Random Walks in the Sky (remembering your steps!)

Marcello Musso

CP^3 - Université catholique de Louvain

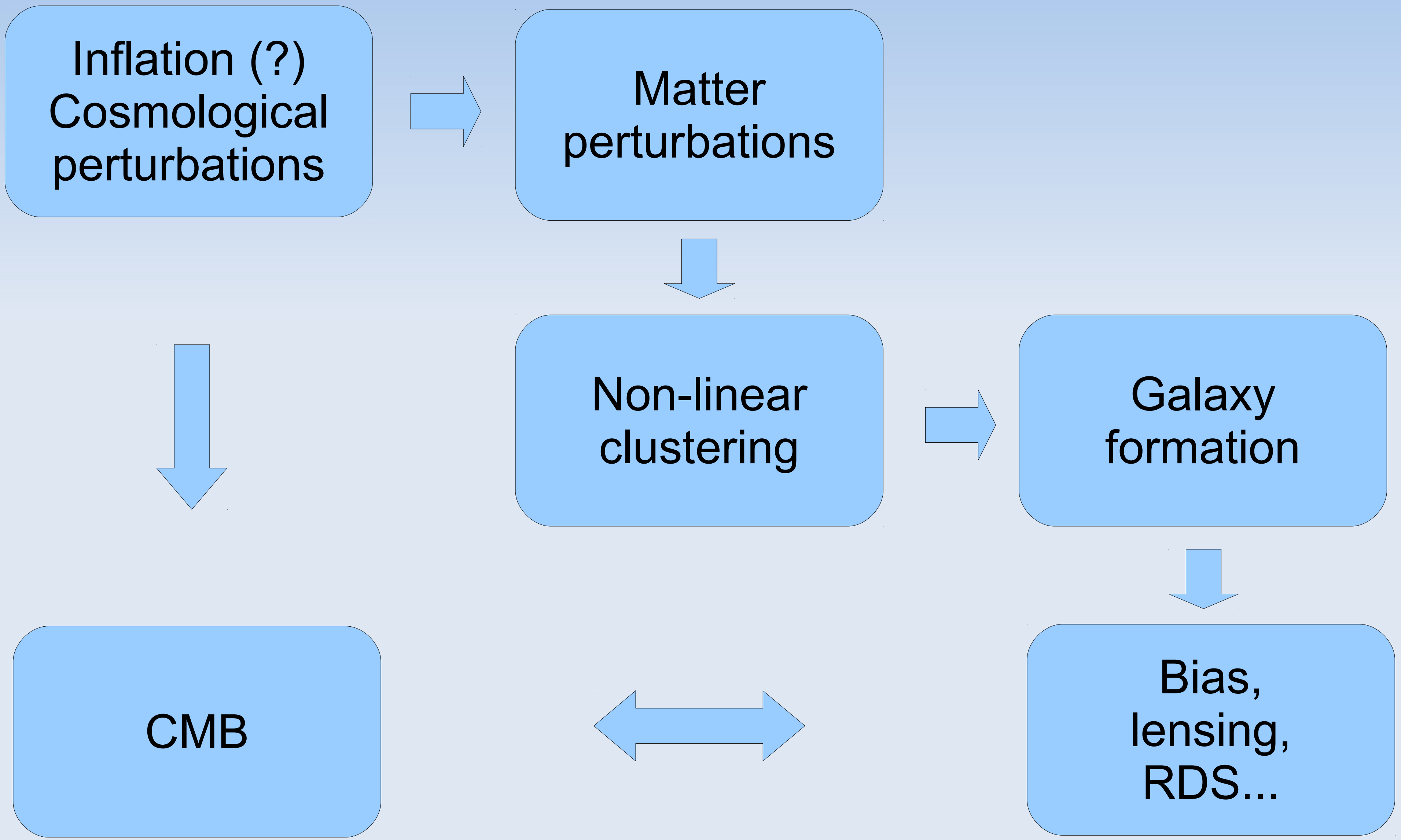
in collaboration with A. Paranjape and R. Sheth
(arXiv: 1201.3876, 1205.3401, 1305.0724, 1306.0551, 1401.8177)

