Modified Gravity and the dRGT Massive Gravity

Fulvio Sbisà

Universidade Federal do Espìrito Santo,

Vitòria, ES, Brazil

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in collaboration with Kazuya Koyama, Gianmassimo Tasinato and Gustavo Niz

Outline

1. The cosmic acceleration and Modified Gravity

Standard cosmology before 2000

- Homogeneity and isotropy
- General Relativity
- SM matter + CDM

⇒

Abundance of elements

CMB

Structure formation (given initial perts.)

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) < 0\,!$$

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The late time acceleration

- Adding a cosmological constant term, observational data on:
  - type IA supernovae
  - CMB anisotropies
  - large scale structure

are best fitted by:

- a universe whose energy density is dominated by the cosmological constant, and
- which has recently entered an accelerated phase

$$\frac{\ddot{a}}{a} > 0 \; !$$

Why is this a problem?

At a fundamental level, we don't understand the $\Lambda\, g_{\mu\nu}$ term:

- Λ may be a second characteristic scale of gravity ... Padmanabhan '06
  - then the universe is very fine tuned: coincidence problem
- ... or the semiclassical manifestation of vacuum energy Weinberg '89
  - observed and predicted values differ by 60 – 120 orders of magnitude
- or there may be both a "bare" and a semiclassical Λ
  - extremely fine tuned cancellation needed $\dfrac{\Lambda - \Lambda_{vac}}{\Lambda} \sim 10^{-60}$

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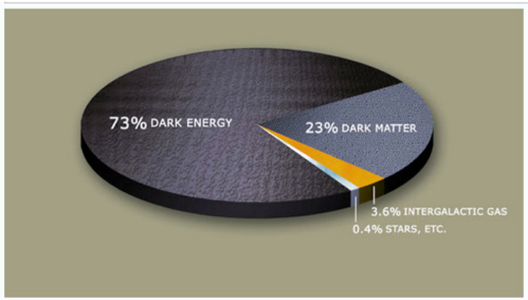
Alternative explanations

- Homogeneity and isotropy → Lemaître-Tolman-Bondi, back-reaction
- General Relativity → Modified Gravity (f(R), braneworlds, massive gravity, . . .)
- SM matter + CDM → Dark Energy (quintessence, k-essence, . . .)

Possibly the solution to the problem is more fundamental. . .

Dark Energy or Modified Gravity?

The energy budget is dominated by a component which is undetected in lab and has exotic properties $\rho + 3\,p < 0$ . . .



. . . or may the data simply suggest that GR is inadequate at cosmological scales?

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Infrared modifications of gravity

- To test the latter idea: build theories of gravity whose predictions differ from GR's on ultra-large scales
  - and reproduce GR predictions where they are well tested
- aim: find "standard" cosmological solutions which flow to de-Sitter
  - explain acceleration as self-acceleration, without fluids with negative pressure
- this may also help with the cosmological constant problem
  - t'Hooft naturalness, degravitation, . . .

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Typical problems in modified gravity

Modifying GR always introduces new degrees of freedom

- they have to be screened on lab/astrophysical scales
  - Vainshtein, Chameleon, Symmetron, ...
- quite often they are ghosts
  - e.g. BD ghost in massive GR, bending mode in s.a. DGP
- ghost $\equiv$ mode with negative kinetic energy
  - catastrophic instability of the vacuum

2. dRGT Massive Gravity

Linearized GR as a free massless spin-2 field

Take free GR action, expand to II order around Minkowski:

$$S^{(2)}_{GR} = \int d^4x \left(-\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h \partial^\lambda h \right)$$

Pretend $h_{\mu\nu}$ is just a field on Minkowski, “forget” about GR

- $h_{\mu\nu}$ transforms in the massless, spin-2 repr. of Lorentz
  - it has 2 propagating degrees of freedom, it is “unique”
- enjoys gauge invariance $h_{\mu\nu} \to h_{\mu\nu} + \partial_{(\mu}\xi_{\nu)}$
  - “unique” quadratic, local, Lorentz inv. action with no more than two derivatives for a symmetric (0,2) tensor

! $S^{(2)}_{GR}$ has very distinct properties besides coming from S_{GR} !

