Modified Gravity and the Cascading DGP model

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Outline

1. The cosmic acceleration and Modified Gravity
2. Braneworlds and the Cascading DGP model
3. The nested branes realization of Cascading DGP
4. The critical tension and ghosts

1. The cosmic acceleration and Modified Gravity

Standard cosmology before 2000

- Homogeneity and isotropy
- SM matter + CDM
- General Relativity

⟹

- Abundance of elements
- CMB
- Structure formation (given initial perts.)

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) < 0\,!$$

The late time acceleration

- Adding a cosmological constant term, observational data on:
  - type IA supernovae
  - CMB anisotropies
  - large scale structure

are best fitted by:

- a universe whose energy density is dominated by the cosmological constant, and
- which has recently entered an accelerated phase

$$\frac{\ddot{a}}{a} > 0 \quad !$$

Why is this a problem?

At a fundamental level, we don't understand the $\Lambda\, g_{\mu\nu}$ term:

- Λ may be a second characteristic scale of gravity ... Padmanabhan '06
  - then the universe is very fine tuned: coincidence problem
- ... or the semiclassical manifestation of vacuum energy Weinberg '89
  - observed and predicted values differ by 60 – 120 orders of magnitude
- or there may be both a "bare" and a semiclassical Λ
  - extremely fine tuned cancellation needed $\dfrac{\Lambda - \Lambda_{vac}}{\Lambda} \sim 10^{-60}$, which is not technically natural

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Alternative explanations

- Homogeneity and isotropy → Lemaître-Tolman-Bondi, back-reaction
- SM matter + CDM → Dark Energy (quintessence, k-essence, . . .)
- General Relativity → Modified Gravity (f(R), braneworlds, massive gravity, . . .)

Possibly the solution to the problem is more fundamental. . .

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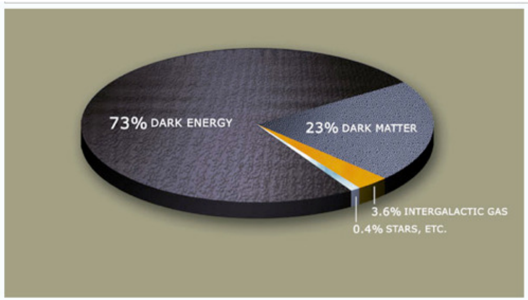
Dark Energy or Modified Gravity?

The energy budget is dominated by a component which is undetected in lab and has exotic properties $\rho + 3\,p < 0$ ...



...or may the data simply suggest that GR is inadequate at cosmological scales?

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Infrared modifications of gravity

- To test the latter idea: build theories of gravity whose predictions differ from GR's on ultra-large scales
  - and reproduce GR predictions where they are well tested
- aim: find "standard" cosmological solutions which flow to de-Sitter
  - explain acceleration as self-acceleration, without fluids with negative pressure
- this may also help with the cosmological constant problem
  - t'Hooft naturalness, degravitation, ...

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Typical problems in modified gravity

Modifying GR always introduces new degrees of freedom

- they have to be screened on lab/astrophysical scales
  - Vainshtein, Chameleon, Symmetron, ...
- quite often they are ghosts
  - e.g. BD ghost in massive GR, bending mode in s.a. DGP
- ghost $\equiv$ mode with negative kinetic energy
  - catastrophic instability of the vacuum

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2. Braneworlds and the Cascading DGP model

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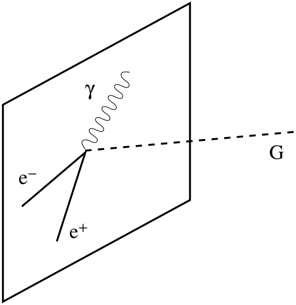
Extra dimensions and braneworlds



Codimension $\equiv$ dim (bulk) - dim (brane)

- braneworld: matter and gauge fields are confined around a submanifold of the ambient spacetime, gravity is not
- realizable in field theory, and essential in string theory
- has a finite thickness and an internal structure, defined by the confination mechanism