UFES - Vitória
May 2014

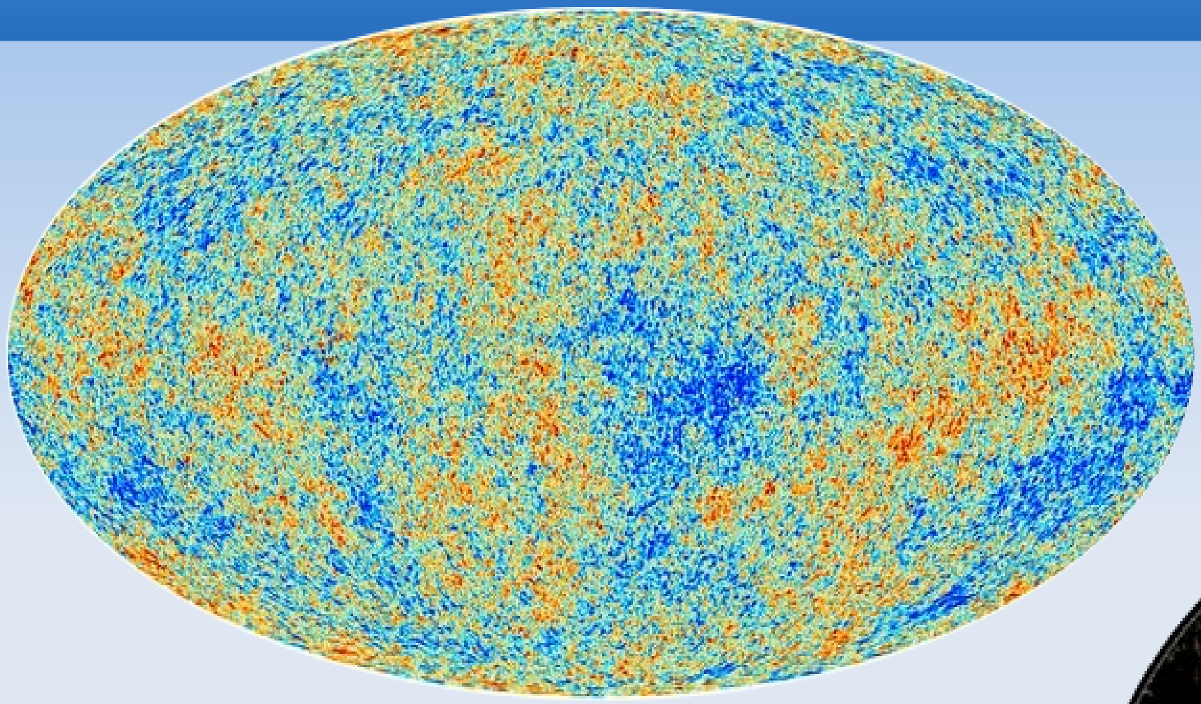
Open questions in Cosmology

- How did the Universe begin?
  Inflation or alternatives
- What fills the Universe today?
  Dark Matter / Dark Energy
  Λ vs Quintessence vs ModGrav
- How do we observe it?
  Bias, cosmic variance, lensing, RDS...

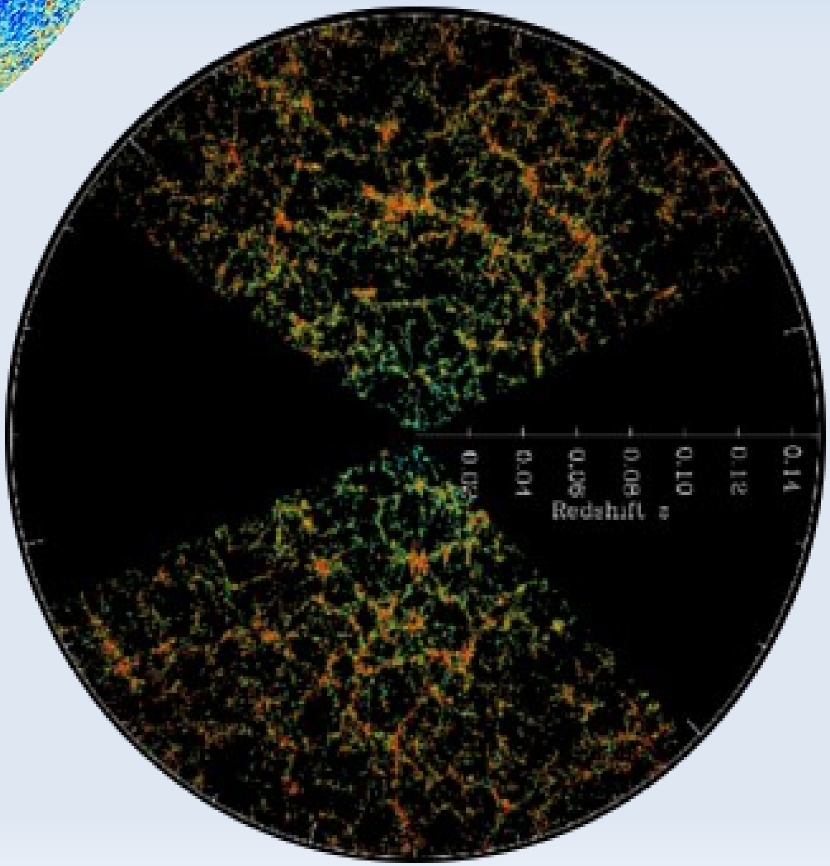
Observables in Cosmology



Formation of structures



Halos evolve from critical overdensities



They trace primordial distribution and energy content

Halo Mass Function and Bias

- How many halos of given mass (mass function)
- Correlation of halo counts with underlying matter field (halo bias)

GOALS:

- Analytical halo statistics reproducing (heavy!) N-body simulations
- Information on initial conditions and matter/energy balance from data

Expected Signatures

- Inflation predicts nearly (not completely!) Gaussian perturbations
- More (less) Dark Energy results in less (more) structures

- Tiny but distinctive deviations due to non-Gaussian initial conditions (on top of a heavily non-linear process!)
- Redshift dependence from small time variation of Λ

Why bother?

- CMB physics is nearly linear, but errors are now dominated by cosmic variance and foregrounds
- LSS has highly biased tracers with boosted signals
- Structure formation is recent, more sensitive to changes in Λ
- Independent probe that breaks degeneracies
- But it is highly non-linear: need to understand the biases
- Bias: not observational problem but theoretical opportunity
- Provides a handle on galaxy formation (very interesting per se)!

Outline of the talk

Part ONE

- The excursion set approach (remembering your steps)
- Analytical progress for mass functions: the UPWARDS approximation (it's always good to step up!)

