Gravitational Waves from Coalescing Binaries

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Vitoria, November 27th 2013
Outline

1. Introduction to GW: sources and observations
   - General Relativity
   - Astrophysics signals

2. GW detector and sensitivity
   - The observatories
   - Data Analysis

3. The physics of GW production
   - How to compute waveforms
   - Bonus slides: How to test gravity

4. Bonus slides: Cosmology with GW’s
Gravitational Wave astronomy

Gravitational waves are produced by the coherent motion of large astrophysical masses $> M_\odot$

- neutron stars (e.g. pulsars)
- stellar mass black holes (e.g. binary x-ray sources)
- supermassive black holes (e.g. in galaxy centers)

or by cosmological production mechanisms

- cosmic strings
- inflation
- phase transitions
- ...
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Einstein equations and the TT-gauge

Weak field approximation, approximately Cartesian coor.:

\[ g_{\mu\nu} \rightarrow \eta_{\mu\nu} + h_{\mu\nu}, \quad ||h_{\mu\nu}|| \ll 1 \]

A gravity wave is the radiative, high frequency part of \( h_{\mu\nu} \) ⊇

1. 4 gauge degrees of freedom
2. 2 physical, radiative degrees of freedom
3. 4 physical, non-radiative degrees of freedom

1&3 propagate with “the speed of thought” (Eddington 1922):

After fixing the diffeomorphism invariance:

\[ h_{\mu\nu} = \left( \begin{array}{cc} -2\Phi & \Xi_i \\ \Xi_i & h_{ij}^{TT} + \theta \delta_{ij} \end{array} \right) \]

\[ \partial_i \Xi^i = h_{ij}^{TT} \delta^{ij} = \partial_i h_{ij} = 0: \text{4 d.o.f.'s eaten by gauge fixing, 6 left} \]

Einstein eq.'s: \[ \nabla^2 \Phi = \nabla^2 \Xi_i = \nabla^2 \Theta = 0 \]
\[ \Box h_{ij}^{TT} = 0 \]
Wave generation: localized sources

Einstein formula relates $h_{ij}$ to the source quadrupole moment $Q_{ij}$

\[ Q_{ij} = \int d^3 x \rho \left( x_i x_j - \frac{1}{3} \delta_{ij} x^2 \right) \]

\[ h_{ij} = \frac{2G_N}{r} \frac{d^2 Q_{ij}}{dt^2} \simeq \frac{2G_N \mu v^2}{r} \cos(2\phi(t)) \]

where:

\[ f = 1\text{kHz} \left( \frac{r}{14\text{Km}} \right)^{-3/2} \left( \frac{M}{m_\odot} \right)^{1/2} \]

\[ v = 0.3 \left( \frac{f}{1\text{kHz}} \right)^{1/3} \left( \frac{m}{m_\odot} \right)^{1/3} < \frac{1}{\sqrt{6}} \]

\[ \frac{dE}{dt} = \frac{r^2}{16\pi G_N} \int d\Omega \langle \dot{h}_+^2 + \dot{h}_x^2 \rangle = \frac{32\eta^2}{5G_N} v^{10} \quad \eta = \mu/M \]
Do GWs exist? The Hulse-Taylor binary pulsar

GW’s have been observed in the NS-NS binary system:

PSR B1913+16

Observation of orbital parameters \( (a_p \sin \iota, e, P, \dot{\theta}, \gamma, \dot{P}) \)

\[ \downarrow \]

determination of \( m_p, m_c \) (1PN physics, GR)

Energy dissipation in GW’s \( \rightarrow \dot{P}^{(GR)}(m_p, m_c, P, e) \), compared with \( \dot{P}^{(obs)} \)

\[
\frac{1}{2\pi} \phi = \int_0^T \frac{1}{P(t)} \, dt \simeq \frac{T}{P_0} - \frac{\dot{P}_0}{P_0^2} \frac{T^2}{2}
\]

