PONTIFICIA UNIVERSIDAD CATOLICA DE VALPARAISO

Modified Gravity 3

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3 ways to hide

$$S = \int \sqrt{-g} d^4x \left[R - Z^{\mu\nu}(\phi, \partial\phi, \cdots)\partial_\mu\phi\partial_\nu\phi - V(\phi) \right] + S_m[e^{2Q\phi} g_{\mu\nu}, \Psi_m]$$

- $Q \ll 1$
  Unnatural like Brans-Dicke theories
- We use the potential $V(\phi)$ (Chameleon mechanism)
  Like $f(R)$-gravity, chamelon gravity, Brans-Dicke with a potential ...
  Nice properties, Simple predictions but the simplest models seem to be ruled out
- We use the non-linearities $Z^{\mu\nu}$ (Vainshtein mechanism)
  Brane models (DGP), massive gravity ...

- Examples
  - K-inflation (C. Armendariz-Picon et al. 1999)

$$S = \int d^4x \Big[R + K(\phi, (\partial\phi)^2)\Big]$$

  - K-essence (T. Chiba et al. 2000)
  - Dirac-Born-Infeld (DBI) (D-branes in string theory) (T. Padmanabhan 2002)

$$S = \int d^4x \Big[R - V(\phi)\sqrt{1 - (\partial\phi)^2}\Big]$$

  - Ghost condensate (N. Arkani-Hamed et al. 2004)

$$S = \int d^4x \Big[R + \frac{1}{2}(\partial\phi)^2 + a(\partial\phi)^4\Big]$$

- Properties
  - Non-gaussianity (D. Seery et al. 2005)
  - Bouncing cosmology (because they violate the null energy condition) (P. Creminelli et al. 2006)
  - Supersymmetric version (J. Khoury et al. 2010)
  - Study of large-scale structure (C. Armendariz-Picon et al. 2005)
  - Study of topological defects, solitons ... (E. Babichev 2008)

- The simplest model is defined with a shift symmetry $(\phi(x) \to \phi(x) + a)$

$$S = \int \sqrt{-g} d^4x \Big[R - \frac{1}{2}(\partial\phi)^2 + \frac{\alpha}{4\Lambda^4}(\partial\phi)^4 + \frac{Q}{M_{Pl}}\phi T \Big]$$

$$\Box\phi - \frac{\alpha}{\Lambda^4}\partial_\mu((\partial\phi)^2\partial^\nu\phi) + \frac{Q}{M_{Pl}}T = 0$$

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We consider a point source $T = -M\delta^{(3)}(x)$

$$\mathrm{div}\Big(\nabla\phi - \frac{\alpha}{\Lambda^4}(\partial\phi)^2\nabla\phi\Big) = \frac{QM}{M_{Pl}}\delta^{(3)}(x)$$

$$\phi' - \frac{\alpha}{\Lambda^4}\phi'^3 = \frac{1}{4\pi r^2}\frac{gM}{M_{Pl}}$$

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$$\frac{\phi'(r)}{\Lambda^2} = \frac{\left(\frac{8\pi}{3}\right)^{1/3}\left(\frac{r}{r_V}\right)^{2/3}}{\left(-9 + \sqrt{81 + 192\pi^2\left(\frac{r}{r_V}\right)^4}\right)^{1/3}} - (72\pi)^{-1/3}\left(-9 + \sqrt{81 + 192\pi^2\left(\frac{r}{r_V}\right)^4}\right)^{1/3}\left(\frac{r_V}{r}\right)^{2/3}$$

(Gabadadze et al. 2012)

$$r_V = \frac{1}{\Lambda}\left(\frac{QM}{M_{Pl}}\right)^{1/2}$$

- In the asymptotic regime
  - For $r \gg r_V$, $\quad \phi'(r) \simeq \dfrac{\Lambda^2}{4\pi}\left(\dfrac{r_V}{r}\right)^2$
  - For $r \ll r_V$, $\quad \phi'(r) \simeq (-\alpha)^{1/3}\Lambda^2\left(\dfrac{r_V}{r}\right)^{2/3}$