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GR as a self-interacting massless spin-2 field

Expand full free GR action around Minkowski in powers of $h_{\mu\nu}$

$$S_{GR} = S_{GR}^{(2)} + \int d^4x \left(\partial^2 h^3 + \partial^2 h^4 + \ldots + \partial^2 h^n + \ldots \right)$$

Start from $S_{GR}^{(2)}$, and add all possible interaction terms

- S_{GR} "singled out" by imposing that gauge invariance holds
  - full gauge invariance $\equiv$ "deformation" of $h_{\mu\nu} \to h_{\mu\nu} + \partial_{(\mu}\xi_{\nu)}$
- Interacting theory still has 2 propagating d.o.f.

  $\longrightarrow$ GR $\sim$ self-interacting theory of a massless spin-2 field
- Substitute $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$: Minkowski metric disappears
  - gauge inv. $\Rightarrow$ we can interpret $g_{\mu\nu}$ as a metric

Why considering massive gravity?

- Example: relativistic scalar field $(\Box - m^2)\,\phi = 0$
  - massless case: $\phi(r) \propto \dfrac{1}{r}$ (Coulomb)
  - massive case: $\phi(r) \propto \dfrac{e^{-mr}}{r}$ (Yukawa)

$\longrightarrow$ for $r \ll r_c \equiv 1/m$ essentially no difference, "Coulombian" profile modified for $r \gtrsim r_c$

- Idea: give gravity a "finite range" $r_c \simeq 1/H_0$ by giving it a mass
  - program: formulate a self-interacting theory of a massive spin-2 field

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Free massive spin-2 field: the Fierz-Pauli action

Action for a symmetric (0,2) tensor field on Minkowski

$$S_{FP}^{(2)} = S_{GR}^{(2)} - \int d^4x \, \frac{m^2}{2} \left(h_{\mu\nu} h^{\mu\nu} - h^2 \right)$$

- $h_{\mu\nu}$ transforms in the massive, spin-2 repr. of Lorentz
  - it has 5 propagating degrees of freedom
- it is “unique”: altering $S_{FP}^{(2)}$ changes number of d.o.f.
  - and generically introduces ghost instabilities
- the FP action does not enjoy gauge invariance
  - the mass term breaks $h_{\mu\nu} \to h_{\mu\nu} + \partial_{(\mu}\xi_{\nu)}$

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The vDVZ discontinuity

Static, spherically symmetric source: test bodies feel

$$ds^2 = -\Big(1 + 2\,\phi(r)\Big)\,dt^2 + \Big(1 - 2\,\psi(r)\Big)\,\delta_{ij}\,dx^i dx^j$$

- For $r \ll r_c$ the grav. potentials correctly scale as $1/r$
  - and decay exponentially for $r \gtrsim r_c$
- But: for $r \ll r_c$, $\phi(r) = 2\,\psi(r)$, while in GR $\phi(r) = \psi(r)$
  - either planet or photon trajectories are wrong (25%) $\rightarrow$ FP does not reproduce GR's results on small scales!
- $m \rightarrow 0$ sends $r_c \rightarrow \infty$, but nothing changes for $r \ll r_c$
  - $m \rightarrow 0$ limit of FP theory is not GR: vDVZ discontinuity

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Massive self-interactions: the Vainshtein mechanism

- Consider *non-linear* GR + FP mass term
  - just one of the possible non-linear extensions of FP
- Non-linearities become important at $r_v = \sqrt[5]{GM/m^4}$
  - plug linear solution in interaction terms, compare amplitude
  - for $r_s \ll r \ll r_v$, linear approx. is valid in GR and not in MG
- For $r < r_v$, non-linearities restore agreement with GR
  - setting $m \sim H_0$, for the sun r_v bigger than the Milky Way!
- $m \to 0$ implies $r_v \to \infty$: vDVZ discontinuity disappears

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Non-linear extensions of FP and the BD ghost

However:

- A generic non-linear extension of FP propagates 6 d.o.f.
  - the sixth degree of freedom is associated with a ghost
- Need to choose interaction terms such that
  - the number of degrees of freedom remains 5
  - Vainshtein mechanism is effective (r_V is model dependent)
- How to do this in practice?
  - theory is not gauge invariant $\rightarrow$ cannot proceed as in GR
- Implement absence of 6th d.o.f., but it is hard...