Test of the

- 1PN conservative
- leading order dissipative dynamics
GW’s from Coalescing Binaries

Introduction to GW: sources and observations

General Relativity

Weisberg and Taylor (2004)

\[ \dot{P}_{GR} - \dot{P}_{exp} \sim 10^{-3} \]

10 pulsars in NS-NS, still \( \sim 100 \text{Myr} \) for coalescence
Burst signals

- Supernovae and collapsing stars, typical amplitude

\[ h \sim 6 \cdot 10^{-21} \left( \frac{E}{10^{-7}m_{\odot}} \right) \left( \frac{1\text{msec}}{T} \right)^{1/2} \left( \frac{1\text{kHz}}{f} \right) \left( \frac{10\text{kpc}}{r} \right) \]

Limits for signals centered at different freq’s

1.7 yr of data, LIGO/Virgo PRD85 (2012)
Asymmetric rotating neutron star

- Pulsar asymmetry $\epsilon \rightarrow h \sim \epsilon (Rf)^2 M/r$

$$L_{GW} = f^6 M^2 R^4 \epsilon^2 \rightarrow t_{SD} \sim \frac{M(Rf)^2}{L_{GW}}$$

If all the spin-down is due to GW emission

$$\epsilon \approx 7 \cdot 10^{-3} \quad \epsilon_{GW} < 1.4 \cdot 10^{-4} \quad \text{Crab}$$
$$\epsilon \approx 1.2 \cdot 10^{-3} \quad \epsilon_{GW} < 5 \cdot 10^{-4} \quad \text{Vela}$$

Beating the spin-down limit!

- Best upper limit $h_{ul} \sim 10^{-24}$ @ 150Hz


- Upper limit on GW’s emitted by Vela pulsar glitch

$h_{ul} < 10^{-20} \quad E_{GW} < 10^{45}$ erg

LIGO PRD 2011
Future detectors (2017+)

Integrated sensitivity ($T_{\text{obs}}=1\text{yr}$) for known pulsars vs. spin-down limit
Coincidence with Gamma Ray Bursts

Search in a $-600 \div +60$ sec window around the GRB event

- 150 analyzed via an un-modeled burst search excluded within $D_{burst} \sim 17 \text{ Mpc}(E_{GW}/10^{-2} M_\odot)^{1/2}$ for signals at 150 Hz
- 24 analyzed via matched filtering with coalescing binary signals
  $D_{NS-NS} \sim 17 \text{ Mpc}$  
  $M_{NS} \sim 1.4 \pm 0.2$
  $D_{BH-NS} \sim 29 \text{ Mpc}$  
  $M_{NS} \sim 1.4 \pm 0.4$  
  $M_{BH} \sim 10 \pm 6$

Distance cumulative distributions:

Looks promising for Adv-LIGO/Virgo (LIGO/Virgo et al. APJ (2012))
Binary coalescence: a tale made of three stories

- Inspiral phase: post-Newtonian approximation: $v/c$
- Merger: fully non-perturbative
- Ring-down: Perturbed Kerr Black Hole

![Graph showing time and a.u. values]
GW’s from Coalescing Binaries

Introduction to GW: sources and observations

Upper limits: coalescences

Low mass (< 25$M_\odot$) binary (inspiral)
Upper limit from previous runs
improved by the 7/2009-10/2010 run

Combined upper limit from old and new runs
compared with astrophysical estimates

LIGO/Virgo PRD 2012
Upper limits for high mass systems

- For high masses $25 < \frac{M}{M_\odot} < 100$ also merger and ring-down are in band and necessary to have complete analytic description of coalescence waveform. Observative bound on coalescence rate of equal mass system with $19 < \frac{(m_1, m_2)}{M_\odot} < 28$:
  $$R_c < 0.3 \text{ Mpc}^{-3} \text{ Myr}^{-1}$$