Part TWO

- Bias from excursion sets (a simple way!)

Excursion set theory

- Halos from "dense enough" patches in the initial matter distribution δ_{in}

$$\delta_R(\mathbf{x}) \equiv \frac{1}{V_R}\int \mathrm{d}^3y\, W_R(\mathbf{y}-\mathbf{x})\delta_{in}(\mathbf{y}) \geq b(s,z) \equiv \delta_c(M)/D(z)$$

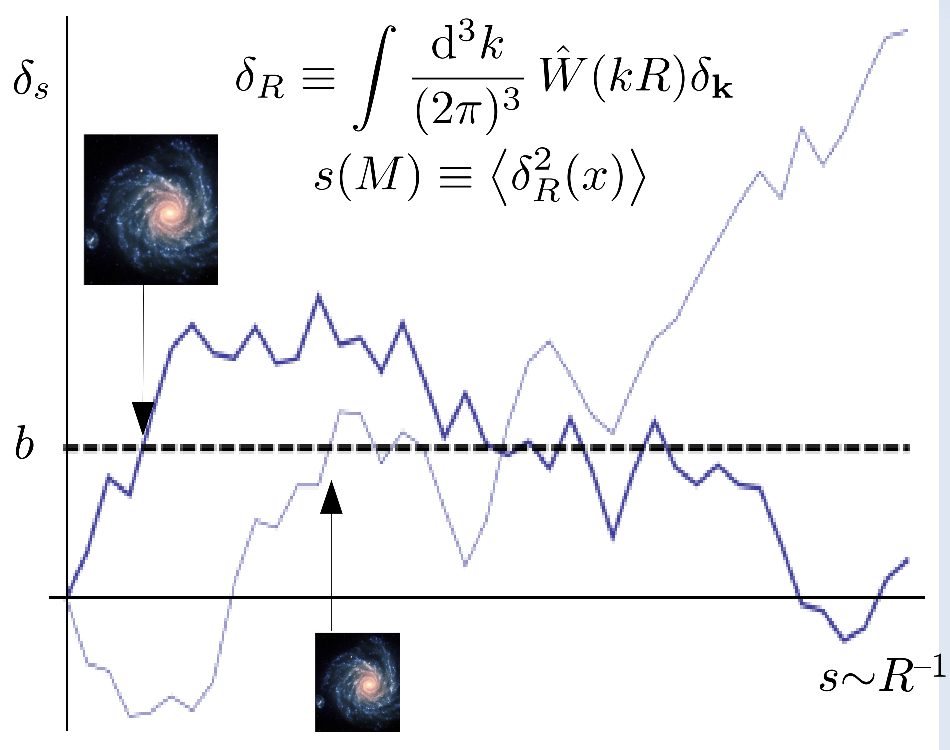
- Different locations realize different random walks:

FIRST PASSAGE PROBLEM!

CORRELATED STEPS!

No known solution

NEED BETTER MATHS



An excursion set dictionary

- Abundance $n(M)$ $\longleftrightarrow$ first crossing probability $f(s)$ at scale $s(M)$
- Barrier $b(s,z) = \delta_c(s)/D(z)$ $\longleftrightarrow$ physics of gravitational collapse and background cosmology (only through D)
- Power spectrum $P(k)$ $\longleftrightarrow$ statistics of the initial matter distribution
- Filter $W_R(\mathbf{x})$ $\longleftrightarrow$ definition of Dark Matter halo
- … let's be as general as possible!

First crossing distribution

- Probability of ANY crossing at s :

$$f(s) = \frac{\mathrm{d}}{\mathrm{d}s} \langle \vartheta(\delta - b(s)) \rangle = \frac{\mathrm{d}}{\mathrm{d}s} \int_{b(s)}^{+\infty} \mathrm{d}\delta \, p(\delta; s)$$

Press & Schechter (1974)

- Not any, but **FIRST** crossing (cloud-in-cloud problem); solution only for Gaussian uncorrelated steps with constant or linear barrier

$$f(s) = \frac{\langle \vartheta(b_1 - \delta_1) \ldots \vartheta(b_{N-1} - \delta_{N-1}) \vartheta(\delta_N - b_N) \rangle}{\Delta s}$$

Bond et al. (1991)

- Can perturb around known solution

Maggiore & Riotto (2010)
Corasaniti & Achitouv (2011)

First crossing distribution

- However: strongly correlated walks are less affected (less zig-zags)

Paranjape, Lam & Sheth (2011)

- Can relax FIRST into simply **UPWARDS**: $\delta = b$; $\delta' \geq b'$

$$f(s) = \left\langle \left[\frac{\mathrm{d}}{\mathrm{d}s}\vartheta(\delta_s - b)\right]\vartheta(\delta'_s - b')\right\rangle = \int_{b'}^{\infty}\mathrm{d}v\,(v - b')\,p(b, v; s)$$

