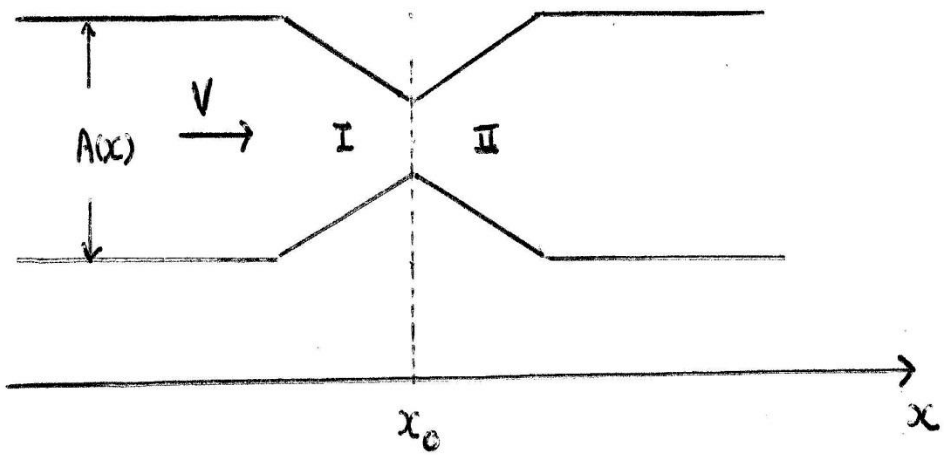
Accretion Processes by Black Holes

The critical and the sonic points



The Laval tube – geometry used in rocket motors

Cylinder with section A(x) that varies along the direction of the flow (x-axis). The minimum value of A(x) is attained at the point x_0

Mass conservation $\longrightarrow$ $A(x)\rho(x)V(x) = cte$

Eq. of motion $\longrightarrow$ $V\frac{dV}{dx} + \frac{1}{\rho}\frac{dP}{dx} = 0$ Take $P = K\rho^{\gamma} \;\Rightarrow\; \frac{dP}{dx} = \gamma K \rho^{\gamma-1}\frac{d\rho}{dx}$

To obtain $\longrightarrow$ $V\frac{dV}{dx} + \frac{a^2}{\rho}\frac{d\rho}{dx} = 0 \quad with \;\; a^2 = \frac{\gamma P}{\rho}$ $(adiabatic\ sound\ velocity)$

Differentiate the mass conservation equation to obtain: $$\frac{1}{\rho}\frac{d\rho}{dx} = -\frac{1}{A}\frac{dA}{dx} - \frac{1}{V}\frac{dV}{dx}$$

Replace into the eq. of motion: $$V\frac{dV}{dx} - a^2\left(\frac{1}{V}\frac{dV}{dx} + \frac{1}{A}\frac{dA}{dx}\right) = 0$$

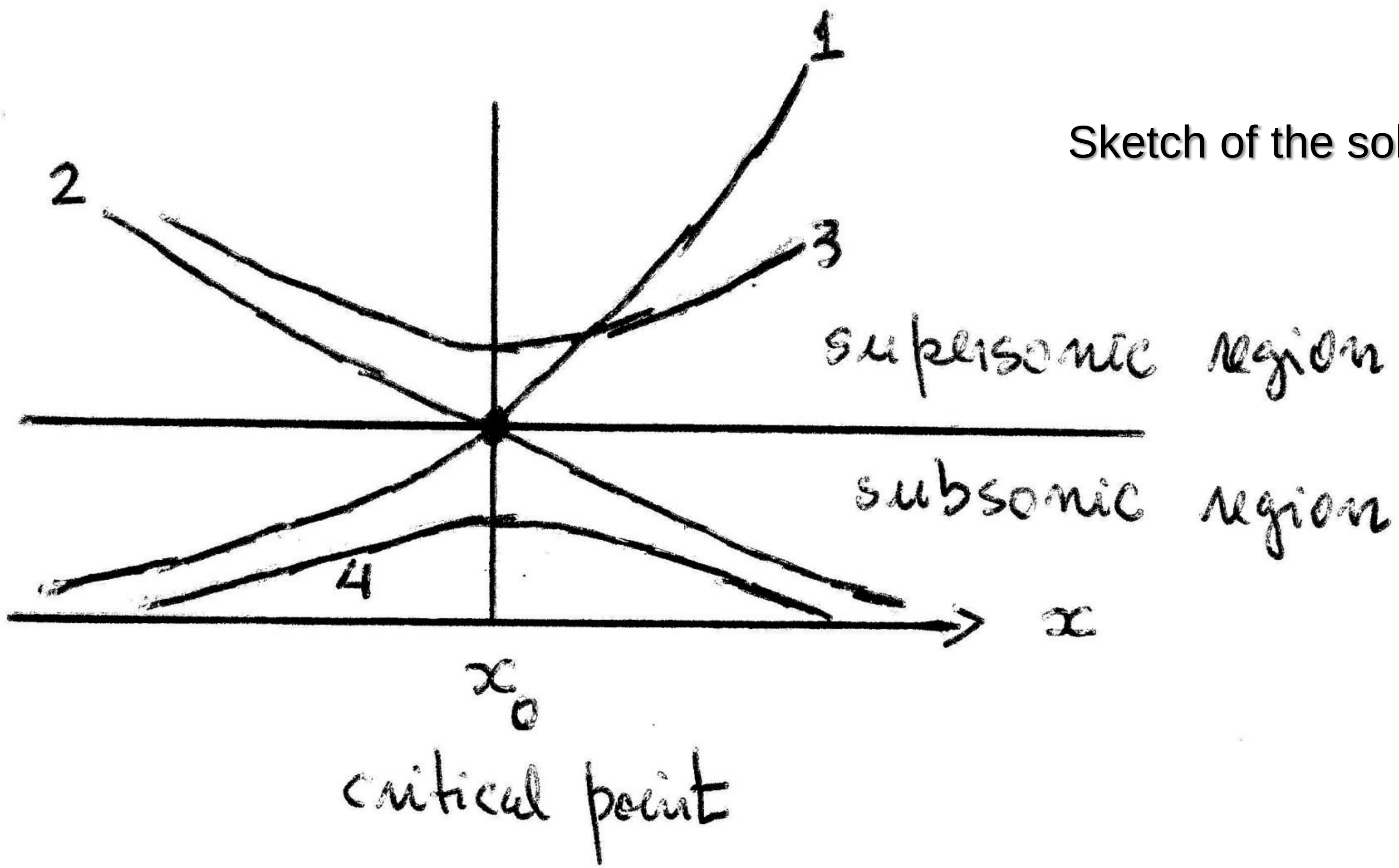
Or finally $\longrightarrow$ $$\left(M^2 - 1\right)\frac{1}{V}\frac{dV}{dx} = \frac{1}{A}\frac{dA}{dx}$$ where $M = V/a$ is the number of Mach

There are four classes of solutions: outside the regions I and II, both sides of the eq. of motion are zero, since *V* and *A* are constants – So only these two regions are important to establish the flow characteristics

Solution type 1 (unique) – the flow is subsonic in region I (*M* < 1). Since *dA/dx* < 0, we have *dV/dx* > 0. When *dA/dx* = 0 at x_0 $\longrightarrow$ V= a (the critical point coincides with the sonic point). In region II, *dA/dx* > 0 and the flow becomes supersonic (this is the rocket solution)

Solution type 2 (unique) – the flow is supersonic in region I (*M* > 1). In this case dV/dx < 0 (flow is decelerated). After the critical = sonic point, the flow becomes subsonic.

Solutions of type 3 and 4 (not unique) do not pass by the sonic point. This means that when $dA/dx = 0 \longrightarrow dV/dx = 0$ implying that the velocity passes by a minimum or a maximum.



