Cosmic and superconducting strings

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Cosmic strings (as field theoretical solutions)...
  form whenever a U(1) symmetry is spontaneously broken (Kibble mechanism)
- topological solitons, i.e. possess a topological charge
- width inversely proportional to mass
- ruled out as (main) source of primordial density perturbations (by CMB data)
- have analogues in condensed matter: flux tubes in type-II superconductors, vortices in superfluid helium
- space-time has deficit angle $\delta = 8\pi GU$, $U$: energy per unit length $\Rightarrow$ gravitational lensing
Cosmic string loops

- form in the evolution of cosmic string networks
- decay due to string tension under emission of gravitational radiation ⇒ important energy loss mechanism ⇒ contribution of strings to total energy density of the universe is kept constant (“scaling regime”)

BUT: loops could be stable if they carry additional internal structure (bosonic or fermionic currents)

⇒ **superconducting strings**
Fundamental (F-) strings (of string theory) ...
- have zero width
- have tension close to the Planck scale
- end on D-branes
- D1-brane = D-string

Connection between cosmic strings and fundamental strings ???

NO: perturbative strings as cosmic strings ruled out (Witten (1985))
**YES**: cosmic superstrings are formed in inflationary models originating from string theory

- D-, F- and bound states of $p$ F-strings and $q$ D-strings ($p$-$q$-strings) are formed in **brane inflation**

Jones, Stoica, Tye (2002); Sarangi, Tye (2002)
... and also: Hybrid inflation (Linde (1994))

- two scalar fields
- inflation ends due to spontaneous symmetry breaking
- cosmic strings form generically at the end of hybrid inflation in Supersymmetric Grand Unified Theories (Jeannerot, Rocher, Sakellariadou (2003))
In agreement with PLANCK 2013?

- Inflationary models
  - Data favours "simple", single-field models
  - BUT: where do these models come from?
  - String inflationary models are **not** ruled out
    (Burgess, Cicoli, Quevedo, JCAP 11 (2013))

- Abelian-Higgs strings
  (Ade et al. [Planck Collaboration], arxiv: 1303.5085)

\[ GU/c^2 \leq 3.2 \cdot 10^{-7} , \quad f_{10} = 0.028 \]

*U*: energy per unit length

*f*_{10}: fraction of contribution to power spectrum at \( \ell = 10 \)
The $U(1) \times U(1)$ model

$U(1) \times U(1)$ gauge field model with Lagrangian density

$$\mathcal{L} = D_\mu \phi (D^\mu \phi)^* - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + D_\mu \xi (D^\mu \xi)^* - \frac{1}{4} H_{\mu\nu} H^{\mu\nu} - V(\phi, \xi)$$

with

$$D_\mu \phi = \nabla_\mu \phi - i e_1 A_\mu \phi \quad , \quad D_\mu \xi = \nabla_\mu \xi - i e_2 B_\mu \xi$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad , \quad H_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

where

$\phi, \xi$: complex scalar fields

$A_\mu, B_\mu$: U(1) gauge fields

$e_1, e_2$: gauge couplings
The potential

\[ V(\phi, \xi) = \frac{\lambda_1}{4} (\phi \phi^* - \eta_1^2)^2 + \frac{\lambda_2}{4} (\xi \xi^* - \eta_2^2)^2 + \lambda_3 (\phi \phi^* - \eta_1^2)(\xi \xi^* - \eta_2^2) \]

\( \eta_1 \neq 0, \eta_2 \neq 0 \)

\( \lambda_1 > 0, \lambda_2 > 0 \): self interaction couplings

\( \lambda_3 > -\frac{1}{2} \sqrt{\lambda_1 \lambda_2} \): interaction coupling
The potential

1. $4\lambda_3^2 < \lambda_1 \lambda_2$
   - Minimum at $(\phi^2, \xi^2) = (\eta_1^2, \eta_2^2)$
   - $\Rightarrow$ spontaneous symmetry breaking of both U(1)s
   - describes two cosmic strings
     - for $\lambda_3 = 0$: two non-interacting Nielsen-Olesen strings (Nielsen, Olesen (1973))
     - $\lambda_3 < 0$: toy model for bound states of $p$ F-strings and $q$ D-strings, i.e. $p$-$q$-strings (Saffin (2005))

2. $4\lambda_3^2 > \lambda_1 \lambda_2$
   - Minimum at $(\phi^2, \xi^2) = (0, \eta_2^2 + \frac{2\lambda_3}{\lambda_2} \eta_1^2)$
     or $(\phi^2, \xi^2) = (\eta_1^2 + \frac{2\lambda_3}{\lambda_1} \eta_2^2, 0)$
   - $\Rightarrow$ spontaneous symmetry breaking of only one of the U(1)s
   - describes superconducting strings
(Interacting) Cosmic strings