Thin limit description of braneworlds

- We just want to study the effect of localization on gravity
  - we don't even have a model of the internal dynamics, which anyway is likely to be very difficult to solve
- and we focus on scales $\gg$ thickness

  ⟶ a thin limit description is very useful (parallel: surface charge in electrostatics)
- the thin limit of codimension-1 branes is well-defined
  - pillbox integration of Einstein equations: Israel junction conditions   Israel '66
- codimension > 1: the thin limit is not well-defined   Geroch, Traschen '87
  - we need to give information about the internal structure

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Brane induced gravity and the DGP model

- The DGP model (cod-1) Dvali, Gabadadze, Porrati '00

$$S = \int_{\mathcal{B}} d^5X \sqrt{-g}\; M_5^3\; R + \int_{\mathcal{C}_1} d^4x \sqrt{-\tilde{g}} \left(M_4^2\; \tilde{R} + \mathscr{L}_M \right)$$

  - induced gravity term on the brane: either phenomenological, or coming from loop corrections

- weak gravity: $r_c \equiv M_4^2 / 2\, M_5^3$ sets the crossover scale
  - gravity is 4D when $r \ll r_c$, is 5D when $r \gg r_c$

- admits self-accelerating cosmological solutions Deffayet '01
  - acceleration happens geometrically when $H \sim 1/r_c$

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The idea is compelling, but. . .

- DGP s.a. solutions are observationally ruled out   Fang et al ’08
  - statistical discrepancy of $\sim 5\,\sigma$
- Perturbations around s.a. solutions contain a ghost mode
  Luty, Porrati, Rattazzi ’03
- Natural extension: higher codimension with induced gravity
  - they may realize the degravitation phenomenon
  - expected to fit the cosmological data better than DGP
- not guaranteed higher codimension helps with ghosts
  - pure cod-2 DGP seems to have more severe ghost problems (still debated and regularization-dependent)

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Pure codimension-2 branes and divergences

- Localized sources produce divergent gravitational field on the thin brane
  Cline et al '03
  - unless a Gauss-Bonnet term in the bulk is included
    Bostock et al '04
- Not unexpected: also in electrostatics the "brane-to-brane" propagator diverges in the thin limit
  - simply due to the dimensionality of the configuration

⟶ finite thickness and internal structure are crucial both

- to have well-behaved gravity (avoid divergences)
- to single out a unique solution (thin limit issue)

...but the analysis becomes very complex and very model dependent

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An inspiring toy model

$$S = \int d^6X \Big[- M_6^4 \, \partial_A\phi \, \partial^A\phi - \delta(z) \, M_5^3 \, \partial_a\phi \, \partial^a\phi - - \delta(z)\delta(y) \left(M_4^2 \, \partial_\mu\phi \, \partial^\mu\phi - \phi \, T \right) \Big] \quad (1)$$

de Rham et al '08

- The propagator can be summed exactly on the $z = 0$ plane:
  - if $M_5^3 \neq 0$, the propagator at $z = y = 0$ is finite
  - if $M_5^3 \to 0$, the propagator at $z = y = 0$ diverges
  - it seems that the 5D piece acts as a regulator!
- Resembles a cod-2 brane embedded inside a cod-1 brane, both equipped with induced gravity
  - in the weak field approximation

A more realistic model

$$S = \int d^6X \left[M_6^4 \sqrt{g_6}\, R_6 + \delta(z)\, M_5^3 \sqrt{g_5}\, R_5 + \right.$$

de Rham et al '08

$$\left. + \delta(z)\delta(y) \sqrt{g_4} \left(M_4^2\, R_4 + \mathscr{L}_m \right) \right] \quad (2)$$

- The propagator for weak $h_{\mu\nu}$ remains finite at $z = y = 0$
  - divergence of $h_{\mu\nu}$ is "regularized" by the localized 5D action
- Weak gravity "cascades": $m_6 = M_6^4 / M_5^3$, $m_5 = M_5^3 / M_4^2$
  - 6D $\rightarrow$ 5D $\rightarrow$ 4D ($m_6 \ll m_5$) or 6D $\rightarrow$ 4D ($m_6 \gg m_5$)
- Localized tension at $z = y = 0$ $\longrightarrow$ critical tension λ_c
  - weak field: ghost for $\lambda < \lambda_c$, ghost-free for $\lambda > \lambda_c$

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Notes of caution

- ? Does the propagator sum to a smooth field for $z > 0$?
  - in (1), (2): propagator summed only at $z = 0$
  - by treating a Dirac delta interaction as a perturbation

- How does these results relate to the general picture?

