Possible wormholes in a Friedmann universe

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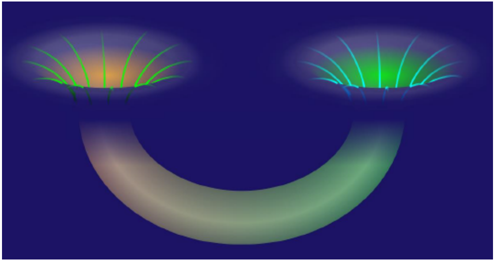

Verão Quantico, Iriri, 09 April 2025

Plan of the talk

- Wormholes in general. Motivation
- Spherically symmetric space-times. Possible dynamic wormholes
- Static wormholes, the necessity of NEC violation
- Dynamic wormholes, different definitions of a throat
- Possible dynamic wormholes without NEC violation
- Lemaître-Tolman-Bondi (LTB) solution with a magnetic field
- Possible throats in LTB space-times
- Matching LTB and Friedmann space-times
- Wormholes in a Friedmann universe: numerical estimates
- Concluding remarks

Wormholes: motivation

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Wormholes, Time Machines, and the Weak Energy Condition

Michael S. Morris, Kip S. Thorne, and Ulvi Yurtsever
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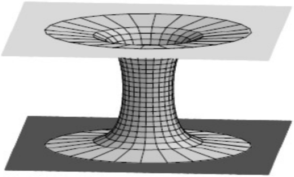
(Received 21 June 1988)

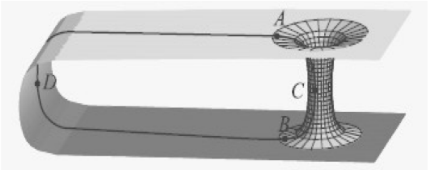
Normally theoretical physicists ask, "What are the laws of physics?" and/or, "What do those laws predict about the Universe?" In this Letter we ask, instead, "What constraints do the laws of physics place on the activities of an arbitrarily advanced civilization?" This will lead to some intriguing queries about the laws themselves.

Wormhole physics has become quite a popular research area, even though nobody has ever seen a real wormhole in Nature. Thus, on 8 April 2025, a search for the term "wormhole" on the site arxiv.org gave 3010 results for all years and 322 results for past 12 months. This popularity looks natural since a wormhole is one more, in addition to a black hole, and even simpler manifestation of a strongly curved space-time which can lead to many effects of great interest both in fundamental physics and astrophysics. To mention a few, these are opportunities of **time machines** and **fast travels**, a possible relation to **quantum entanglement** and potential traces in **astronomical observations**.

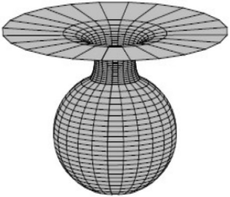
Wormhole types and topologies



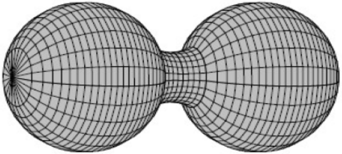
Sph: twice asymptotically flat wormhole (topology $S^2 \times R$).
Cyl: topology $S^1 \times R \times R$



A wormhole as a "handle", a **shortcut** between remote parts of the Universe (or a **time machine** if times at A and B are essentially different). The topology depends on that of the ambient world



A "hanging drop" wormhole.
Topology:
R^3 for both Sph and Cyl



A "dumbbell" wormhole
Sph: topology S^3
Cyl: topology $S^2 \times R$

Brief wormhole history

General relativity (A. Einstein, 1915-16): **gravity = curvature**

If space-time is curved, it can be very strongly curved !!!