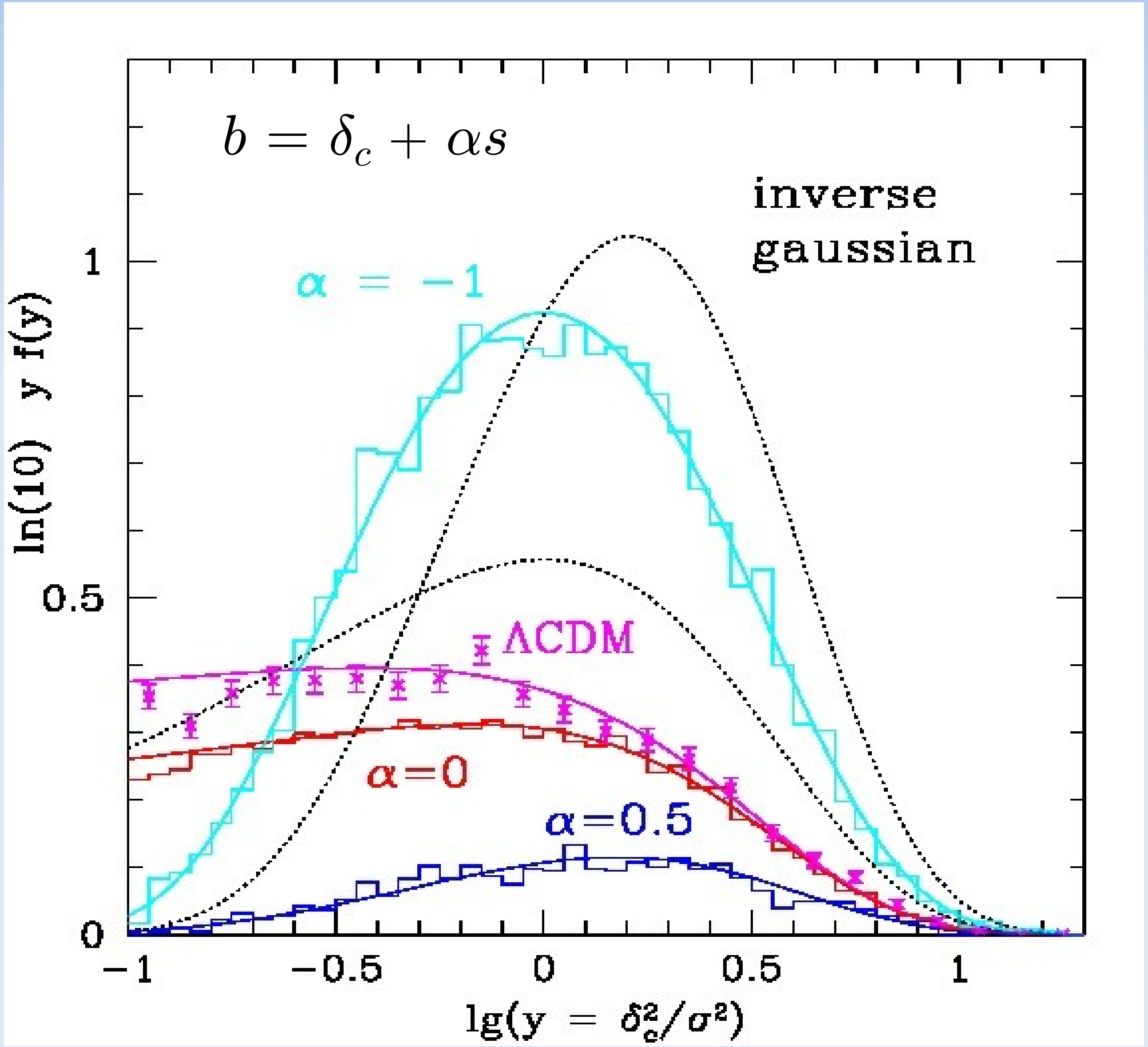
MM & Sheth (2012)

$$f(s) = -\left(\frac{b}{\sqrt{s}}\right)'\frac{e^{-b^2/2s}}{\sqrt{2\pi}}\left[\frac{1 + \mathrm{erf}(X/\sqrt{2})}{2} + \frac{e^{-X^2/2}}{2X\sqrt{2\pi}}\right]$$