dRGT Massive Gravity

$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} \left(R[\mathbf{g}] - \frac{m^2}{2} \left(\mathcal{U}_2[\mathcal{K}] + \alpha_3 \mathcal{U}_3[\mathcal{K}] + \alpha_4 \mathcal{U}_4[\mathcal{K}] \right) \right) + \mathscr{L}_M[\mathbf{g}, \psi_{(i)}] \right]$$

$$\mathcal{K}^\mu{}_\nu = \delta^\mu_\nu - \left[\sqrt{\mathbf{g}^{-1} \cdot \boldsymbol{\eta}} \right]^\mu{}_\nu \qquad\qquad \mathcal{U}_i[\mathcal{K}] \supset tr(\mathcal{K}^n)^m$$

- At linear order in $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$, S reduces to S_{FP}
- The theory propagates 5 degrees of freedom
  - the Boulware-Deser mode is absent
- The background structure $\eta_{\mu\nu}$ do not disappear
- The theory does not enjoy gauge invariance
  - $g_{\mu\nu}$ is a tensor field on Minkowski $\Leftrightarrow$ absolute space

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The Stückelberg trick

"Restore diff. invariance": construct a *different* action which is diff. invariant and physically equivalent to the previous one.

- Substitute $\eta_{\mu\nu} \to \Sigma_{\mu\nu} \equiv \frac{\partial \phi^\alpha}{\partial x^\mu} \frac{\partial \phi^\beta}{\partial x^\nu} \eta_{\alpha\beta}$, ϕ^α : scalar functions
  - $\Sigma_{\mu\nu}$ transforms as a (0,2) tensor under reparametrizations
- ϕ^α encodes how we are positioned in relation to the absolute reference
  - usual to write $\phi^\alpha(x) = x^\alpha - \pi^\alpha(x)$. If $\pi^\alpha(x) = 0$, we are in the absolute reference (called "unitary gauge")
- Changing coordinates now leave the action invariant in form
  - but excites the Stückelberg fields $\pi^\alpha(x)$

3. The Vainshtein mechanism in dRGT Massive Gravity

Spherically symmetric solutions

- Most general static, spherically symmetric ansatz

$$ds^2 = -C^2(r)\,dt^2 + A^2(r)\,dr^2 + 2D(r)\,dtdr + B^2(r)d\Omega^2$$

- Two branches of solutions: diagonal ($D = 0$) and non-diagonal ($D \neq 0$) (in unitary gauge)
  - the Birkhoff theorem does not hold
- The two branches are physically distinct
  - without Stückelbergs: no reparametrization invariance
  - or: mapping one branch into the other excites Stückelbergs
- Non-diagonal branch has asymptotically non-flat solutions
  - we consider only the diagonal branch

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Focusing on the Vainshtein mechanism

- Rescale $r \to \rho(r)$: $ds^2 = -(1 + n(\rho))\, dt^2 + (1 - f(\rho))\, d\rho^2 + \rho^2 d\Omega^2$
  - $H(\rho) \equiv B(r(\rho))$ remains in the equations of motion
  - define $1 + h(\rho) \equiv r(\rho)/H(\rho)$
- At the Vainshtein radius $\rho_V = \sqrt[3]{GM/m^2}$ only h goes non-linear ($\sim$ Vainshtein paper)
  - if $r_c \gg r_s$, we get the hierarchy $r_s \ll \rho_V \ll r_c$
- We then focus on scales $GM \ll \rho \ll 1/m$
  - and take the linear approximation for n, f, while keeping all non-linear terms in h

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The quintic equation

$$f = -2\,GM/\rho - (m\rho)^2 \left(h - \alpha h^2 + \beta h^3\right) \qquad \alpha = 1 + 3\,\alpha_3$$

$$\dot{h} = 2\,GM/\rho^2 - m^2\rho \left(h - \beta h^3\right) \qquad \beta = \alpha_3 + 4\,\alpha_4$$

$$q(h,\rho;\alpha,\beta) = \frac{3}{2}\,\beta^2\,h^5 - \left(\alpha^2 + 2\beta\right) h^3 + 3\left(\alpha + \beta\,(\rho_v/\rho)^3\right) h^2 - \frac{3}{2}\,h - (\rho_v/\rho)^3 = 0$$