Sketch of the solutions

Flow of the solar corona

Simplest model – Parker's one-fluid isothermal corona (EoS): $P = 2nkT$

Eq. of motion: $V\frac{dV}{dr} + \frac{1}{\rho}\frac{dP}{dr} + \frac{GM}{r^2} = 0$ and mass conservation: $\frac{dm}{dt} = 4\pi r^2 \rho V$

As before, use EoS and differentiate mass conservation to replace into the eq. motion

$$\frac{1}{V}\frac{dV}{dr}\left(V^2 - a_{is}^2\right) = \left(\frac{2a_{is}^2}{r} - \frac{GM}{r^2}\right)$$ where $a_{is}^2 = \frac{2kT}{m_H}$ square of the isothermal sound velocity

Solutions are similar to the flow inside the Laval nozzle

Critical point : $r_c = \frac{GMm_H}{4kT}$ critical velocity: $V_c = a_{is}$

Critical point coincides with sonic point - Solution type 4 – "breeze"

Radiative acceleration: Thomson scattering

Radiative force per volume unit: $$f_r = \frac{1}{c}\int \sigma_\nu n \phi_\nu d\nu \;\Rightarrow\; f_r = \frac{\sigma_T \rho}{\mu m_H c}\phi$$

For an optically thin atmosphere $\longrightarrow$ $$\phi = \frac{L}{4\pi r^2}$$

Eq. of motion: $$V\frac{dV}{dr} + \frac{1}{\rho}\frac{dP}{dr} + \frac{GM}{r^2} - \frac{1}{\rho} f_r = 0$$

Consider an isothermal atmosphere and use mass conservation to obtain

$$\frac{1}{V}\left(1 - \frac{a_{is}^2}{V^2}\right)\frac{dV}{dr} = \frac{2a_{is}^2}{r} - \frac{GM}{r^2}(1-\Gamma)$$ where $$\Gamma = \frac{L}{L_E} \quad and \quad L_E = \frac{4\pi G M m_H c}{\sigma_T}$$

Critical radius: $r_c = \frac{GM}{2a_{is}^2}(1-\Gamma)$ hence in the subsonic, region the effective gravity must be **positive**

Winds of Massive Stars

Radiative force on lines – From the Sobolev approximation, the force per unit volume is

$$f_r = \frac{g}{2c^2}\zeta(r)\phi \quad \Rightarrow \quad g = \sum_{ij}\frac{\nu_{ij}\phi_{ij}}{\phi} \quad and \quad \zeta(r) = \left[1-\left(1-\frac{R^2}{r^2}\right)^2\right]\frac{dV}{dr}$$

Near the star surface, where is located the critical point, $\zeta(r) \approx dV/dr$

In this case, the eq. of motion (coupled to the mass conservation eq.) becomes

$$V\frac{dV}{dr}\left[1-\frac{\alpha\dot{M}}{4\pi r^2}-\frac{a_{is}^2}{V^2}\right] = \frac{2a_{is}^2}{r}-\frac{GM}{r^2} \quad \Rightarrow \quad \alpha = \frac{g}{2c^2}\phi$$

In the radiative winds of massive stars, the critical point does not coincides with the sonic point

$$V_c = a_{is}\left[1-\frac{\alpha\dot{M}}{\pi}\frac{a_{is}^4}{(GM)^2}\right]^{-1}$$

Accretion theory of Hoyle-Lyttleton-Bondi

Euler equation $\longrightarrow$ $V\dfrac{dV}{dr}+\dfrac{1}{\rho}\dfrac{dP}{dr}+\dfrac{GM}{r^2}=0$ and EoS $\longrightarrow$ $P=K\rho^{\gamma}$

Combining these eqs.$\longrightarrow$ $V\dfrac{dV}{dr}+\gamma K\rho^{\gamma-2}\dfrac{d\rho}{dr}+\dfrac{GM}{r^2}=0$

Equivalently: $\dfrac{d}{dr}\left[\dfrac{1}{2}V^2+\dfrac{a^2}{(\gamma-1)}-\dfrac{GM}{r}\right]=0 \quad with \quad a^2=\dfrac{\gamma P}{\rho}$

Integrating (Bernoulli equation): $\dfrac{1}{2}V^2+\dfrac{a^2}{(\gamma-1)}-\dfrac{GM}{r}=\dfrac{a_\infty^2}{(\gamma-1)}$

Combining the mass conservation eq. with the Euler eq.

$$V\frac{dV}{dr}\left[1-\frac{a^2}{V^2}\right]=\left[\frac{2a^2}{r}-\frac{GM}{r^2}\right]$$

Critical conditions $\longrightarrow$ $$r_c = \frac{GM}{2a^2} \quad and \quad V_c = a$$

Insert the critical conditions into the Bernoulli eq. in order to obtain the sound and the flow velocity at the critical point in terms of the sound velocity at "infinity"

$$V_c = a = \left(\frac{2}{5-3\gamma}\right)^{1/2} a_\infty \quad \Rightarrow \quad r_c = \left(\frac{5-3\gamma}{4}\right)\frac{GM}{a_\infty^2}$$

There is **no critical point** if γ = 5/3

The accretion rate derived from conditions at the critical point: $$\frac{dm}{dt} = 4\pi r_c^2 V_c \rho_c$$

The density at this point is needed: $$\frac{a^2}{a_\infty^2} = \frac{P}{P_\infty}\frac{\rho_\infty}{\rho} = \left(\frac{\rho}{\rho_\infty}\right)^{\gamma-1} = \frac{2}{(5-3\gamma)}$$

Finally: $$\frac{dm}{dt} = \pi\left(\frac{2}{5-3\gamma}\right)^{(5-3\gamma)/2(\gamma-1)} \frac{\rho_\infty}{a_\infty^3}(GM)^2$$

Radiation efficiency

Radiation efficiency of the accreting envelope $\longrightarrow$ $\eta = L / \dot{M}c^2$

Luminosity due to free-free emission (bremsstrahlung)

$$L = 4\pi \int_{2r_g}^{\infty} \varepsilon(\rho, T) \sqrt{1 - \frac{r_g}{r}} \exp\left[-\tau(r,\infty)\right] r^2 dr$$

The above eq. includes redshift effect and Thomson scattering: $\tau(r,\infty) = \int_r^{\infty} \frac{\sigma_T}{\mu m_H} \rho(r') dr'$

Numerical solutions for: $T_\infty = 10^4 K$ *and* $\gamma = 1.2$ Grid for different BH masses and ρ_∞

Best fit: $L = 1.92 \times 10^{22} n_\infty^{1.726} (M / M_\odot)^{2.721}$ erg / s

This is valid for accretion rates less than $\longrightarrow$ $\frac{dm}{dt} = 1.82 \times 10^{-5} \left(M / M_\odot\right) \; M_\odot yr^{-1}$

Efficiency: $\eta \simeq 2.6 \times 10^{-7} \tau_s^{0.733}$ High efficiencies need large optical depth in contradiction with the flow model

Relativistic Accretion Theory
(Michel 1972)

Consider a non-rotating (static) black hole and a steady spherical flow – the metric is

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega^2$$

The horizon (or horizons) is (are) given by the zero (zeros) of the lapse function *f(r)*

Conservation eqs.: mass flux conservation $\longrightarrow \quad \nabla_k J^k = 0 \quad with \quad J^k = mnu^k$

where J^k is the mass-current density.