K. Schwarzschild, 1916 - First exact solution, now termed a **black hole**

L. Flamm, 1916: the Schwarzschild space as a kind of tunnel

A. Einstein, N. Rosen, 1935: attempt to create a particle model based on GR (Einstein-Rosen bridge)

J.A. Wheeler, 1955,… - quanta, space-time foam, charge without charge, mass without mass, the term **"wormhole"**

1973 and so on, **up to now** (H. Ellis, K.B., G. Clement, …) - numerous exact wormhole solutions in different theories, analysis of their properties, including stability

K. Thorne et al., 1988, V.P. Frolov, I.D. Novikov, 1989........: beginning of a broad discussion of time machines, shortcuts etc,

Wormholes: definitions

A **wormhole:** a tunnel or shortcut between different universes or distant regions of the same universe (a Lorentzian, two-way traversable wormhole).
(We leave aside quantum or Euclidean wormholes, as well as "nontraversable" or "one-way traversable" wormholes with horizons = black holes.)

Let us, for simplicity, assume **spherical symmetry** (although other kinds of wormholes are also widely discussed):

$$ds^2 = A(x,t)dt^2 - B(x,t)dx^2 - r^2(x,t)d\Omega^2, \quad d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2,$$

where x is a radial coordinate, and $\underline{r(x,t)}$ is the spherical radius.

Definition. A **throat in a spatial section** $t = \mathrm{const}$ is **a regular minimum** of the function $r(x,t)$.

Definition. A **wormhole** is **a space-time region** containing at least one throat with $r = r_{\rm th}$ and radii $\boxed{r(x,t) \gg r_{\rm th}}$ on both sides of the throat(s).

These definitions directly extend those used for static metrics. For dynamic ones there is **ambiguity in the choice of spatial sections**, and there are some other definitions (*Hochberg and Visser (1998), Hayward (1999), etc*) that rest on the behavior of null congruences. In this work, however, there are physically preferable spatial sections related to the comoving reference frame, and the above definitions are suitable and intuitively clear.

Wormholes: general observations

For **static wormholes**, the above definition naturally refers to the static reference frame, in which the difference $\binom{0}{0} - \binom{1}{1}$ of the Einstein equations in suitable coordinates reads

$$\frac{r''}{r} \sim -(\rho + p_r) \qquad (\rho\text{=density, } p_r\text{=radial pressure})$$

On a throat $r' = 0, \ r'' > 0$, hence $\rho + p_r < 0 \ \Rightarrow$ **NEC violation**, "exotic matter".

Dynamic wormholes defined as above are attached to a specific set of spatial sections of a spherically symmetric space-time, they, in general, exist in a finite time interval, but do not require NEC violation and are not excluded in space-times with normal, non-exotic matter.

A regular minimum requires $r' = \partial r/\partial x = 0$. On the other hand, the equality

$$r^{,\alpha} r_{,\alpha} \equiv g^{tt}\dot{r}^2 + g^{xx} r'^2 = (1/A)\dot{r}^2 - (1/B)r'^2 = 0$$

indicates an apparent horizon, so that r may be used as a time coordinate if $r^{,\alpha} r_{,\alpha} > 0$ (a *T-region*) and as a spatial one if $r^{,\alpha} r_{,\alpha} < 0$ (an *R-region*). Thus a throat, where $r' = 0$, gets into a T-region if $\dot{r} \neq 0$. This excludes many opportunities.

Examples of wormholes without NEC violation

- Axial symmetry, wormholes with **ring singularities:** stationary Kerr and Kerr-Newman solutions, Zipoy-Voorhees static solution *(D. Zipoy, 1968; KB, J.C. Fabris, 1997).*
- Rotating cylindrical wormhole space-times, matching to Minkowski on both sides from the throat *(KB, V.G. Krechet, 2017).*
- Dynamic wormholes without NEC violation, different sources. Examples with nonlinear ED *(A. Arellano and F. Lobo, 2006; KB, 2018)*. (Though, with Lagrangians containing terms like $(F_{\mu\nu}F^{\mu\nu})^{1/4}$).
- Lemaître-Tolman-Bondi (LTB) solution, wormholes with possible inclusion of an electromagnetic field and a cosmological constant. **To be further discussed.**