LIGO/Virgo PRD 2011

- For higher masses $100 < \frac{M}{M_\odot} < 500$ signals are burst-like: Best upper limit for equal mass system with $M \sim 170M_\odot$:
  Merger Rate $R_m < 0.13 \text{ Mpc}^{-3} \text{ Myr}^{-1}$

LIGO/Virgo PRD 2012
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Detector locations

All are now being upgraded to their Advanced version due to to start data taking in 2015
2017+ for design sensitivity

Old coincident runs terminated in October 2010.
GW’s from Coalescing Binaries

GW detector and sensitivity

The observatories

Advanced detectors

Sensitivity vs. signals @ 200Mpc w. optimal orientation
Distance reach for compact binary coalescence
Observational rate estimates

LIGO/Virgo Advanced Observatories will detect

\[(SNR = 8, \text{optimal orientation})\]

<table>
<thead>
<tr>
<th>Distance (Mpc)</th>
<th>Rates MWEG(^{-1})Myear(^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>NS-NS</td>
<td>450Mpc</td>
</tr>
<tr>
<td>BH-BH</td>
<td>1Gpc</td>
</tr>
<tr>
<td>10 (M_{\odot})</td>
<td>1 (\div 10^3)</td>
</tr>
<tr>
<td></td>
<td>(4 \cdot 10^{-2} \div 100)</td>
</tr>
</tbody>
</table>

\[N = 0.011 \times \frac{4}{3\pi} \left(\frac{D_H/\text{Mpc}}{2.26}\right)^3 \text{MWEG}\]

Realistic case:

\[R_{NS-NS} \sim 10\text{yr}^{-1} \quad R_{BH-BH} \sim 10^2\text{yr}^{-1}\]

for LIGO/Virgo at design sensitivity
GW’s from Coalescing Binaries
GW detector and sensitivity
The observatories

Advanced LIGO/Virgo goals

- Make first “direct” detection of GWs from neutron stars and/or black holes
- Probe intermediate mass black hole range: $M \sim 100M_\odot$
- Measure rate of binary coalescences
- Measure pulsar parameters
- Possible probe of neutron star interior/nuclear matter at high density
- Verify association between short GRB’s and GW’s
- Combine EM and GW detection
- Make strong tests of GR
- Use coalescing binaries as standard sirens for cosmology
Data analysis technique: Matched filtering

An experimental apparatus output: time series

\[ O(t) = h(t) + n(t) \quad h(t) = D_{ij} h_{ij}(t) \]

Noise is conveniently characterized by its spectral function

\[ \langle \hat{n}(f)\hat{n}^*(f') \rangle = \delta(f - f') S_n(f) \quad [Hz^{-1}] \]

Matched filter enhances the sensitivity

\[
\frac{1}{T} \int_0^T O(t) h(t) \, dt = \frac{1}{T} \int_0^T h^2(t) \, dt + \frac{1}{T} \int_0^T n(t) h(t) \, dt \sim \]

\[ h_0^2 + \sqrt{\frac{T_0}{T}} n_0 h_0 \]
Hunting for tiny signals

Detector’s output is flooded with noise:

Noise + GW signal from 2+12 $M_\odot$ system at 50 Mpc distance
Matched filtering

Matched filtering enhances sensitivity:

\[ O(t) \rightarrow MF(t) \propto \int \frac{O(f)h^*(f)}{S_n(f)} e^{2\pi ift} df \]

MF with \( h' \neq h \)

but requires good model of the signal \( h \)

or a complete bank of \( h' \)'s
GW detection

***Inspiral*** \( h = A \cos(\phi(t)) \) \[ \frac{\dot{A}}{A} \ll \dot{\phi} \]

**Virial relation:**

\[
\nu \equiv (G_N M \pi f_{GW})^{1/3} \quad \nu = \frac{m_1 m_2}{(m_1 + m_2)^2}
\]

\[
E(\nu) = -\frac{1}{2} \nu M v^2 \left(1 + \#(\nu)v^2 + \#(\nu)v^4 + \ldots\right)
\]

\[
P(\nu) \equiv -\frac{dE}{dt} = \frac{32}{5G_N} v^{10} \left(1 + \#(\nu)v^2 + \#(\nu)v^3 + \ldots\right)
\]