- gravitational potentials trivial to find once we know $h(\rho)$
- At fixed ρ, h obeys an *algebraic* equation (quintic)
  - impossible to write the general solution in radicals
- Remarkable symmetry connects behaviour at different α, β

$$q\left(\frac{h}{k}, \sqrt[3]{k}\,\rho; k\,\alpha, k^2\,\beta\right) = \frac{1}{k}\,q\big(h,\rho;\alpha,\beta\big)$$

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Local solutions, asymptotic and inner

Local existence of solutions $\Leftrightarrow$ Implicit Function theorem:

$$\left[q(h_0, \rho_0) = 0 \, , \frac{\partial q}{\partial h}(h_0, \rho_0) \neq 0 \right] \Rightarrow \left[q(h, \rho) = 0 \Leftrightarrow h = f(\rho) \right] \; ; \; f'(\rho_0) = \frac{\partial q}{\partial \rho}\Big|_0 \left(\frac{\partial q}{\partial h}\Big|_0 \right)^{-1}$$

- Define *shape function* $s_\rho(h) \equiv q(h, \rho)$ (1 parameter family)
  - simple root of $s_{\bar{\rho}}(h) \Leftrightarrow$ local solution $h(\rho)$ of the quintic in a neighbourhood of $\rho = \bar{\rho}$
- Asymptotic solutions $\rho \to \infty$: two types of solutions
  - "Coulombian" vDVZ solution $(1/\rho)$; non-flat solution
- Inner solutions $\rho \gtrsim 0$: two types of solutions
  - "Coulombian" GR solution $(1/\rho)$; self-shielding solution

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Solution matching in the phase space

- Solutions $h(\rho)$ of the quintic $\Leftrightarrow$ flow of simple roots of $s_\rho(h)$
  - Local solution extends until the root remains simple
- Global solutions: root corresponding to an asymptotic sol. flows to the root corresponding to an inner sol.
  - remaining a simple root all the way down
- Strategy: instead of trying to find solutions of the quintic, we study the flow of roots of $s_\rho(h)$
  - study analytically the evolution of $s_\rho(h)$ with ρ
  - ask Mathematica for which α, β double roots appear
  - plot $s_\rho(h)$ and change ρ "continuously" (Mathematica)

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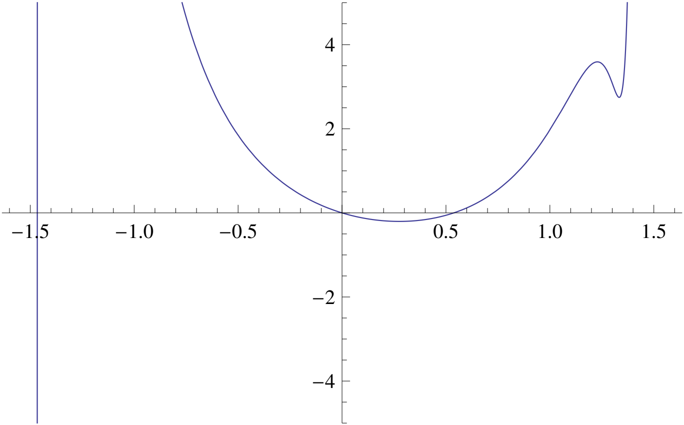
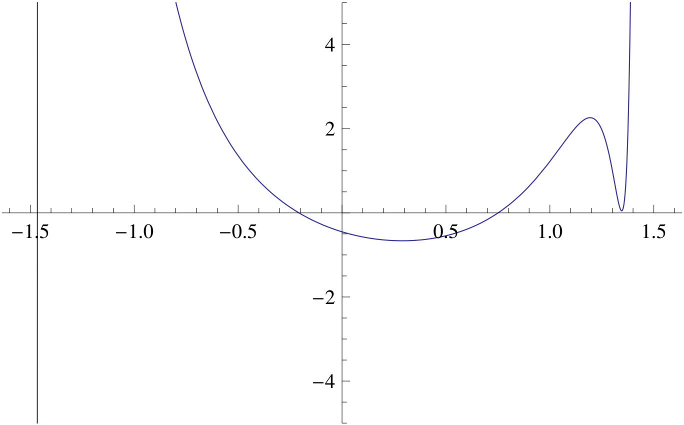
Phenomenology: creation and annihilation of solutions



Three asymptotic solutions and one inner solutions, but no global solutions!