- Cascading configuration $\Leftrightarrow$ vast variety of different internal structures
  - is the cod-1 brane an efficient “regulator” in general?
  - is the thin limit well-defined for nested configurations?

- To settle this: recover (2) as thin limit of a covariant action
  - with generic internal structures

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The (6D) Cascading DGP model

$$S = M_6^4 \int_{\mathcal{B}} d^6X \sqrt{-g}\, R + M_5^3 \int_{\mathcal{C}_1} d^5\xi \sqrt{-\tilde{g}}\; M_5^3\, \tilde{R} + \int_{\mathcal{C}_2} d^4\chi \sqrt{-g^{(4)}} \Big(M_4^2\, R^{(4)} + \mathscr{L}_M \Big) \qquad (3)$$

de Rham et al '08

- If $\mathcal{C}_1$ and $\mathcal{C}_2$ are thin: as it stands (3) does not define a unique theory
  - to derive the e.o.m. we need to know how the fields behave at $\mathcal{C}_1$ and $\mathcal{C}_2$ (cannot assume beforehand they are smooth)
- If $\mathcal{C}_1$ and $\mathcal{C}_2$ are thick:
  - how to equip them with truly 5D and 4D ind. grav. terms?
  - how are the internal structures made/related?

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3. The nested branes realization of Cascading DGP

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How to approach this problem?

- We could consider compact higher dimensional regularizations of $\mathcal{C}_1$ and $\mathcal{C}_2$
  - introduces extra d.o.f. which remain alive in the thin limit
  - very interesting, but different from the original model

Instead of working with the most general class of nested configurations, we look for specific subsets of configurations hoping that the thin limit can be defined rigorously

- if it can be done, introducing the ind. grav. terms is trivial
- we look for *realizations* of the Cascading DGP idea

Nested branes and thickness hierarchy

- Thickness hierarchy: consider $\mathcal{C}_1$ much thinner than $\mathcal{C}_2$
  - $\sim$ thin cod-1 brane with a thick "ribbon" brane inside (not a "string")
- In this case, we can describe the system taking the (well defined) thin limit of the cod-1 brane
  - The system is described by the Einstein eq. in the bulk and by the Israel junction conditions at $\mathcal{C}_1$

$$2M_6^4 \left(\tilde{K}_{ab} - \tilde{K}\, \tilde{g}_{ab} \right) + M_5^3\, \tilde{G}_{ab} = \tilde{T}_{ab}^{(ribbon)}$$

- The hierarchy of scales permits to get rid of the $\mathcal{C}_1$ internal structure, but the internal structure of $\mathcal{C}_2$ is still there

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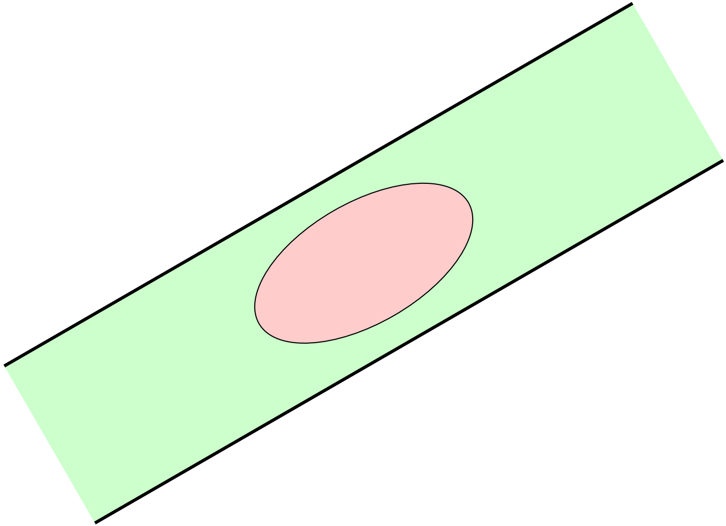
Thickness hierarchy: pictorial view