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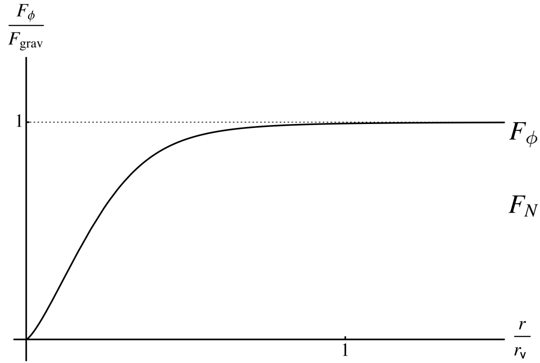

$$F_\phi = \frac{Q}{M_{Pl}}\nabla\phi = \frac{Q}{M_{Pl}}\phi'(r)$$

$$F_N = \frac{M}{8\pi M_{Pl}^2}\frac{1}{r^2} = \frac{\Lambda^2}{8\pi M_{Pl}}\left(\frac{r_V}{r}\right)^2$$

- Generalizations (Babichev et al. 2009)

$$S = \int \sqrt{-g} d^4x \Big[R - \frac{1}{2}(\partial\phi)^2 + \sum_{i=2}^{n} \frac{c_i}{\Lambda^{4i-4}} (\partial\phi)^{2i} + \frac{Q}{M_{Pl}} \phi T \Big]$$

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If $n \to \infty$, we have DBI $F_\phi / F_N \simeq r^2$ (de Rham et al. 2014)

- (D)BI (Burrage et al. 2014)
  Negative tension brane embedded in a 5-dimensional space (with 2 time-like directions), at the lowest order, we have

$$S = \int \sqrt{-g} d^4x \left[R + \Lambda^4 \sqrt{1 - \frac{(\partial\phi)^2}{\Lambda^4}} + \frac{Q}{M_{Pl}} \phi T \right]$$

$$\phi'(r) = \frac{\Lambda^2}{\sqrt{1 + 16\pi^2 (r/r_V)^4}}$$

- $F_\phi \simeq 1/r^2$ (for $r \gg r_V$)
- $F_\phi / F_N \simeq (r/r_V)^2$ (for $r \ll r_V$)

General picture

$$S = \int \sqrt{-g} d^4x \left[R - \frac{1}{2} Z^{\mu\nu}(\phi) \partial_\mu \phi \partial_\nu \phi + \frac{Q}{M_{pl}} \phi T \right]$$

$$Z_{\mu\nu} \approx g_{\mu\nu} + \frac{1}{\Lambda^3} \partial_\mu \partial_\nu \phi + \frac{1}{\Lambda^6} \left(\partial_\mu \partial_\nu \phi \right)^2 + \cdots$$

- At low energy $Z_{\mu\nu} \approx g_{\mu\nu}$
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The field is strongly coupled to itself and becomes weakly coupled to external sources

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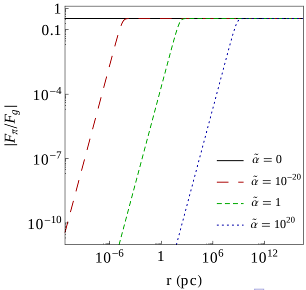
$$R_V \simeq (\alpha r_S)^{1/3}$$
$$\tilde\alpha = H_0^2\alpha$$



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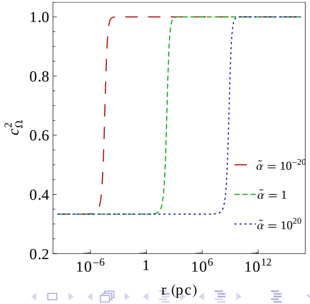
Considering perturbations around this solution
$\phi = \bar{\phi} + \delta\phi$

$$S = \int d^4xQ\Big[(\partial_t\delta\phi)^2 - c_r^2(\partial_r\delta\phi)^2 - c_\Omega^2(\partial_\Omega\delta\phi)^2\Big]$$