Energy-momentum conservation $\longrightarrow \quad \nabla_k T_i^k = 0$

where the fluid energy tensor is $\longrightarrow \quad T_i^k = (P+\varepsilon)u_i u^k - P\delta_i^k$

In the present case, the only non-null components of the 4-vector velocity are:

$$u = u^1 = \frac{dr}{ds} \qquad and \qquad u^0 = \frac{dt}{ds}$$

Since $\longrightarrow$ $u_i u^i = u_0 u^0 + u_1 u^1 = -1 \;\; \Rightarrow \;\; u_0 = -fu^0 \;\; and \;\; u_1 = \dfrac{u^1}{f}$

One obtains that $\longrightarrow$ $u_0^2 = f + u^2$

Under spherical and steady state conditions, one obtains from the energy-momentum conservation:

$$(P+\varepsilon)u_0 u^1 \sqrt{-g} = (P+\varepsilon)u\sqrt{f+u^2}\sqrt{-g} = C_1$$

Use now the mass current conservation to get (space component): $nu\sqrt{-g} = C_2$

From these two eqs. $\longrightarrow$ $\dfrac{(P+\varepsilon)}{n}\left(f+u^2\right)^{1/2} = \dfrac{C_1}{C_2} = \Delta$

This last equation is the relativistic version of the Bernoulli equation (RBE)

Derivate the RBE with respect the radial coordinate and use the mass-flux current

$$\frac{d\lg u}{d\lg r}\left[u^2 - V^2 F(r,u)\right] = \left[2V^2 F(r,u) - \frac{r}{2}\frac{df}{dr}\right]$$

where: $V^2 = \frac{d\lg(P+\varepsilon)}{d\lg n} - 1$ (generalized sound velocity) and $F(r,u) = f(r) + u^2$

Critical conditions: $u_c^2 = V_c^2 F(r_c, u_c) \quad and \quad r_c = \frac{4u_c^2}{f'(r_c)}$

Working these eqs: $u_c^2 = \frac{V_c^2}{(1-V_c^2)} f(r_c) \qquad and \qquad r_c = \frac{4V_c^2}{(1-V_c^2)}\frac{f(r_c)}{f'(r_c)}$

Steps – 1) define the EoS and compute the generalized sound velocity

2) compute the critical radius

3) compute the flow velocity at the critical point

Example – **accretion of non relativistic matter by a Schwarzschild black hole**

Metric ⟶ $f(r) = 1 - \frac{r_g}{r}$ and $f'(r) = \frac{r_g}{r^2}$

EoS corresponding to an adiabatic flow: $P = Kn^{\Gamma}$

The energy density is: $\varepsilon = mnc^2 + \frac{P}{(\Gamma - 1)}$

With these relations compute the generalized sound velocity:

$$V^2 = \frac{a^2}{\left[1 + \frac{a^2}{(\Gamma - 1)}\right]}$$

where $a^2 = \Gamma \frac{P}{mnc^2}$ (adiabatic sound velocity with respect to light velocity)

Next, compute the constant Δ in the RBE by imposing null flow velocity when $r \to \infty$

$$\Delta = \frac{(P+\varepsilon)_\infty}{n_\infty} = mc^2 + \frac{\Gamma P_\infty}{(\Gamma - 1) n_\infty} = mc^2 \left[1 + \frac{a_\infty^2}{(\Gamma - 1)}\right]$$

Get the conditions at the critical point

$$\frac{r_c}{r_g} = \frac{(1+3V_c^2)}{4V_c^2} \qquad and \qquad u_c^2 = \frac{V_c^2}{(1+3V_c^2)}$$

Use the RBE at the critical point to compute the relation between the sound velocity at this point and "infinity"

$$\left[1+\frac{a_c^2}{(\Gamma-1)}\right]\frac{1}{\left(1+3V_c^2\right)^{1/2}} = 1+\frac{a_\infty^2}{(\Gamma-1)}$$

To the first order $\longrightarrow$ $a_c^2 \simeq \dfrac{2a_\infty^2}{(5-3\Gamma)}$ and $\longrightarrow$ $\dfrac{r_c}{r_g} \simeq \dfrac{(5-3\Gamma)}{8a_\infty^2} + SRC$

The critical radius differs only by small relativistic corrections from the Newtonian value

The critical velocity $\longrightarrow$ $u_c \simeq a_c - \dfrac{(3\Gamma-2)}{2(\Gamma-1)}a_c^3$

In the relativistic flow, the critical point differs slightly from the sonic point

The accretion rate is computed using conditions at the critical point:

$$\frac{dm}{dt} = 4\pi r_c^2 (T_0^r)_c \qquad \text{where} \qquad T_0^r = (P+\varepsilon)_c u_0 u_c$$

To the first order:

$$(P+\varepsilon)_c = mn_c c^2 \left[1 + \frac{a_c^2}{\Gamma - 1}\right] \approx \rho_\infty c^2 \left(\frac{2}{5-3\Gamma}\right)^{1/(\Gamma-1)}$$

$$u_0 \approx 1 - \frac{3a_\infty^2}{(5-3\Gamma)} \qquad and \qquad u_c \approx \sqrt{\frac{2}{(5-3\Gamma)}}\, a_\infty$$

and $$r_c^2 \approx r_g^2 \left[\frac{(5-3\Gamma)}{8a_\infty^2}\right]^2 = \frac{4(GM)^2}{c^4}\left[\frac{(5-3\Gamma)}{8a_\infty^2}\right]^2$$ finally, the accretion rate

$$\frac{dm}{dt} = \pi \left[\frac{2}{(5-3\Gamma)}\right]^{\frac{(5-3\Gamma)}{2(\Gamma-1)}} (GM)^2 \rho_\infty a_\infty^{-3} \quad + O(a_\infty^6)$$

To the first order, the relativistic accretion rate compares with the Newtonian value

Schwarzschild black hole – accretion of a relativistic fluid

The relations for the critical point are the same but the EoS is now given by

$$P = Kn^{4/3} \quad \Rightarrow \quad P + \varepsilon = 4P = 4Kn^{4/3}$$

Hence $\longrightarrow$ $$V^2 = \frac{d\lg(P+\varepsilon)}{d\lg n} - 1 = \frac{4}{3} - 1 = \frac{1}{3} \quad \Rightarrow \quad V = \frac{1}{\sqrt{3}}$$

and the critical conditions $\longrightarrow$ $$\frac{r_c}{r_g} = \frac{(1+3V_c^2)}{4V_c^2} = \frac{3}{2} \qquad and \qquad u_c^2 = \frac{V_c^2}{(1+3V_c^2)} = \frac{1}{6}$$

A relativistic fluid crosses always **subsonically** the critical point (critical and sonic points)

This is a general result valid for all relativistic inflow – the sonic point is located near the horizon

$$1 < \frac{r_S}{r_g} < \frac{3}{2}$$

Use the fact that $\longrightarrow \quad (P+\varepsilon)=\frac{4}{3}\varepsilon \quad and \quad n \propto \varepsilon^{3/4}$

To write the RBE as $\longrightarrow \quad K_* \varepsilon^{1/4}\left[1-\frac{r_g}{r}+u^2\right]^{1/2}=\Delta$

when $\longrightarrow \quad r \to \infty \quad and \quad u \to 0 \quad \Rightarrow \quad \Delta = K_* \varepsilon_\infty^{1/4}$

Compute the RBE at the critical point to get $\longrightarrow \quad \varepsilon_c = 4\varepsilon_\infty$

Compute the accretion rate $\longrightarrow \quad \frac{dm}{dt}=4\pi r_c^2 (T_0^r)_c = 4\pi r_c^2 \left(P+\varepsilon\right)_c (u_0 u^r)_c$

To get $\longrightarrow \quad \frac{dm}{dt}=32\sqrt{3}\,\pi \frac{(GM)^2}{c^5}\varepsilon_\infty$

Compare with the result using the relativistic cross section capture : $\sigma_c = \frac{27\pi}{4} r_g^2$

$$\frac{dm}{dt}=\sigma_c \phi_\infty = 27\pi \frac{(GM)^2}{c^5}\varepsilon_\infty$$

Accretion by a Reissner-Nordström Black Hole

Lapse function $\longrightarrow$ $f(r) = 1 - \dfrac{2M}{r} + \dfrac{Q^2}{r^2}$ Zeros give the inner and the event horizon