\(E(\nu)(P(\nu))\) known up to 3(3.5)PN

\[
\frac{1}{2\pi} \phi(T) = \frac{1}{2\pi} \int^{T} \omega(t) dt = -\int^{v(T)} \frac{\omega(\nu)}{P(\nu)} \frac{dE}{dv} dv
\]

\[
\sim \int \left(1 + \#(\nu)v^2 + \ldots + \#(\nu)v^6 + \ldots\right) \frac{dv}{v^6}
\]
GW detection

\[ N_{cycles} \simeq 1.6 \cdot 10^4 \left( \frac{10\text{Hz}}{f_{\text{min}}} \right)^{5/3} \left( \frac{1.2M_\odot}{M_c} \right)^{5/3} \]

Sensitivity \( \propto M_c^{5/3} \sqrt{N_{\text{cycles}}} \propto M_c^{5/6} \)

\( f_{\text{Max}} \propto M^{-1}, \quad M_c \equiv \eta^{3/5}(m_1 + m_2) \)

Important to know the phase at \( O(1) \) when taking correlation of detector’s output and model waveform
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The EFT point of view on the 2-body problem: how to compute waveform

Different scales in EFT:

- Very short distance \( \lesssim r_s \)
  negligible up to 5PN (effacement principle)

- Short distance: orbital scale
  \( r \sim r_s/v^2 \), potential
  gravitons \( k_\mu \sim (v/r, 1/r) \)

- Long distance: GW’s
  \( \lambda \sim r/v \sim r_s/v^3 \)
  \( k_\mu \sim (v/r, v/r) \) coupled to
  point particles with moments
Theory at large distances

Extended object coupled to gravity via multipoles

\[ S_{\text{ext}} \int dt \, M + L_{ab} \omega_0^{ab} + Q_{ij} E_{ij} + J_{ij} B_{ij} + O_{ijk} \partial_k E_{ij} + \ldots \]

Emission amplitude

\[ A_h(k) = \sqrt{G_N k^2} \epsilon_{ij} Q_{ij} \rightarrow P \propto |A_h|^2 \]

In order to make prediction we need to know what are the multipoles:

Need to match with the theory at the orbital scale:

\[ Q_{ij} = \int \dddot{T}_{00} x_i x_j \]
Divergences

Theory of extended object has UV divergencies: assuming source concentrated at \( r = 0 \) gives divergencies, in the amplitude

\[
\left| \frac{A_{h,v^6}}{A_{h,v^0}} \right|^2 = (G_N M \omega)^2 \left( \frac{1}{d-3} + \log \frac{k^2}{\mu^2} + \ldots \right)
\]

in the radiation-reaction

\[
\delta \ddot{x}_{ai}(t) = -\frac{8}{5} x_{a j}(t) G_N^2 M \int_{-\infty}^{t} dt' Q^{(7)}_{ij}(t') \log \frac{t-t'}{\mu}
\]
Log terms give running

\[
\begin{align*}
\mu \frac{dM}{d\mu} &= -\frac{214}{105} (G_N m\omega)^2 I_{ij}^{(R)} \\
\mu \frac{dI_{ij}}{d\mu} &= \frac{2(G_N M\omega)^2}{5} \left( 2 I_{ij}^{(5)} I_{ij}^{(1)} - 2 Q_{ij}^{(4)} I_{ij}^{(2)} + I_{ij}^{(3)} I_{ij}^{(3)} \right)
\end{align*}
\]