old PS result

$$X = \frac{\text{mean of } p(v|b) - b'}{\sqrt{\text{var of } p(v|b)}}$$

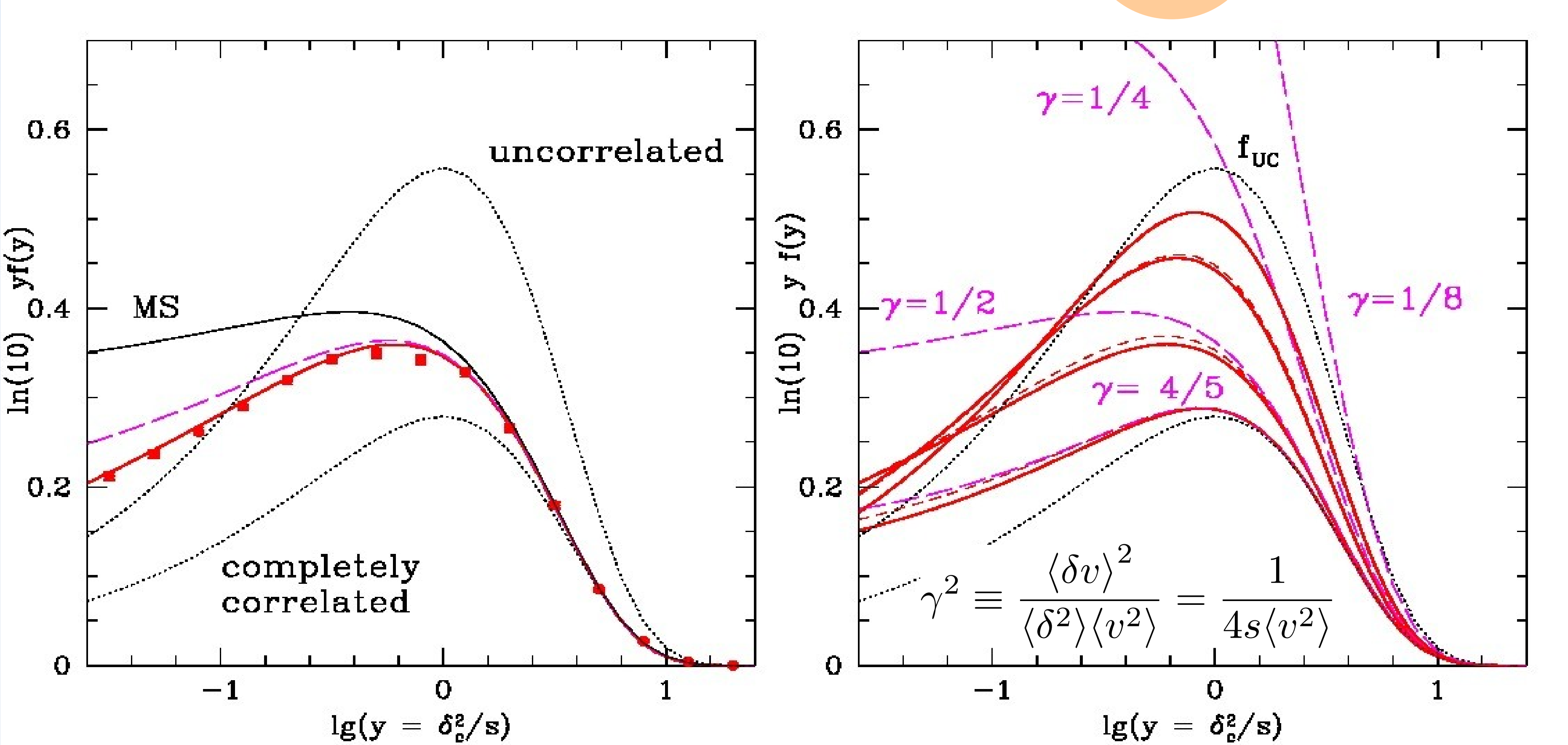
First crossing distribution



MM & Sheth (2012)

Upward mobility, back-substitution

$$p(\delta \geq b, s) = \int_0^s \mathrm{d}S f(S) p(\delta \geq b, s | \mathrm{up}, S)$$



MM & Sheth (2013)

- Upcrossing captures $f(s)$ for all power spectra, filters and barriers

Upward mobility, back-substitution

$$p(\delta \geq b, s) = \int_0^s \mathrm{d}S f(S) p(\delta \geq b, s|\mathrm{up}, S)$$

Chicago Tribune

SPORTS

Front Page | News | Sports | Business | Lifestyles | Opinion | A&E

Home > Featured Articles > Wisconsin

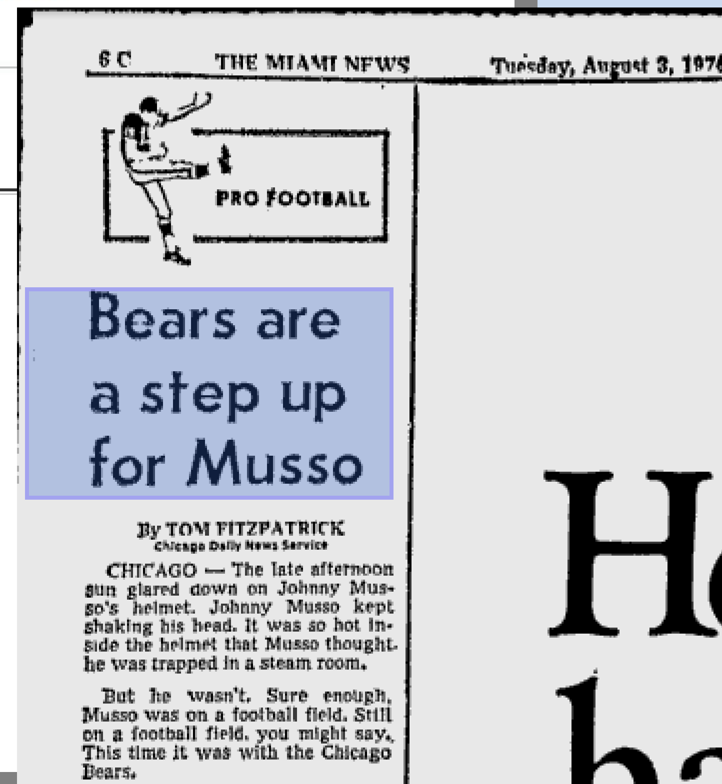
Another Musso steps up

November 13, 2003 | By Bill Jauss.

Recommend | Tweet 0 | Share 51 | 8+1 0

Pittasorre

The latest "Italian Stallion" from the Musso family in Hinsdale, Brad Musso, has helped Wheaton College to 18 straight victories and a No. 4 ranking. The junior wide receiver has eight TD catches and has scored twice on punt returns this year. He's fourth in Division III with a 17.9 yards average per return and 24th with 97.2 receiving yards per game. Musso's father Johnny "Italian Stallion" Musso was an All-American for coach Bear Bryant's Alabama teams in 1970 and '71 and played for the Bears. Brad Musso's older brothers, Scott and Brian, both starred as receivers and return men at Northwestern. Wheaton (9-0) plays at Augustana (7-2) on Saturday for the College Conference of Illinois and Wisconsin title.