Horizons $\longrightarrow$ $r_\pm = \dfrac{r_g}{2}\left[1 \pm \sqrt{1-\beta^2}\right]$ $\quad with \quad \beta = \dfrac{Q}{M}$

Sign "-" corresponds to the Cauchy and sign "+" to the event horizon

The case β= 1, extreme RN black hole – both horizons coincide

Critical radius: $$\frac{r_c}{r_g} = \frac{(1+3V_c^2)}{8V_c^2}\left\{1 \pm \left[1 - \frac{8\beta^2 V_c^2 (1+V_c^2)}{(1+3V_c^2)}\right]^{1/2}\right\}$$

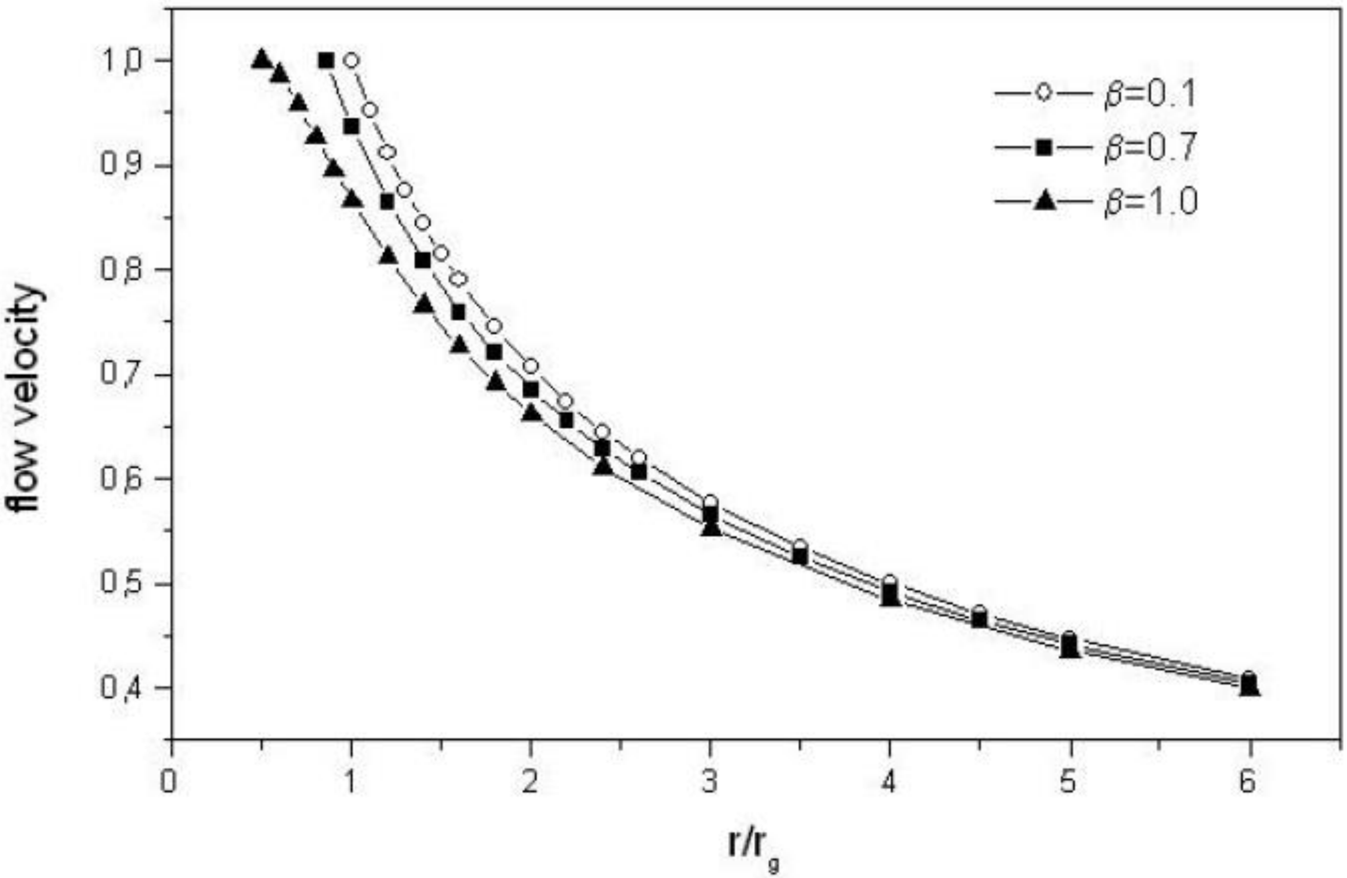
Two solutions: sign "+" $\longrightarrow$ critical radius larger than event horizon

sign "-" $\longrightarrow$ critical radius between both horizons

Accretion of non-relativistic matter:

Critical radius $\longrightarrow$ $$\frac{r_c}{r_g} \simeq \frac{(5-3\gamma)}{8a_\infty^2}\left[1+\frac{(6-4\beta^2)a_\infty^2}{(5-3\gamma)}\right]$$

dominant term in the charge represents a small correction to the critical radius



Flow velocity profile for different values of the charge parameter (γ =1.2)

The flow crosses the event horizon always at light velocity.

Flow velocity at the critical point is about 12 km/s

In non-relativistic flows (V_c << 1), real solutions exist only if $\longrightarrow$ $\beta^2 < \frac{(1+3V_c^2)^2}{8V_c^2(1+V_c^2)} \approx \frac{1}{8V_c^2}$

The extreme case when $\beta^2 \approx 1/8V_c^2 >> 1$ corresponds to a "naked" singularity.

No solution exists for the flow beyond the critical radius $\longrightarrow$ $r_c \approx r_g /(8V_c^2)$

It is possible that when β > 1 an ideal fluid does not accrete onto a naked singularity.

Instead a static atmosphere will be formed $\longrightarrow$ $\frac{\partial P}{\partial r} + (P+\varepsilon)\frac{\partial \lg\sqrt{f(r)}}{\partial r} = 0$

Solution of hydrostatic equilibrium indicates that matter density increases as the singularity is approached, attains a maximum and then decreases to zero.

Such a behavior (Rayleigh-Taylor unstable) can be explained by the fact that the

effective gravity $g_{ef} = \partial \lg\sqrt{f(r)} / \partial r$ (negative faraway the BH) becomes positive

for $r < 0.5\beta^2 r_g$. The repulsive force is a consequence of the modification of the space curvature due to the presence of the electric charge (**protection of the singularity?**)

Relativistic flow – critical radius ⟶ $\frac{r_c}{r_g} = \frac{3}{4}\left[1 \pm \sqrt{1 - \frac{8}{9}\beta^2}\right]$

A critical point exists only if $\beta \le 3/\sqrt{8}$ $(=1.06066)$ - Extreme BH may accrete

When β =1 the inner and the event horizon coincide and are equal to the gravitational radius