- Cylindrically symmetric Ansatz

\[ \phi(\rho, \varphi) = \eta_1 h(\rho) e^{in\varphi} , \quad \xi(\rho, \varphi) = \eta_2 f(\rho) e^{im\varphi} \]

\[ A_\mu d\mathbf{x}^\mu = \frac{1}{e_1} (n - P(\rho)) d\varphi , \quad B_\mu d\mathbf{x}^\mu = \frac{1}{e_2} (m - R(\rho)) d\varphi \]

$n, m$: winding numbers

- magnetic fields \( \vec{B}_i = B_{i,z} \vec{e}_z \), \( i = 1, 2 \) with

\[ B_{1,z} = - \frac{1}{e_1} \frac{dP/d\rho}{\rho} , \quad B_{2,z} = - \frac{1}{e_2} \frac{dR/d\rho}{\rho} \]

- quantized magnetic fluxes

\[ \Phi_1 = - \frac{2\pi n}{e_1} , \quad \Phi_2 = - \frac{2\pi m}{e_2} \]
\( \lambda_3 = 0: \) Nielsen-Olesen strings

- scalar core width \( \sim (\text{Higgs mass})^{-1} = M_{H,i}^{-1} = (\sqrt{2\lambda_i \eta_i})^{-1}, \quad i = 1, 2 \)
- width of flux tubes \( \sim (\text{gauge boson mass})^{-1} = M_{W,i}^{-1} = (e_i \eta_i)^{-1}, \quad i = 1, 2 \)
- \( T_0^0 = T_Z^Z \): energy per unit length \( U = \text{string tension} \ T \)
- energy per unit length \( U = U_1 + U_2 \) where

\[
U_1 = 2\pi \eta_1^2 n g_1(M_{H,1}^2/M_{W,1}^2), \quad U_2 = 2\pi \eta_2^2 m g_2(M_{H,2}^2/M_{W,2}^2)
\]

\( g_i = O(1) \) and weak dependence on \( M_{H,i}^2/M_{W,i}^2 \)
**λ₃ = 0: Nielsen-Olesen strings**

- for $\lambda_1 = 2e_1^2$, $\lambda_2 = 2e_2^2$ ⇒ **BPS limit:**
  - width of flux tube = width of scalar core
  - saturate energy bound $U_1 = 2\pi \eta_1^2 n$ and $U_2 = 2\pi \eta_2^2 m$
  - Equations of motion: 1.order
- Typical solution: BPS and $n = 1$, $m = 2$
\( \lambda_3 < 0: \ p-q\text{-strings} \)

(Saffin (2005))

- \( p \rightarrow n, \ q \rightarrow m \)
  - \Rightarrow\) interaction of \( n \)-string with \( m \)-string

- no longer BPS

- binding increases with
  - decreasing \(-1 < \lambda_3 < 0\)
  - increasing \( n \) and \( m \)

- formation of bound states \(\Rightarrow\) effective energy loss
  mechanism to reach scaling regime for \((p, q)\) networks
Gravitational effects

- Action
  \[ S = \int d^4x \sqrt{-g} \left( \frac{R}{16\pi G} + \mathcal{L} \right) \]

- Ansatz for the metric
  \[ ds^2 = N^2(\rho)dt^2 - d\rho^2 - L^2(\rho)d\varphi^2 - N^2(\rho)dz^2 \]

- weak gravitational fields
  \[ ds^2 = dt^2 - d\tilde{\rho}^2 - \tilde{\rho}^2 d\tilde{\varphi}^2 - dz^2 \]

where \( \tilde{\varphi} = (1 - 4GU)\varphi \) with \( 0 \leq \tilde{\varphi} < 2\pi(1 - 4GU) \)
  - locally identical to flat space-time
  - globally: deficit angle \( \delta = 8\pi GU \)
globally regular for $\delta < 2\pi$, i.e. $G \lesssim (8\pi \eta_1^2 n)^{-1}$

**String solutions**

$$N(\rho \to \infty) \to c_1, \quad L(\rho \to \infty) \to c_2\rho + c_3$$

where $c_1$, $c_2$, $c_3$ constants and $\delta = 2\pi(1 - c_2)$

**Melvin solutions** (have no flat space-time counterpart):

$$N(\rho \to \infty) \to a_1 \rho^{2/3}, \quad L(\rho \to \infty) \to a_2 \rho^{-1/3}$$

where $a_1$, $a_2$ constants
Gravitational effects

- For $G > (8\pi \eta_1^2 n)^{-1}$: singular solutions
  - supermassive string solutions \((\text{Garfinkle, Laguna (1989); Ortiz (1991)})\)

  $$\delta > 2\pi \quad \text{with} \quad L(\rho = \rho_s) = 0 \quad , \quad N(\rho = \rho_s) \text{ finite}$$