6 C THE MIAMI NEWS Tuesday, August 3, 1976

PRO FOOTBALL

Bears are a step up for Musso

By TOM FITZPATRICK
Chicago Daily News Service

CHICAGO — The late afternoon sun glared down on Johnny Musso's helmet. Johnny Musso kept shaking his head. It was so hot inside the helmet that Musso thought he was trapped in a steam room.

But he wasn't. Sure enough, Musso was on a football field. Still on a football field, you might say. This time it was with the Chicago Bears.

IT'S ALWAYS GOOD TO STEP UP!

Back to Cosmology !

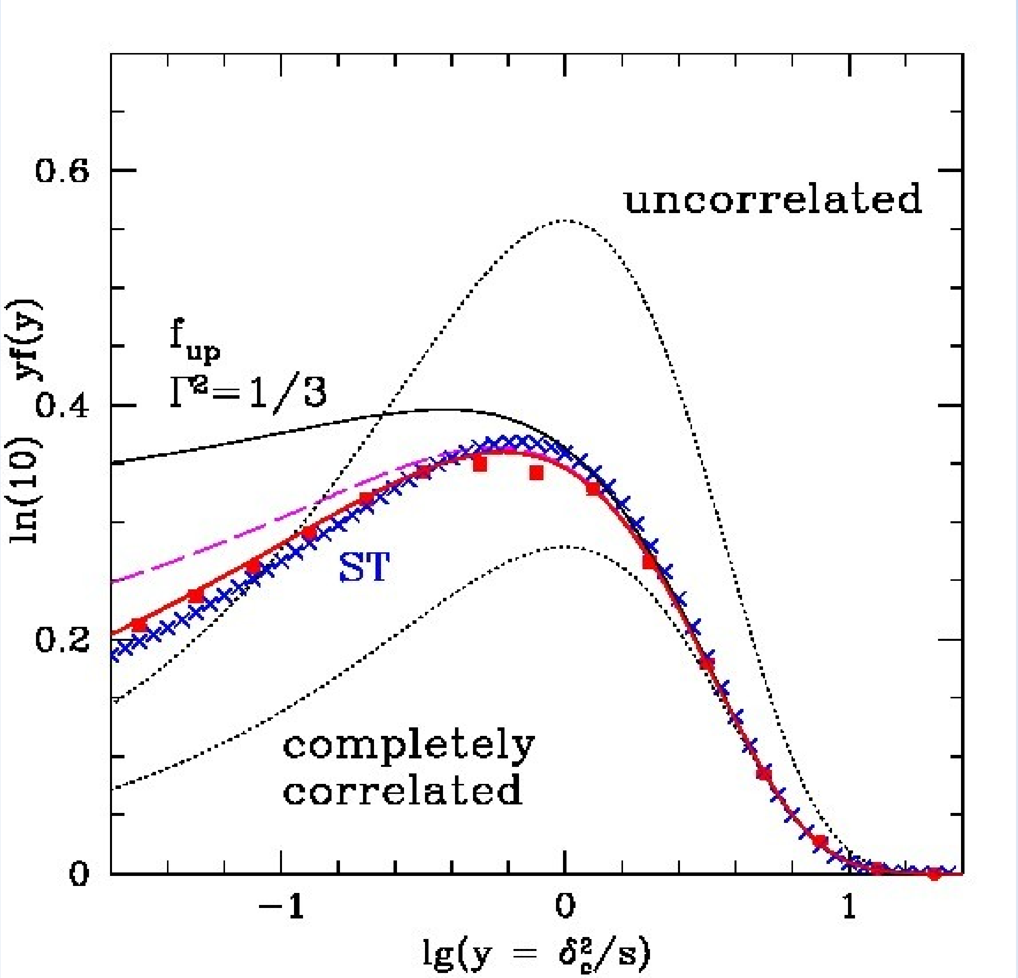
- Does this $f(s)$ reproduce N-body mass function? Not really...

HOWEVER:

- Halos form near density peaks
- Off-peak walks must have lower threshold
- Rescaled “mean” barrier:

$$b = \delta_c \to \sqrt{0.7}\delta_c$$

IT WORKS!!



MM & Sheth (2013)

Adding non-Gaussianity: MF

- Same formalism for non-Gaussian initial conditions:

$$f(s) = \left[\frac{\mathrm{d}}{\mathrm{d}s}\int_{b(s)}^{\infty}\mathrm{d}\delta\, p(\delta;s)\right]\left[\frac{1+\mathrm{erf}(X/\sqrt{2})}{2} + \frac{e^{-X^2/2}}{2X\sqrt{2\pi}} + \dots\right]$$

old PS result — Gaussian structure — Small! (computed)

$$X = \frac{\text{mean of } p(v|b) - b'}{\sqrt{\text{var of } p(v|b)}}$$

full NG pdf

MM & Sheth (2013)

- Non-perturbative in NG parameters: $p(\delta;s)$ is the exact NG pdf!

$$p(b;s) = \frac{1}{\sqrt{2\pi}}\exp\left[-\frac{b^2}{2s} + \mu\frac{b^3}{3!} + \dots\right] \qquad \left[\mu = \frac{\langle\delta^3\rangle}{s^3} \sim f_{\mathrm{NL}}\right]$$

- Residual NG corrections are perturbative, sometimes not there at all (e.g. NG from shear field)

MM & Sheth (2014)

Light vs Mass: Halo Bias

- Relation between halo abundance δ_h and underlying DM density:

$$\delta_h = b_1 \delta_0 + b_2 \delta_0^2 + \dots$$

- Usually, computed from $p(\delta_h;s|\delta_0;s_0)$. Is there a better way?

