusion plot





$$\Psi(r) = -\frac{GM}{r}(1 + Qe^{-r/\lambda})$$

L. Taddei, L.A., 2014

f(R)

$\lambda^{-2} = m^2 = \frac{R}{3}(\frac{f_{,R}}{Rf_{,RR}} - 1).$

Q=1/3

Conclusion

Is space flat?
How is the universe expanding?
How are perturbations growing?
Is GR correct at all scales and epochs?

Next generation experiments like Euclid and LSST can answer these questions in a model-independent way