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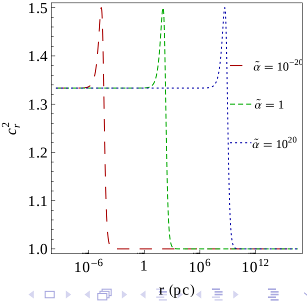

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There is always a direction in which the speed of propagation is superluminal

- Galileons
  - Boundary effective theory on the DGP brane comes as (A. Nicolis, R. Rattazzi 2004)

$$S = \int d^4x\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 + \alpha(\partial\phi)^2\Box\phi + \phi T\right]$$

- Generalized galileons (Horndeski) (G. W. Horndeski 1974)
  - Most general action with a scalar field and second order differential equation

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    - Self-accelerating solution
    - Superluminal behavior but causal
    - Self-accelerating solution is unstable (R.G., M. Sami 2010), we need generalization
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$$S = \int d^4x\sqrt{-g}\Big[K(\phi,X) - G_3(\phi,X)\Box\phi + G_4(\phi,X)R + G_{4,X}(\phi,X)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$
$$+ G_5(\phi,X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5,X}(\phi,X)\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu)^2 + 2\nabla^\mu\nabla_\alpha\phi\nabla^\alpha\nabla_\beta\phi\nabla^\beta\nabla_\mu\phi\right]\Big]$$

Origin of the Vainshtein mechanism

- Massive gravity
  - Quadratic action for massive spin-2 particle (M. Fierz, W. Pauli 1939)

$$S = \int d^4x \Big[-\frac{1}{2}\partial_\alpha h_{\mu\nu}\partial^\alpha h^{\mu\nu} + \partial_\mu h_{\nu\alpha}\partial^\nu h^{\mu\alpha} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\alpha h \partial^\alpha h - \frac{m^2}{2}\left(h_{\mu\nu}h^{\mu\nu} - h^2\right)\Big]$$

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Wave equation for 5 propagating polarizations

- Solution for a point source

$$h_{00} = \frac{2M}{3}\frac{1}{4\pi}\frac{e^{-mr}}{r}$$
$$h_{ij} = \frac{M}{3}\frac{1}{4\pi}\frac{e^{-mr}}{r}\delta_{ij}$$

$\Rightarrow$ The PPN parameter is $\gamma = \frac{h_{11}}{h_{00}} = \frac{1}{2}$ like Brans-Dicke

- Even in the limit of zero mass, gives predictions which are order one different from linearized GR
- This is the vDVZ (van Dam, Veltman, Zakharov) discontinuity (H. van Dam and M. J. G. Veltman; V. I. Zakharov 1970)

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- The Stückelberg trick
  - Formalism to expose the origin of this discontinuity
  - Taking $m \to 0$ in the equations does not yield to a smooth limit, because degrees of freedom are lost
  - To find the correct limit, the trick is to introduce new fields into the theory in a way that does not alter the theory and then the limit can be found in which no degrees of freedom are gained or lost
- The first part of the action is invariant under linear diffeomorphism $h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu\xi_\nu + \partial_\nu\xi_\mu$
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- But it opens the door to degravitation (N. Arkani-Hamed et al. 2002): a large cosmological constant does not strongly curve space-time

- Because of these problems, massive gravity has been extended
  - Background metric $g^{(0)}_{\mu\nu}$ is dynamical, bi-metric theory (Hassan et al. 2012)
  - Multi-metric theory (Hinterbichler et al. 2012)
  - The mass is not constant but it's a scalar field, quasi-dilaton massive gravity (D'Amico et al. 2013)
    - Cosmology studied in (R.G. et al. 2013). The model has bouncing, turnaround or cyclic solutions

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Any theory that exhibits degravitation must reduce to a theory of massive/resonance gravity (continuum of massive gravitons) at linearized level (G. Dvali et al. 2007)

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