Observer at different distance disagree on the value of $M, Q_{ij}$

Goldberger, Ross PRD 2010
Goldberger, Rothstein, Ross 1211.6095
Foffa & RS PRD 2012

\[
P \propto G_N \int \omega^6 \left( Q_{ij}^2 + \frac{16}{45} J_{ij}^2 + \frac{5}{189} \omega^2 I_{ijk}^2 \\
+ a(G_N M\omega)Q_{ij}^2 + b(G_N M\omega)^2 Q_{ij}^2 + \ldots \right)
\]
Conservative dynamics

To compute emitted flux (and the energy) the theory at orbital scale is needed: heavy, classical sources coupled to gravitons

$$\exp [iS_{eff}(x_a)] = \int \mathcal{D}h(x) \exp [iS_{EH}(h) + iS_{pp}(h, x_a)]$$

$$S_{pp} = -\sqrt{G_N}m \int dt \left( h_{00}/2 + v_i h_{0i} + v^i v^j h_{ij}/2 + \ldots \right)$$

$$S_{EH} = \int d^4x \left[ (\partial_i h)^2 - (\partial_t h)^2 + \sqrt{G_N} h(\partial h)^2 + \ldots \right]$$

Power counting to integrate out potential gravitons in the NR limit

$$\int d^{3+1}k \frac{e^{ik \cdot x}}{k^2} \sim \int d^3 k \delta(t) \frac{1}{k^2} \left( 1 - \frac{\partial_t^2}{k^2} + \ldots \right)$$

$$\sim \int d^3 k \delta(t) \frac{e^{ik \cdot x}}{k^2} \left( 1 + \frac{(kv)^2}{k^2} + \ldots \right)$$

Manifest scaling
The 1PN potential

Scaling: \( L \) \( v^2 \sim G_N M/r \)

Using virial theorem \( v^2 \sim G_N M/r \)

\[
V = -\frac{G m_1 m_2}{2r} \left[ 1 - \frac{G_N m_1}{2r} + \frac{3}{2} (v_1^2) - \frac{7}{2} v_1 v_2 - \frac{1}{2} v_1 \hat{r} v_2 \hat{r} \right] + 1 \leftrightarrow 2
\]
Divergences and quantum effects

Graviton loops are quantum corrections suppressed by
\( \frac{\hbar}{L} \sim 10^{-76} \left( \frac{M}{M_\odot} \right)^{-2} (v/0.1) \)

Harmless power law divergence: infinite mass renormalization absorbed in the bare coupling

\[ \propto \int d^d k \frac{1}{k^2} \]

EFT does not predicting all physical parameters, some are inputs

\( \log \) divergencies calls for UV completion of the theory
The 3PN computation automatized

- **Topologies**
- **Graphs**
- **Amplitudes**
- **Evaluation**

\[ A = G_N m_i v_i \int d^d k d^d k_1 \frac{1}{k^2(k - k_1)^2} \ldots \]

Analytic integral in a database

S. Foffa & RS PRD 2011
Feynman diagrams at 3PN order

\[ G_N u^6 \]

\[ G^2_N u^4 \]
Feynman diagrams at 3PN order: $G^3_N v^2$
Feynman diagrams at 3PN order: $G^4_N$

Final result matches previous derivation of 3PN Hamiltonian, see eq. (174) of Blanchet’s Living Review on Relativity
The 4 PN status

Recovered the 3PN Hamiltonian

Blanchet et al. PRD ’03

At 4PN we have to compute:

- 3 graphs $@ G_N v^8$ order
- $23 @ G_N^2 v^6$
- $202 @ G_N^3 v^4$
- $307 @ G_N^4 v^2$
- $50 @ G_N^5$

Foffa & RS in preparation
The 4 PN on the way

The major obstruction to the computation is one of the $G^{5}_{N}$ graphs equivalent to the 4-loop quantum field theory diagram in a massless, euclidean theory in $d = 3$

Result at $G^{5}_{N}$ obtained with traditional method (different gauge)