LTB solution with Λ and a magnetic field

Matter = neutral dust $\Rightarrow$ comoving reference frame, the metric

$$ds^2 = d\tau^2 - \mathrm{e}^{2\lambda(R,\tau)}dR^2 - r^2(R,\tau)d\Omega^2, \qquad (1)$$

with τ = proper time along particle trajectories,
R = radial coordinate (specified up to $R \mapsto F(R)$),
$r(R,\tau)$ = time-dependent radius of the Lagrangian sphere "number" R.
SET of dustlike matter: only $T^0_0 = \rho \neq 0$.

Radial (external) electromagnetic field: $T_\mu^{\nu\,[\mathrm{em}]} = \dfrac{q^2}{8\pi G r^4}\,\mathrm{diag}(1,\ 1,\ -1,\ -1)$,
$q = \mathrm{const}$ = magnetic charge (for certainty) .
Einstein equations, $R^\nu_\mu - \frac{1}{2}\delta^\nu_\mu R + \delta^\nu_\mu \Lambda = -8\pi G T^\nu_\mu \quad \Rightarrow$

$$2r\ddot{r} + \dot{r}^2 + 1 - e^{-2\lambda} r'^2 = \frac{q^2}{r^2} + \Lambda r^2, \qquad (2)$$

$$\frac{1}{r^2}(1 + \dot{r}^2 + 2r\dot{r}\dot{\lambda}) - \frac{\mathrm{e}^{-2\lambda}}{r^2}(2rr'' + r'^2 - 2rr'\lambda') = 8\pi G\rho + \frac{q^2}{r^4} + \Lambda, \qquad (3)$$

$$\dot{r}' - \dot{\lambda} r' = 0. \qquad (4)$$

Conservation law:

$$\dot{\rho} + \rho\left(\dot{\lambda} + \frac{2\dot{r}}{r}\right) = 0, \quad \Rightarrow \quad \rho = \frac{F'(R)}{r^2 r'}, \quad F(R) = \text{arbitrary function.} \qquad (5)$$

LTB solution with Λ and a magnetic field — 2

Integrate Eq. (4) in $\tau \quad \Rightarrow$

$$e^{2\lambda} = \frac{r'^2}{1+f(R)}, \qquad f(R) = \text{arbitrary function}, \ f(R) > -1. \tag{6}$$

Using that, integrate Eq. (2) in $\tau \quad \Rightarrow$

$$\dot r^2 = f(R) + \frac{F(R)}{r} - \frac{q^2}{r^2} + \frac{\Lambda}{3} r^2, \tag{7}$$

where $F(R)$ is the same as in (5). According to (5),

$$F(R) = 2GM(R) = 8\pi G \int \rho r^2 r' dR \quad (M(R) = \text{mass function}). \tag{8}$$

$f(R) \mapsto$ initial velocity distribution. At $\Lambda = 0$, from (7) it follows:

$f > 0$ — hyperbolic motion, making possible $r(R,\tau) \to \infty$.
$f = 0$ — parabolic motion,
$f < 0$ — elliptic (finite) motion.

At $\Lambda \neq 0$, things are more involved, the boundary of finite motion is shifted.

Further integration of Eq. (7) with $\Lambda \neq 0$ leads to elliptic integrals. In what follows we assume $\Lambda = 0$, so that only elementary functions are needed.