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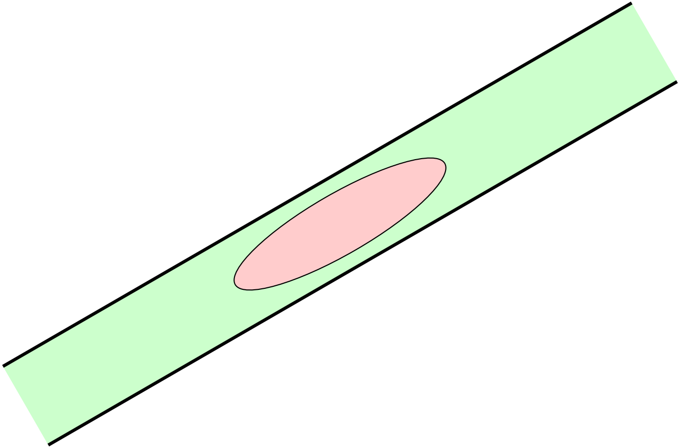

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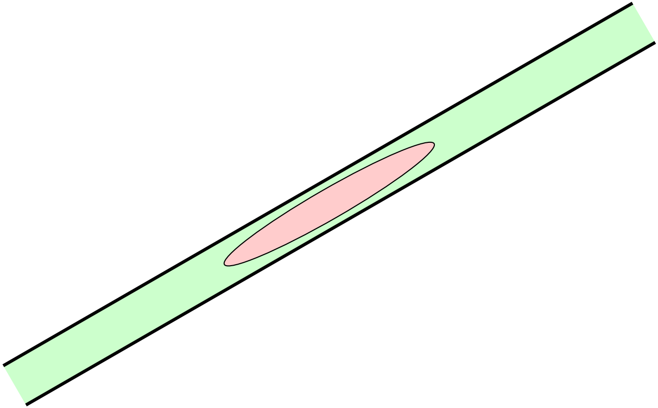

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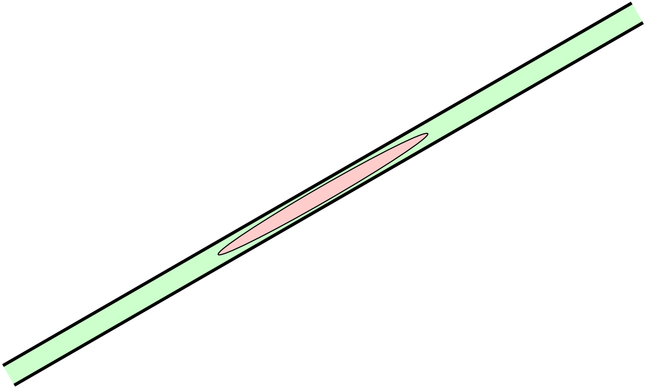

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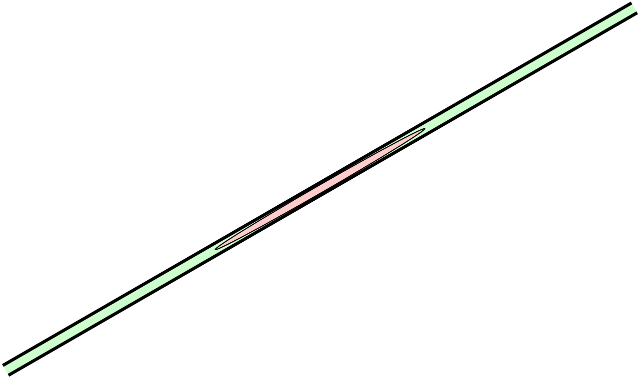

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Thin limit of the ribbon brane