- Kasner solutions \((\text{Christensen, Larsen, Verbin (1999)})\)

  $$L(\rho = \rho_k) = \infty \quad , \quad N(\rho = \rho_k) = 0$$

- for $\lambda_1 = 2e_1^2$, $\lambda_2 = 2e_2^2$, $\lambda_3 = 0$ are still BPS \((\text{Linet (1987)})\)
Gravitating \( p-q \)-strings

(B.H. & J. Urrestilla (2008))

- \( G \) fixed
  - singular solutions can be made regular by decreasing \( \lambda_3 \)
  - \( \delta \) decreases for decreasing \( \lambda_3 \), i.e. increasing interaction
- \( \lambda_3 \) fixed
  - \( \delta \) increases for \( G \) increasing

\[
G = \frac{1}{16\pi\eta_1^2}
\]

\[
\lambda_3 = -0.95
\]

\[
8\pi G \eta_1^2 \lambda_3 = -0.95
\]
Cosmic strings in de Sitter

(B.H. & Y. Brihaye (2008))

- only one string: $R = f \equiv 0$, $m = 0$, $\lambda_2 = \lambda_3 = \eta_2 = 0$
- de Sitter background space-time

$$ds^2 = \left(1 - \frac{r^2}{l^2}\right) dt^2 - \left(1 - \frac{r^2}{l^2}\right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

- de Sitter radius $l = \sqrt{3/\Lambda}$, $\Lambda$ positive cosmological constant
- For $\Lambda > 24 M_W^2$, $\Lambda > 2/3 M_H^2$

$$P(\rho \gg 1) = P_0 \rho^a$$
$$f(\rho \gg 1) = 1 - f_0 \rho^b$$

$P_0, F_0, a, b$ : parameter dependent constants
Cosmic strings in de Sitter

(B.H. & Y. Brihaye (2008))

- For $\Lambda < 24M_W^2$, $\Lambda < 2/3M_H^2$
  
  \[ P(\rho >> 1) = P_0 \rho^{-1/2} \sin \left( \sqrt{-R_1/2} \log \rho + \phi_1 \right) \]
  
  \[ f(\rho >> 1) = 1 - f_0 \rho^{-3/2} \sin \left( \sqrt{-R_2/2} \log \rho + \phi_2 \right) \]

$P_0, f_0, R_1, R_2, \phi_1, \phi_2$: parameter dependent constants

\Rightarrow oscillating

- deficit angle

\[ \delta \approx 8\pi GU \left( 1 + \frac{16}{9} \frac{\Lambda}{M_H^2} \right) \]

increases for $\Lambda$ increasing
Superconducting strings

- let $e_2 = 0$, only one $U(1)$ is gauged
- Ansatz for “standard” string

$$\phi(\rho, \varphi) = \eta_1 h(\rho) e^{in\varphi}, \quad A_\mu dx^\mu = \frac{1}{e_1} (n - P(\rho)) \, d\varphi$$

- “neutral” current carrier field

$$\xi(\rho, z, t) = \eta_1 f(\rho) e^{ikz - \omega t}, \quad k \text{ integer}$$

$f(\rho \to \infty) \to 0 \Rightarrow$ unbroken $U(1)$

$f(\rho = 0) \neq 0 \Rightarrow$ “condensate” inside the string
Conserved quantities

- Conserved Noether current in $z$-direction $J_z = k \int dx^3 f^2$
- Conserved Noether charge $Q = \omega \int dx^3 f^2$
- Conserved momentum $P_z = \omega J_z = \omega k \int dx^3 f^2$
Parameter restrictions

- Only one U(1) broken $\Rightarrow 4\lambda_2^2 > \lambda_1 \lambda_2$
- Minimum of potential at $(\phi^2, \xi^2) = (\eta_1^2 + \frac{2\lambda_3}{\lambda_1} \eta_2^2, 0)$
  $\Rightarrow \lambda_1 \eta_1^4 > \lambda_2 \eta_2^4$
- **New** restriction (B.H. & B. Carter (2008)):
  Non-vanishing condensate inside string, i.e. $f(0) \neq 0$ only for
  $c\lambda_3 \eta_1^4 < \lambda_2 \eta_2^4$

where $c = O(1)$ and $c \lesssim 2.8$
Stability of superconducting strings

Stability criterion (B. Carter (1989)):

*speed of transversal (T) & longitudinal (L) perturbations real*

\[ c_T^2 = \frac{T}{U} > 0 \quad , \quad c_L^2 = -\frac{dT}{dU} > 0 \]

- energy per unit length

\[ U = 2\pi \int \rho d\rho T^{00} \]

with \( T^{00} = 2\omega^2 f^2 - \mathcal{L} \)

- tension

\[ T = -2\pi \int \rho d\rho T^{zz} \]

with \( T^{zz} = 2k^2 f^2 + \mathcal{L} \)
Stability of superconducting strings

Question: how does

\[ L = 2\pi \int \rho d\rho \mathcal{L} \]

depend on \( \omega \) and \( k \)?