A throat is, by our definition, a regular minimum of $r(R, \tau)$ at fixed time τ. In terms of a well-behaved radial coordinate R ($e^{\lambda} > 0$), we have $r' = 0$, $r'' > 0$. According to (6), $e^{-2\lambda} r'^2 = 1 - f(R)$, hence on the throat $R = R_{\rm th}$

$$e^{-2\lambda} r'^2 = 1 + f(R_{\rm th}) = 0 \quad \Rightarrow \quad f(R_{\rm th}) = -1, \quad \text{or} \quad h(R_{\rm th}) = 1, \qquad (9)$$

where $h(R) = -f(R)$. It immediately follows that **only elliptic models with $h(R) > 0$ are compatible with throats.** To keep the metric (1) nondegenerate, it must be in general $1 + f = 1 - h > 0 \quad \Rightarrow \quad h = 1$ occurs at a single point $R = R_{\rm th}$, so that at $R = R_{\rm th}$ we have a maximum of $h(R)$: $h'(R_{\rm th}) = 0, \quad h''(R_{\rm th}) < 0$.

Calculation of r'' from (6) leads to $r''\Big|_{R=R_{\rm th}} = -\dfrac{h'}{2r'}\Big|_{R=R_{\rm th}} > 0$.

A finite density $\rho > 0$ requires $F'/r' > 0 \ \Rightarrow \ F'$ should vanish together with r'.

We summarize that on a regular throat $R = R_{\rm th}$ with $\rho > 0$ it holds

$$r' = 0, \qquad h = 1, \qquad h' = 0, \qquad h'' < 0; \qquad \frac{h'}{r'} < 0;$$
$$F' = 0, \qquad \frac{F'}{r'} > 0, \qquad F^2 - 4hq^2 > 0. \qquad (10)$$

(the last inequality also follows from the equations of motion.)

LTB space-times and Friedmann models ($\Lambda = 0$)

Further integration of (7) in τ for elliptic models gives

$$\pm[\tau - \tau_0(R)] = \frac{1}{h}\sqrt{-hr^2 + Fr - q^2} + \frac{F}{2h^{3/2}} \arcsin \frac{F - 2hr}{F^2 - 4hq^2}, . \qquad (11)$$

the arbitrary function $\tau_0(R)$ $\mapsto$ clock synchronization at different R. A well-known convenient parametrization of the solution (for $\tau_0(R) \equiv 0$):

$$\begin{aligned} r &= \frac{F}{2h}(1 - \Delta\cos\eta), \qquad \Delta = \sqrt{1 - \frac{4hq^2}{F^2}}, \qquad 0 < \Delta \leq 1, \\ \tau &= \frac{F}{2h^{3/2}}(\eta - \Delta\sin\eta). \end{aligned} \qquad (12)$$

At $\Delta = 1$ (LTB solution with **pure dust**), there are singularities $r = 0$ at $\eta = 0, 2\pi$, etc., while at $q \neq 0$, $\Delta < 1$, the value $r = 0$ is never achieved.

At $q{=}0$ ($\Delta{=}1$), under the assumptions (e.g., *Landau-Lifshitz, Field Theory*)

$$F(\chi) = 2a_0 \sin^3\chi, \qquad h(\chi) = \sin^2\chi, \qquad a_0 = \mathrm{const} \qquad (13)$$

(using the radial coordinate $R = \chi$ of angular nature), the solution describes **Friedmann's closed isotropic universe** filled with dust, such that

$$r = r(\eta, \chi) = a(\eta)\sin\chi, \qquad a(\eta) = a_0(1 - \cos\eta) = \text{scale factor}. \qquad (14)$$

Wormholes in LTB space-times ($\Lambda = 0$)

At a throat, the quantity $-\lim\limits_{R\to R_{\rm th}} [Fh'/(F'h)] = B > 0$ is finite. Then we obtain