- Yes, there is! Expanding halo correlation functions in terms of DM correlation functions

Light vs Mass: Halo Bias

- Take the most generic dependence on the matter field:

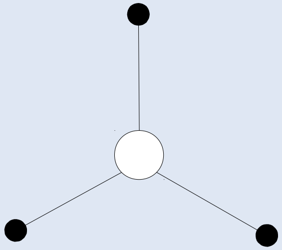
$$\delta_h(\mathbf{x}) = \sum_{k=0}^{\infty} \frac{1}{k!} \int \mathrm{d}^3y_1 \dots \mathrm{d}^3y_k \, b_k(\mathbf{x}-\mathbf{y}_1, \dots, \mathbf{x}-\mathbf{y}_k)\delta(\mathbf{y}_1)\dots\delta(\mathbf{y}_k)$$

Bernardeau et al (2008); Matsubara (2011)...

- Compute connected correlation functions of δ_h and δ :

$$\langle \delta_h(\mathbf{x})\delta(\mathbf{z}_1)\dots\delta(\mathbf{z}_n)\rangle_c = \int \mathrm{d}^3\mathbf{x}_1 \dots \mathrm{d}^3\mathbf{x}_n \underbrace{\left\langle \frac{\delta^n \delta_h(\mathbf{x})}{\delta\delta(\mathbf{x}_1)\dots\delta\delta(\mathbf{x}_n)} \right\rangle}_{\text{Bias functions}} \prod_{j=1}^{n} \langle \delta(\mathbf{x}_j)\delta(\mathbf{z}_j)\rangle$$

- δ_h acts as an effective vertex for δ :



Halo Bias from Excursion Sets

- Use the UPWARDS approximation. Only two variables!

$$1+\delta_h = \frac{1}{f(s)}\left[\frac{\mathrm{d}}{\mathrm{d}s}\vartheta(\delta_s - b)\right]\vartheta(\delta'_s - b')$$

$$\frac{\delta}{\delta\delta(\mathbf{y})} \to -W_R(\mathbf{y})\frac{\partial}{\partial b} - W'_R(\mathbf{y})\frac{\partial}{\partial b'}$$

- The real space bias functions become easy:

$$c_1(m,\mathbf{y}) = c_{10}W_R(\mathbf{y}) + c_{11}W'_R(\mathbf{y})$$

$$c_n(m,\mathbf{y}_1,\dots,\mathbf{y}_n) = \prod_{i=1}^{n} W_R(\mathbf{y}_i)\left[c_{n0} + c_{n1}\sum_{j=1}^{n}\frac{W'_R(\mathbf{y}_j)}{W_R(\mathbf{y}_j)} + \dots\right]$$

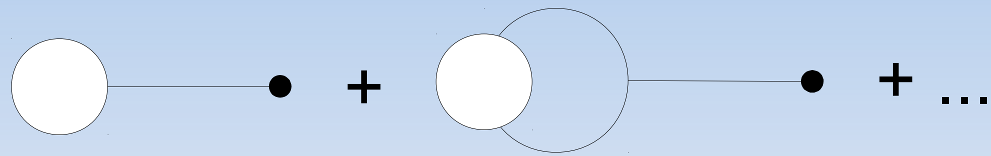
$$c_{nk} = \frac{(-1)^n}{f(s)}\frac{\partial^{n-k}}{\partial b^{n-k}}\frac{\partial^k}{\partial b'^k}f(s)$$

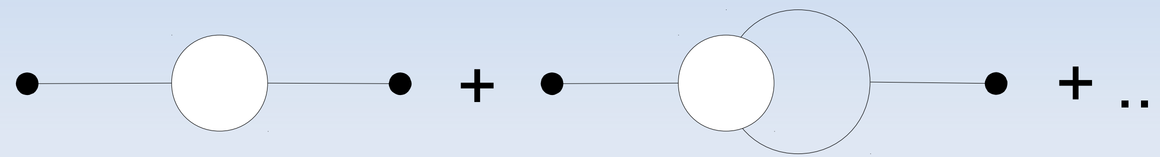
MM, Paranjape & Sheth (to appear)

- Mass function $n(M)$ is the generator of the bias coefficients (not specific to excursion sets)

Adding non-Gaussianity: Bias

- Expand halo-matter n-point functions in matter polyspectra

$\langle \delta_h(\mathbf{x})\delta(\mathbf{z}_1)\rangle_c =$ 

$\langle \delta_h(\mathbf{x})\delta(\mathbf{z}_1)\delta(\mathbf{z}_2)\rangle_c =$ 