• Suggestion (P. Peter (1992))

\[ L = -m^2 - \frac{1}{2} m^2 \ln(1 + \delta^2 w) \quad , \quad w = k^2 - \omega^2 \]

• \( \delta^* = 1/ m_\xi \) with \( m_\xi = 2\lambda_3 \eta_1^2 - \lambda_2 \eta_2^2 \) mass of carrier field

• \( m^2 = -L_{w=0} \)
A logarithmic equation of state

- Confirmation and evaluation of parameters
  (B.H. & B. Carter, PRD (2008))

\[ m^2 = \pi \eta_1^2 \ln \left( \frac{M_H}{\sqrt{2}M_W} \right), \quad m_*^2 = \pi m_\xi^2 \frac{\lambda_2 \eta_2}{\lambda_1 \lambda_2 \eta_1} \]

valid for \( M_W \ll M_H \)
Semilocal strings

Cosmic strings in a $SU(2)_{\text{global}} \times U(1)_{\text{local}}$ model
(T. Vachaspati & A. Achucarro (1991))

$$\mathcal{L} = D_\mu \Phi (D^\mu \Phi)^\dagger - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\lambda}{4} (\Phi^\dagger \Phi - \eta^2)^2$$

- $\Phi = (\phi, \xi)^T$ complex doublet invariant under global SU(2) transformations
- $D_\mu \Phi = \partial_\mu \Phi - ieA_\mu \Phi$ with $A_\mu : U(1)$ gauge field
Stability of semilocal strings

(B.H. & P. Peter, PRD (2012))

semilocal strings in $SU(2)_{\text{global}} \times U(1)_{\text{local}}$ are always unstable

$$c_L^2 \equiv -\frac{dT}{dU} \leq 0$$
Gravitational effects

For weak gravitational fields

\[ ds^2 = (1 + 2\psi(\tilde{\rho})) \, dt^2 - (1 - 2\psi(\tilde{\rho})) \, dz^2 - d\tilde{\rho}^2 - (1 - 2G(U + T))^2 \tilde{\rho}^2 \, d\varphi^2 \]

where \( \psi(\tilde{\rho}) = 4G(U - T) \ln(\tilde{\rho}/\tilde{\rho}_0) \)

• **deficit angle** \( \delta = 4\pi G(U + T) \)

• **attractive force** towards string \( \propto G(U - T)/\tilde{\rho} \)
**Gravitational effects**

Full gravitating system (B.H. & F. Michel, PRD (2012))

- **No** boost symmetry along z-axis
  \[
  ds^2 = N^2(\rho) dt^2 - d\rho^2 - L^2(\rho) d\varphi^2 - K^2(\rho) dz^2 , \quad K(\rho) \neq N(\rho)
  \]

- bigger currents possible for gravitating superconducting strings

![Graph showing current vs. W = k^2 - \omega^2 with \kappa = 8\pi G\eta_1]
Summary

- Link between cosmic strings $\leftrightarrow$ fundamental strings
- to understand observational effects it is important to ...  
  - ... understand gravitational effects (gravitational lensing,...)
  - ... know which type of strings form and how string networks evolve (CMB data, gravitational waves,...)
- in view of this ...
  - ... gauge and Higgs fields describing a cosmic string oscillate for all accepted values of the (positive) cosmological constant
  - ... gravitating $p$-$q$-strings can be made regular by increasing the interaction between the $p$- and the $q$-string
  - ... superconducting strings can be described by a logarithmic equation of state
  - ... semilocal strings are always unstable $\rightarrow$ cannot form stable vortons
Detection of cosmic strings?

- CMB data (power and polarization spectra)
  (e.g. P. Ade et al. [Planck Collaboration], arxiv: 1303.5085)

- motion of test particles in the space-time of cosmic strings
Introduction
The $U(1) \times U(1)$ model
Cosmic strings
Superconducting strings
Summary and Outlook

Selected References

- B. Hartmann, J. Urrestilla: *Gravitating (field theoretical) cosmic (p,q)-superstrings*, JHEP **0807** (2008) 006
- B. Hartmann, F. Michel: *Gravitating superconducting strings with timelike or spacelike currents*, Phys. Rev. D **86** (2012) 105026

...
Beware: vortices are everywhere!

Muito obrigada pela sua atenção!