Phenomenology: creation and annihilation of solutions



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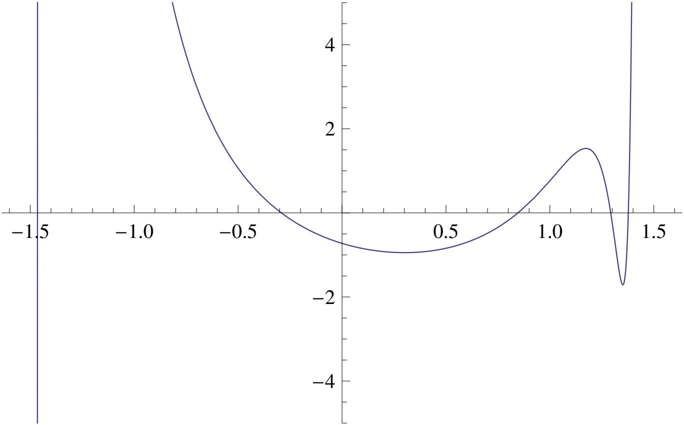

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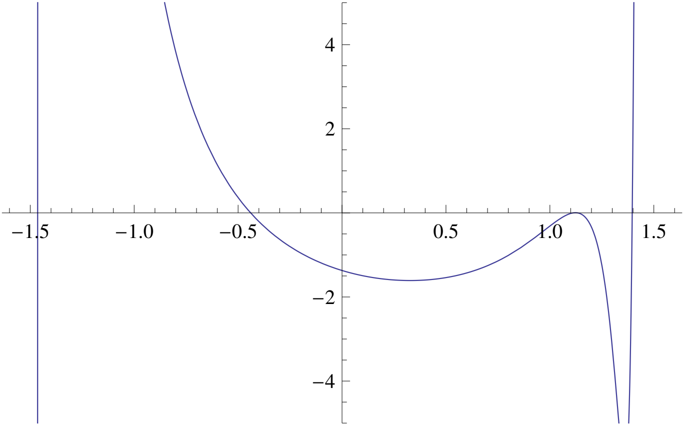

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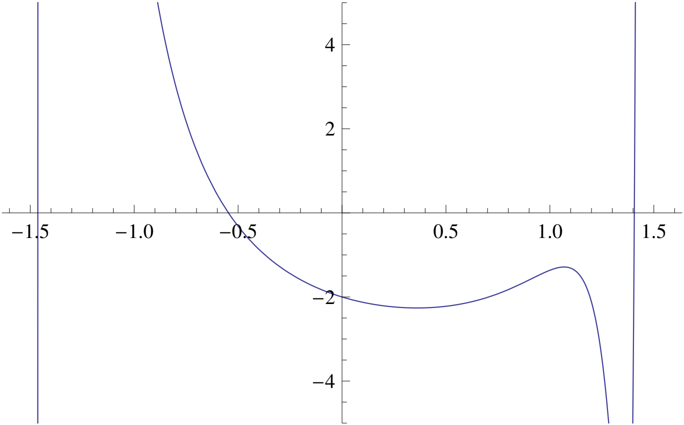

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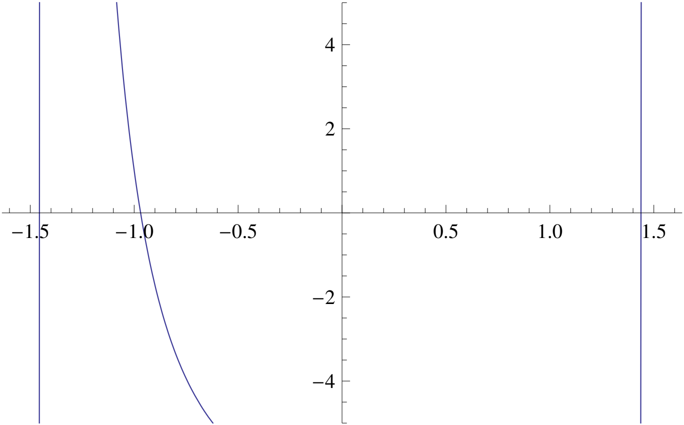