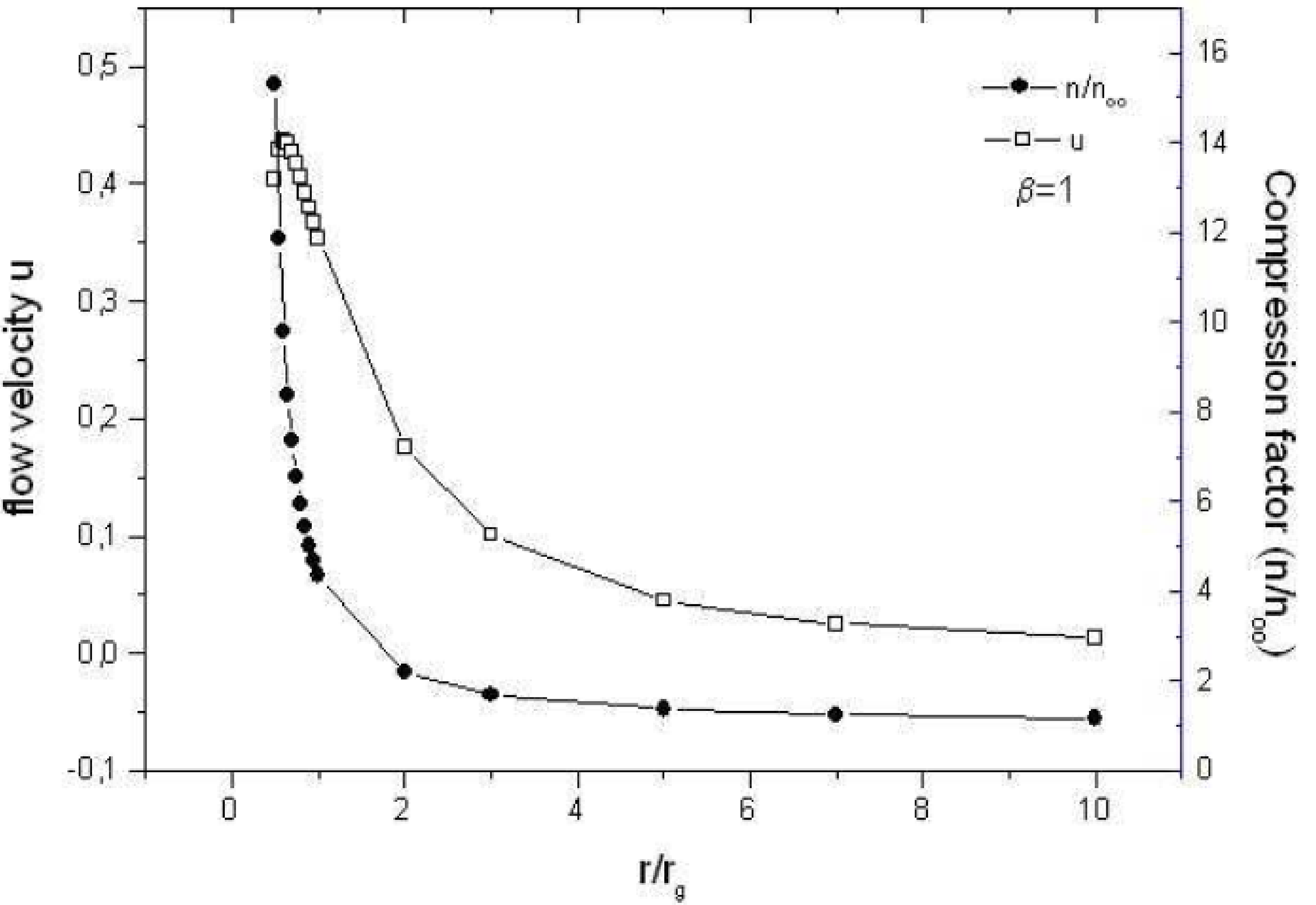
Two solutions for the critical radius

$$"-" \Rightarrow r_c = r_g / 2$$

$$"+" \Rightarrow r_c = r_g$$

The "+" solution implies that the critical radius coincides with the horizon and the flow velocity at this point is $u_c = 1/\sqrt{8}$

The "-" solution implies that the critical radius is inside the horizon and $u_c = 0$



flow velocity u
Compression factor (n/n∞)
r/rg
n/n∞
u
β=1
0,5
0,4
0,3
0,2
0,1
0,0
-0,1
0
2
4
6
8
10
16
14
12
10
8
6
4
2
0

Dark Matter Accretion

Dark matter is supposed to be a collisionless fluid (only gravitational effects)

In the absence of highly dissipative mechanisms, the phase-space density is conserved (Peirani & de Freitas Pacheco 2008; Piattella et al. 2013)

Phase-space density indicator ⟶ $Q = \dfrac{n}{<\sigma^2>^{3/2}}$

To the first order ⟶ $\varepsilon = mnc^2 + \dfrac{1}{2}mn\sigma^2 \quad and \quad P = \dfrac{1}{3}mn\sigma^2$

From these eqs. ⟶ $(P+\varepsilon) = mnc^2 + \dfrac{5}{6}mn\sigma^2 = mnc^2 + \dfrac{5}{6}m\dfrac{n^{5/3}}{Q^{2/3}}$

and ⟶ $\dfrac{d\lg(P+\varepsilon)}{d\lg n} = \dfrac{\left(1+\frac{25}{18}X\right)}{\left(1+\frac{5}{6}X\right)} \qquad with \quad X = \dfrac{n^{2/3}}{Q^{2/3}c^2}$

Up to second order, the generalized sound velocity is: $V^2 \approx \frac{5}{9}X - \frac{25}{54}X^2$

Hence, using the preceding formulae, one obtains for the critical conditions:

$$\frac{r_c}{r_g} \simeq \frac{9}{20X_c} = \frac{9}{20}\left(\frac{Q}{n_c}\right)^{2/3} c^2 \quad and \quad u_c \simeq \frac{\sqrt{5}}{3}X_c^{1/2} = \frac{\sqrt{5}}{3c}\left(\frac{n_c}{Q}\right)^{1/3}$$

To obtain the relation between the density at the critical point and at "infinity" use RBE

$$\left[1+\frac{5}{6}X_c\right]\left[1+3V_c^2\right]^{-1/2} = \left[1+\frac{5}{6}X_\infty\right]$$

Expanding and comparing the lowest order terms $\longrightarrow$ $X_c^2 \simeq \frac{4}{5}X_\infty \Rightarrow \frac{n_c}{n_\infty} \simeq \left(\frac{4}{5}\right)^{3/4}\left(\frac{c}{\sigma_\infty}\right)^{3/2}$

Critical conditions in terms of values at "infinity"

$$\frac{r_c}{r_g} \simeq \frac{9\sqrt{5}}{40}\left(\frac{c}{\sigma_\infty}\right) \quad and \quad u_c \simeq \frac{\sqrt{5}}{3}\left(\frac{4}{5}\right)^{1/4}\left(\frac{\sigma_\infty}{c}\right)^{1/2}$$

If the black hole is in the center of a dark matter halo, the flow is possible if the critical radius is smaller than the influence radius $r_* = 2GM / \sigma_\infty^2$

For a 10^8 solar mass black hole and $\sigma_\infty = 200\ km/s$

the influence radius is **6.67 x 10^{19} cm** while the critical radius is **2.24 x 10^{16} cm**