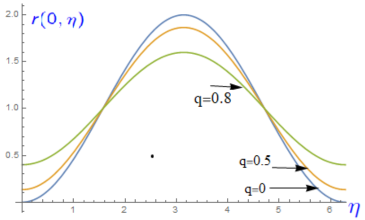
$$r'\Big|_{R_{\rm th}} \approx \frac{F'(2N_2 - BN_1)}{4\Delta\,(1-\Delta\,\cos\eta)}, \tag{15}$$

$$\rho\Big|_{R_{\rm th}} = \frac{F'}{8\pi G r^2 r'}\Big|_{R_{\rm th}} = \frac{\Delta(1-\Delta\,\cos\eta)}{2\pi G r^2 (2N_2 - BN_1)}\Big|_{R_{\rm th}}. \tag{16}$$

$$N_1(R,\eta) = \cos\eta - 3\Delta + 3\Delta^2(\cos\eta + \eta\sin\eta) + \Delta^3(-2+\cos^2\eta),$$

$$N_2(R,\eta) = -\cos\eta + 2\Delta - \Delta^2(\cos\eta + \eta\sin\eta) \tag{17}$$

Example: $\boxed{h(R) = \dfrac{1}{1+R^2},\ \ F(R) = b(1+R^2),\ \ b = \mathrm{const}}$ (length scale)



Time dependence of the throat radius $r(0,\eta)$ for $b=1$, $q = 0, 0.5, 0.8$. The singular value $r = 0$ is reached only if $q = 0$.

There are also shell-crossing singularities close to $\eta = 0$ and $\eta = 2\pi$ connected with $N_* = 2N_2 - BN_1 \to 0$, where $r \neq 0$, $r' \to 0$, and $\rho \to \infty$. Thus even with $q \neq 0$ the wormholes have a finite lifetime. It is quite a general property of all such models.

Boundaries and matching

For space-time regions with different dust distributions, separated by a particular value of R, the well-known Darmois-Israel junction conditions lead to the requirements

$$[r] = 0, \quad [\mathrm{e}^{-\lambda} r'] = 0 \quad \Rightarrow \quad [f] = 0, \quad [F] = 0. \qquad (18)$$

In other words, to match two particular solutions, we must simply identify the values of $F(R)$ and $h(R)$ on the junction surface $R = R_*$. The parameters q, Λ should also coincide on both sides of $R = R_*$.

If $\rho = 0 \iff F = \mathrm{const}$ in a certain region, then it is (electro)vacuum with the Reissner-Nordström-(anti) de Sitter metric written in geodesic coordinates and comoving to a certain set of electrically neutral test particles. By matching with it, we can obtain models of evolving finite dust configurations along with their exteriors.

Further on, we will consider matching of LTB regions representing wormholes to other special LTB regions, namely, isotropic cosmological models.

Suppose that a wormhole region is matched to a Friedmann universe described by Eqs. (13), (14). Hence, we are dealing with $q = 0$.

To obtain a particular model, we must choose $h(R)$ and $F(R)$. We take into account: **(a)** the throat conditions (10), **(b)** the freedom of choosing the R coordinate, and **(c)** the assumption of wormhole symmetry with respect to its throat $(R = 0)$. We can make **without loss of generality** a particular choice of $h(R)$; then, the choice of $F(R)$ will be significant.

Wormhole: $\boxed{h(R) = \dfrac{1}{1+R^2}, \quad F(R) = 2b(1+R^2)^k,} \quad b, k = \mathrm{const} > 0.$

Friedmann: $\boxed{h(\chi) = \sin^2\chi, \quad F(\chi) = 2a_0 \sin^3\chi.}$

If the two metrics are matched at $\chi = \chi_* \ll 1$, then, putting $\sin\chi_* \approx \chi_*$,

$$1 + R_*^2 = \chi_*^{-2}, \qquad F(R_*) = 2b\chi_*^{-2k}, \qquad \frac{b}{a_0} = \chi_*^{2k+3}. \tag{19}$$

We also have in the wormhole solution $r(R,\eta) = b(1+R^2)^{k+1}(1-\cos\eta)$, therefore the constant $2b =$ maximum value of the wormhole throat radius $r_{\rm th} = r(0,\eta)$.