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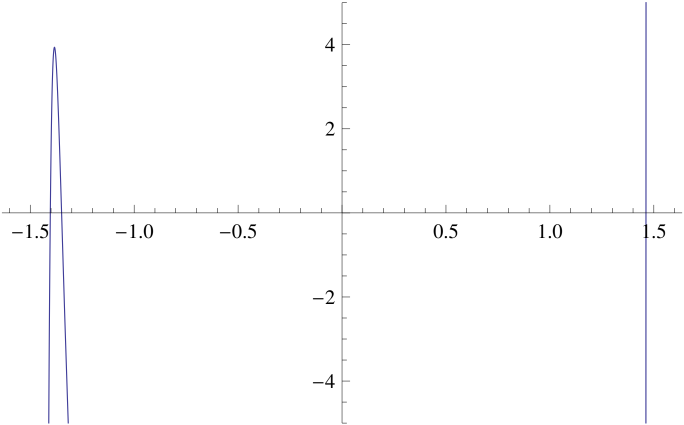

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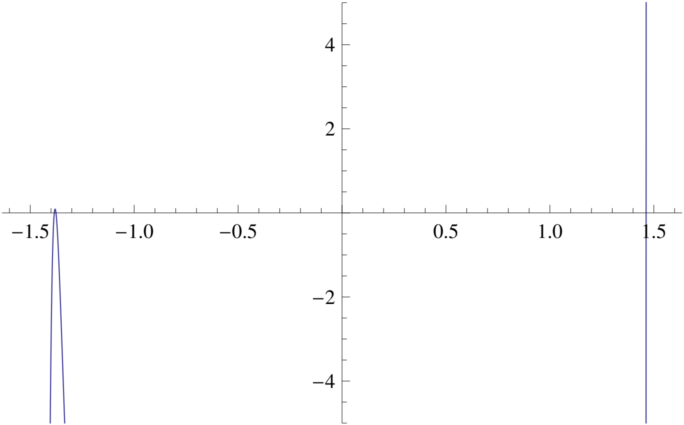

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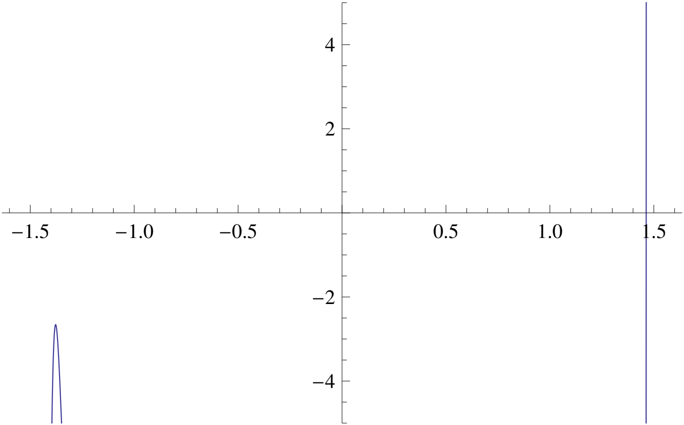
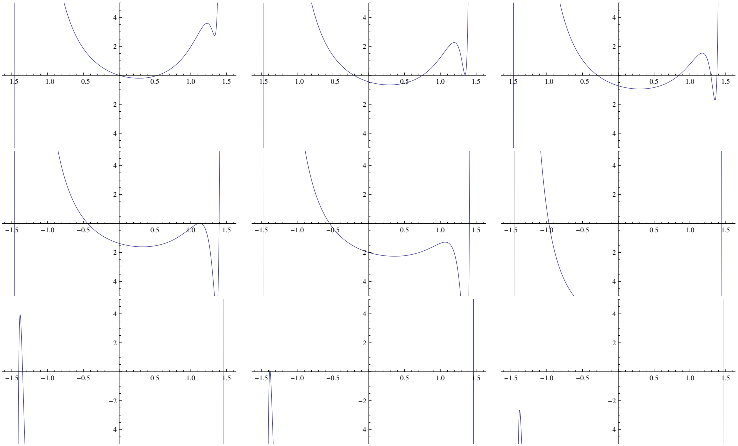
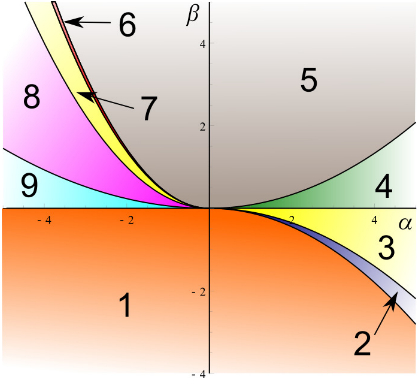


Three asymptotic solutions and one inner solutions, but no global solutions!