- Is the thin limit of the ribbon brane *inside the thin cod-1 brane* well-defined? If yes,
  - the thin limit for the whole system is well-defined
  - trivial to equip $\mathcal{C}_1$ and $\mathcal{C}_2$ with 5D and 4D ind. grav. terms
  - realization of Cascading DGP!!
- Ribbon thin limit $\Leftrightarrow$ pillbox integration of the Israel j.c.
  - send $l_2 \to 0$ keeping $\int_{-l_2}^{+l_2} \hat{T}_{\mu\nu}^{(ribbon)} d\hat{\xi} = T_{\mu\nu}^{(4)}$ fixed
- How do metric and embedding behave in this limit?
  - The equations don't say this: need to propose an ansatz and verify a posteriori if it is consistent

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Ansatz for the behaviour of the fields

- Consider a thick pure tension source
  - vacuum energy concentrated in the ribbon brane
  - traslational symmetry in the 4D directions
- Exact solution: flat bulk, non-trivial embedding profile
  - thin limit well-defined: embedding develops a cusp at $\mathcal{C}_2$
  - singularity supported purely by the embedding
- General ansatz: $g_{AB} \rightarrow$ smooth, $\varphi^A \rightarrow$ cuspy
  - where g_{AB} needs not be flat, and the opening of the cusp may vary along the 4D directions

Nested branes realization of Cascading DGP

At linear order around pure tension solutions:

- the ansatz is consistent for every choice of $T^{(4)}_{\mu\nu}$ Sbisà and Koyama '14 (I)
- The thickness hierarchy indeed permits to define the thin limit for the whole system!!
  - bulk Einstein eqs. (free) + cod-1 junction conditions (free) + cod-2 junction conditions (sourced)
- We can equip $\mathcal{C}_1$ and $\mathcal{C}_2$ with 5D and 4D ind. grav. terms
  - Nested branes realization of the Cascading DGP model

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Geometrical explanation of regularization

- By our ansatz, g_{AB} and φ^A converge to continuous functions
  - cod-1 and cod-2 induced metrics are continuous, so gravity on the thin cod-2 brane is finite
- 4D scalar sector: keeping only the diverging pieces

$$-2M_6^4\,\delta\hat{K}_{\xi\xi}\,\eta_{\mu\nu} + M_5^3\,\delta\hat{G}_{\mu\nu} = \hat{T}^{(ribbon)}_{\mu\nu}$$

  - if $M_5^3 = 0$, the ansatz is consistent only if $\hat{T}^{(ribbon)}_{\mu\nu} \propto \eta_{\mu\nu}$
- The 5D brane regulates the divergence because it allows to accomodate a localized e. m. tensor with a continuous configuration!
  - geometrical understanding of gravity regularization

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4. The critical tension and ghosts

Searching for the ghost

- The ghost was found among linear perturbations around pure tension solutions de Rham et al '08, '10
  - in a 4D scalar-vector-tensor decomposition, it belongs to the scalar sector
  - bending modes in the extra dimensions are 4D scalars
- We study the scalar sector in the nested branes realization
  - use gauge-invariant variables both for metric and bending
- master variables of the scalar sector: π, $\delta\varphi_{\perp}$
  - metric: $h_{\mu\nu} = \mathscr{H}_{\mu\nu} + \partial_{(\mu} V_{\nu)} + \pi\, \eta_{\mu\nu} + \partial_\mu \partial_\nu \varpi$
  - bending: $\delta\varphi_{\perp} = \bar{n}_A\, \delta\varphi^A$

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The critical tension and 4D effective ghosts

- π and $\delta\varphi_{\perp}$ obey a system of coupled differential equations
  - on the cod-2 brane: master equation for π alone
- $\partial_{\bar{\mathbf{n}}}\, \pi$ is present: still need to solve the 6D problem
  - 4D limit: $|m_6\, \partial_{\bar{\mathbf{n}}}| \ll |\Box_4|$, $|m_5\, \partial_{\bar{\mathbf{n}}}| \ll |\Box_4|$ (decouples $\mathcal{C}_2$)

$$3M_4^2 \left[1 - \frac{3}{2} \frac{m_5}{m_6} \tan\left(\frac{\lambda}{4M_6^4} \right) \right] \Box_4\, \pi = \mathcal{T}$$