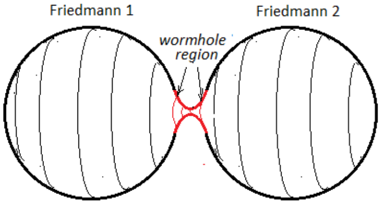
Numerical estimates

The table shows some estimates, assuming $a_0 = 10^{28}\,\mathrm{cm} \approx$ size of the observed part of the Universe and $b = 10\,\mathrm{m}$, 10 000 km, or 1 pc.

Parameter	$b = 10$ m, $k = 1$	$b = 10$ m, $k = 0.1$	$b = 10^4$ km $k = 0.1$	$b = 1$ pc, $k = 0.1$
χ_*	10^{-5}	1.5×10^{-8}	1.1×10^{-6}	1.05×10^{-3}
r_*	30 kpc	50 pc	4 kpc	3.5 Mpc
ρ_*, $\mathrm{g/cm^3}$	2.2×10^{-30}	2.2×10^{-31}	2.2×10^{-31}	2.2×10^{-31}
$\rho_{\rm th}$, $\mathrm{g/cm^3}$	$2.2\ \times10^{20}$	2.2×10^{19}	2.2×10^{7}	2.5×10^{-12}

The wormhole region has the size of parsecs or larger even for small throats.

Near the throat, the density is $\rho_{\rm th} \gg \rho_{\rm nuclear}$ for $b = 10\,\mathrm{m}$.
It is of white-dwarf order near a throat of planetary size, and quite small near a throat of 1 pc.



At the junction, the density ρ_* is of mean cosmological order of magnitude.

Wormhole lifetime and traversability

Total lifetime of any shell:

$$\tau_{\rm tot} = \pi F(R)/h^{3/2}(R).$$

Junction surface:

$$\tau_{\rm tot} = \tau_{\rm tot}[\text{universe}].$$

Throat: $\tau_{\rm th} = 2\pi b$,
where $r = 2b$ is the throat's maximum radius.

The lifetime ends with a singularity (dust shells shrink to points)

Examples:

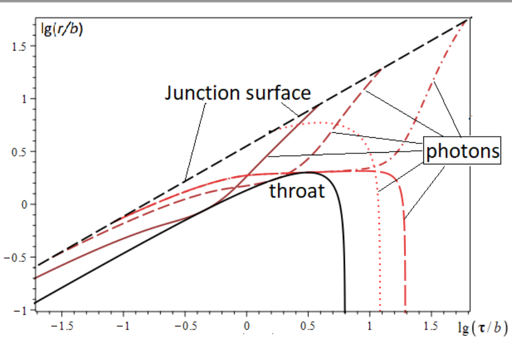
$b = 10^{11}\,\mathrm{cm} \sim R_\odot$

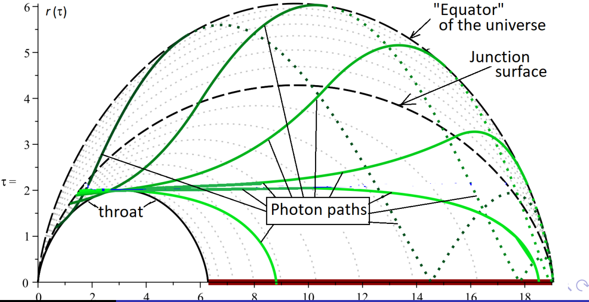
$\Rightarrow \quad \tau_{\rm th} \sim 20$ seconds,

$b = 10^{18}\,\mathrm{cm} \sim 1\,\mathrm{pc}$

$\Rightarrow \quad \tau_{\rm th} \sim 20$ years

Still the wormholes are **traversable** for photons emitted at proper times (see the figures).





The density at junction

Assuming that in the wormhole solution $F'/h' < 0$ (as at $R = R_{\rm th}$), we obtain

$$\rho_*\Big|_{\eta=\pi} < \frac{1}{32\pi G a_0^2} \approx 5.5\times 10^{-30}\,\frac{\rm g}{\rm cm^3} \qquad (20)$$

if $a_0 \sim 10^{28}$ cm, approx. the size of the visible part of the Universe.