Shape function and flow of the roots



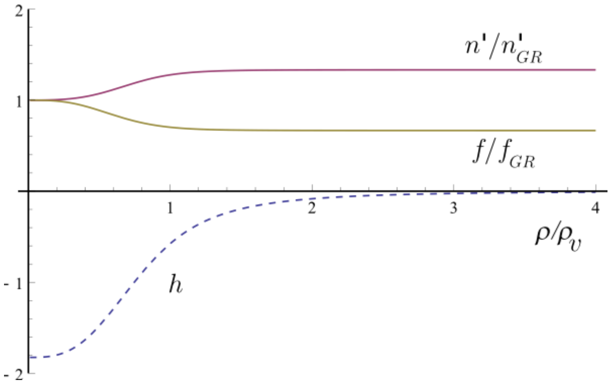
Phase space diagram of global solutions



- Asymptotically flat Vainshtein solutions: 4 & 5
  - Non asymptotically flat Vainshtein solutions: 5, 6, 7, 8 & 9
- Self-shielding, asympt. flat solutions: all but 2 and 7
  - region 2: NO global solutions

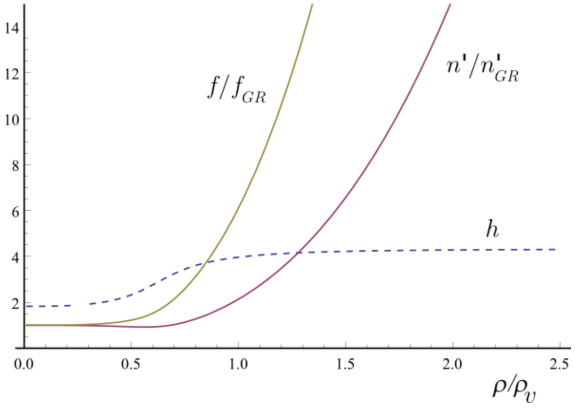
Numerical solutions: Vainshtein mechanism (flat)

Ratio with the gravitational potentials in GR, asymptotically flat spacetime



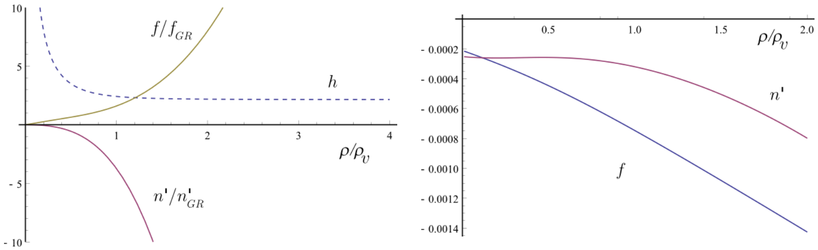
Numerical solutions: Vainshtein mechanism (non flat)

Ratio with the gravitational potentials in GR, non asymptotically flat spacetime



Numerical solutions: self-shielding

Non asymptotically flat with self-shielding at the origin



Ratio with the GR potentials

The potentials for $\rho \gtrsim 0$

4. Conclusions and present status

Conclusions and present status: sph. symm. sol.

Diagonal branch:

- The Vainshtein mechanism works in a wide part of the phase space
  - we characterized exactly where this happens
- However, all these solutions are unstable
  - self-shielding solutions are ruled out

Non-diagonal branch:

- Also here, solutions are unstable

Conclusions and present status: cosmology

- FLRW solutions exist, mass terms mimics exactly a CC in the entire cosmic history
  - background evolution is marvellous
- All these solutions are unstable
  - vanishing kinetic terms for vector and scalar perturbations, higher order analysis reveals energy unbounded from below
  - ghost-like instability: but no BD ghost (5 d.o.f.)!
- Stable cosmological solutions: inhomogeneous and/or anisotropic
  - our universe may be in a “Vainshtein region” of an inhomogeneous universe until $r_V \sim r_H$

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Conclusions and present status: extensions

Plently of ways to extend the theory. Some examples:

- Non-flat absolute geometry
  - usually de Sitter or Anti-de Sitter
- Massive bi-gravity models
  - by promoting the absolute metric to a dynamical object
- Add scalar fields
  - mass varying Massive Gravity, quasi-dilaton Massive Gravity
- Modify the coupling to matter
  - matter can couple to a mixture of the physical and absolute metrics

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Thank you very much!