- critical tension $\lambda_c = 4M_6^4 \arctan\left(2m_6/3m_5\right)$

Sbisà and Koyama '14 (II)

  - π is an effective ghost for $\lambda < \lambda_c$, is healthy for $\lambda > \lambda_c$

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Geometrical interpretation of the critical tension

At linear order, $\mathcal{T}$ excites π via two separate channels

- directly, via the 4D induced gravity term
  - in a ghostly and λ-independent way
- indirectly, via $\delta\varphi_{\perp}$ and the 5D induced gravity term
  - in a healthy and λ-dependent way
  - $\mathcal{T} \rightarrow$ cod-2 brane moves $\rightarrow$ cod-1 brane moves $\rightarrow$ $\delta\varphi_{\perp}$ excites π on cod-1 brane $\rightarrow$ π excitation on cod-2 brane
- λ_c originates from the competition between the channels
  - the most effective channel decides if π is a ghost

Ghost-free regions in parameters space

- Our λ_c differs from the literature's when $m_6 \gtrsim m_5$

$$4M_6^4 \arctan\left(2m_6/3m_5\right) \quad \text{vs} \quad 8\, m_6^2\, M_4^2/3$$

Sbisà and Koyama '14 (II)

(A) de Rham et al '08
(B) de Rham, Khoury, Tolley '10

- the tension cannot exceed $\lambda_M = 2\pi M_6^4$
  - for geometrical reasons, deficit angle $< 2\pi$
- our result: $\lambda_c < \lambda_M$ for every m_6, m_5
  - always possible to choose λ s.t. theory is ghost-free
- dRKT's: $\lambda_c < \lambda_M$ only if $m_6 \lesssim m_5$
  - half of the parameters space is phenomenologically excluded

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Understanding the difference . . .

- (A): propagator summation with branes kept fixed
  - gauging away the bending modes needs a singular transformation: the bulk metric becomes discontinuous. The summation is suspicious!
- (B): pure tension source: exact solution with straight branes, the bulk metric becomes discontinuous
  - . . . in the thin limit of the cod-2 brane: consistent with our ansatz
- In fact our background solution and (B)'s are connected by a gauge trasformation
  - which indeed proves we are studying the same set-up
  - the difference must show up at perturbation level

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Subtleties in the pillbox integration

- culprit: the cod-2 junction conditions
  - $\equiv$ thin limit of the pillbox integration across the cod-2 brane
- different hypothesis about the singular structure
  - we assume the bending modes $\delta\varphi^z$, $\delta\varphi^y$ are continuous
  - dRKT assumes the normal projection $\delta\varphi_\perp$ is continuous
- test: consider exact solutions for the perturbations when the ribbon brane is thick
  - perform the integration numerically, for several thicknesses, and study limit $\to 0^+$
  - compare numerical integration with our result and dRKT

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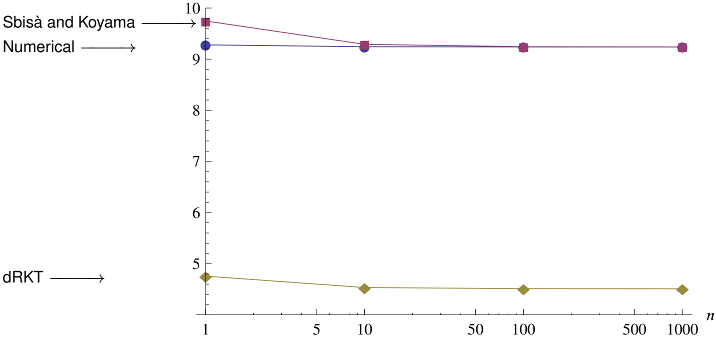
Numerical result

Pure tension perturbation: exactly solvable both inside and outside the ribbon brane



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Thank you very much!

ArXiv :1406.3384 [hep-th] (PhD thesis)

ArXiv :1404.0712 [hep-th] (gravity regularization in Cascading DGP)

ArXiv :1405.7617 [hep-th] (critical tension in Cascading DGP)

ArXiv :1406.4550 [hep-th] (review on ghosts)

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