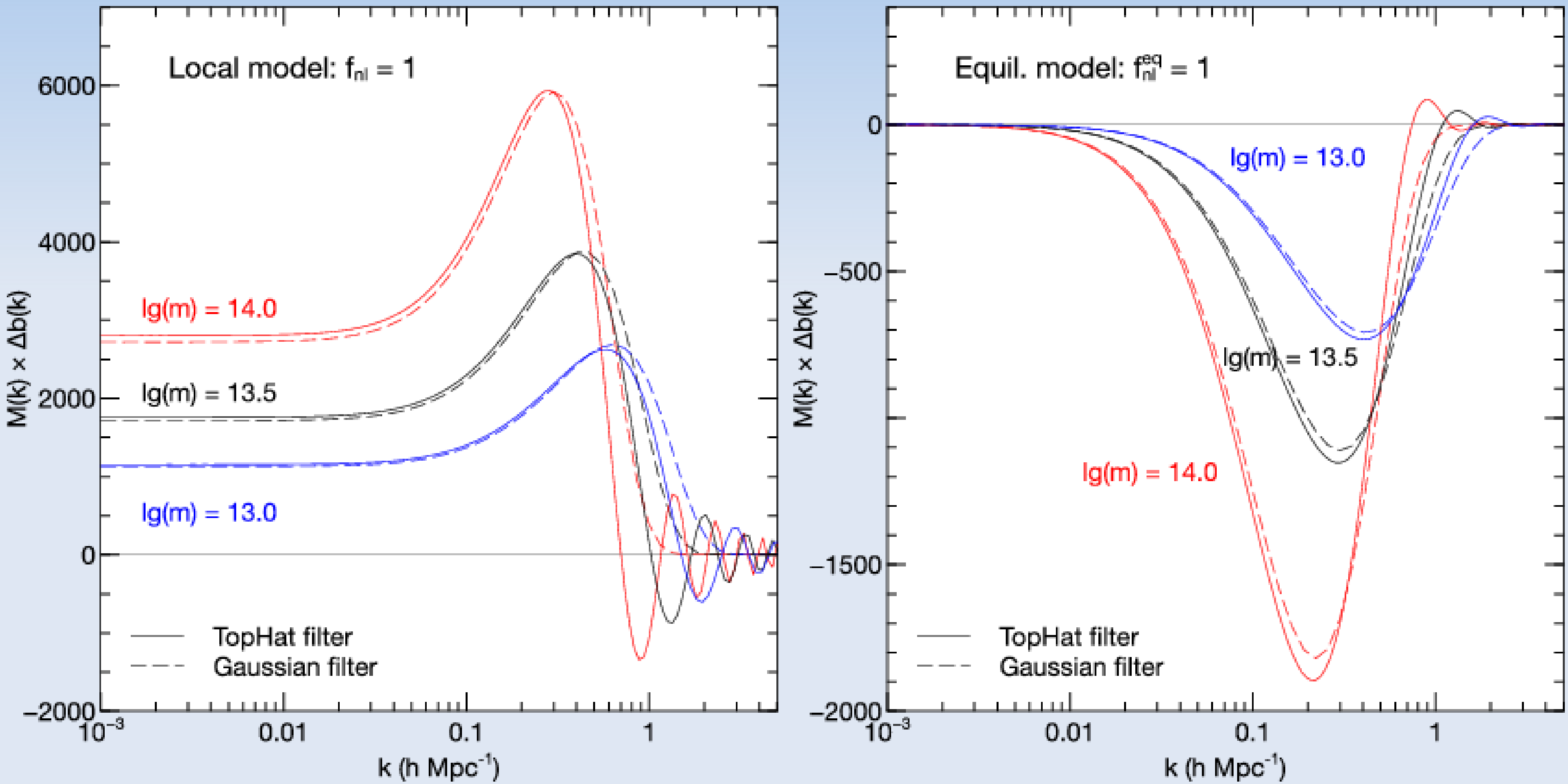
- Excursion sets with UPWARDS approximation:

$$\langle \delta_h \delta_0 \rangle \simeq c_{10}\,\langle \delta\delta_0\rangle + c_{11}\,\langle \delta'\delta_0\rangle + \frac{1}{2}\left[c_{20}\,\langle \delta^2\delta_0\rangle + 2\,c_{21}\,\langle \delta'\delta\delta_0\rangle + c_{22}\,\langle \delta'^2\delta_0\rangle\right] + \dots$$

$$\Rightarrow \quad \Delta c_1(k) = \frac{2 f_{\rm NL}^{\rm local}}{k^2 T(k)}\left[s\,c_{20} + c_{21} + \left\langle(\delta')^2\right\rangle c_{22} + \mathcal{O}(k^2)\right]$$

MM, Paranjape & Sheth (to appear)

Adding non-Gaussianity: Bias



MM, Paranjape & Sheth (to appear)

Profile, assembly bias, merger trees

- Bias depends on parameters other than δ_S at $S < s$:

$$f(s|\delta_S, V_S)$$

While δ is not Markovian, V nearly is. No other variables are relevant, nor other scales! At fixed concentration, formation history does not depend on the environment

MM and Sheth (2014)

- Insight on halo profile (actually, on the full pdf of matter around the halo) by reversing the perspective:

$$p(\delta_S|\delta_h(m))$$

- Analytical merger trees including halo concentration are also straightforward:

Work in progress...

Conclusions

- Full understanding of the random walk problem for any barrier
- Rescaling the spherical collapse barrier reproduces the N-body mass function
- Straightforward inclusion of NG of any type
- Self-contained prediction of bias functions, new strategies to measure them in simulations
- To do: check bias against N-body simulations, study space correlation between walks (peaks?)
- Interesting possibilities (?) for primordial non-Gaussianity
- Applications to RDS, merger trees, 21-cm from reionization, finance...

Thanks!!