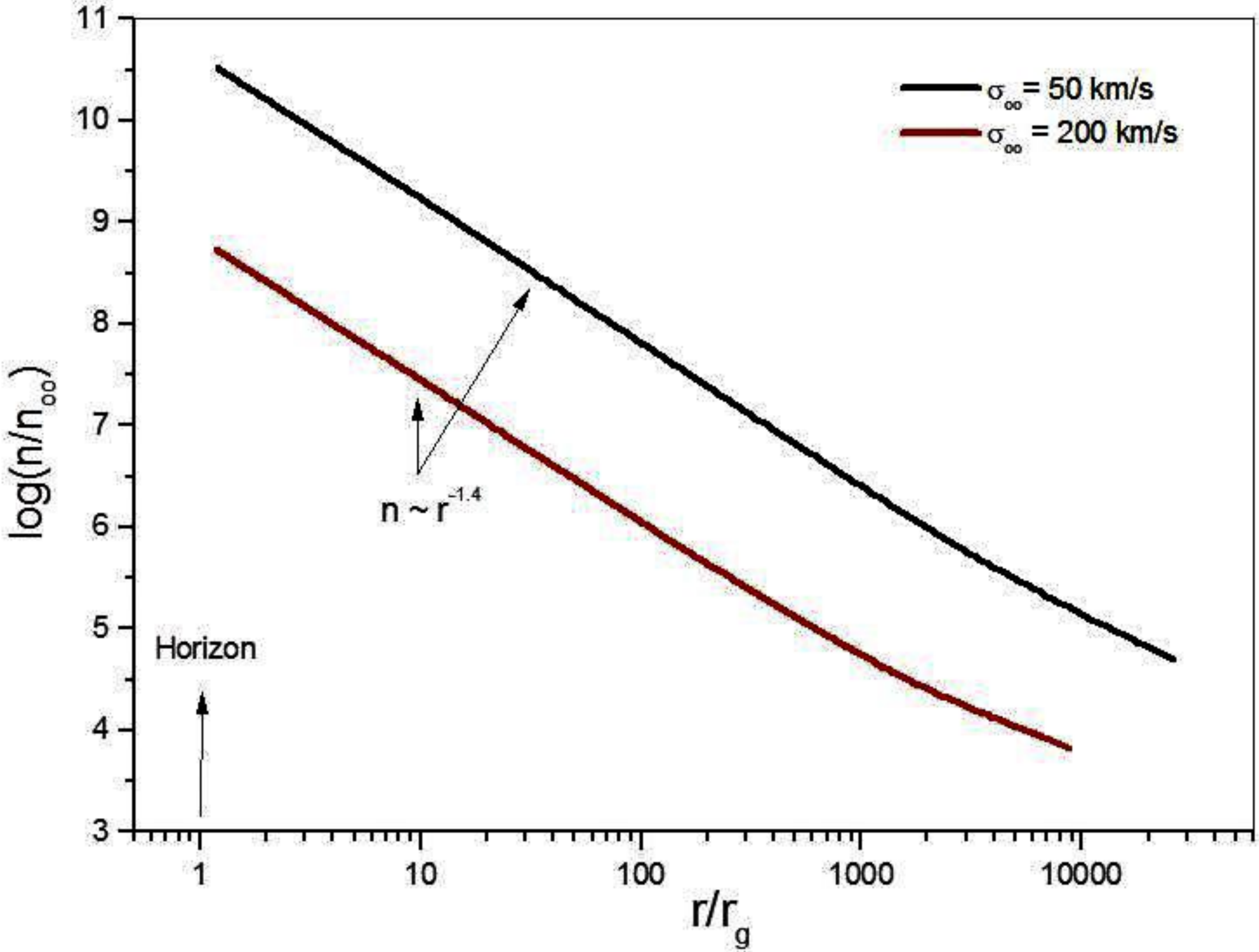
Notice that the flow velocity at the critical point will be $u_c \sim 0.0182c$

Accretion rate: $$\frac{dm}{dt} = 4\pi r_c^2 (T_0^r)_c = \frac{27\sqrt{5}}{25}\pi(GM)^2 \frac{\rho_\infty}{\sigma_\infty^3} = \frac{27\sqrt{5}}{25}\pi(GM)^2 Q$$

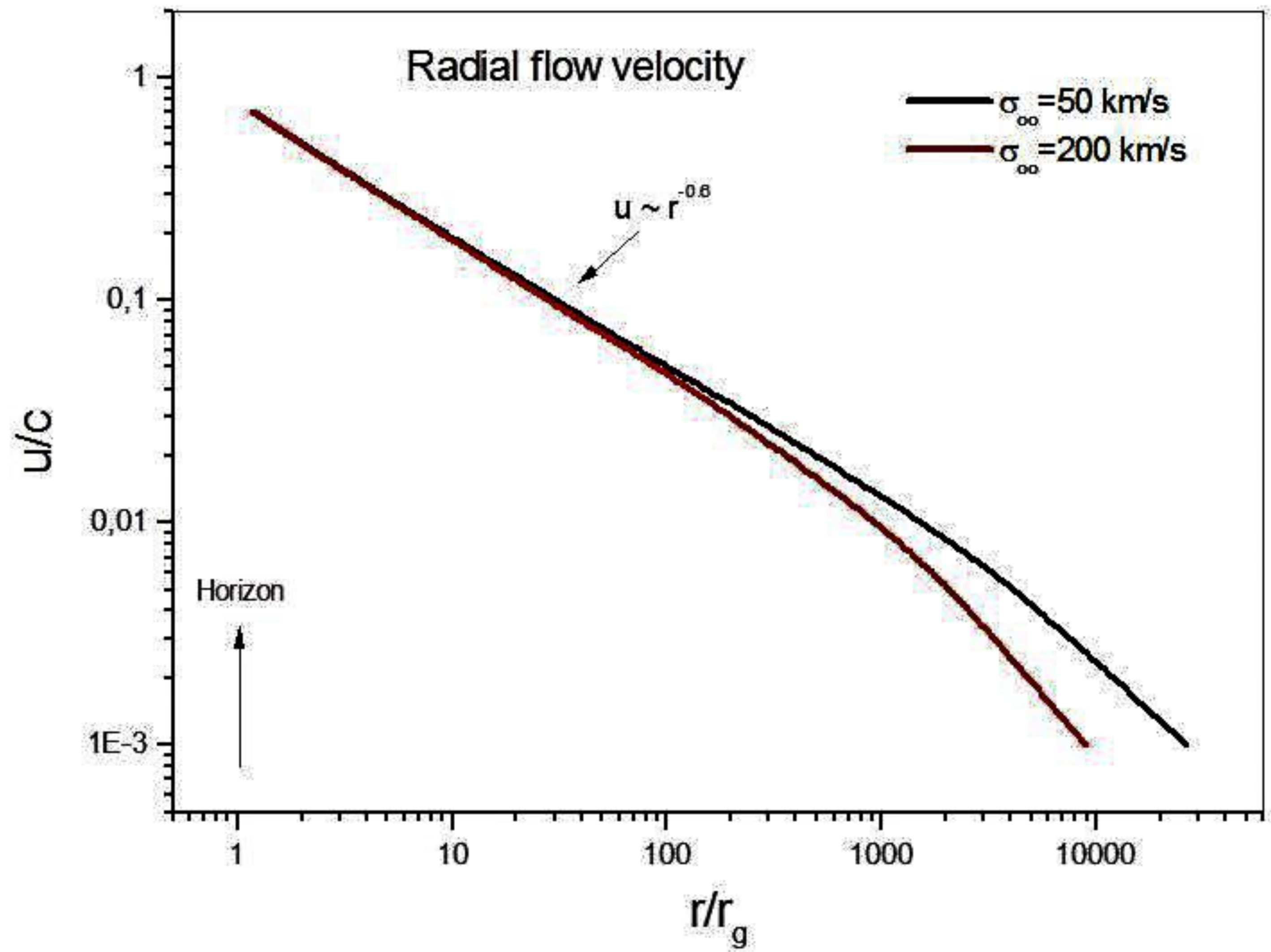
Compare with the "classical" particle capture rate: $$\frac{dm}{dt} = 4\pi r_c^2 \rho_\infty V_\infty = 16\pi\left(\frac{GM}{c}\right)^2 \frac{\rho_\infty}{V_\infty}$$

$$\frac{present}{classical} = \frac{27\sqrt{5}}{400}\left(\frac{c^2 V_\infty}{\sigma_\infty^3}\right) \approx 84000$$ for V_∞ = 50 km/s and σ_∞ = 200 km/s