Jaranowski, Schäfer PRD 2013
Testing GR

\[ \phi(t) = v^{-5} \sum_{n=0}^{7} \left( \phi_n + \phi_n^{(l)} \log(v) \right) v^n \]

Bayesian model selection between two hypothesis

- \( \mathcal{H}_{\text{GR}} \)
- \( \mathcal{H}_{\text{modGR}} \): one or more \( \phi_i \)'s are not as predicted by GR

Odds ratio:

\[ O_{\text{modGR}}^{\text{GR}} \equiv \frac{P(\mathcal{H}_{\text{modGR}}|d,I)}{P(\mathcal{H}_{\text{GR}}|d,I)} \]

In absence of noise \( O_{\text{modGR}}^{\text{GR}} \geq 0 \) favours \( \text{modGR} \) over \( \text{GR} \)

Li, RS et al. JPCS (2012, PRD (2012)
Parameter estimation bias

Waveform match (fitting factor)

$$FF = \int df \frac{h_1^*(f)h_2(f) + h_1(f)h_2^*(f)}{S_n(f)}$$

for $\delta \chi_3$ injections varying $\eta$ (maximized over other params)

GR deviation do not prevent detection, but considerable bias!
Simulated signals in noise

**Constant shift** in $\phi_3 \rightarrow \phi_3(1 + \delta \chi_3)$  
SNR’s limited to 8-25

Single sources with noise:
Odds ratio overlaps

15-sources catalogs can disentangle fundamental effects
Future directions for testing GR

- Are waveforms accurate enough? In early inspiral yes
- For neutron stars: are finite size and matter effects important? Mostly after 450Hz (see Hinderer et al. PRD (2010))
- Is the effect of spin important? Not for neutron stars, but for BH it has to be considered
- Are internal calibration errors under control? Yes
  Vitale et al. PRD85 (2012)
- What about inclusion of merger and ring-down? Work in progress
- Can the computational challenge be satisfied? Maybe yes...
New era is about to start (∼ 4 years)

- GW astronomy will open a new window onto the Universe, both astrophysically and cosmologically
- Observational test of strong gravity field dynamics not available so far: GW detection will give access to strong gravity phenomena
- Field theory method tuned out to be useful in classical gravity
Spare slides
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Measuring $H_0$

Coalescing binary systems are standard sirens:

$$h(t) = \frac{G_N \eta M^{5/3} f_s^{2/3}}{D} \cos[\phi(t)]$$

In cosmological settings source and observer clocks tick differently:

$$dt_o = (1 + z)dt_s \quad f_o(1 + z) = f_s$$

$$h(t_o) = \frac{G_N \eta f_o^{2/3} M^{5/3}(1 + z)^{2/3}}{a(t_o)D} \cos[\phi(t_s(t_o))]$$
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$$M \frac{d\phi(t_s/M)}{dt_s} = (1 + z) M \frac{d\phi((1 + z)t_s(t_o)/M(1 + z))}{dt_o}$$

$$\phi(t_o/M) = \phi(t_s/M) \quad \mathcal{M} \equiv M(1 + z)$$
Determining $H_0$

Hubble law: $z = H_0 d_L$

$D_L$ can be measured, $z$ degenerate with $M$, however if

- the source in the sky has been localized ($\alpha, \delta$)
- GW sources are in the galaxy catalog with known red-shift

$$P(z, D_L | c_i) = \int dM \, d\theta \, d\alpha \, d\delta \, P(D_L M, \theta, \alpha, \delta | c_i) \pi(z, |\alpha, \delta)$$

Schutz, Nature '86
W. Del Pozzo, arXiv:1108.1317
Stochastic background

\[ \Omega_{GW} = \frac{f \, d\rho_{GW}}{\rho_c \, df} \]

\[ S_{GW} = \frac{3H_0^2 \, \Omega_{GW}}{10\pi^2 \, f^3} \]

Between 50 and 150 Hz