ArXiv :1406.3384 [hep-th] (PhD thesis)

ArXiv :1204.1193 [hep-th] (Vainshtein in dRGT Massive Gravity)

ArXiv :1406.4550 [hep-th] (review on ghosts)

Scalar-tensor sector of dRGT massive gravity in the decoupling limit ($\Lambda_3 = \sqrt[3]{M_P\, m^2}$)

$$\mathcal{L} = -\frac{1}{2}\, h^{\mu\nu} \mathcal{E}_{\mu\nu}{}^{\rho\sigma} h_{\rho\sigma} + h^{\mu\nu} X^{(1)}_{\mu\nu} - \frac{\alpha}{\Lambda_3^3}\, h^{\mu\nu} X^{(2)}_{\mu\nu} - \frac{\beta}{\Lambda_3^6}\, h^{\mu\nu} X^{(3)}_{\mu\nu} + \frac{1}{M_P}\, h_{\mu\nu} T^{\mu\nu}$$

where, indicating $\Pi_{\mu\nu} = \partial_\mu \partial_\nu \pi$ and with [ ] the cyclic trace

$$X^{(1)}_{\mu\nu} = [\Pi]\, \eta_{\mu\nu} - \Pi_{\mu\nu}$$

$$X^{(2)}_{\mu\nu} = \Big([\Pi]^2 - [\Pi^2] \Big)\, \eta_{\mu\nu} - 2[\Pi]\Pi_{\mu\nu} + 2\Pi^2_{\mu\nu}$$

$$X^{(3)}_{\mu\nu} = \Big([\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^3] \Big)\, \eta_{\mu\nu} - 3\Big([\Pi]^2 - [\Pi^2] \Big)\, \Pi_{\mu\nu} + 6[\Pi]\Pi^2_{\mu\nu} - 6\Pi^3_{\mu\nu}$$

Note that π does not have a kinetic term of its own, but is kinetically mixed with $h_{\mu\nu}$. The field redefinition

$$h_{\mu\nu} = \hat{h}_{\mu\nu} + \pi\, \eta_{\mu\nu} - \frac{\alpha}{\Lambda_3^3}\, \partial_\mu \pi\, \partial_\nu \pi$$

almost diagonalizes the Lagrangian, leaving one coupling (which cannot be removed by a local field redefinition).

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We get

$$\mathcal{L} = \mathcal{L}_{\hat{h}} + \mathcal{L}_{\pi} + \mathcal{L}_{coup}$$

where

$$\mathcal{L}_{\hat{h}} = -\frac{1}{2}\,\hat{h}^{\mu\nu}\mathcal{E}_{\mu\nu}{}^{\rho\sigma}\hat{h}_{\rho\sigma} + \frac{1}{M_P}\,\hat{h}_{\mu\nu}T^{\mu\nu}$$

$$\mathcal{L}_{\pi} = 6\,\mathcal{L}_2^{gal} + \frac{a(\alpha,\beta)}{\Lambda_3^3}\,\mathcal{L}_3^{gal} + \frac{b(\alpha,\beta)}{\Lambda_3^6}\,\mathcal{L}_4^{gal} + \frac{c(\alpha,\beta)}{\Lambda_3^9}\,\mathcal{L}_5^{gal} + \\ + \frac{1}{M_P}\,\pi T - \frac{\alpha}{M_P\Lambda_3^3}\,\partial_\mu\pi\partial_\nu\pi T^{\mu\nu}$$

$$\mathcal{L}_{coup} = -\frac{\beta}{\Lambda_3^6}\,\hat{h}^{\mu\nu}X^{(3)}_{\mu\nu}(\pi)$$

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Small perturbations around non-trivial spherically symmetric background:

$$\pi(t, r, \Omega) = \pi_0(r) + \varphi(t, r, \Omega)$$

Effective kinetic term for the perturbations

$$S_\varphi = \frac{1}{2} \int d^4x \left[K_t(r)(\partial_t \varphi)^2 - K_r(r)(\partial_r \varphi)^2 - K_\Omega(r)(\partial_\Omega \varphi)^2 \right]$$