Flow Characteristics



Flow Characteristics

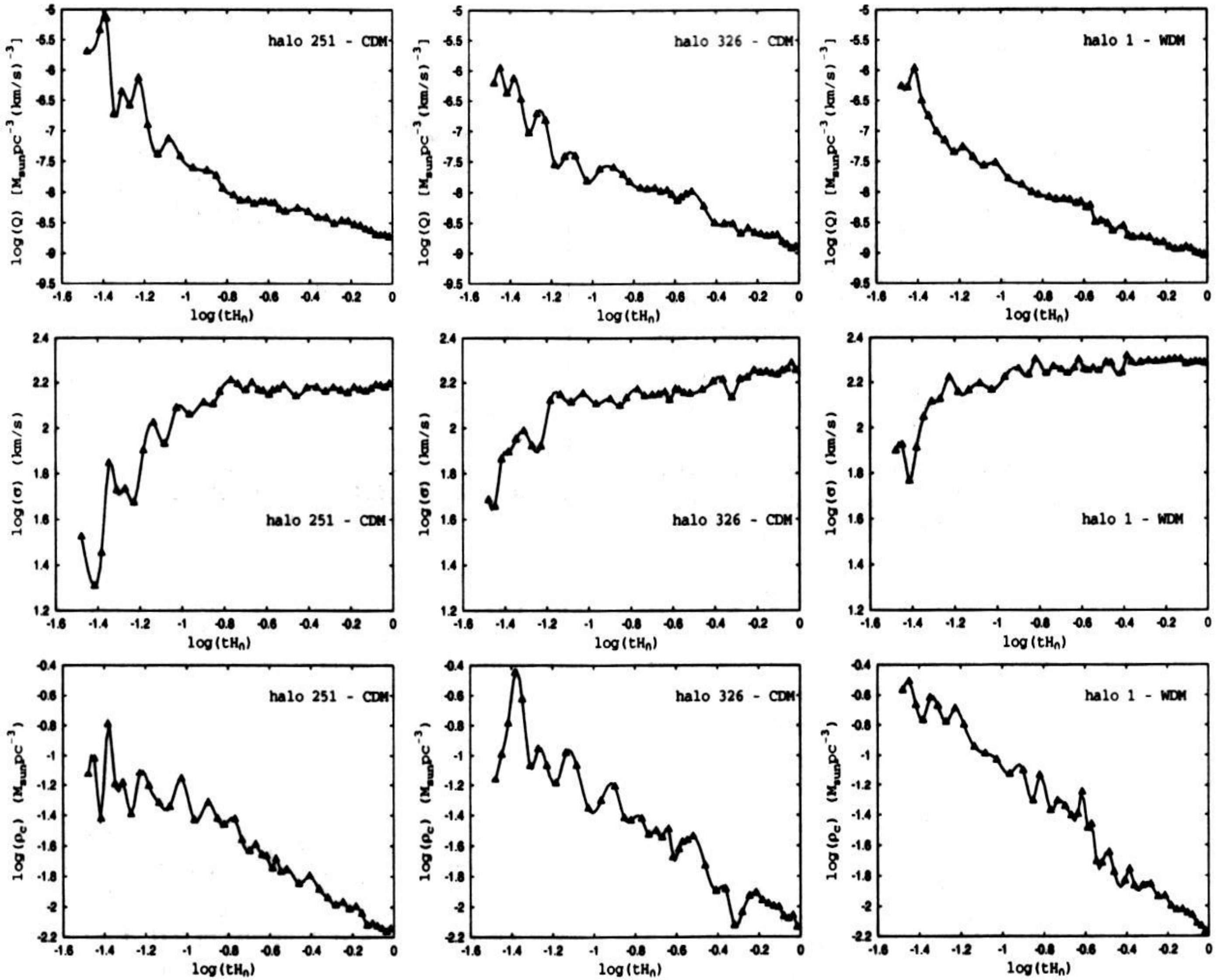


Examples of Halo Evolution

phase-space density

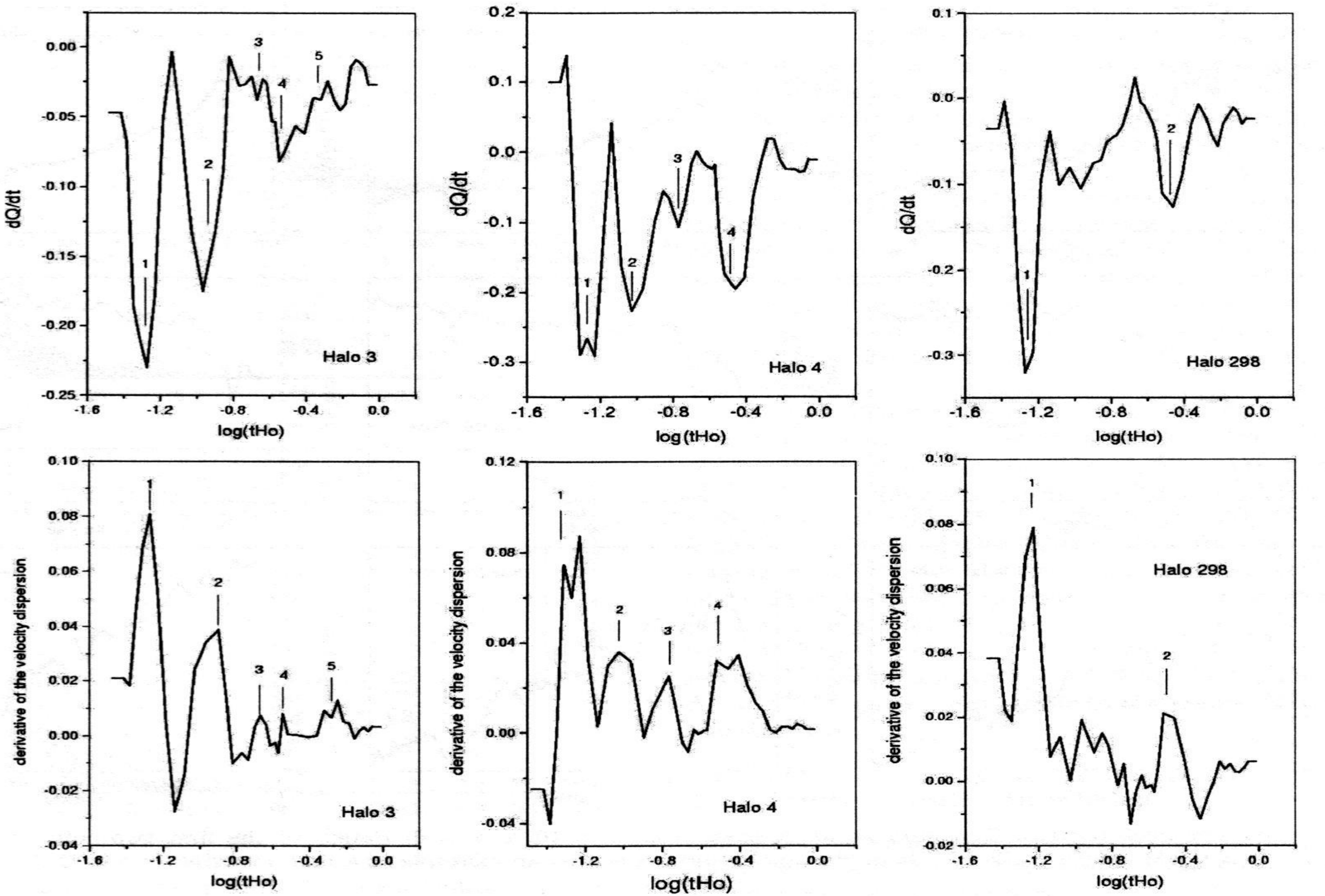
dispersion velocity

central density



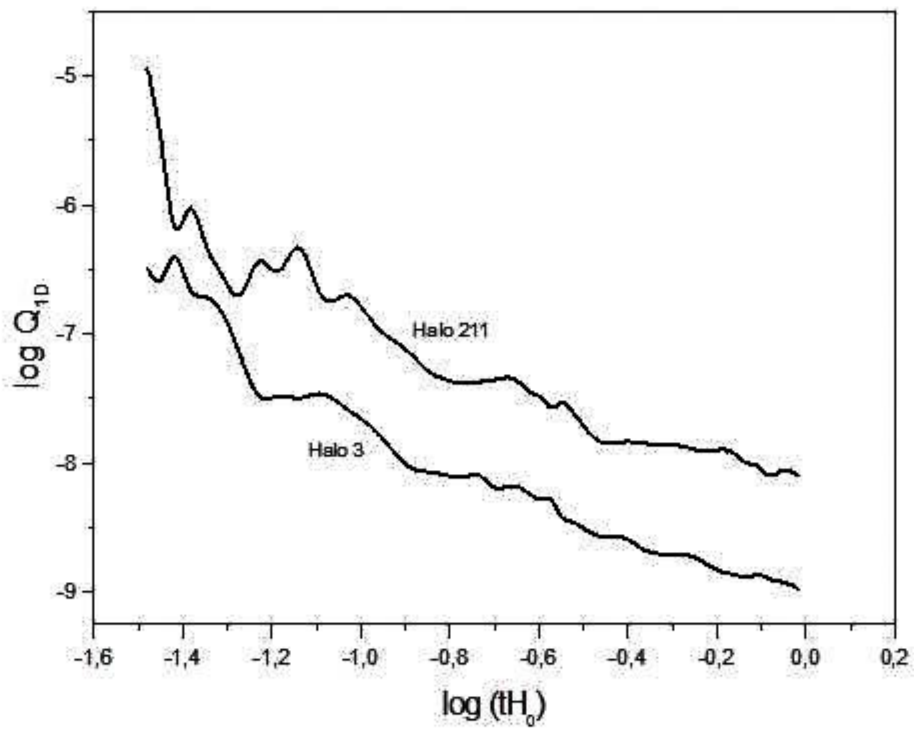
Phase Space Density Evolution

(Peirani, Durier & de Freitas Pacheco MNRAS 2006)



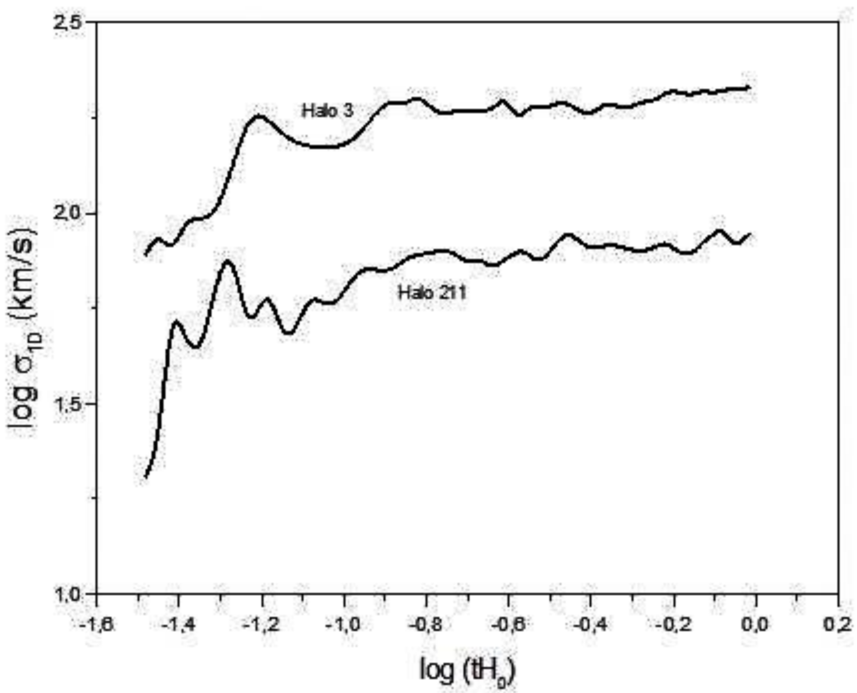