On the other hand, in Friedmann's solution (13), (14) the density is

$$\rho_{\rm Fr}(\eta) = \frac{3}{4\pi G a_0^2 (1-\cos\eta)^3}, \qquad \rho_{\rm Fr}\Big|_{\eta=\pi} = \frac{3}{32\pi G a_0^2}. \qquad (21)$$

Thus according to (20), the wormhole matter density at the junction surface Σ is not only very small, but it is even a few times smaller (by at least a factor of three) than the cosmological matter density.

In other words, the wormhole region is, at least close to Σ, a region of smaller density, resembling a void. This observation was made for the instant $\eta = \pi$, but it remains true at all times since the η dependence is the same for the wormhole and cosmological solutions.

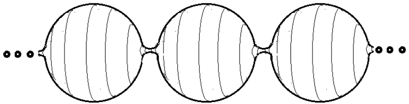
Cosmological constant and a magnetic field

Small values of Λ cannot qualitatively change such local phenomena as the existence of wormholes. However, the global evolution with $\Lambda > 0$ must lead to accelerated expansion, which must also encompass the wormhole region.

Inclusion of small $q \neq 0$ also cannot strongly change the local picture of a wormhole. However, globally:

(i) The Universe cannot be precisely homogeneous and isotropic due to a vector field;

(ii) A charge on one "pole" inevitably leads to an opposite charge on the other, where the lines of force again converge. There can be a similar wormhole and one more universe beyond it, and so on. The whole picture will resemble a "churchkhela", the wonderful Georgian dessert.

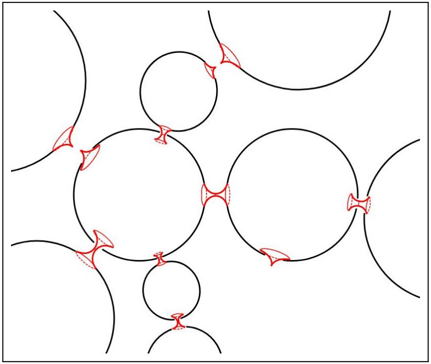


More complex structures

Our examples of wormholes were symmetric with respect to the throat, but this is certainly not necessary. For an asymmetric model we can suggest, e.g.,

$$h(R) = \frac{1}{1+R^2}, \qquad F(R) = 2b(1+R^2+cR^3)^k, \qquad b, c, k = \text{const} > 0.$$

Also, two mouths of a wormhole may open in a single universe. A still more complex picture emerges if there are multiple wormholes in a multi-universe, like this:



Concluding remarks

- We have constructed models of dynamic traversable wormholes on the basis of the LTB solution of general relativity, connecting two closed Friedmann universes filled with dust, or two distant parts of the same universe.
- Dynamic wormholes having **finite lifetime**, evolving together with the whole Universe, are possible **without exotic matter**. The throats have a short life, but the wormhole region survives up to the universe age, making it possible to catch signals from far away or from another universe.
- Wormhole throats of macroscopic or even planetary size require very large matter densities. The size of a wormhole region matched to cosmology is many orders of magnitude larger than that of the throat.
- Close to the junction surface, a wormhole region turns out to be a region of smaller density than its cosmological environment, resembling a void.
- Inclusion of Λ makes it possible to obtain such wormholes in a closed universe with **accelerated expansion**, or in the ΛCDM model (closed version). Constructuon of such models can be a problem for future studies.
- The possible magnetic field of evolving wormholes may substantially complicate the global geometry and topology.

Outlook

- A study of possible properties of wormholes emerging at different stages of the universe evolution.
- Analysis of possible wormhole solutions with matter other than dust or in addition to dust.
- Analysis of possible observational consequences of single or multiple wormholes: geodesics, lensing, shadows etc.
- ...

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A further work on possible observational signatures is in progress.

Thanks for your attention!