Properties of DM-Halos

halo 3 mass: $8.06 \times 10^{12}\ M_\odot$
halo 11 mass: $7.46 \times 10^{11}\ M_\odot$
(masses at z = 0)

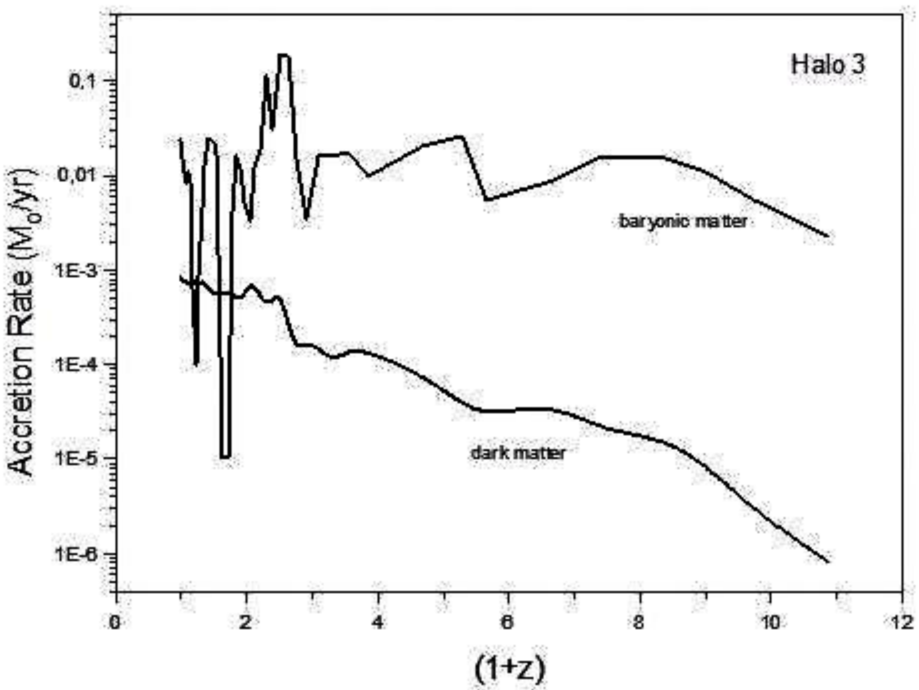


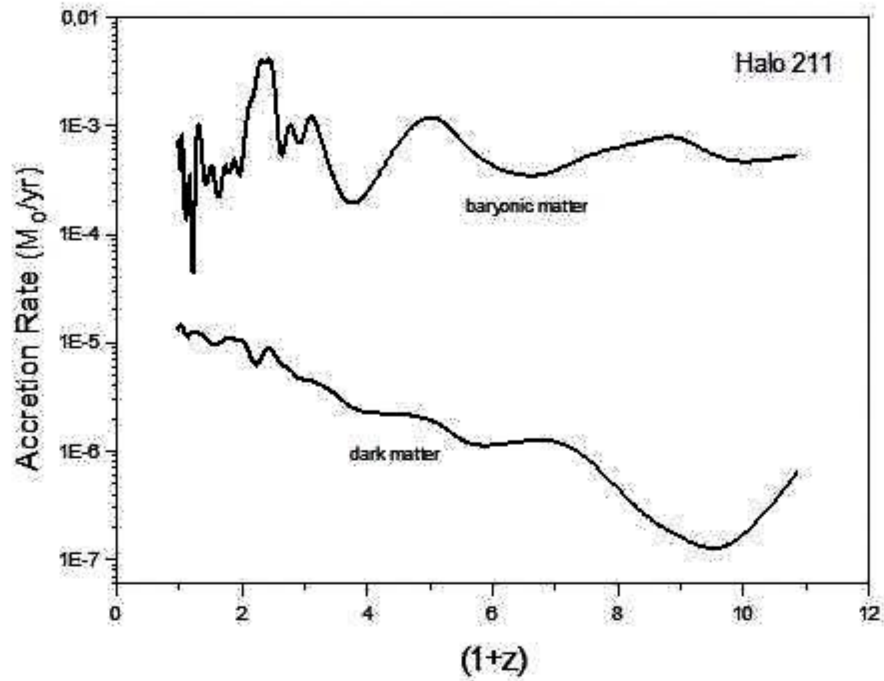
Evolution of the Phase Density

Evolution of the 1-D dispersion velocity



Comparison – Baryonic & DM accretion rates





Black Hole Masses

DM Fraction